COE782- ML - Lista2 -Parte prática - E1

Inferência Baysiana Sequencial

Vivian de Carvalho Rodrigues

DRE:125228569

In [1]:

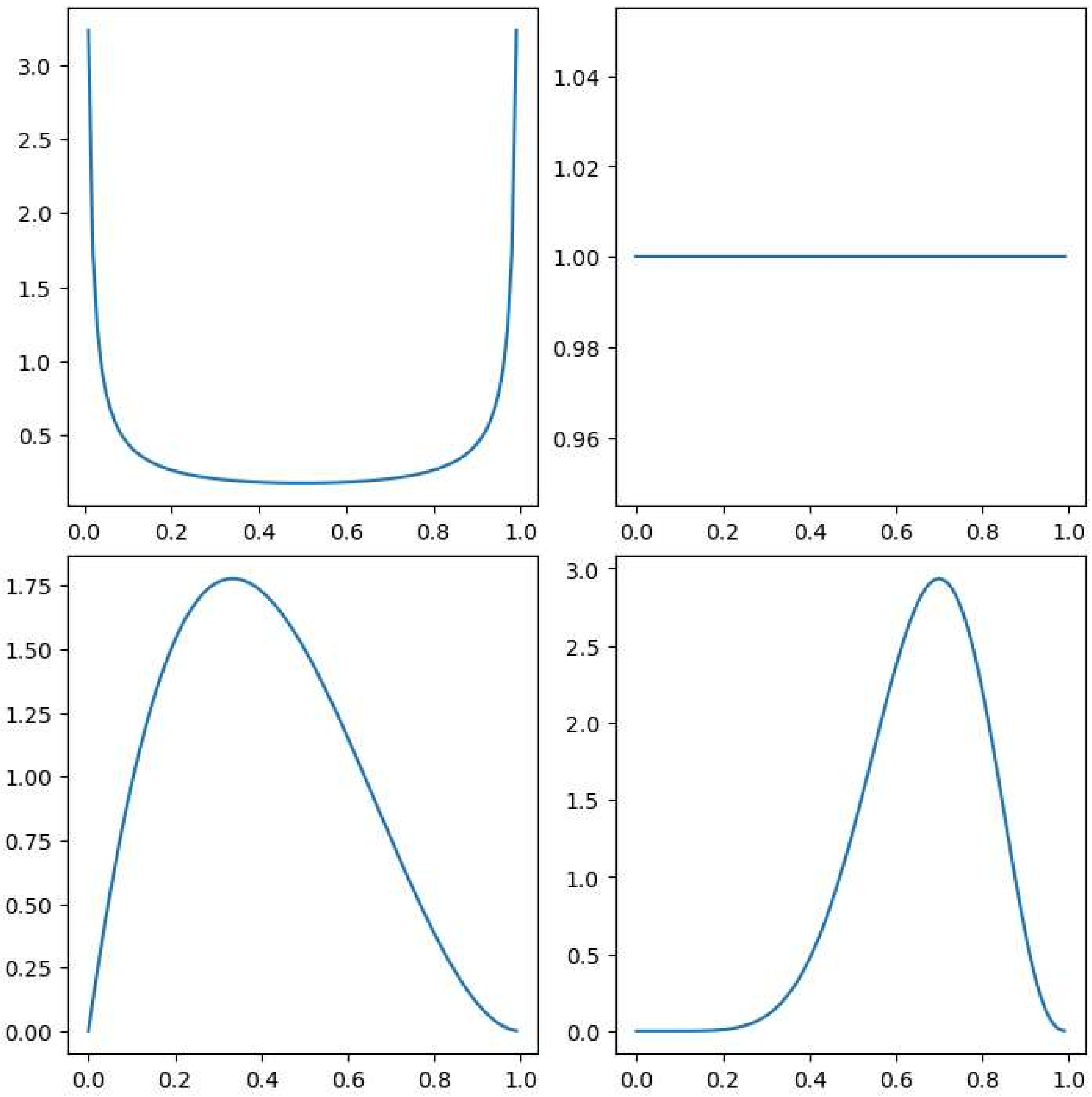
```
#Referências:
#https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.beta.html#scip
#https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.binom.html#sci

import numpy as np
from scipy.stats import beta, binom
import matplotlib.pyplot as plt
```

In [2]:

```
#Distribuição Beta
a = [0.1, 1, 2, 8]
b = [0.1, 1, 3, 4]
```

In [3]:

```
#fig 2.2 do Bishop
Nx = 100
mub = np.arange(0., 1., 1/Nx).reshape(Nx,1)

fig, axs = plt.subplots(2, 2, figsize=(7, 7), layout='constrained')

axs[0,0].plot(mub, beta.pdf(mub, a[0], b[0]))
axs[0,1].plot(mub, beta.pdf(mub, a[1], b[1]))
axs[1,0].plot(mub, beta.pdf(mub, a[2], b[2]))
axs[1,1].plot(mub, beta.pdf(mub, a[3], b[3]))
plt.show()
```

