Noise Fundamentals

Vanessa Kwong

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Abstract

Our objective in studying Johnson noise is to confirm the Boltzmann constant a value relating amount of kinetic energy in joules (J) to a degree of temperature in Kelvin (K) in a particle to be $k_B = 6.6 * 10^{-23} \pm 1.731^{-23}$ *J/K from temperature-dependence, and* $7.1096 * 10^{-22} \pm 1.6624 * 10^{-21}$ *J/K from resistance dependence. We notice the linear increasing trend between resistance and Johnson noise spectral density, while temperature generally has a negative effect. Following, we focus on Shot noise by calculating the electron charge to be* $2.963 * 10^{10}$ *Coulombs (C) and photodiode current dependence from output voltage with a varying photocurrent. There is an increase with noise spectral density as photocurrent increases.*

I Introduction

In the process of data collection and experimentation, there is often interference or an unwanted signal that affects the collected numerical or observational results, defined as noise. Here, we examine two types of common noise: Johnson noise, which is caused by thermal interference in electron flux during flow, and Shot noise, which is caused by a difference in electrical charge. [1]

Examining Johnson noise, we relate to Nyquist's theorem, derived from the frequency-dependent Planck distribution. Nyquist's theorem states that

$$\langle V_J^2(t) \rangle = 4 k_B R T \Delta f \tag{1}$$

From Eq. (1), we find that thermal voltage $\langle V_J^2(t) \rangle$ is scales linearly with resistance R, temperature T, and bandwidth Δf. We aim to observe the trend between resistance, R, and charge spectral density calculated with thermal voltage, and confirm the value of Boltzmann's constant uncertainty based on this data.

Constrastingly, Shot noise resulting from charge quantization affecting a current can be derived from Schottky's theorem, then eliminating time dependence by applying the Nyquist-Shannon theorem to give:

$$I_{shot}^2 = 2 e I_{dc} \Delta f \tag{2}$$

where e is the electron charge, I_{dc} is the DC current measurement, and Δf is the bandwidth [1].

II Experimental Procedure

The general experimental setup consists of first, a low-level electronics (LLE) box consisting of a pre-amplifier module, voltage supply tuning for bulb involved in Shot noise, and a temperature module which connects to a low temperature probe to determine varying cryogenic temperature for evaluating Johnson noise. The pre-amplifier power output from the LLE box is inputted to the high-level electronics (HLE) box, where adjustments to the signal can be made through applying various high-pass and low-pass filters to apply cutoff frequencies, such as needed in bandwidth-varying data. Gain and a multiplier is added to amplify the signal to be probed with a voltmeter. An oscilloscope displays the noise signal following gain application. [2]

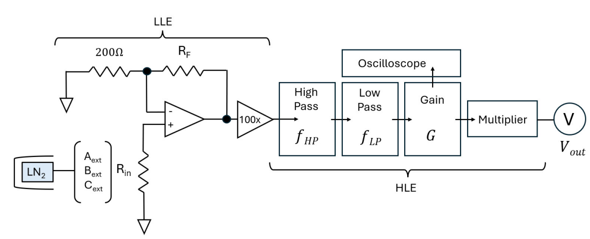

Figure 1: Schematic setup of temperature-dependent Johnson noise measurement

Johnson noise is measured for its output voltage first over various adjusted values of R_{in}, then various cryogenic temperatures by the setup indicated in Fig. 1, noted through measured voltage in temperature probe with the tip submerged in liquid nitrogen. Our parameters include a high-pass filter of 11Hz and a low-pass filter of 10kHz. The applied gain value from the HLE box is added in addition to the gain from the pre-amplifier module. From the data, we can extract the resistance dependence and temperature dependence of Johnson noise, and calculate a value for k_B using Eq. 1, with set parameters substituted in. [2]

Corrections are made for the change in capacitance when performing temperature-dependent measuremnts through probing the temperature probe breakout box. The noise signals from the pre-amplifier are initially affected by the low temperature. However, factoring in capacitive coupling through adding the following equation:

$$G_C = \frac{1}{\sqrt{1+(f/f_C)^2}} \tag{3}$$

into calculation for bandwidth and integrate across all positive frequency values with the following:

$$\Delta f = \int_0^\infty (G_C G_{LP} G_{HP})^2 df \tag{4}$$

to make up for the nonzero frequency for the initial signal. [1] To calculate the noise spectral density S, we use the following relations:

$$S = \frac{\langle V_J^2 \rangle}{\Delta f}, S = \frac{\langle V_S^2 \rangle}{\Delta f} \tag{5}$$

The LLE box is then reconfigured for Shot noise measurements, involving a photodiode converting light from an inner incandescent light bulb into a current source to be passed through the HLE and measured. With an applied low pass filer of 10000Hz and only the natural high pass filter of 16Hz from the pre-amplifier, we first observe the trend of Shot noise as a function of bandwidth by adjusting f_{LP}. Then, we observed the output voltage through tuning light intensity measured by $V_{Monitor}$ where $I_{dc} = 10\mu A$ when $V_{Monitor} = -100mV$. Kirchhoff's voltage law is used to attain the output voltage measurements from the current:

$$V_0 = -R_f I_{PD} \tag{6}$$

III Results and Analysis

In calculations for finding the Boltzmann's constant through Johnson noise, the relation between output voltage V_{out} and k_B are given by the equation:

$$V_{out} = (\langle V_J^2 \rangle + \langle V_n^2 \rangle)(G_1 G_2)^2/10V \tag{7}$$

where the relation given by Nyquist's theorem (Eq. 1) can be substituted into thermal voltage $\langle V_J^2 \rangle$. Since $\langle V_J^2 \rangle$ is minimal, we calculate $\langle V_J^2 \rangle$ using Eq. 2 where pre-amplifying gain $G_1 = 600$ and applied gain $G_2 = 600$. Applied to Nyquist's theorem in Eq.1, we find the calculated resistance-dependent Boltzmann's constant to be approximately $7.1096 * 10^{-22}$J/K with an uncertainty of $\pm 1.6624 * 10^{-21}$J/K.

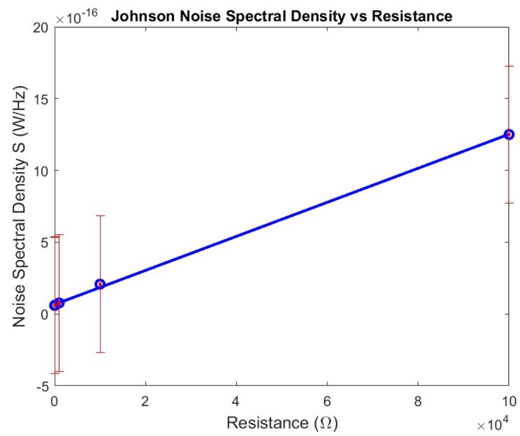

Figure 2: Plot of resistance-dependent Johnson noise spectral density given by Eq. (5) with calculated error

With increasing resistance applied, Johnson noise spectral density generally increases linearly, as expected from theoretical expectations with Nyquist's theorem (Eq. 1).

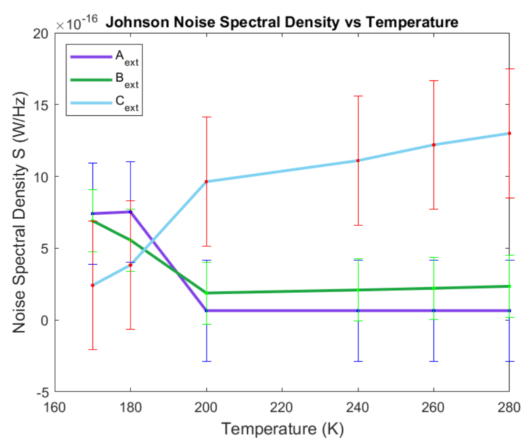

Figure 3: Plot of temperature-dependent Johnson noise spectral density given by Eq. (5) with calculated error

We observe lesser noise spectral density with increase of temperature with resistance values of 60Ω and 10kΩ for settings A_{ext} and B_{ext}. There is an increased noise spectral density for resistor C_{ext}, with a resistance value of $100k\Omega$. From the data, we calculate the temperature-dependent Boltzmann's constant to be $6.6 * 10^{-23} \pm 1.731^{-23}$J/K. Nyquist's theorem (Eq. 1) suggests that Johnson noise increases with temperature, consistent with our findings.

In evaluating data for Shot noise, we use the following relation for calculating electron charge e from taken values for V_{out} and DC current I_{dc}:

$$V_{out} = (R_f \langle I_{shot}^2 \rangle + \langle V_n \rangle^2)(100 * G_2)^2/10V \quad (8)$$

Our calculated charge for an electron e from Eq. 2 is approximately $2.963 * 10^{10}$ C with an uncertainty of $\pm 1.952 * 10^{10}$ C.

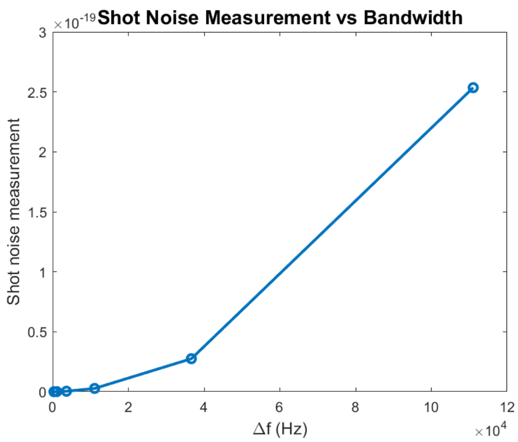

Figure 4: Plot of bandwidth-dependent Shot noise

From carefully decreasing f_{LP}, bandwidth Δf is increased. The Shot noise measurement is seen to increase exponentially with this tuned bandwidth, in contrast to the expected linear increase as suggested in Schottky's theorem (Eq. 2).

In comparison to Schottky's theorem (Eq. 2), this trend is realistic, as there is an increasing relation. From the aforementioned equation, Shot noise theoretically should have a linear increase with the photocurrent, resulting in higher noise spectral density.

IV Conclusion

In conclusion, we have successfully demonstrated the linearity between both temperature resistance value with noise spectral density of Johnson noise and confirm the value of Boltzmann's constant, experimentally. Boltzmann's constant is approximately $1.381 * 10^{-23}$J/K according to literature [3], falling into the range of uncertainty for our calculated values from both resistance and temperature

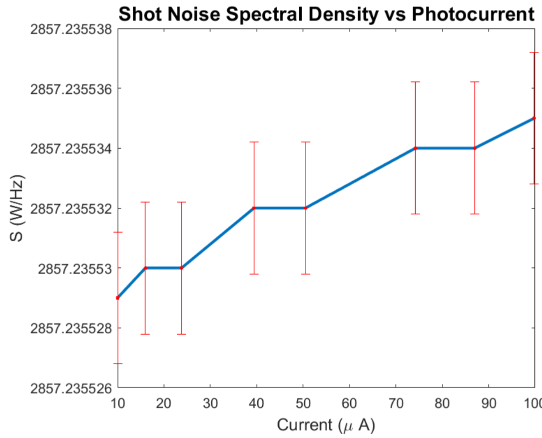

Figure 5: Plot of photocurrent-dependent Shot noise spectral density given by Eq. (5) with calculated error

measurements. We also observe the relation between bandwidth values and Shot noise, found to be increasing as expected. However, the data shows an exponential increase rather than linear. There is positive trend of noise spectral density as a function of photodiode current as expected. The calculated electron charge is $e = 2.963 * 10^{10} \pm 1.952 * 10^{10}$C, where the literature value $e = 1.602 * 10^{-19}$C [4] falls into the range of uncertainty.

Across many measurements done, especially for temperature-dependent Johnson Noise measurements and Shot noise measurements, the voltmeter had a large varying output from the HLE. The raw data taken for reported measurements may have human error. These values are attempted to be kept consistent by taking the average seen value on the voltmeter. For future experiments, we aim to keep the environment consistent, as many voltmeter readings seemed to alter through outside sound or interference in the lab area. Having several trials, testing Johnson noise across a larger range of resistances and temperatures, along with taking more data points for Shot noise in the selected range of photocurrent may improve future investigations.

References

[1] W. P. Beyermann, *Noise Fundamentals*. 2015.

[2] D. V. Baak, "Noise fundamentals," pp. 22–140, TeachSpin, 2010.

[3] G. Fasol, "Boltzmann constant k," *Eurotechnology Japan, KK*, 2014.

[4] J. B. Marion, *Physics in the Modern World (Second Edition)*. University of Maryland, College Park: Elsevier Inc., 1981.