

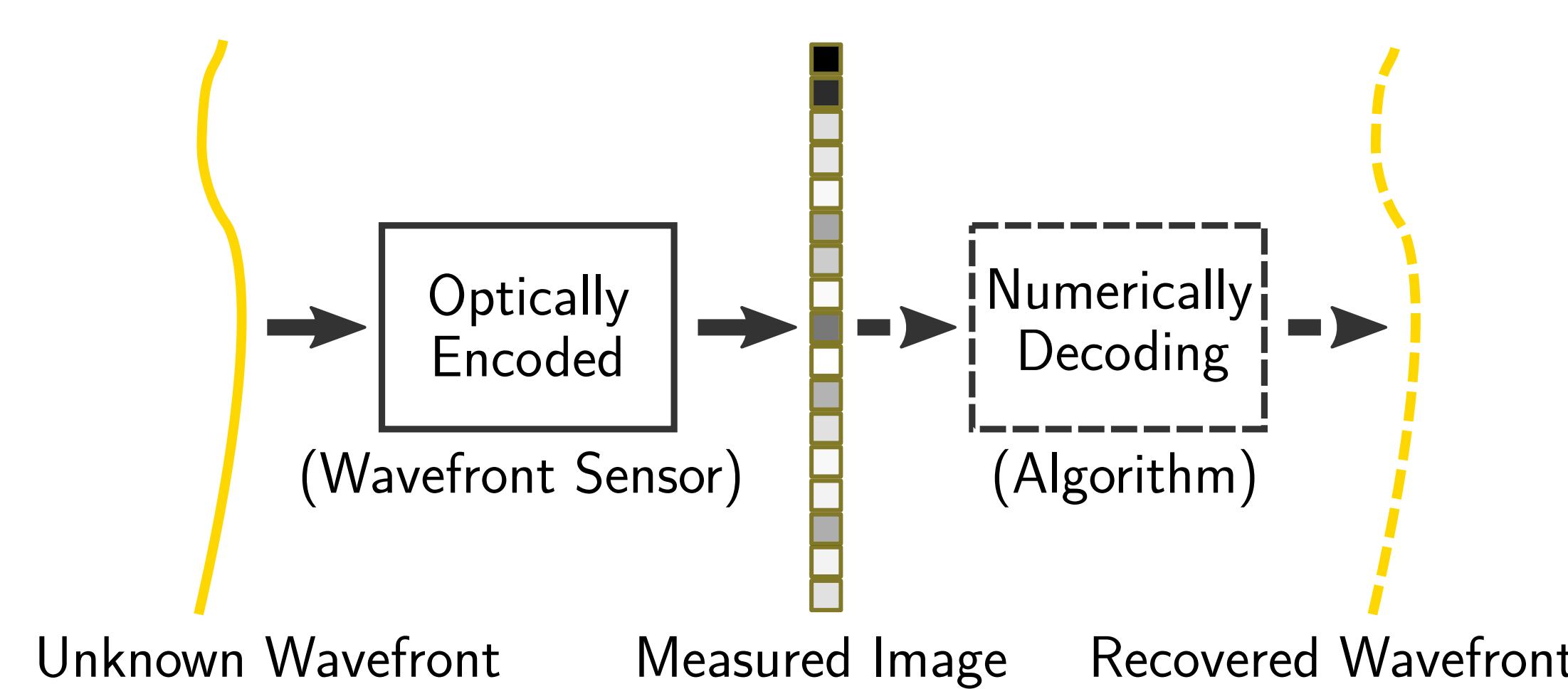
# Ultra-High Resolution Coded Wavefront Sensor