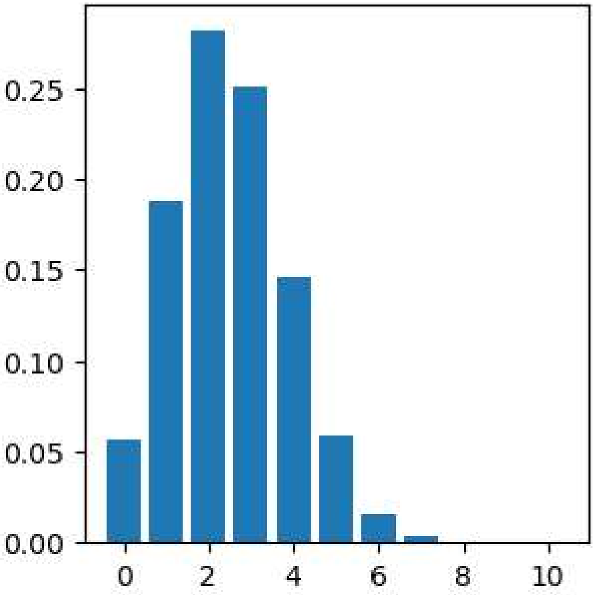


In [4]:

```
#distribuição binomial
N, mu = 10, 0.25
m = np.arange(11)

fbin = []

fbin += [binom.pmf(i, N, mu) for i in range(len(m))]
```

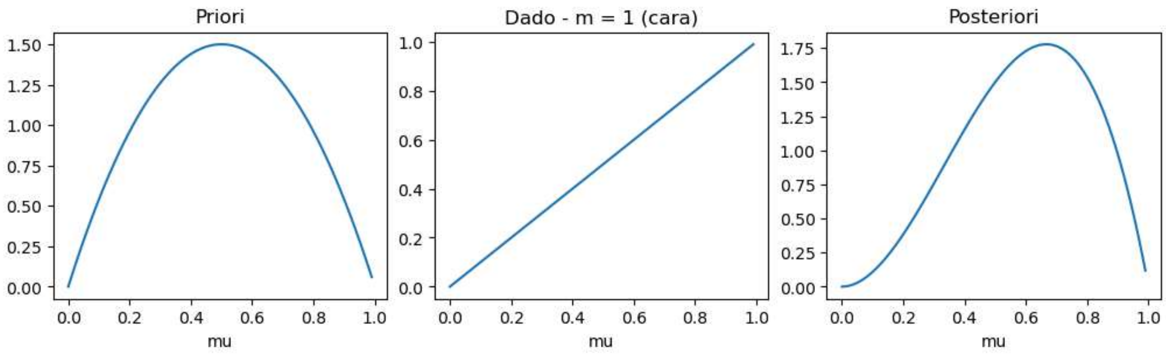
In [5]:

```
#fig 2.1 do Bishop
fig, axs = plt.subplots(1, 1, figsize=(3, 3), layout='constrained')
axs.bar(m, fbin)
plt.show()
```



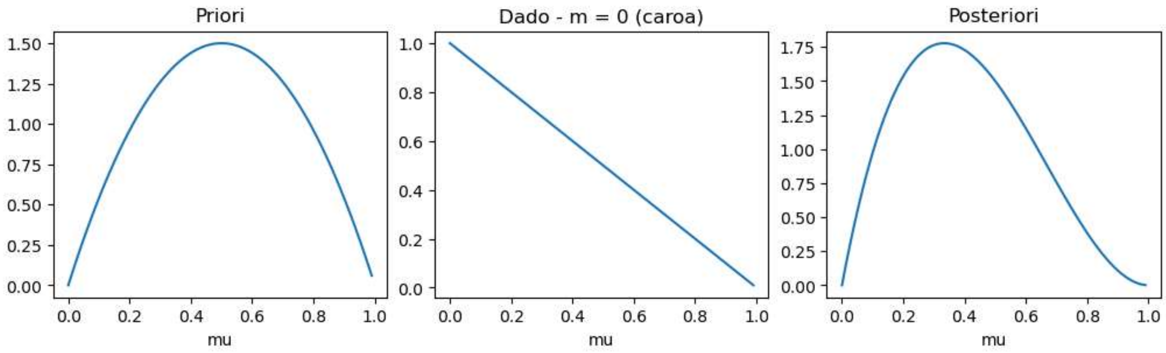
In [6]:
```
#exemplo da figura 2.3 Bishop
N = 1                                  #1 jogada apenas
mu_b = np.arange(0., 1., 1/Nx)

a= 2
b= 2
```

