Graphical Representation of the Significant 6R KUKA Robots Spaces

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Abstract—**Trajectory planning for serial 6 degree of freedom (DOF) machinery systems is demanding due to complex kinematic structure which affects the machinery tool fame, position, orientation and singularity. These three characteristics represent the key elements for production planning and layout design of the manufacturing systems. Both, simple and complex machine trajectory is defined as series of connected points in 3D space. Each point is defined with its position and orientation related to the machine's base frames. To visualize the machine's reachable space, the work envelope is calculated and graphically presented as very well known machine's property. A methodology to predetermine regions of feasible tool orientation (work window) is analytically and graphically presented. The work envelope boundary is generated using the filtering points algorithm, while work window is generated using either empirical or analytical methods. The singularity regions are calculated by finding the determinant of the reconfigurable Jacobian matrix. These three tool path characteristics represent three different spaces which are identified both analytically and graphically and plotted in Cartesian space using MATLAB tools. The singularity regions will be represented within the workspace and work window for a single machinery kinematic structure. The KUKA KR robot family is used as a case study.**

I. Introduction

Today's manufacturing environment is dominated by market change and global competition. Reconfigurable manufacturing systems are identified as a means of improving the production outcome by quickly adjust its production capacity and functionality in response to sudden market changes and needs, [1]. To achieve the above said characteristics, effective and robust systems are required. This particularly applies to machine and robot cells. It is important to know whether the robot end-effector can reach a particular point in its workspace at a desired orientation to allow modification or change in the placement or configuration before setting up the robot on the shop floor. Currently, this reach problem is solved by visual inspection, simulation packages, by manually operating a teach pendant and by visually analyzing the workspace of the robot. Applications will require the tool to be at a certain nonsingular position and orientation to perform the tasks necessary. The work space of a kinematic structure can be defined as the set of all points that it can reach in space. The robot workspace does not capture the effect of tool orientation and singularity regions. It is therefore important to introduce a parameter that can capture the effect of orientation for multiple robots and configurations. This is called the functional work space or workwindow, which is a subset of the work envelope. Beside these two spaces, the singularity conditions must be included for the complete definition of the robot's available space related to the selected application.

A. Literature Review

The 'boundary of space' model representing all possible positions which may be occupied by a mechanism during its normal range of motion (for all positions and orientations) is called the work envelope. In the robotic domain, it is also known as the robot operating envelope or workspace [2]. Several researchers have investigated workspace boundaries for different degrees of freedom (DOF), joint types and kinematic structures utilizing many approaches [3-6]. A recent example utilizes a reconfigurable modeling approach, where the 2D and 3D boundary workspace is created by using a method identified as the Filtering Boundary Points (FBP) algorithm, [7]. In [8], the authors first defined the work window as the functional subset of the work space. Previous research investigations were on developing Jacobian by using different methods, [9]. These methods are compared in terms of their computational efficiency since the Jacobian must be computed in real time for control. Using three different methods for calculating Jacobian of 6 DOF robots with rotary of sliding joints are presented by [10] and applied on the PUMA robot. By employing symbolic reduction techniques in conjunction with the separation of the resultant equations into on-line, temporary variables and off-line constraints, efficient formulations of the Jacobian, inverse Jacobian, and inverse Jacobian multiplied by a vector have been developed by [11]. Jacobian can be obtained by simple method explained in [12]. The singularity research has been done for 6DOF robot manipulators with different kinematic structures, [13-17]. An important step for robot singularity analysis is task decoupling. This concept has been known since Pieper's pioneering work, [18]. The complete analysis of task decoupling was done by [19]. The structural synthesis and the singularity analysis of six different families of orthogonal anthropomorphic 6R robotic manipulators were presented by [20]. The kinematics singularities of each family were analyzed and interpreted both algebraically and geometrically. The singular configurations of 6R robots with spherical wrist in general and the KUKA KR-15/2 industrial robot in particular, are analytically described and classified by [21]. Several researchers worked on the KUKA direct and inverse kinematic problem using different approaches and applications [22-24]. The Kuka Control Toolbox (KCT) has been developed for motion control of KUKA robot manipulators using MATLAB [25]. The analysis and synthesis of many kinetic structures has been previously done in [26], [27].

The manual and analytical procedures for generating the robot functional workspace, named Workwindow has been presented in [28]. The paper [28] represents an initial research which is extend and applied for the KUKA KR robot family. The fundamental theoretical information related to this research can be found in [29-31].