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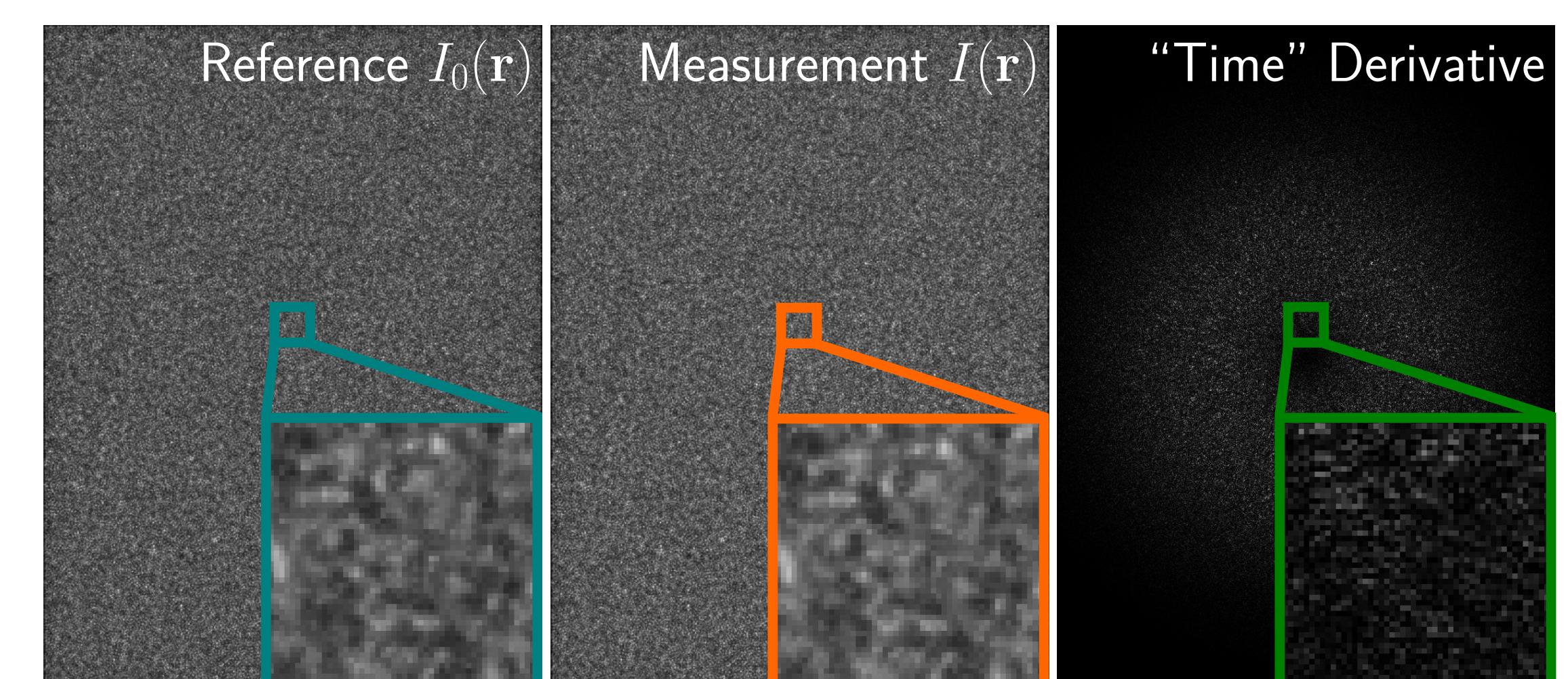
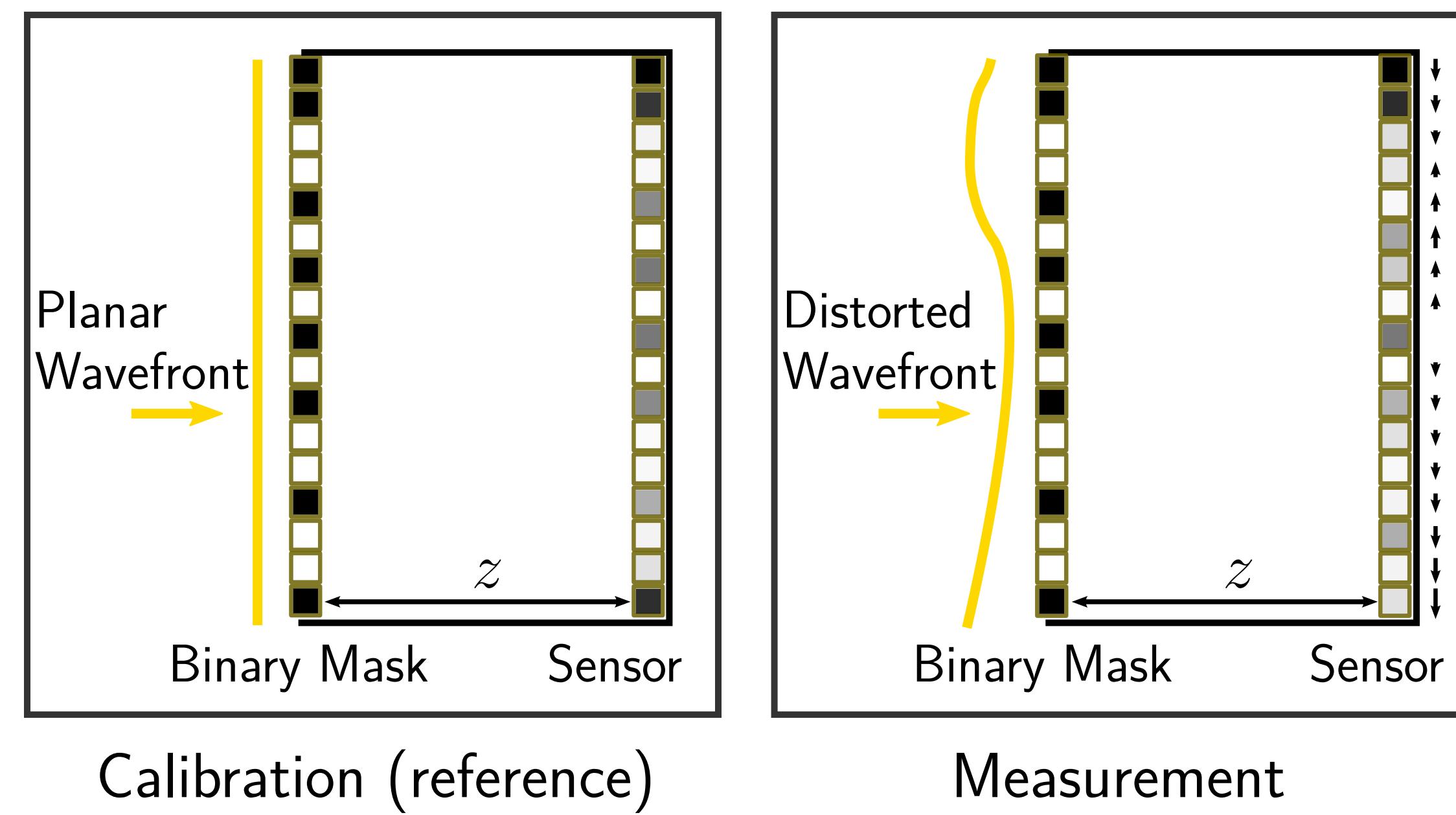
## The Wavefront Sensing Problem

Wavefront sensing is an old yet fundamental problem in optics: Direct phase measurement is not feasible through intensity sensors, and thus requires a joint design of both the hardware and the software.

- Traditional wavefront sensors [1, 2] are limited to using conventional optical components and simple algorithms, suffering a tradeoff between temporal, spatial resolution and the wavefront range.
- We introduce the *Coded Wavefront Sensor*, which is easy to fabricate and calibrate, with high wavefront range and accuracy, and of high temporal-spatial wavefront resolution as well.



## The Coded Wavefront Sensor: Principle & Implementation



## Results

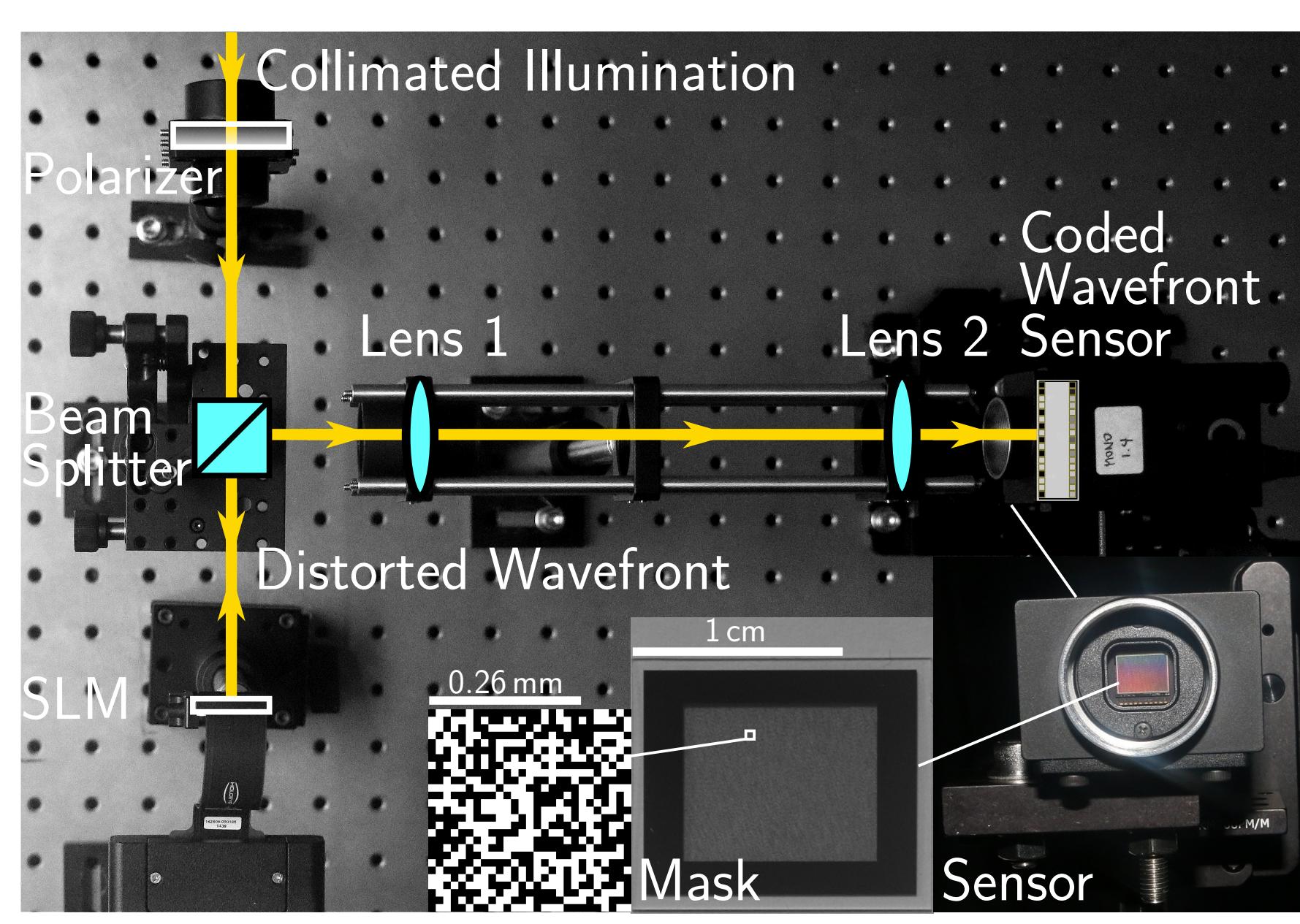


Figure: Experimental setup for accuracy validation.

### Quantitative Wavefront Measurement:

We evaluated the Coded Wavefront Sensor by using it to measure known wavefronts that are generated by a SLM. A distant point white light source serves as illumination. A telescope system ensures the wavefront sensor and the SLM are in conjugate. Results are shown on the right.