In [7]:
```
#Cara = 1 (1 jogada)
m = N    #no exemplo m = 1 (cara = 1 por exemplo)
fpri= []                                #priori
lh = []                                 #verossimilhança
fpos = []                               #posteriori
prob = 0.5                              #probabilidade de cara/coroa (denominador)

fpri += [beta.pdf(mu_b[i], a, b) for i in range(len(mu_b))]
lh += [binom.pmf(m, N, mu_b[i]) for i in range(len(mu_b))]
fpos += [binom.pmf(m, N, mu_b[i])*beta.pdf(mu_b[i], a, b)/prob for i in range(le


#fig 2.3 do Bishop
fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='constrained'
ax1.plot(mu_b, fpri)
ax1.set_xlabel("mu")
ax1.set_title("Priori")

ax2.plot(mu_b,lh)
ax2.set_xlabel("mu")
ax2.set_title("Dado - m = 1 (cara)")


ax3.plot(mu_b, fpos)
ax3.set_xlabel("mu")
ax3.set_title("Posteriori")

plt.show()
```



In [8]:

```
#Coroa = 0 (1 jogada)
m = 0
fpri= []                           #priori
lh = []                            #verossimilhança
fpos = []                          #posteriori
prob = 0.5                         #probabilidade de cara/coroa (denominador - p

fpri += [beta.pdf(mu_b[i], a, b) for i in range(len(mu_b))]
lh += [binom.pmf(m, N, mu_b[i]) for i in range(len(mu_b))]
fpos += [binom.pmf(m, N, mu_b[i])*beta.pdf(mu_b[i], a, b)/prob for i in range(le


#fig 2.3 do Bishop adaptado
fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='constrained'
ax1.plot(mu_b, fpri)
ax1.set_xlabel("mu")
ax1.set_title("Priori")

ax2.plot(mu_b,lh)
ax2.set_xlabel("mu")
ax2.set_title("Dado - m = 0 (caroa)")

ax3.plot(mu_b, fpos)
ax3.set_xlabel("mu")
ax3.set_title("Posteriori")

plt.show()
```



In [9]:

```
#gera as jogadas (Dado m) com distribuição Bernoulli
#https://numpy.org/doc/stable/reference/random/generated/numpy.random.Generator.

m = np.random.default_rng().geometric(p=0.7, size=5)
for i in range(len(m)):
    if m[i] > 1:
        m[i] = 0
m
```