II. SIGNIFICANT 6R KUKA ROBOTS SPACES

The machine path generation depends on its kinematic structure which is limited by the workspace, workwindow and singularity conditions. These three kinematic related spaces are calculated for the KUKA KR robot family, and graphically presented in this paper. The kinematic chain for the KUKA KR robot family is presented in Fig. 1. The related D-H (Denavit-Hartenberg, [32]) parameters are given in Table 1.

This kinematic structure is generated using the manufacturers' information presented in Fig. 2 and Fig. 3, http://www.kuka-robotics.com, and D-H rules, [32]. Using the right-hand-rule all z-axis are place in the kinematic chain such that they satisfy all joint positive directions predefined by the KUKA manufacturer. The orientation of the z-axis relative to each other is controlled by the twist angles α_i. These angles are fixed to the whole KUKA KR family.

The KUKA KR family kinematic structure comparing to the Fanuc and ABB kinematic structures has some similarities and some differences. The main difference is the orientation of first coordinate frame, which has z_0 axis pointing down, while the Fanuc and ABB robots has z_0 axis pointing up. The positive directions for the joint 2 and joint 3 are forward, for the ABB and KUKA robots. The Fanuc robots joint 2 positive direction is forward, while joint 3 positive direction is backwards. The path planning depends on the robot kinematic structure which is limited according to the three significant spaces: Workspace, Workwindow and singularity space, which are calculated and graphicaly presented below. The shape and the size of all three spaces depend on the D-H parameters.

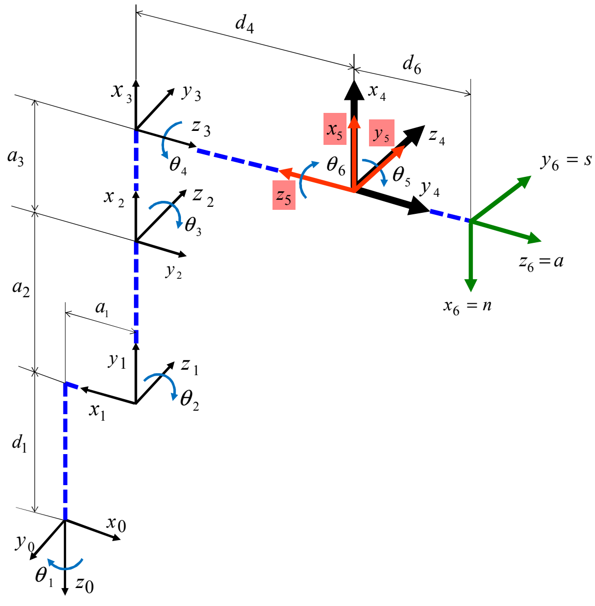

Figure 1. Kinematic chain for KUKA KR family

TABLE I.
D-H PARAMETERS

i	d_i	θ_i	a_i	α_i
1	d_1	180°	a_1	-90°
2	0	90°	a_2	0°
3	0	0°	a_3	-90°
4	d_4	0°	0	90°
5	0	0°	0	90°
6	d_6	180°	0	180°

A. Workspacce for the KUKA KR family

For the workspace calculation, the first three joints are used. By varying their joint limits from minimum to maximum, the complete 3-D reachable workspace is described. Most of the 6R industrial robots have linked Joint 2 and Joint3, but KUKA KR family does note have. The 2D Workspace is generated using the manufacturers' information given in Fig. 3. The numerical values for the general D-H parameters are: $d_1 = -865$, $d_4 = 1200$, $d_6 = -210$, $a_1 = -410$, $a_2 = 1000$, $a_3 = 45$ and joint limits are: $J_{2\min} = -40$, $J_{2\max} = 93$, $J_{3\min} = -210$ and $J_{3\max} = 210$.

The graphical representation of the 2D Workspace is shown in Fig. 4. It is clear that the model is correct by comparing the Fig. 3 and Fig. 4.

The 3D Workspace has been generated by varying the Joint 1 limits between the minimum and maximum values. See Fig. 5.