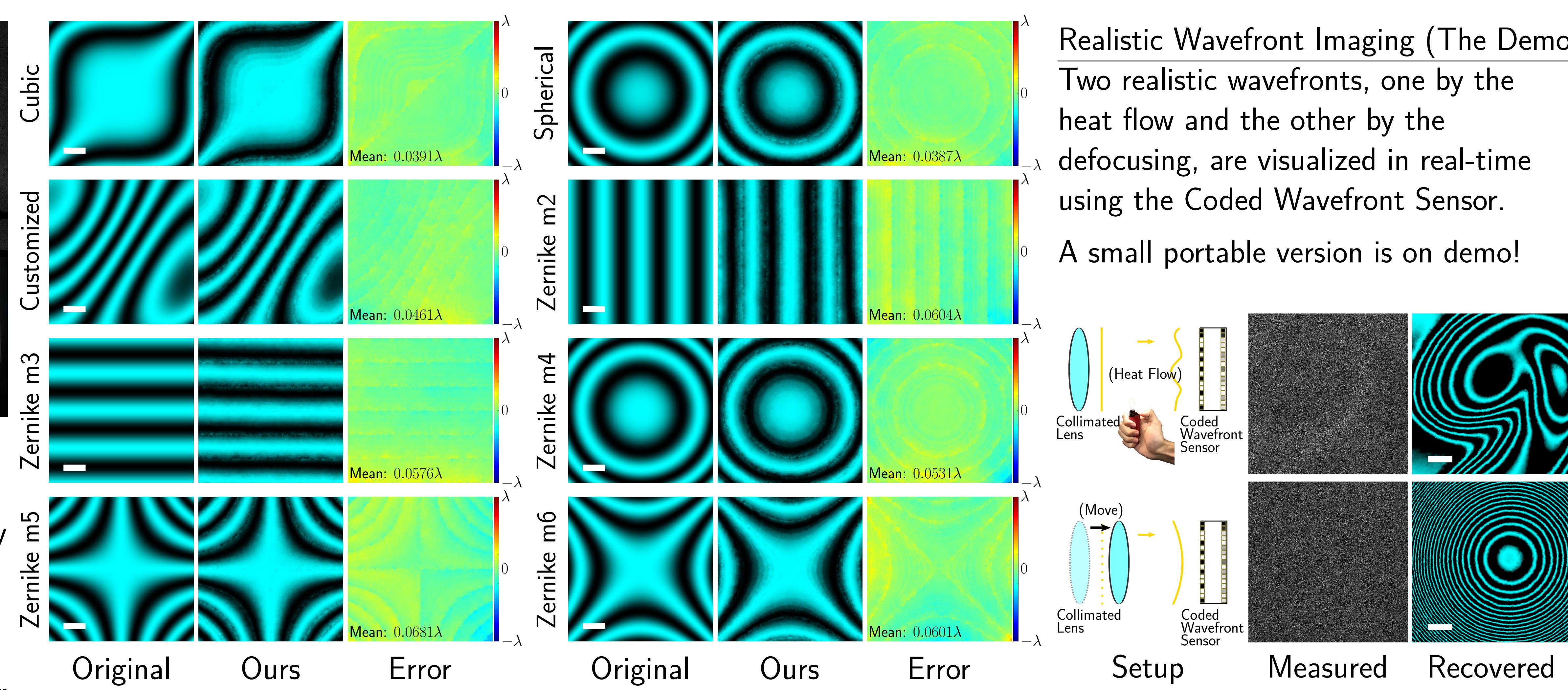


Figure: Selected quantitative experimental results. The original ground truth, our reconstruction, and the errors are shown. All wavefronts are shown in interference fringes where one fringe maps to wavefront difference of  $\lambda = 632.8$  nm. Scale bar is 1 mm.

For small  $z$  (e.g. 1.5 mm), one can show using the Rayleigh-Sommerfeld diffraction formula [3]:

$$I(\mathbf{r}) \approx I_0(\mathbf{r} - (z/k)\nabla\phi(\mathbf{r})),$$

where  $k$  is the wave number and  $\nabla\phi$  is the wavefront gradients. A direct linearization leads to the so-called optical flow method [4]:

$$\frac{z}{k}\nabla\phi(\mathbf{r}) \cdot \nabla I_0(\mathbf{r}) + I(\mathbf{r}) - I_0(\mathbf{r}) = 0.$$

We devise our own reconstruction method, adding a wavefront smoothness regularizer, and solve for the wavefront directly. In linear algebra:

$$\underset{\phi}{\text{minimize}} \| \mathbf{G} \mathbf{M} \nabla\phi + \mathbf{g}_t \|_2^2 + \alpha \| \nabla\phi \|_2^2,$$

where  $\mathbf{G} = [\text{diag}(\mathbf{g}_x) \ \text{diag}(\mathbf{g}_y)]$  is a concatenated diagonal matrix with the image derivatives ( $(\mathbf{g}_x, \mathbf{g}_y) = \nabla I_0(\mathbf{r})$ ) on the diagonal, a “time” derivative  $\mathbf{g}_t = I(\mathbf{r}) - I_0(\mathbf{r})$ , and  $\mathbf{M}$  is a binary diagonal matrix that selects only the visible pixels from the wavefront samples. Our solver employs ADMM [5], and each updating step enjoys closed-form solution and is parallelizable on GPU.

Table: Timing performance of our CUDA implementation solver on a GeForce GTX TITAN X (Pascal) GPU.

Unknown Size	Performance
1024 × 1024	19.54 ms / 51.18 frames/s
1024 × 768	15.24 ms / 65.62 frames/s
640 × 480	10.97 ms / 91.16 frames/s
512 × 512	5.77 ms / 173.31 frames/s
256 × 256	2.95 ms / 338.98 frames/s
128 × 128	2.17 ms / 460.83 frames/s

## Conclusion

We introduce the Coded Wavefront Sensor, a novel sensor design that is physically implemented by a single binary masked sensor to encode the incoming wavefront, and is numerically implemented by an efficient optimization decoding algorithm, such that high spatio-temporal resolution wavefront reconstruction is achieved within sub-wavelength accuracy.