Out[9]:

```
array([1, 1, 0, 0, 1], dtype=int64)
```

In [10]:

```
len(m)
```

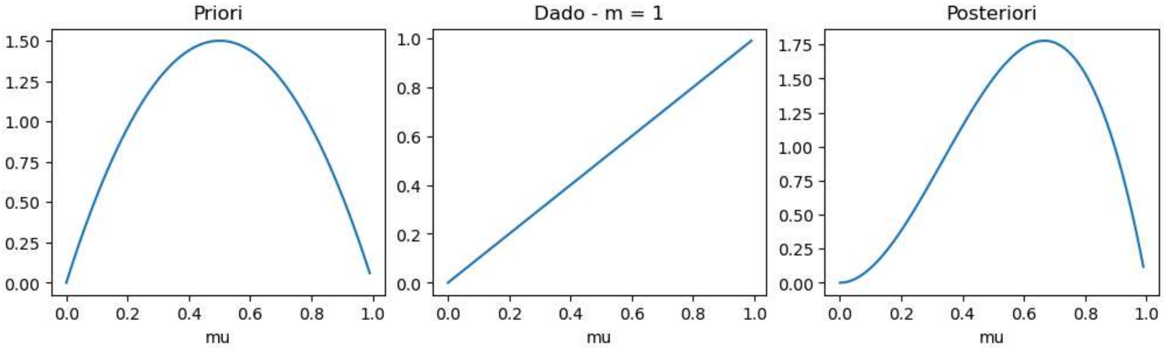
Out[10]:

```
5
```

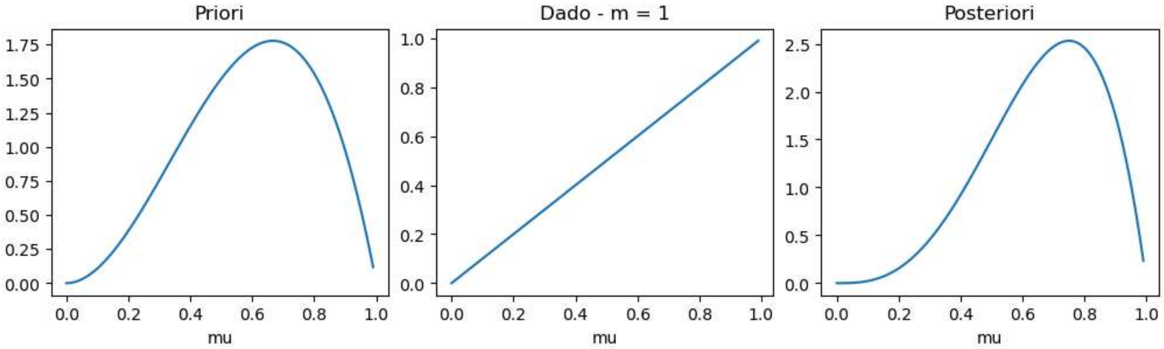
In [11]:

```
#Considerando 5 jogadas aleatórias
fpri= []                            #priori
lh = []                             #verossimilhança
fpos = []                           #posteriori
prob = 0.5                          #probabilidade de cara/coroa (denominador - p


for k in range (len(m)):
    if k == 0:
        fpri += [beta.pdf(mu_b[i], a, b) for i in range(len(mu_b))]
        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*beta.pdf(mu_b[i], a, b)/prob for i

        #fig 2.3 do Bishop adaptado
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()


        fpri = fpos                         #atualiza priori
        lh = []                             #zera joga para próxima rodada
        fpos = []                           #zera posteriori

    else:

        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*fpri[i]/prob for i in range(len(mu_

        #fig 2.3 do Bishop adaptado
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()


        fpri = fpos                                 #atualiza priori
        lh = []                                     #zera joga para próxima rodada
        fpos = []                                   #zera posteriori
```