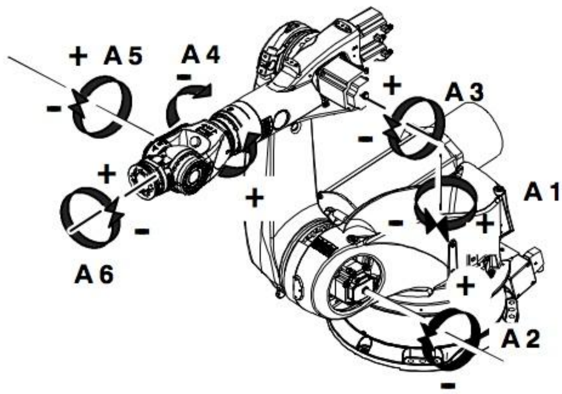

Figure 2. KUKA KR family joints positive directions (http://www.kuka-robotics.com)

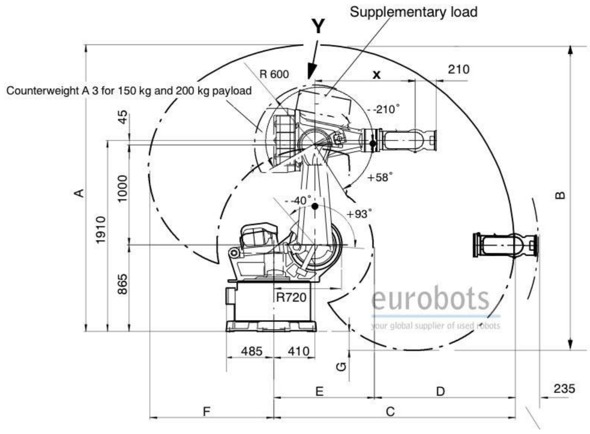

Figure 3. KUKA KR family limits (http://www.kuka-robotics.com)

B. *Workwindow for the KUKA KR family*

The work window is defined as the valid functional space for a configuration to allow a kinematic structure (robot, machine tool, etc.) to follow a path for a set of conditions relating to the system configuration, tooling, fixture location and so forth [8]. A serial link manipulator is a series of links, which connects the end-effector to the base, with each link connected to the next by an actuated joint. If a coordinate frame is attached to each link, the relationship between two links can be described with a homogeneous transformation matrix using D-H rules, and they are named ${}^{i-1}A_i$, where i is number of joints.

$$ {}^{i-1}A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1} $$

The robot can now be kinematically modeled by using the link transforms:

$$ {}^0A_n = {}^0A_1 {}^1A_2 {}^2A_3 {}^3 \ldots {}^{n-1}A_n . \tag{2} $$

Where ${}^0A_n$ is the pose of the end-effector relative to

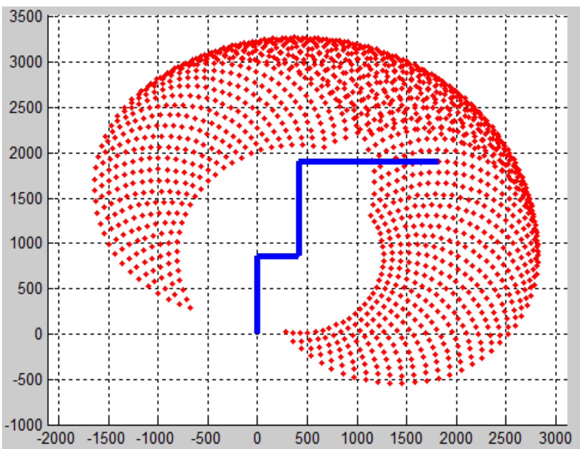

Figure 4. 2D Workspace for KUKA KR family

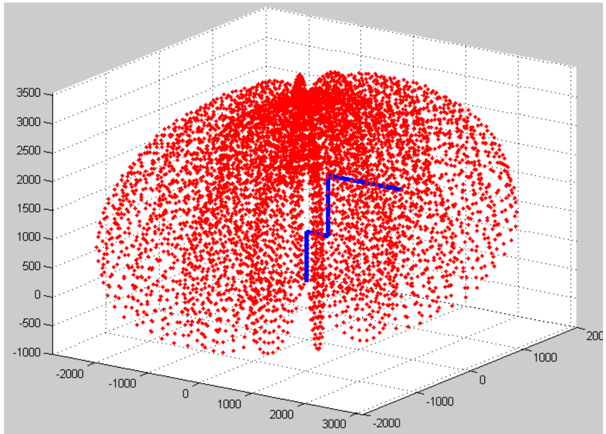

Figure 5. 3D Workspace for KUKA KR family

base; ${}^{i-1}A_i$ is the link transform for the i^{th} joint; and n is the number of links. For the KUKA KR robot family, six homogeneous transformation matrices have been developed using Maple 16 symbolic manipulation software. The pose matrix of the end-effector relative to base is presented in (3).

$$ {}^0A_6 = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3} $$

The end-effector orientation is defined with the rotation matrix in (4).

$$ {}^0R_6 = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}. \tag{4} $$

The orientation of the end-effector, relative to the robot base frame, is defined with the three vectors: n, s, and a. For the end-effector approach vector pointing down, see Fig. 6, the rotational matrix is given with (5).