## Funding & Acknowledgement

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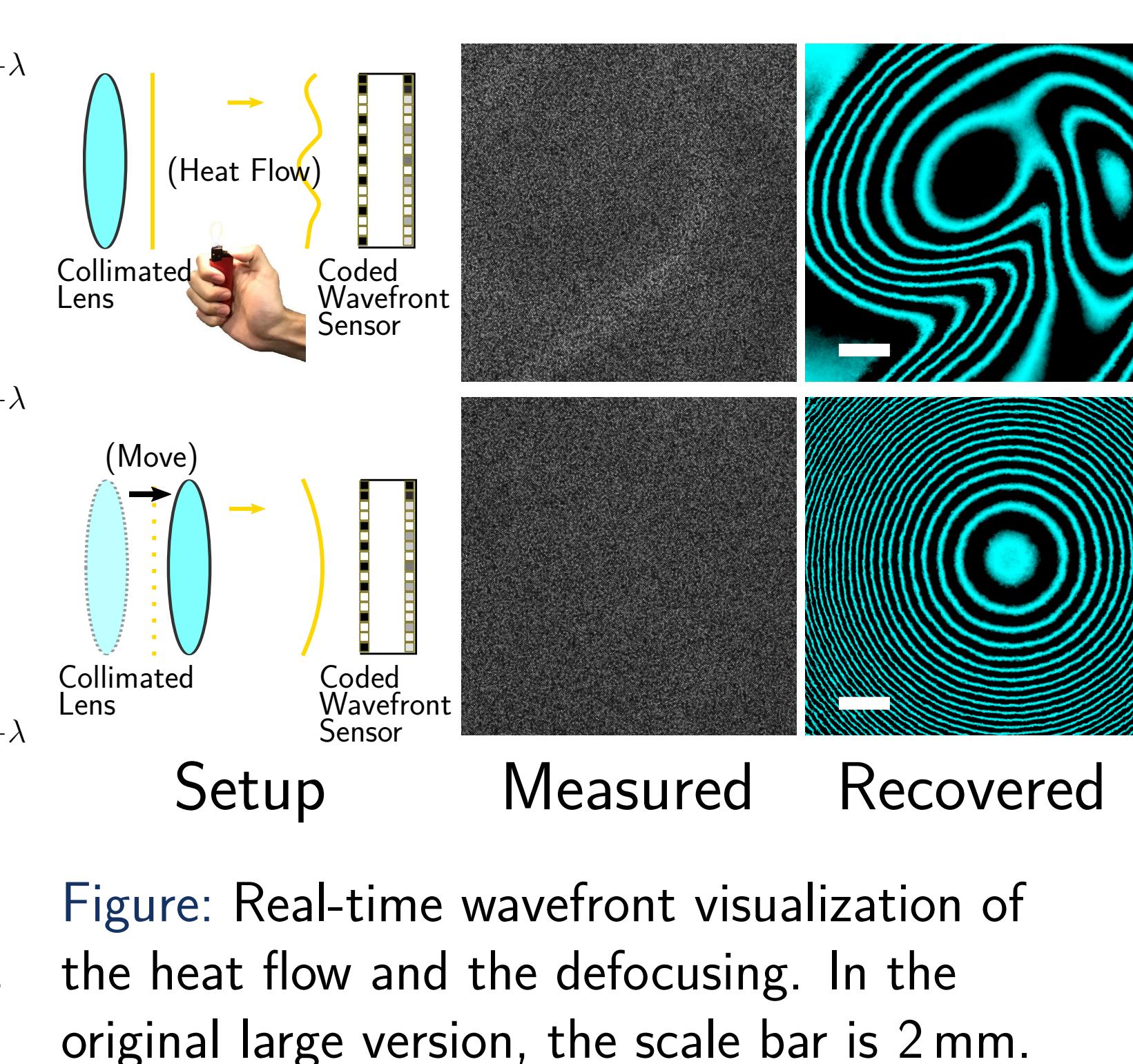


Figure: Real-time wavefront visualization of the heat flow and the defocusing. In the original large version, the scale bar is 2 mm.