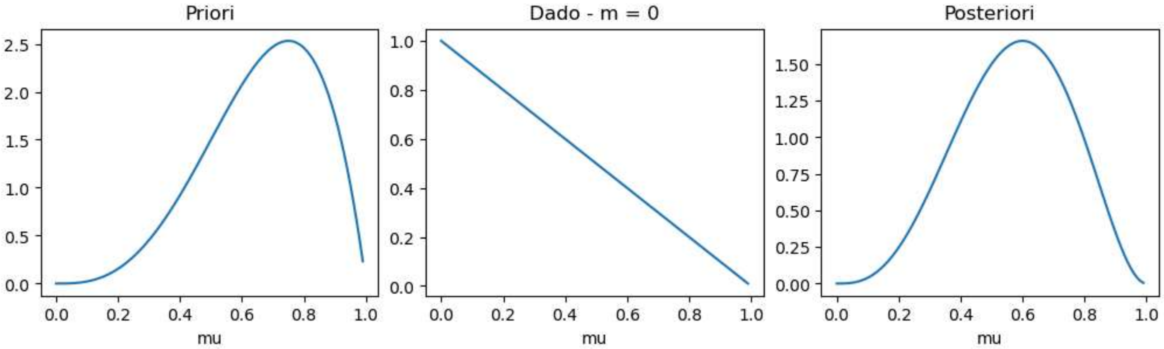
```
Jogada 1 com resultado = 1
```



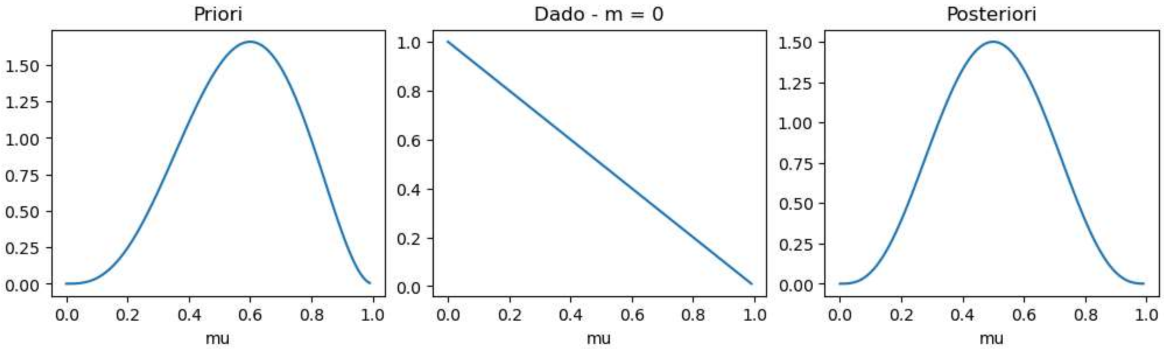
```
Jogada 2 com resultado = 1
```



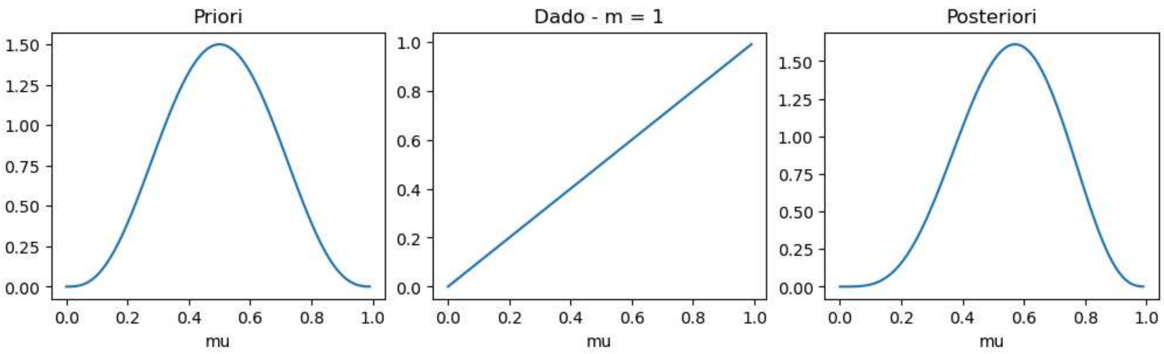
```
Jogada 3 com resultado = 0
```



```
Jogada 4 com resultado = 0
```



```
Jogada 5 com resultado = 1
```


Priori

Dado - m = 1

Posteriori

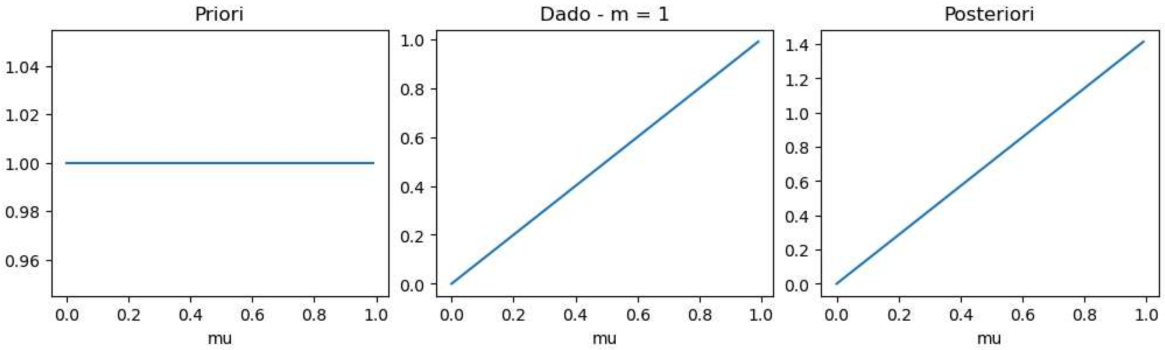
In [12]:

```
#Exercício E1
#1º Caso
a= 1
b= 1
prob = 0.7                         #probabilidade de cair cara(denominador - p(x
```

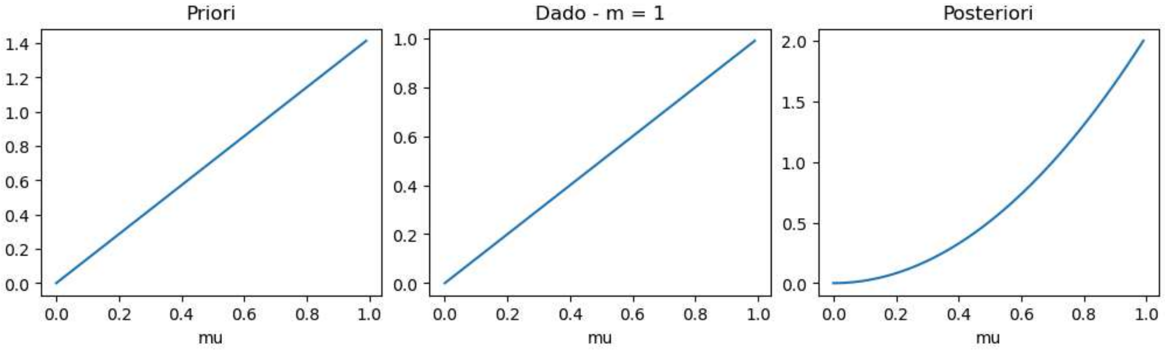
In [13]:

```
#Considerando 5 jogadas aleatórias
fpri= []                           #priori
lh = []                            #verossimilhança
fpos = []                          #posteriori



for k in range (len(m)):
    if k == 0:
        fpri += [beta.pdf(mu_b[i], a, b) for i in range(len(mu_b))]
        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*beta.pdf(mu_b[i], a, b)/prob for i

        #plota resultados
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()


        fpri = fpos                        #atualiza priori
        lh = []                            #zera joga para próxima rodada
        fpos = []                          #zera posteriori

    else:

        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*fpri[i]/prob for i in range(len(mu_

        #plota resultados
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()



        fpri = fpos                        #atualiza priori
        lh = []                            #zera joga para próxima rodada
        fpos = []                          #zera posteriori
```