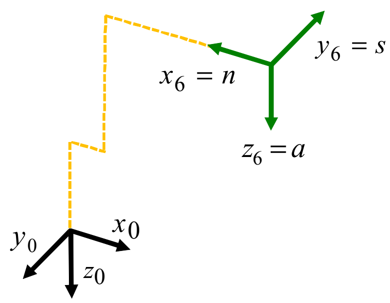

Figure 6. Relation between end-effector and base frames for KUKA KR robot family

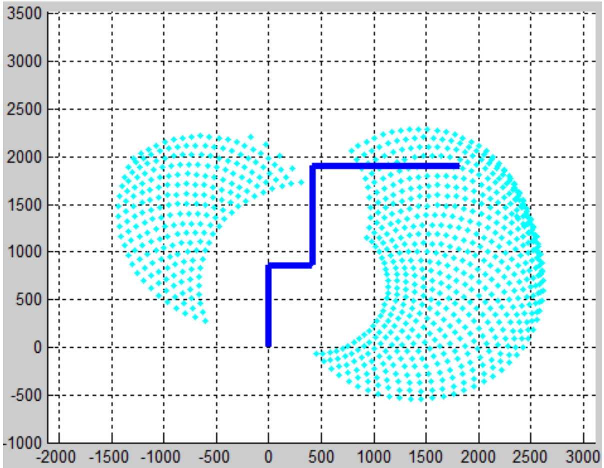

Figure 7. 2D Workwindow with normal approach vector for KUKA KR family

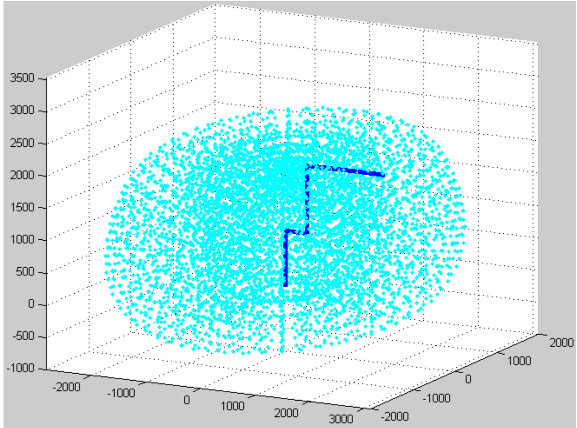

Figure 8. 3D Workwindow with normal approach vector for KUKA KR family

$$ {}^{0}R_6 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

Comparing the end-effector frame orientation in Fig. 6 and the (5), it is clear that they match each other. The calculation of the 2D end-effector orientation is dependent on the three joint angles: Joint 2, Joint 3 and Joint 5. The formula for Joint 5 is generated by assigning initial values for the Joint 1, Joint 4 and Joint 6 in the forward kinematic solution. The rotational matrix in that case is calculated and shown in (6).

$$ {}^{0}R_6 = \begin{bmatrix} \cos(\theta_2+\theta_3)\cos\theta_5 - \sin(\theta_2+\theta_3)\sin\theta_5 & 0 & \cos(\theta_2+\theta_3)\sin\theta_5 + \sin(\theta_2+\theta_3)\cos\theta_5 \\ 0 & -1 & 0 \\ \cos(\theta_2+\theta_3)\sin\theta_5 + \sin(\theta_2+\theta_3)\cos\theta_5 & 0 & \sin(\theta_2+\theta_3)\sin\theta_5 - \cos(\theta_2+\theta_3)\cos\theta_5 \end{bmatrix}. \tag{6}$$

By combining (5) and (6) the formula for the Joint 5 angle is generated.

$$\theta_5 = a\tan 2\left(\frac{\sin(\theta_2+\theta_3)}{\cos(\theta_2+\theta_3)}\right). \tag{7}$$

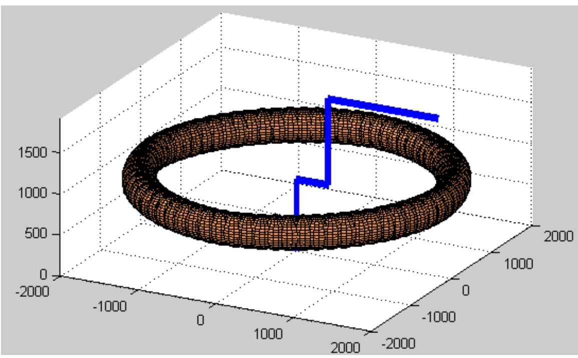

Figure 9. 3D Singularity space for KUKA KR 150

To be able to generate complete work window, the Joint 2 and Joint 3 angles must vary between their given limits for the desired increment value Δ and using the forward kinematic solution to generate the solution for Joint 5. The workwindow for the KUKA KR 150 robot with approach vector pointing normal to the floor is given in Fig. 7. The numerical values for the general D-H parameters are: $d_1 = -865$, $d_4 = 1200$, $d_6 = -210$, $a_1 = -410$, $a_2 = 1000$, $a_3 = 45$.

The 3D Workwindow has been generated by varying the Joint 1 limits between the minimum and maximum values. See Fig. 8.

C. *Singularity space for the KUKA KR family*

The American National Standard for Industrial Robots and Robot Systems — Safety Requirements (ANSI/RIA R15.06-1999) defines a singularity as "a condition caused by the collinear alignment of two or more robot axes resulting in unpredictable robot motion and velocities." [33]. The Jacobian matrix is calculated for the KUKA KR family shown in Fig. 1. This type of kinematic structure consists a 3-DOF forearm with a 3-DOF spherical wrist. This robot is in the singular position if and only if the (8) is true.

$$\det(J) = 0. \tag{8}$$

The Jacobian matrix for the KUKA KR family model has been calculated using the Recursive Newton-Euler method. The boundary singularity is give with the (9).

$$C_b = -a_3 \sin(\theta_3) + d_4 \cos(\theta_3) = 0. \tag{9}$$

The interior singularities are given with the (10).

$$C_i = a_1 + a_2\cos(\theta_2) + a_3\cos(\theta_2+\theta_3) + d_4\sin(\theta_2+\theta_3) = 0. \tag{10}$$

The wrist singularities can be identified by checking (11).

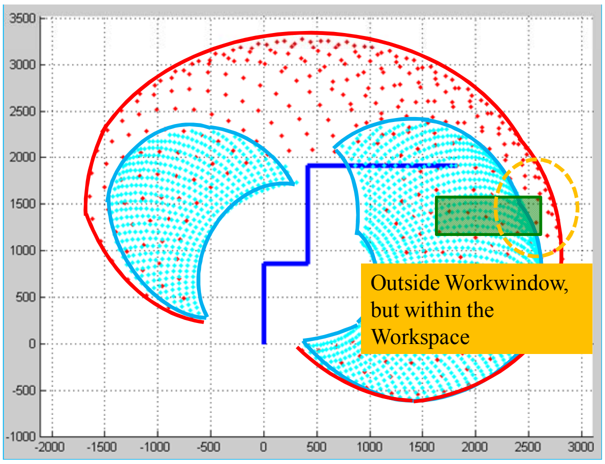

Figure 10. Benefits of 3D visualization

$$\sin(\theta_5) = 0 \,. \tag{11}$$

The equations for the graphical representation of the boundary and interior singularity conditions for the KUKA KR family have been generated from (10) and (11). See (12) and (13).

$$\theta_2 = a\tan 2\left(\frac{a_1 - a_2 - a_3\cos\theta_3 + d_4\sin\theta_3}{a_3\sin\theta_3 + d_4\cos\theta_3}\right). \tag{12}$$

$$\theta_3 = a\tan 2\left(\frac{-d_4}{a_3}\right). \tag{13}$$

For one set point, an ellipisoid representation of the singularity region is generated.

To represent a complete singularity region θ_1 must be incremented between -180° and $+180^\circ$. Using MATLAB tools, a singularity region is derived for a KUKA KR 150 robot, see Fig. 9.

D. Visual comparison of three significat space for the KUKA KR family robots

In Fig. 10, the encircled region cannot be reached for the ‘normal to the base’ orientation for a KUKA KR 150 robot although the part is within the envelope.

The actual work window conditions will vary based on the part, travel path tool parameters, geometry, machine kinematic composition and the application.

For machining operations, the work window will vary for each unique orientation/ tooling /fixture combination. Structural reconfigurations, such as adding or removing an axis (DOF and type of joints) either virtually or physically, will also impact the work window.

The assessment of system elements is necessary in order to examine the feasibility and practicality of a set of kinematic structures in a manufacturing scenario. This does not exist at this time. There is a direct relation between the kinematic structures, their manipulability and constraints, to the work window.

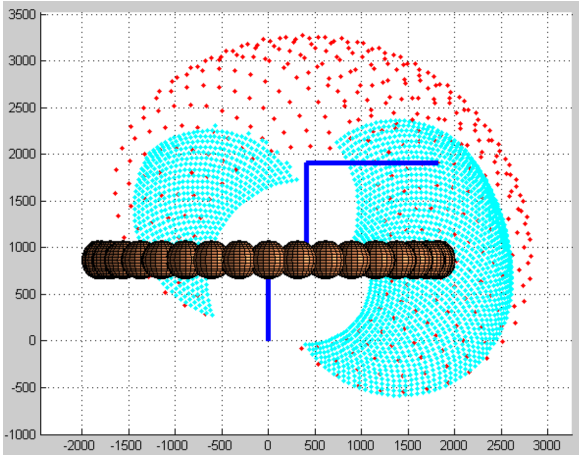

Figure 11. 2D Visual comparison of KUKA KR 150 Workspace, Workwindsow and Singularity space

The actual work window conditions will vary based on the part, travel path tool parameters, geometry, machine kinematic composition and the application.

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The assessment of system elements is necessary in order to examine the feasibility and practicality of a set of kinematic structures in a manufacturing scenario. This does not exist at this time. There is a direct relation between the kinematic structures, their manipulability and constraints, to the work window. Proper assessment of system elements will help in optimization of a system and help in deciding various parameters such as number of degrees of freedom, placement of machines, robots and other supporting mechanisms (in case of a work cell consisting of multiple kinematic chain elements) and so forth. The methodology needs to be applicable for the reconfigurable type of machinery. The relation between the three spaces in 2D and 3D are presented in Fig. 11 and 12.

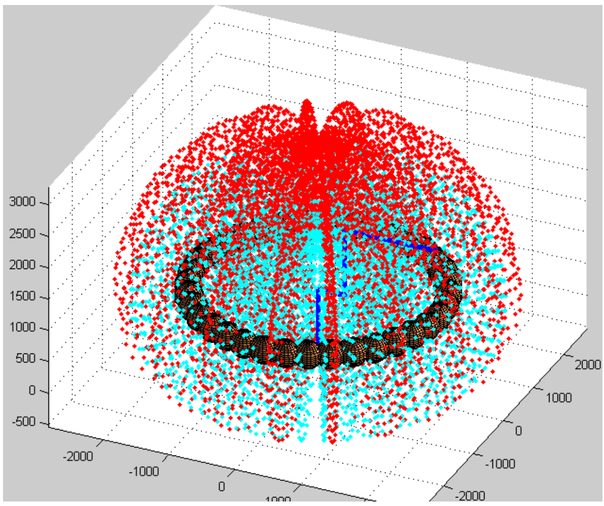

Figure 12. 3D Visual comparison of KUKA KR 150 Workspace, Workwindsow and Singularity space

III. CONCLUSIONS

A methodology to predetermine regions of feasible operation for selected kinematic chain mechanisms is presented. The feasible operating space represents the difference between workwindow and singularity space that belongs to the robot workspace. After the basic region of valid operation space is determined, assessment of the part location and travel paths can be performed. Changing a reconfiguration can be executed virtually, and these methods employed to determine feasibility prior to physical setups or modification in the manufacturing environment.

A visual representation of these zones will help process designers develop valid travel paths in a timely fashion. Additionally, designers will be able to develop travel paths in regions that are insensitive to singularities when modifications due to (i) in field adjustment, (ii) new product changes, or (iii) modifications due to a new kinematic structure are required. These regions are identified both analytically, and graphically. This study has been done for the KUKA KR robot family.

This research will be extend and applied for the aerial robot presented in [34].

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