Name:

Short Quiz 4A

8 December 2025

Question 1: Let $f\colon \mathbb{R}^d \to \mathbb{R}$. Write the first order Taylor expansion of f at x_0, using the gradient $\nabla f(x_0)$.

Answer: $f(x_0 + h) =_{h \to 0} f(x_0) + \langle \nabla f(x_0), h \rangle + o(\|h\|)$

Question 2: Compute the gradient of $x \mapsto \langle u, Mx \rangle$ at x_0, where $u \in \mathbb{R}^d$ and $M \in \mathbb{R}^{k \times d}$. *Hint: switch M from the right to the left of the scalar product using the formula seen last time, and you will fall back on an example from the lecture.*

Answer:

$$\langle u, M(x_0 + h) \rangle = \langle M^\top u, x_0 + h \rangle$$
$$= \langle M^\top u, x_0 \rangle + \langle M^\top u, h \rangle.$$

Therefore, the gradient at x_0 is $M^\top u$.

Question 3: What is the idea of bagging?

Answer: The idea of bagging is to train several independent weak learners (for example random trees) by bootstrapping the train dataset: we construct new datasets by picking samples in the train dataset, with replacement, and we train a different weak learner on each of these new datasets. Then, we average the predictions of the weak learners. The errors of the weak learners, that are independent, cancel out, leading to a better prediction.