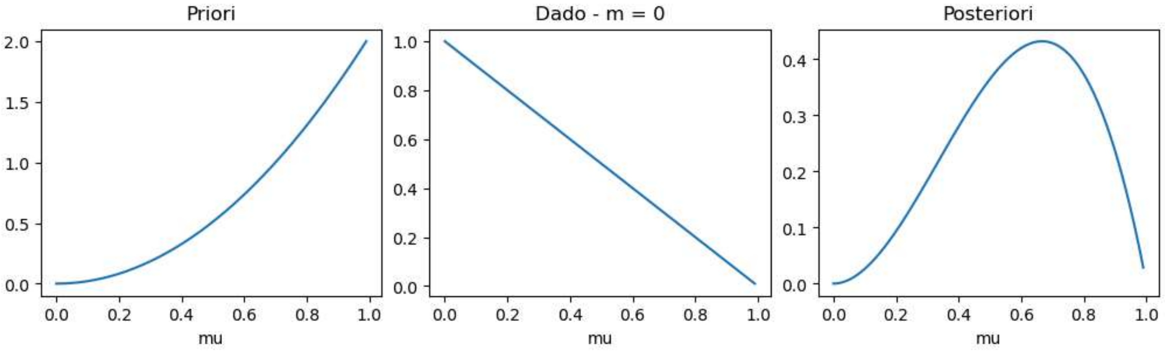
```
Jogada 1 com resultado = 1
```



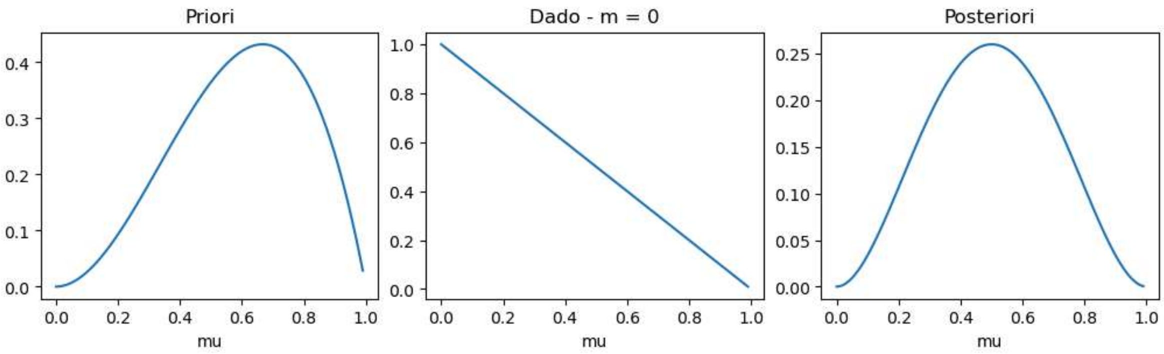
```
Jogada 2 com resultado = 1
```



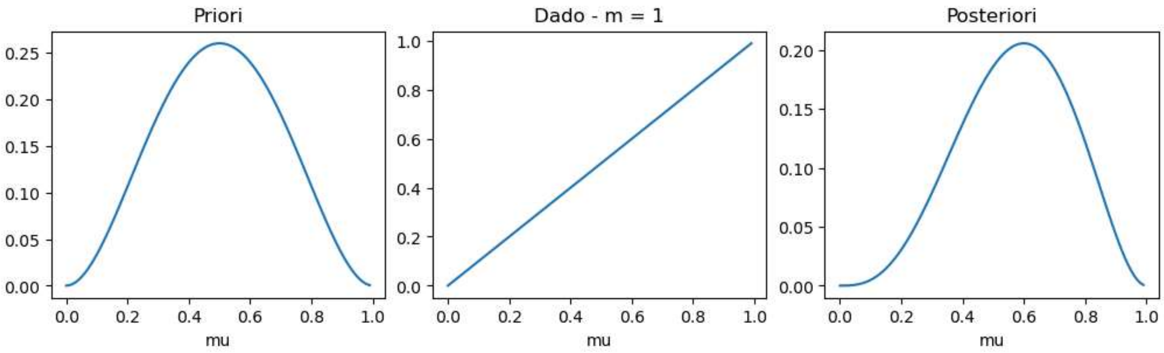
```
Jogada 3 com resultado = 0
```



```
Jogada 4 com resultado = 0
```



```
Jogada 5 com resultado = 1
```



