This problem set has 11 questions, for a total of 100 points. Answer the questions below and mark your answers in the spaces provided. If the question asks for showing your work, you must provide details on how your answer was calculated.

Your Name:

- 1. For each of the following, give an exact formula T(n) for the number of times the line // op is run. Show your work and justify your answer. Assume i increments by 1 at each iteration unless otherwise specified.
 - (a) [5 points]

```
for (int i = 0 ; i < 4*n ; i ++) {
// op
}
```

If n= 3, i goes from 0 until 12 but since 12 is not less than 12 the loop only executes from 0-11 which is 12 times

(a) _____

(b) [5 points]

If n= 3, i goes from 1 until 27 and since 27 is less than or equal to 27, the loop executes from 1-27 which is 27 times

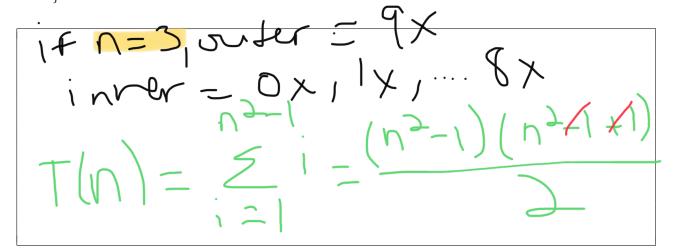
(b) _____

(c) [5 points]

(c) _____

(d) [5 points]

```
for (int i = 0 ; i < n*n ; i++) {
   for (int j = 0 ; j < i ; j++) {
      // op
   }
}</pre>
```



(d) _____

(e) [5 points]

(e) _____

(f) [5 points] Hint: the formula should work with even and odd values of n.

$$i \in N = \{0 \in \}$$
 $i \in N = \{0 \in \}$
 $i \in$

(f) _____

(g) [5 points]

(g) _____

(h) [5 points]

```
int m = std::pow(2, n);
for (int i = 1 ; i <= m ; i *= 2) {
    // op
}</pre>
```

$$i + n = 3, m = 3 = 8$$

 $i = 1, 2, 4, 8$
 $i = 1, 2, 4, 4, 16, 32, 44, 128$
 $i = 1, 2, 4, 4, 16, 32, 44, 128$
 $t = 1, 4, 4, 16, 32, 44, 128$

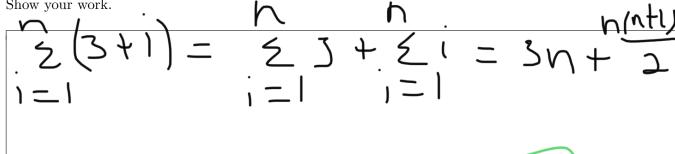
(h) _____

(i) [5 points] Hint: Assume n is a power of 2

$$ifn = 2, i = 2$$
 $ifn = 16, i = 16, 4, 4, 4, 1$
 $ifn = 16, i = 16, 4, 4, 4, 1$
 $ifn = 16, i = 16, 4, 4, 4, 1$
 $ifn = 16, i = 16, 4, 4, 4, 1$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4, 4$
 $ifn = 16, i = 16, 4, 4$
 $ifn = 16, i = 16, 4$
 $ifn = 16, 4, 4$
 $ifn = 16, 4$
 $ifn = 16,$

(i) _____

2. [5 points] Rewrite the following expression into its closed form (i.e. without the sigma): $\sum_{i=1}^{n} (3+i)$. Show your work.

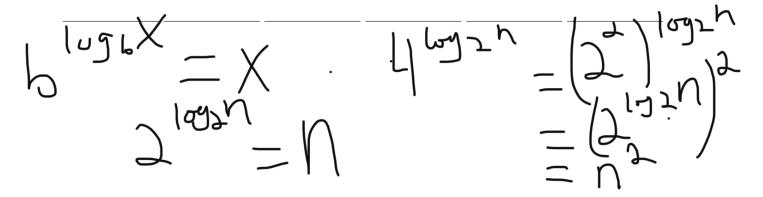


A. $3 + \frac{n*(n+1)}{2}$ B. $3 - \frac{n*(n-1)}{2}$ C. $3n - \frac{n*(n+1)}{2}$ D. $3n + \frac{n*(n-1)}{2}$ E. $3n + \frac{n*(n+1)}{2}$

2. ____

3. [5 points] Rank the following functions by their asymptotic growth rate in ascending order.

$$\log \log n$$
 $2^{\log_2 n}$ 2^{100} 4^n $n^2 \log n$ $4^{\log_2 n}$



4.	[10]	points	Mark	each	of	the	follo	owing	as	true or	false.
	1 - 0	POILION	11100111	Cacii	01	OIL			COL	or are or	iaioc.

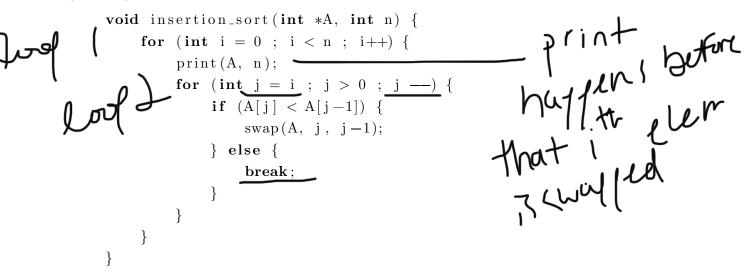
oint	oints] Mark each of the following as true or false.								
Т	(n)	Big O	T/F	Big Omega	T/F	Big Theta	T/F		
$\frac{n^2}{10}$	$\frac{n^2}{n^2} + 10n \log n$	$O(n \log n)$	1	$\Omega(n \log n)$	7	$\Theta(n \log n)$	F		
2r	$n^2 + n \log n$	$O(n^2)$	$\overline{\tau}$	$\Omega(n)$	T	$\Theta(\log n)$	F		
$\frac{n}{2}$	$\log n + 4n$	$O(2^n)$	\top	$\Omega(n \log n)$	T	$\Theta(n \log n)$	T		
10	$0\sqrt{n} + 2\log n$	$O(\log n)$	F	$\Omega(n)$	F	$\Theta(\log n)$	F		
3,	$\sqrt{n} + 10 \log n$	$O(\sqrt{n})$	T	$\Omega(1)$	Τ	$\Theta(\sqrt{n})$	T		

5. [10 points] Complete the following table using Big Θ notation with respect to the number of comparisons.

		1	l /
Algorithm	Best Case	Average Case	Worst Case
Selection Sort	N,	トナ	n,
Insertion Sort	n	U	NT
Maximum of an Unsorted Array	N	n	
Median of a Sorted Array	\	1	
Mode of a Sorted Array	<u> </u>	\cap	n

) hard to he w/o actu paper so just term is fire

6. [5 points] Consider the implementation of insertion-sort below.



Given the array A with elements [2,44,21,9,1] and assuming that print sends the current values of A to the standard output. Show what is printed at every iteration of the outer loop.

i = 0	7	4	71	9	
i = 1	d	4	7	9	
i = 2	4	44	71	9	1
i = 3	4	21	44	9	1
i = 4	7	9	2-1	44	1

log I goes that every elem mile
up I goes buckward from elem
from loop I & every time
elem in loop I is maller than
its frey it sways it else it
breaks out of loop I

7. [5 points] Consider the implementation of selection-sort below.

Given the array A with elements [2,44,21,9,1] and assuming that print sends the current values of A to the standard output. Show what is printed at every iteration of the outer loop.

i = 0	2	44	عـا	9	
i = 1	1	44	7-1	9	2
i = 2	l	7	74	9	44
i = 3		2	9	21	44
i = 4	l	7	9	71	44

wy I goes thru each elem while lost of finds smallet elem in rest of list to then swort windex from lost

- 8. An inversion is any pair of two elements that are out of order. How many inversions are present in each of the following arrays?

5 (a) _____

(b) ____

(c) [1 point] [1, 2, 3, 4, 5]

(c) _____

(d) _____

- $\overset{\text{(e)}}{\textbf{Q}}\overset{\text{[1 point]}}{\textbf{N}}\overset{\text{[6,9,1,4,10]}}{\textbf{Q}}\overset{\text{C,1}}{\textbf{L}}\overset{\textbf{L}}{\textbf{L}}$
- 9. Consider the following functions.

}

int foo (int x, int *y) { x = x + 10; *y = x * 2; return x; x = x + 10; *y = x * 2; return x; x = x + 10; x = x

new x = 10+10=20 and y = 20 +1=40

For each of the questions below, what are the values of x and y after running the provided line of code. If you think the code may trigger an error at any point indicate the reason. Do not use a computer for solving this question.

- (a) [2 points] int x = 2, y = 3; x = foo(x, &y);
- (b) [2 points] int x = 10, y = 20; x = foo(x, &y);

 $\begin{array}{c}
X = 1 \\
Y = 1 \\$

- (c) [2 points] int x = 0, y = 0; x = foo(x, &y);
- Y = 20
- (d) [2 points] int x = 1, y = 3; int *z = bar(y); x = *z;
- CALO C
- (e) [2 points] int x = 1, y = 0; int *z = bar(y); x = *z;

Optional Questions

The following questions are **optional**, and will not be graded. They are simple problems that should serve as decent practice for solving problems on paper.

- 10. Write an algorithm in $\Theta(n)$ time that calculates the missing number in an array A of integers. The length of the array is n-1 and every element A[i] in the array is such that $1 \leq A[i] \leq n$. For example, given A = [3, 2, 1, 5] the output sould be 4.
- 11. Write an algorithm that removes all duplicate integers from an input array A. The length of A is n and every element A[i] is such that $1 \le A[i] \le 100$. For example, given A = [12, 2, 2, 3, 4, 2, 5], the algorithm should return [12, 2, 3, 4, 5]. Can you make your algorithm run in $\Theta(n)$ time?

Town x = 0+10=10 Sume y = 10+2=20 Done in Lab St Lo find sum st I-N tract arrivem Subtract arrivem Olem missing in

ont return at re