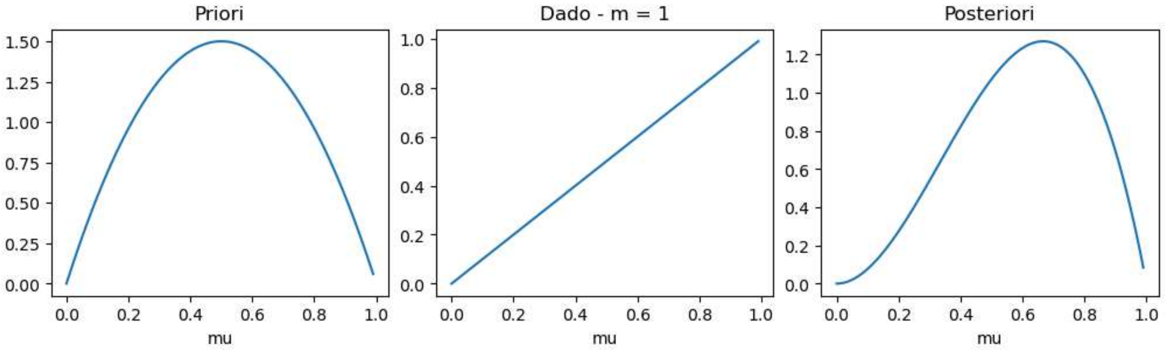
In [14]:

```
#2º Caso
a = 2
b = 2
```

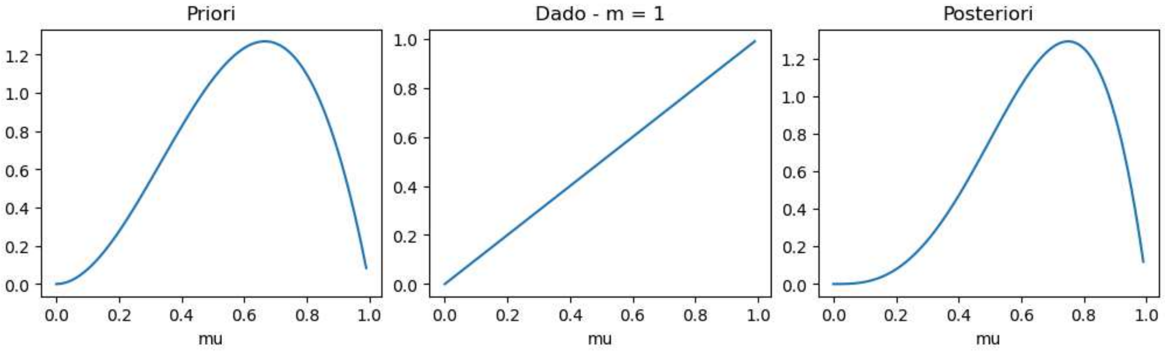
In [15]:

```
#Considerando 5 jogadas aleatórias
fpri= []                                   #priori
lh = []                                    #verossimilhança
fpos = []                                  #posteriori




for k in range (len(m)):
    if k == 0:
        fpri += [beta.pdf(mu_b[i], a, b) for i in range(len(mu_b))]
        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*beta.pdf(mu_b[i], a, b)/prob for i

        #plota resultados
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()


        fpri = fpos                                #atualiza priori
        lh = []                                    #zera joga para próxima rodada
        fpos = []                          #zera posteriori

    else:

        lh += [binom.pmf(m[k], N, mu_b[i]) for i in range(len(mu_b))]
        fpos += [binom.pmf(m[k], N, mu_b[i])*fpri[i]/prob for i in range(len(mu_

        #plota resultados
        print('Jogada', k+1 ,'com resultado =', m[k])
        fig, (ax1, ax2, ax3)  = plt.subplots(1, 3, figsize=(10, 3), layout='cons
        ax1.plot(mu_b, fpri)
        ax1.set_xlabel("mu")
        ax1.set_title("Priori")

        ax2.plot(mu_b,lh)
        ax2.set_xlabel("mu")
        ax2.set_title("Dado - m = "+ str(m[k]))

        ax3.plot(mu_b, fpos)
        ax3.set_xlabel("mu")
        ax3.set_title("Posteriori")
        plt.show()


        fpri = fpos                        #atualiza priori
        lh = []                            #zera joga para próxima rodada
        fpos = []                          #zera posteriori
```

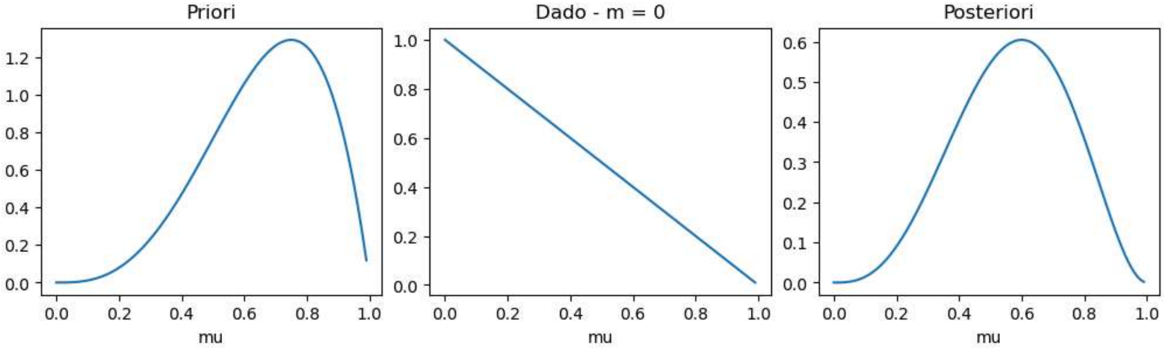
```
Jogada 1 com resultado = 1
```



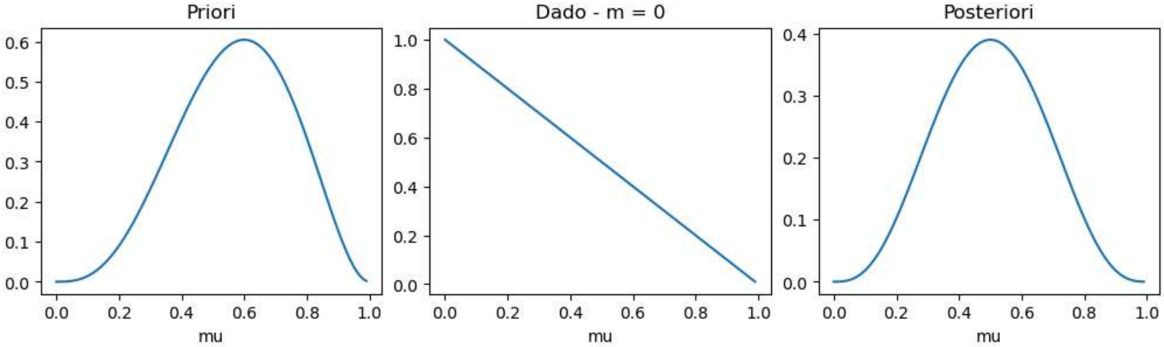
```
Jogada 2 com resultado = 1
```



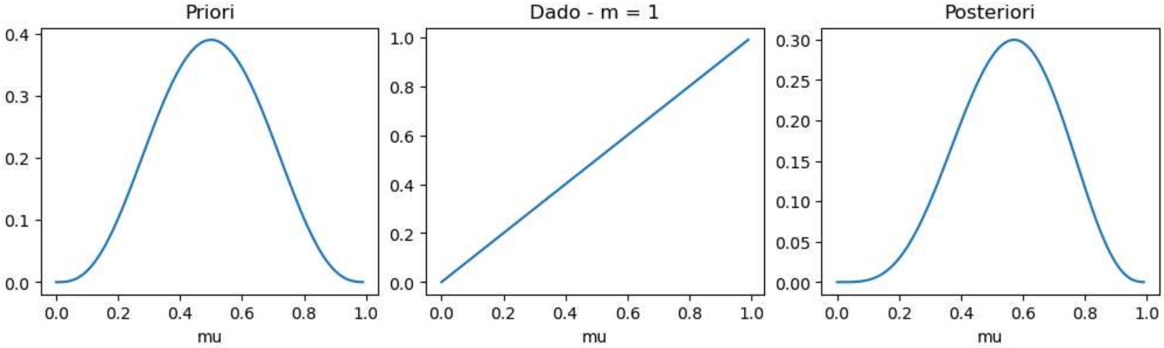
```
Jogada 3 com resultado = 0
```



```
Jogada 4 com resultado = 0
```



```
Jogada 5 com resultado = 1
```



Conclusão : A distribuição a priori influencia apenas a posteriori para 1 jogada. A medida que o númeoro de jogadas aumenta, os dois cados convergem para a mesma distribuição a posteriori.

In [ ]: