CS 247 – Scientific Visualization
Lecture 15: Volume Rendering, Pt. 2 [preview]

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Reading Assignment #8 (until Mar 21)

Read (required):

- Real-Time Volume Graphics, Chapter 1
  (*Theoretical Background and Basic Approaches*),
  from beginning to 1.4.4 (inclusive)
- Real-Time Volume Graphics, Chapter 4 (Transfer Functions)
  until Sec. 4.4 (inclusive)

- Look at:
  Nelson Max, Optical Models for Direct Volume Rendering,
  IEEE Transactions on Visualization and Computer Graphics, 1995
  `http://dx.doi.org/10.1109/2945.468400`

Quiz #2: Mar 23

Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

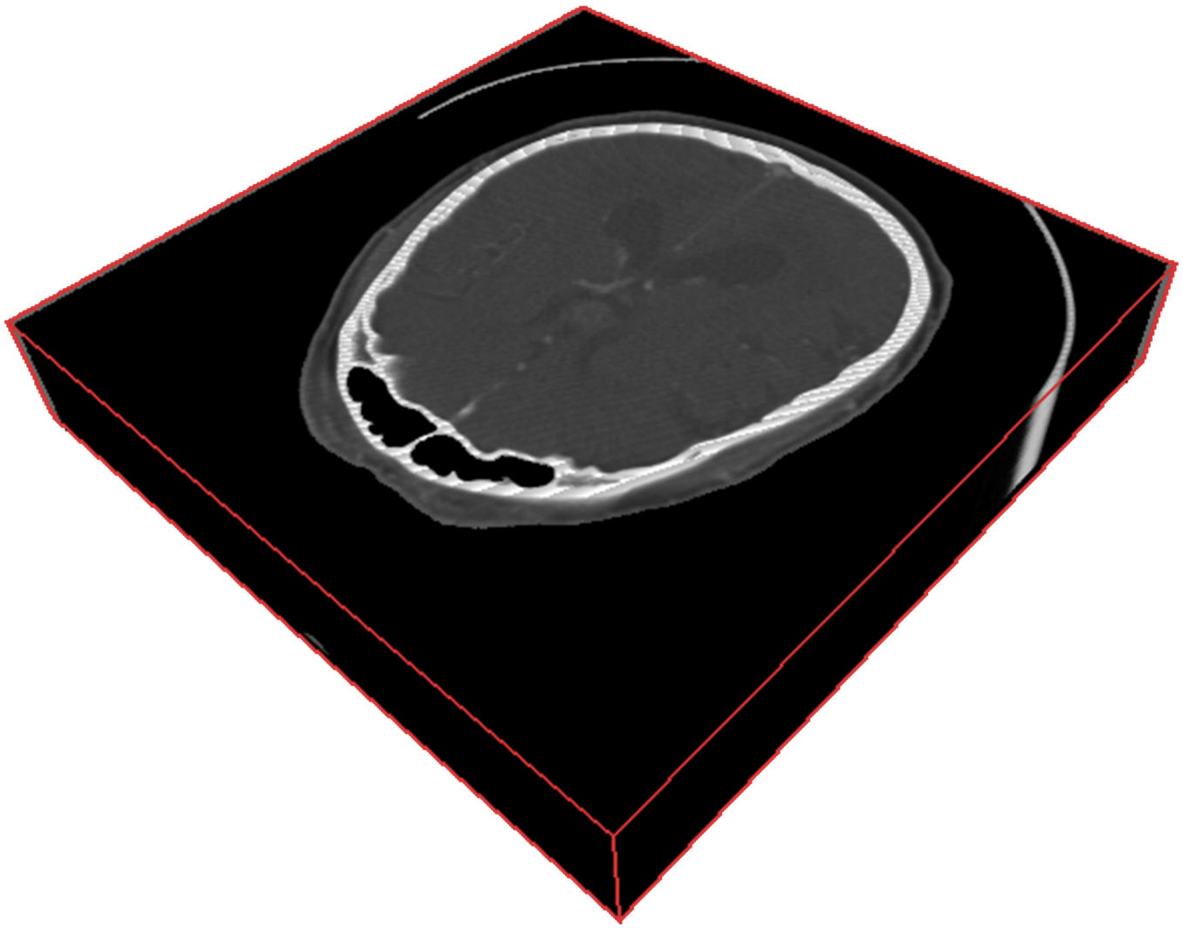
Content of questions

- Lectures (both actual lectures and slides)
- Reading assignments (except optional ones)
- Programming assignments (algorithms, methods)
- Solve short practical examples

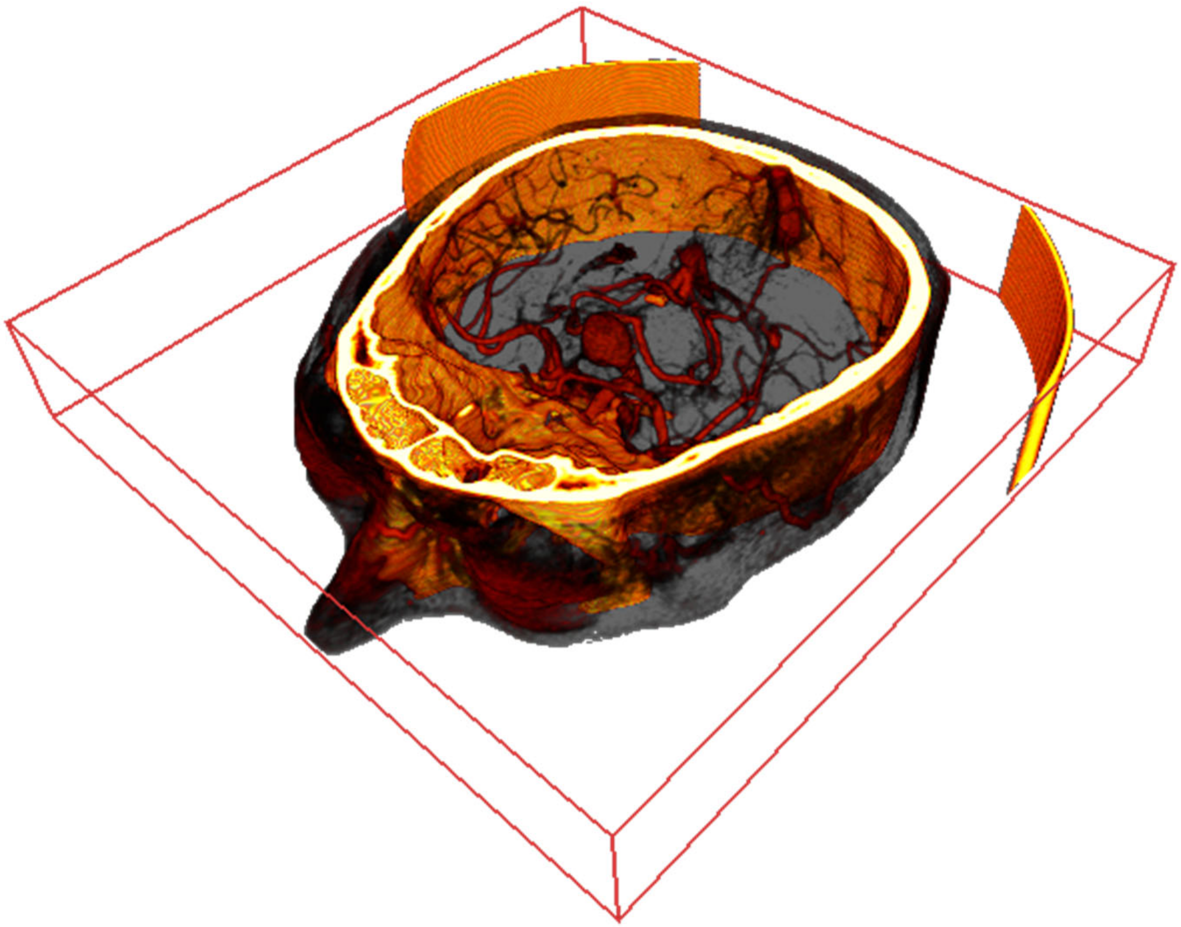
Volume Visualization

VolVis: Theory

Direct Volume Rendering

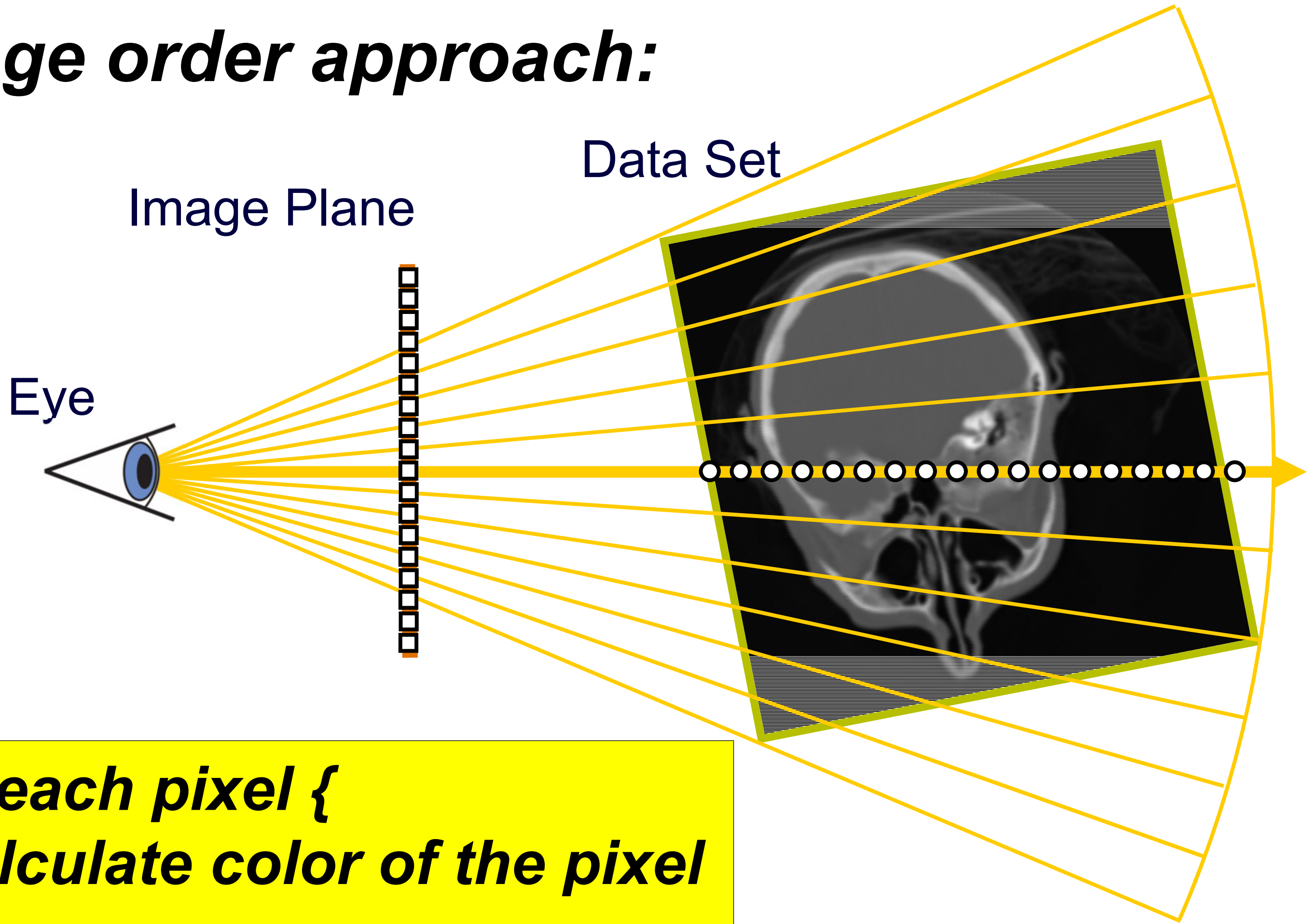


Direct Volume Rendering



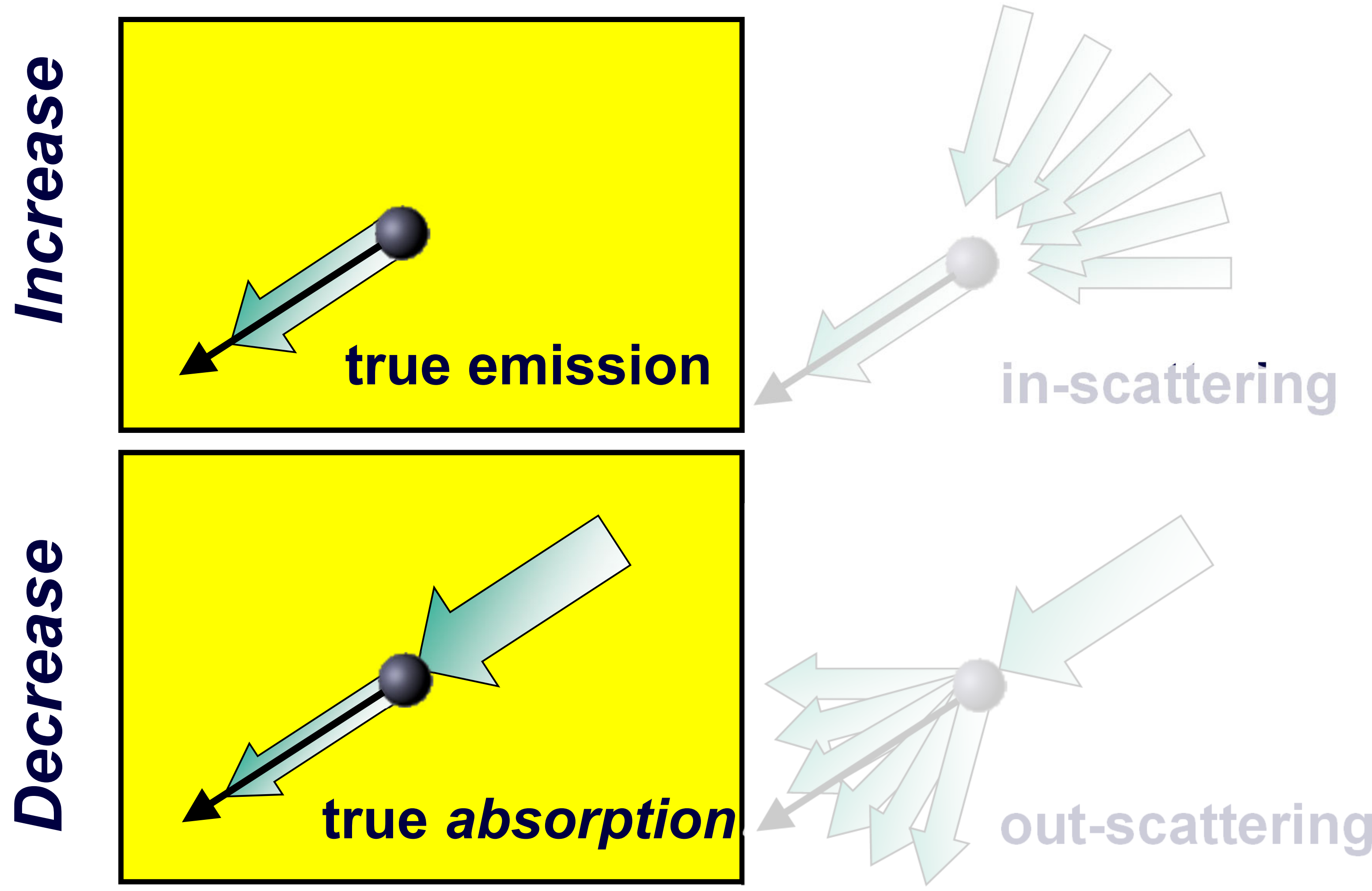
Direct Volume Rendering

Image order approach:



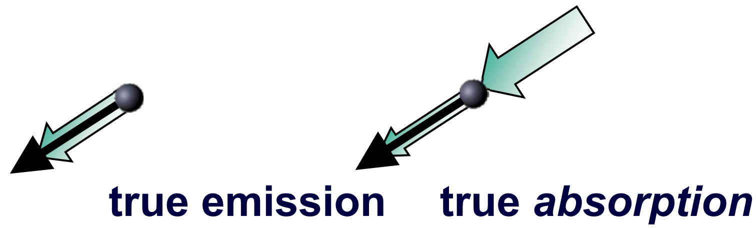
For each pixel {
calculate color of the pixel
}

Physical Model of Radiative Transfer



Volume Rendering Integral Summary

Volume rendering integral
for *Emission Absorption* model



$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

Numerical solutions:

Back-to-front compositing

$$C'_i \;=\; C_i + (1 - A_i) C'_{i-1}$$

Front-to-back compositing

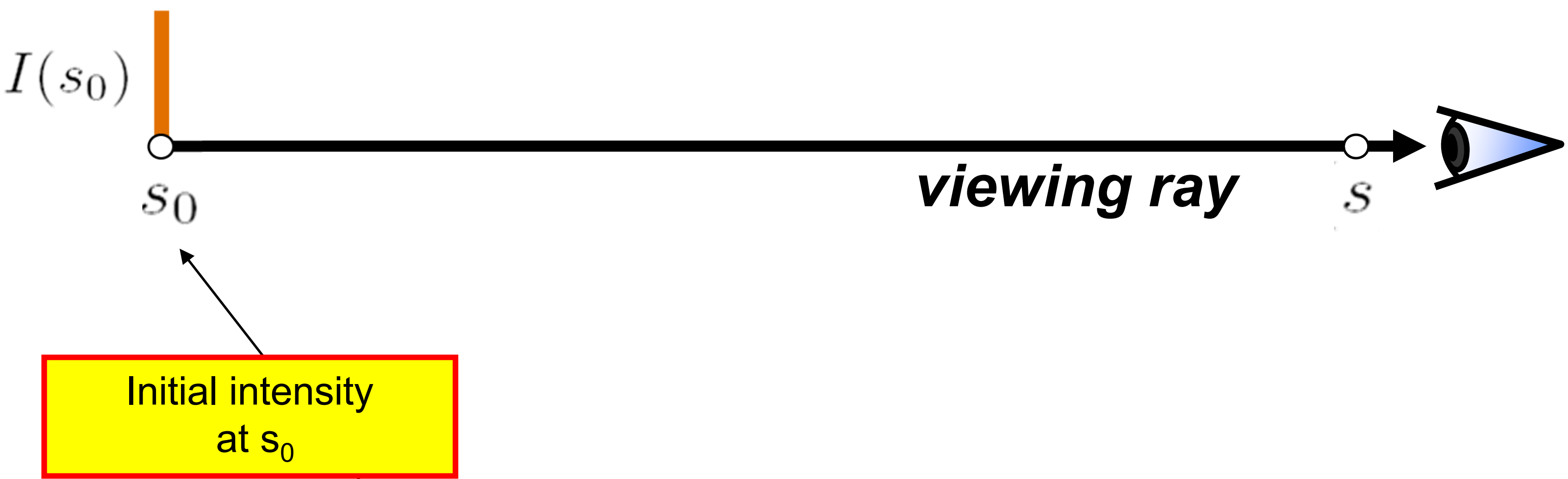
$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1}) C_i$$
$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1}) A_i$$

here, all colors are associated colors!

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

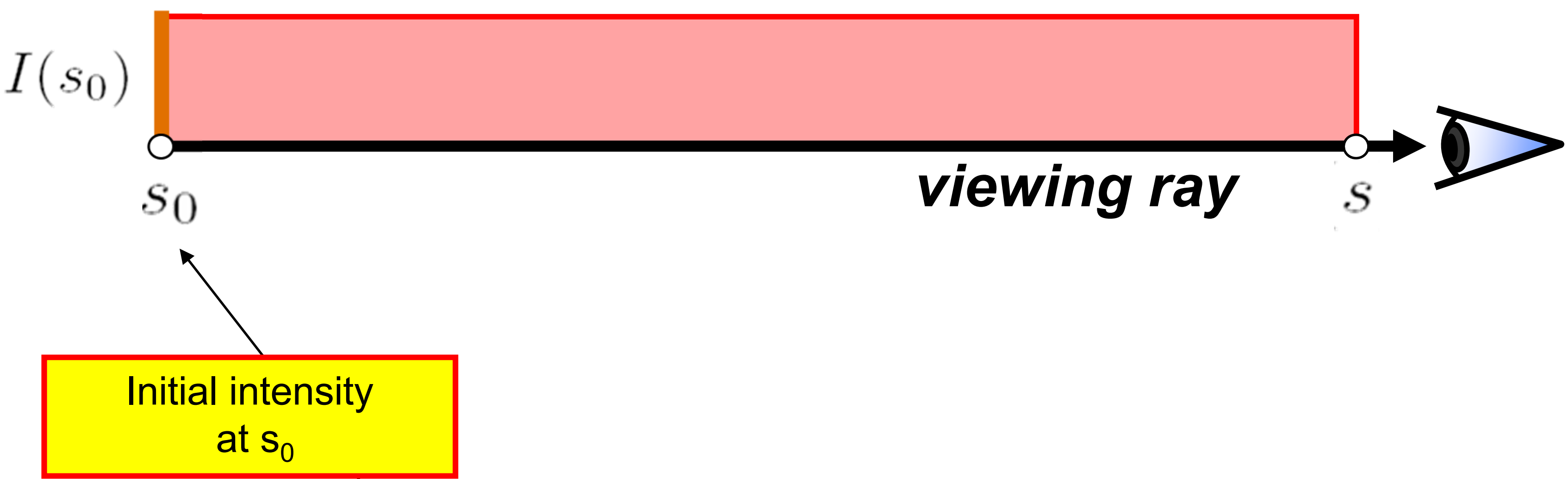


$$I(s) = \boxed{I(s_0)}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



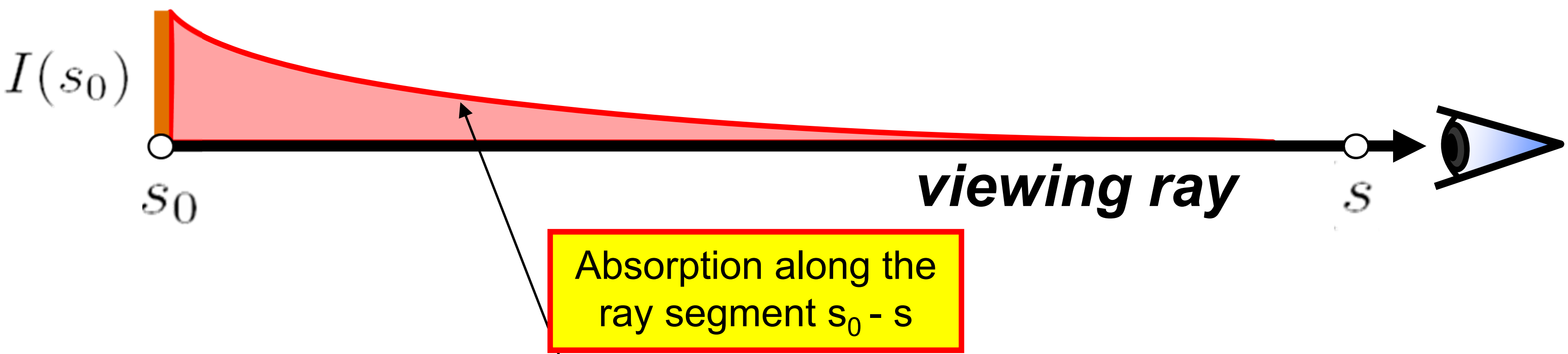
$$I(s) = \boxed{I(s_0)}$$

Without absorption all the initial radiant energy would reach the point ***s.***

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

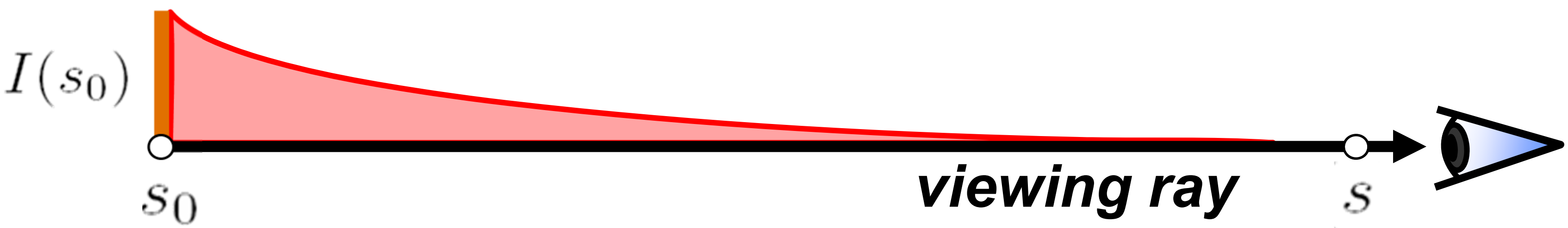


$$I(s) = I(s_0) \boxed{e^{-\tau(s_0,s)}}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Optical depth τ

Absorption κ

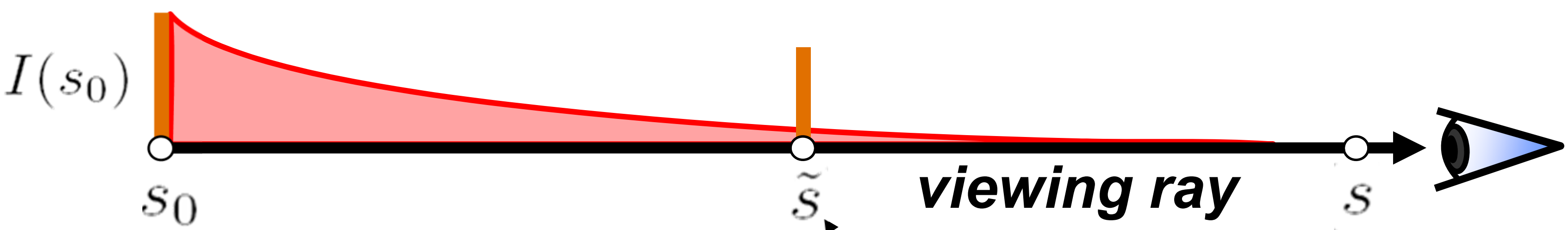
$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

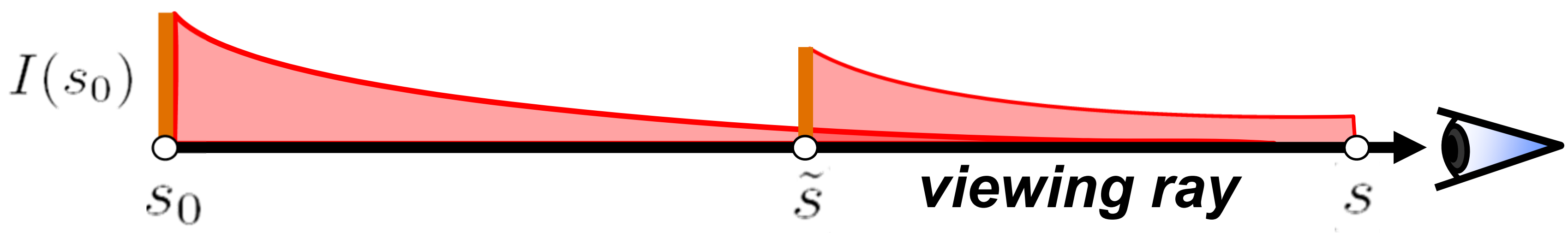
Active emission at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \boxed{q(\tilde{s})}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

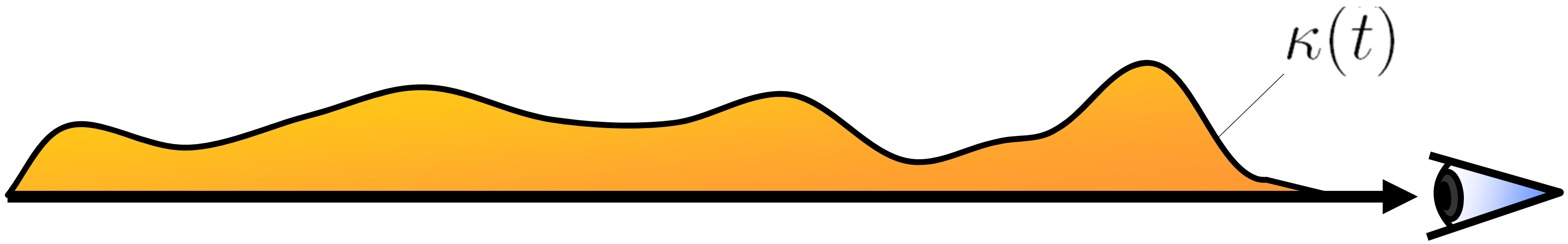
Physical model: emission and absorption, no scattering



Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

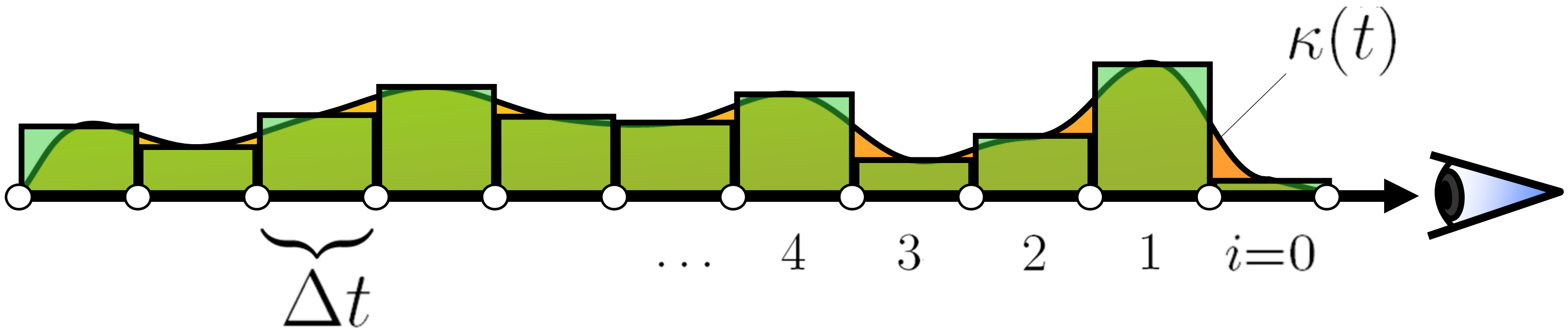
$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} \;+\; \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0,t) \quad = \quad \int_0^t \kappa(\hat{t})\, d\hat{t}$

Volume Rendering Integral: Numerical Solution

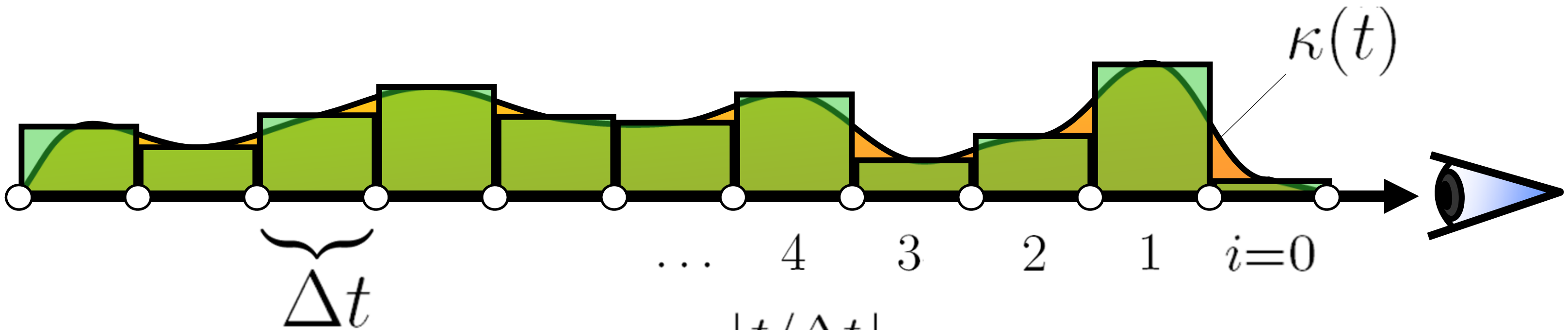


Optical depth: $\tau(0,t) \;=\; \int_0^t \kappa(\hat{t})\, d\hat{t}$

Approximate Integral by Riemann sum:

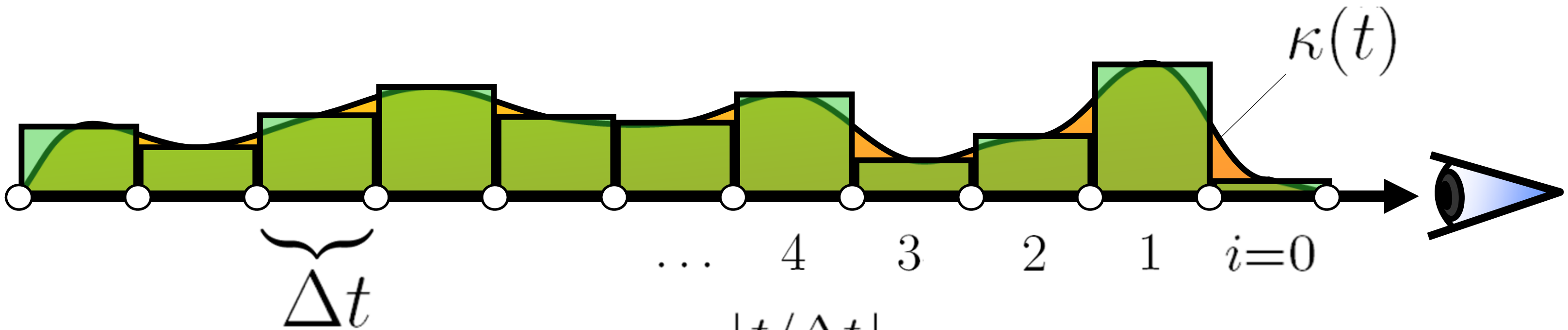
$$\tau(0,t) \;\approx\; \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

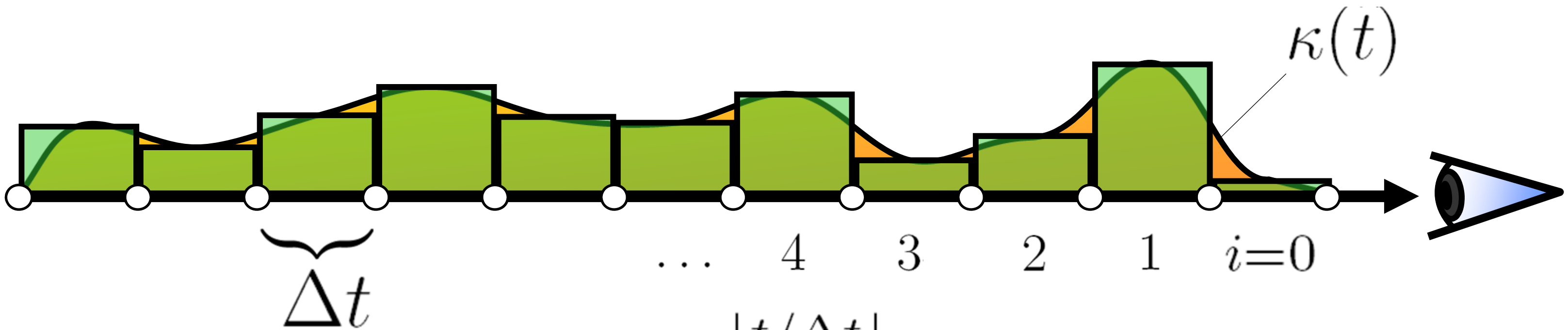
Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

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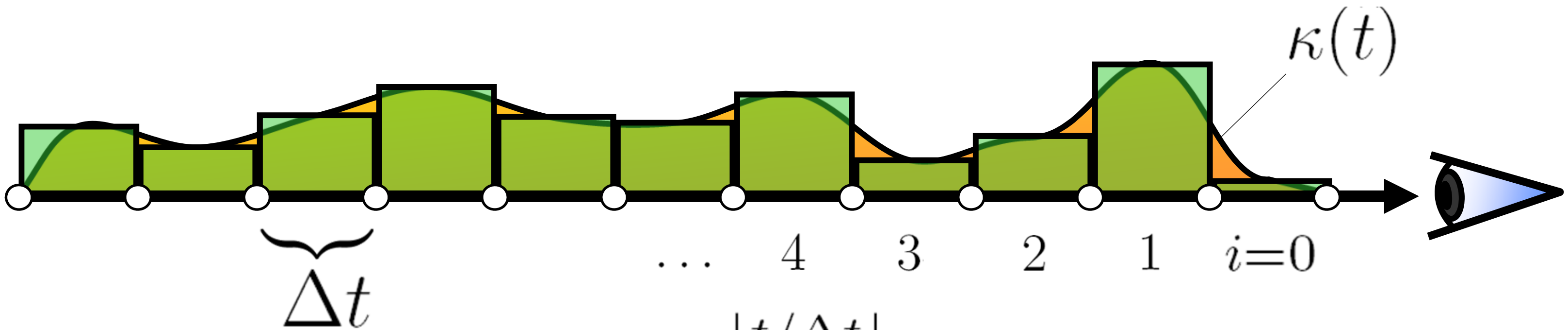
Volume Rendering Integral: Numerical Solution



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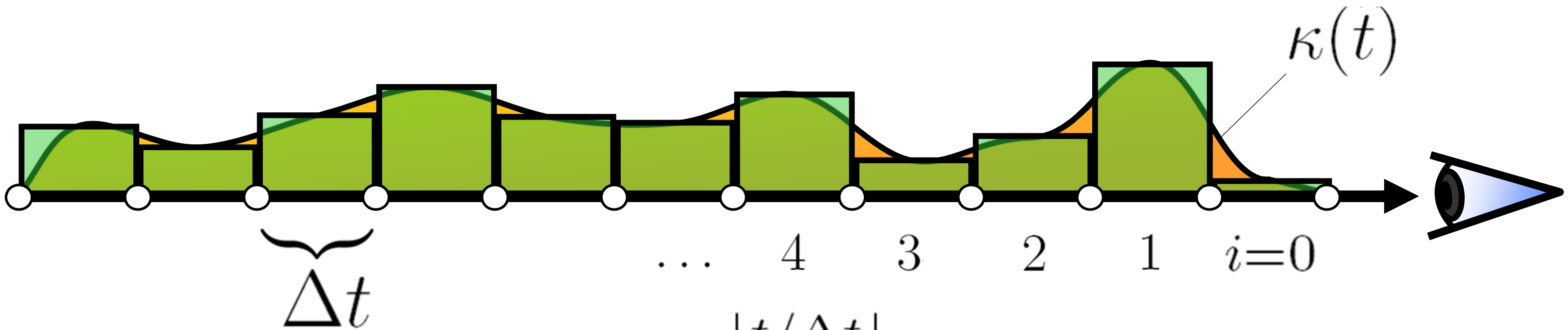
Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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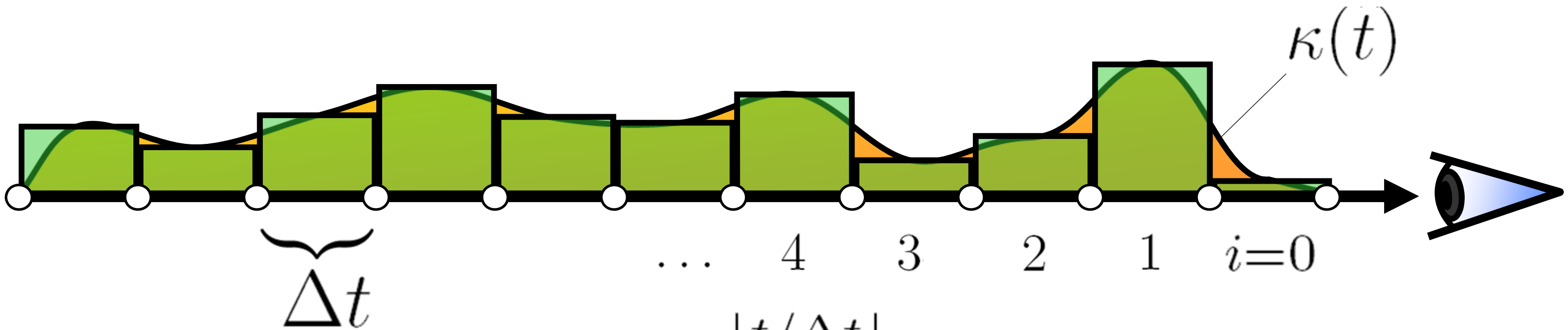
Now we introduce *opacity*:

$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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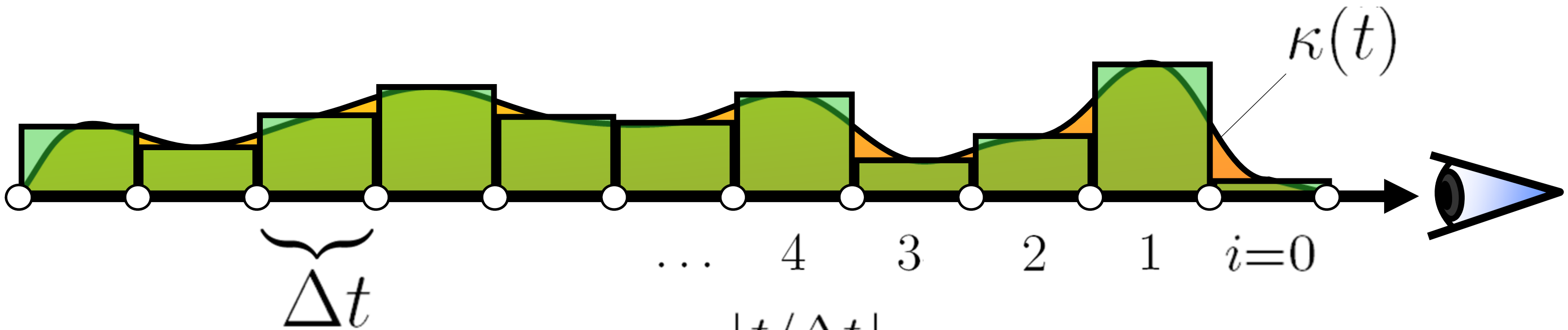
Volume Rendering Integral: Numerical Solution



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$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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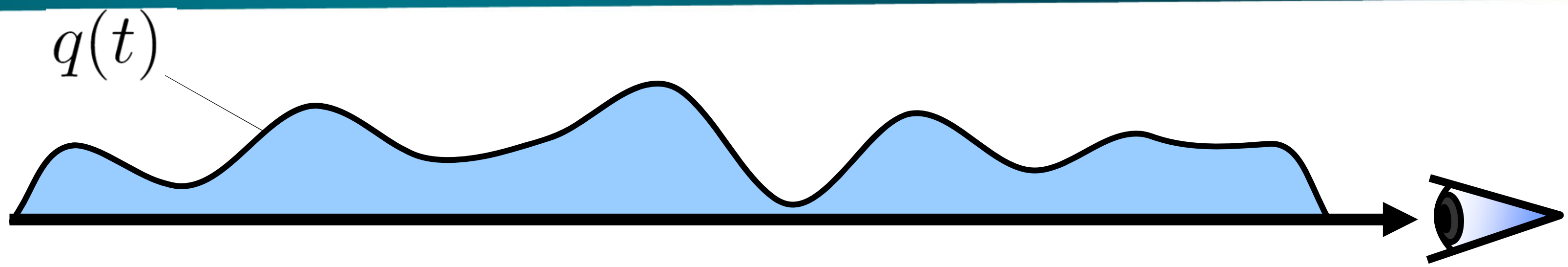
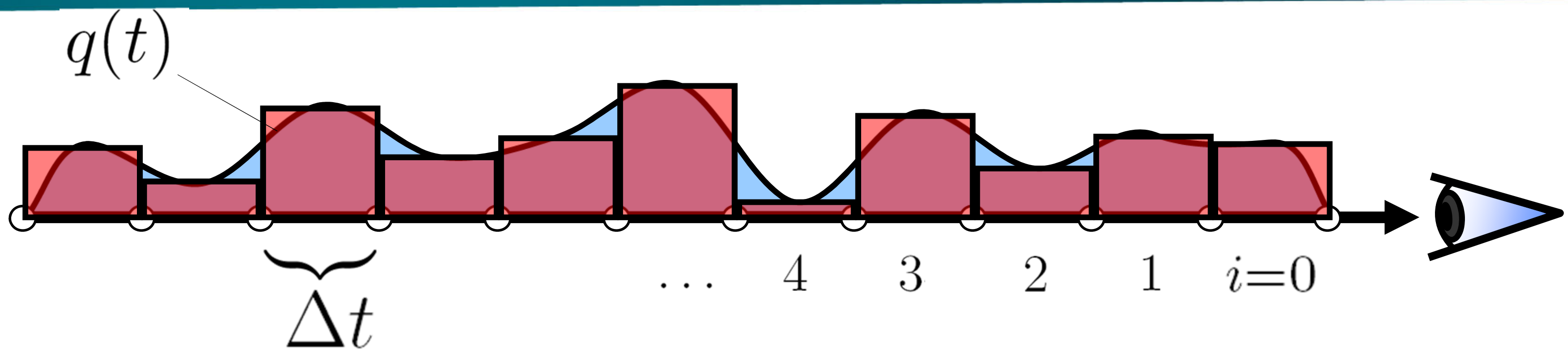
$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



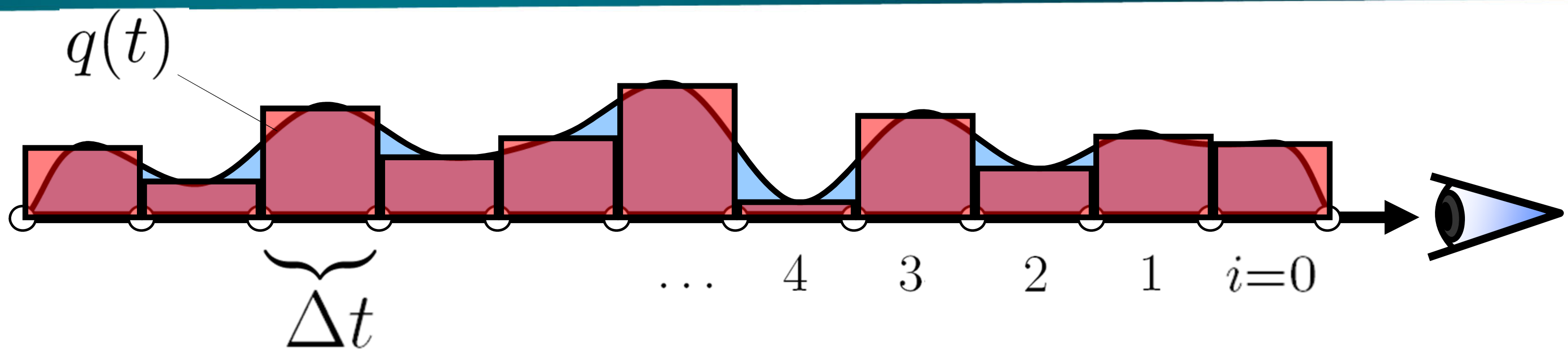
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

Volume Rendering Integral: Numerical Solution

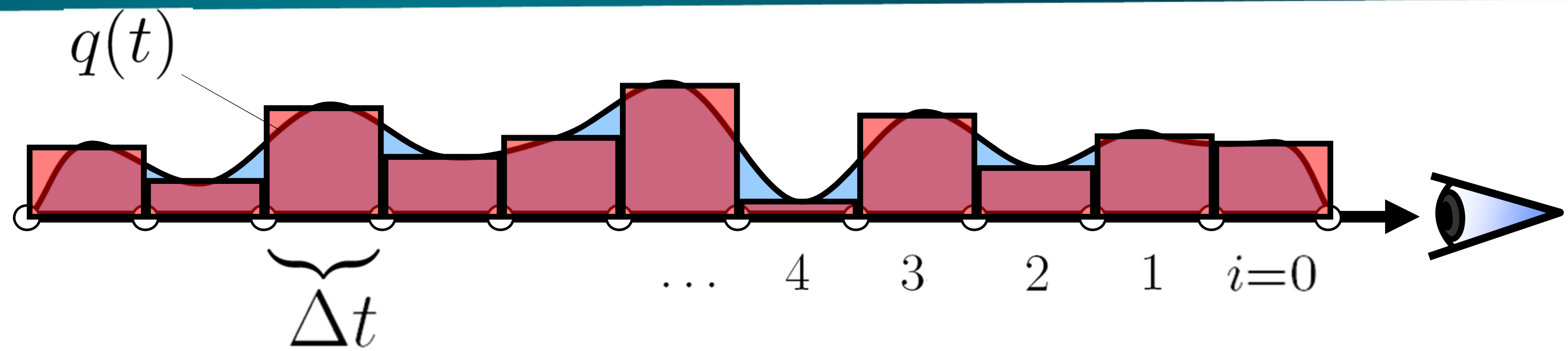


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\, \Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution



$$\boxed{e^{-\tilde{\tau}(0,t)}} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

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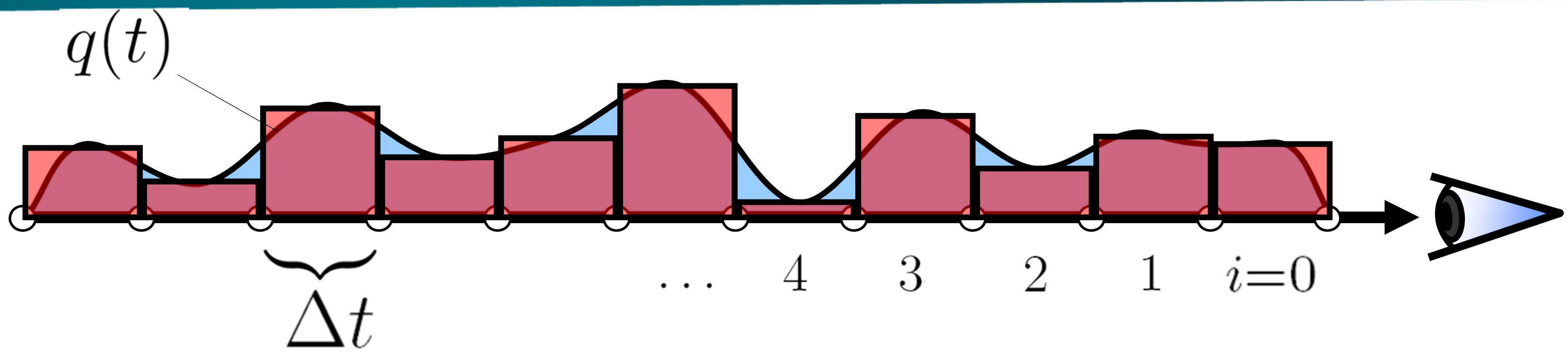
Volume Rendering Integral: Numerical Solution



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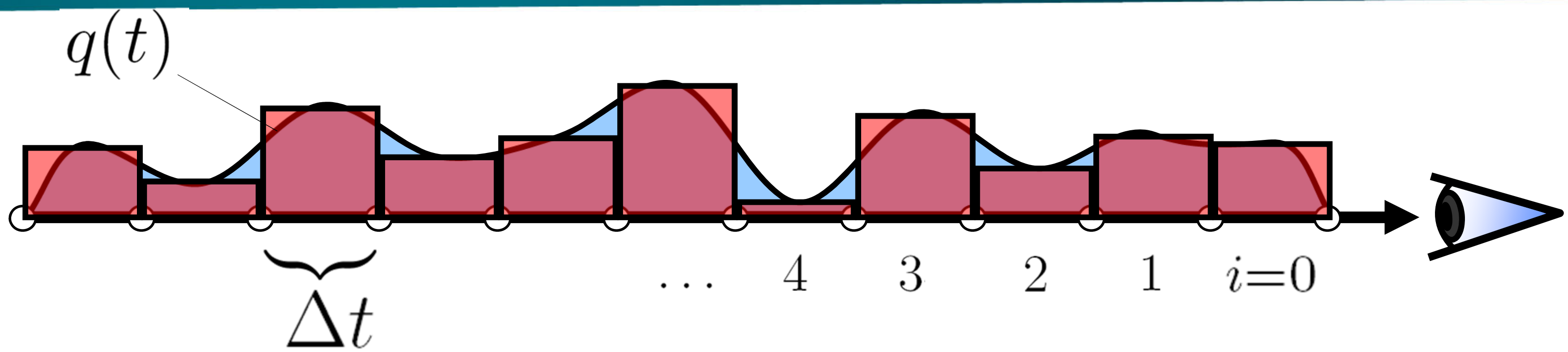
$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

Volume Rendering Integral: Numerical Solution



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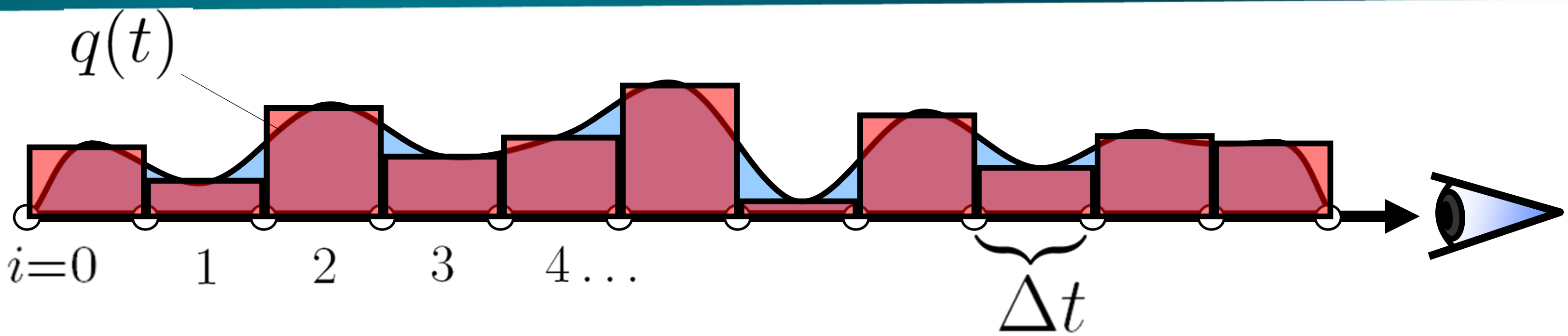
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively/iteratively!

Volume Rendering Integral: Numerical Solution



Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !

can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

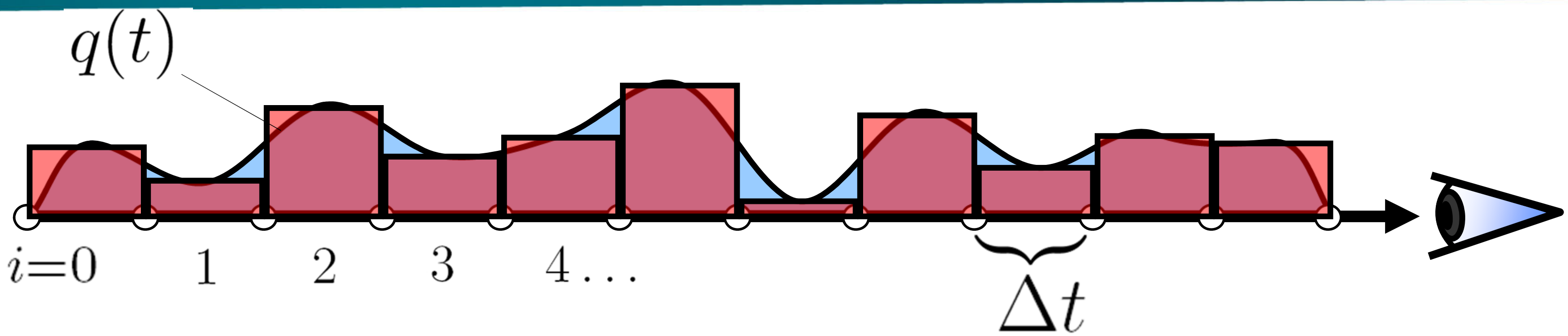
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Iterate from *i*=0 (back) to *i*=max (front): *i* increases

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

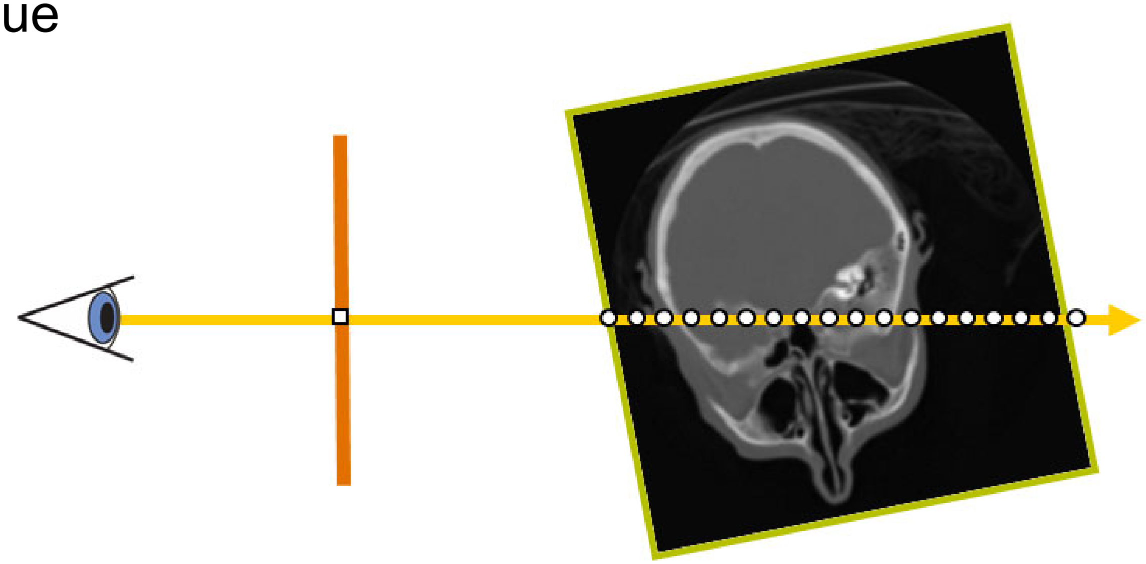
VolVis: Implementation

Implementation

Ray setup

Loop over ray

- Resample scalar value
- Classification
- Shading
- Compositing



Implementation

Ray setup

Loop over ray

- Resample scalar value
- Classification
- Shading
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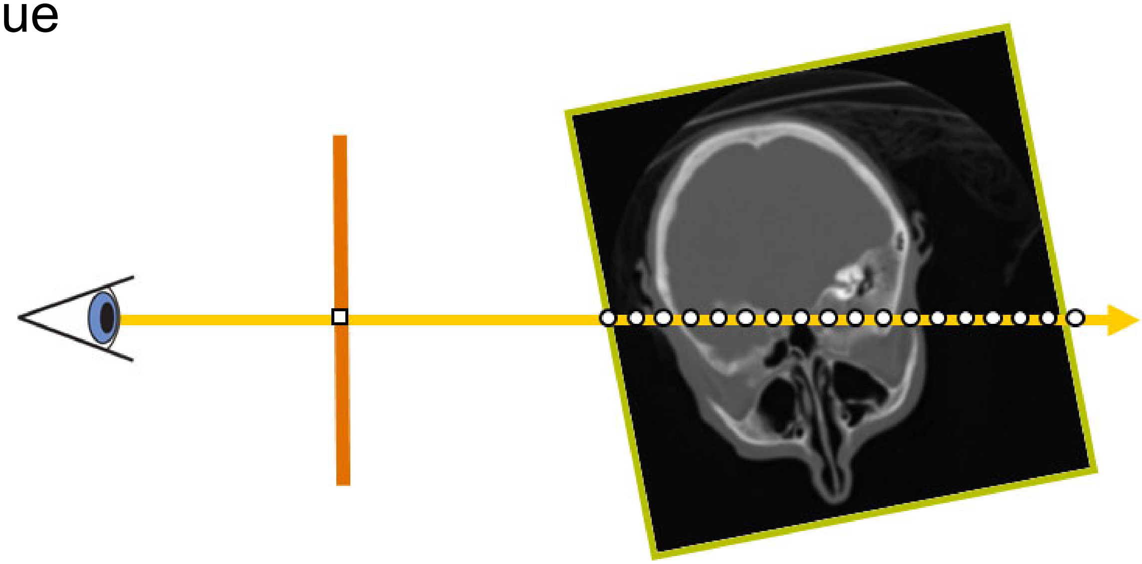


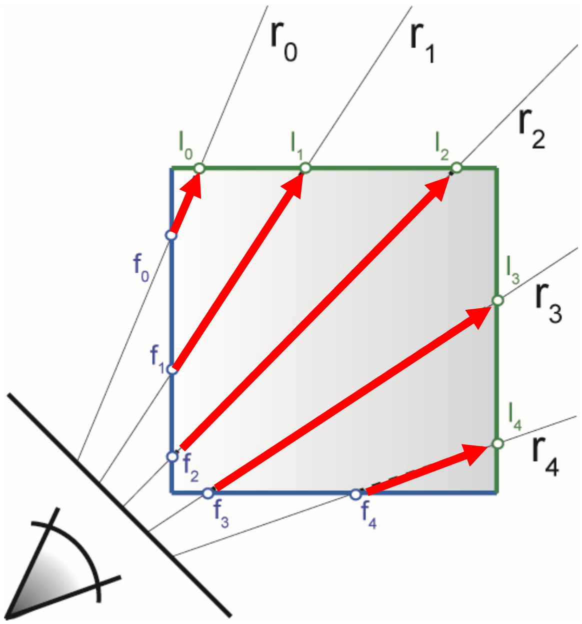
Ray Setup

Two main approaches:

- Procedural ray/box intersection
  [Röttger et al., 2003], [Green, 2004]
- Rasterize bounding box
  [Krüger and Westermann, 2003]

Some possibilities

- Ray start position and exit check
- Ray start position and exit position
- Ray start position and direction vector

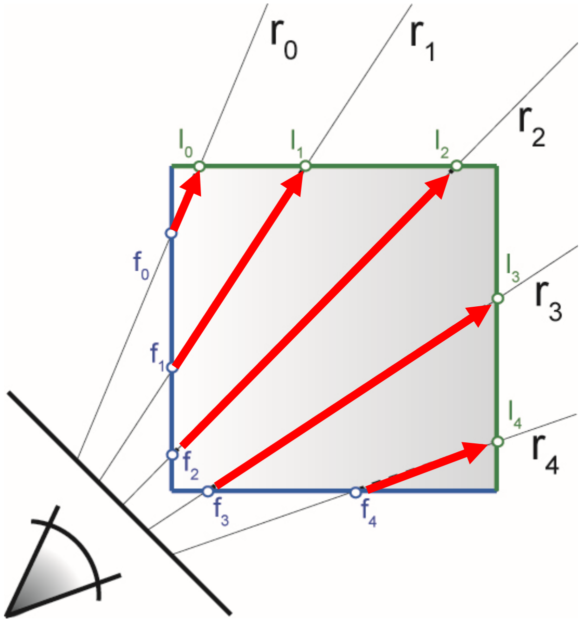


Procedural Ray Setup/Termination

- Everything handled in the fragment shader / CUDA kernel
- Procedural ray / bounding box intersection

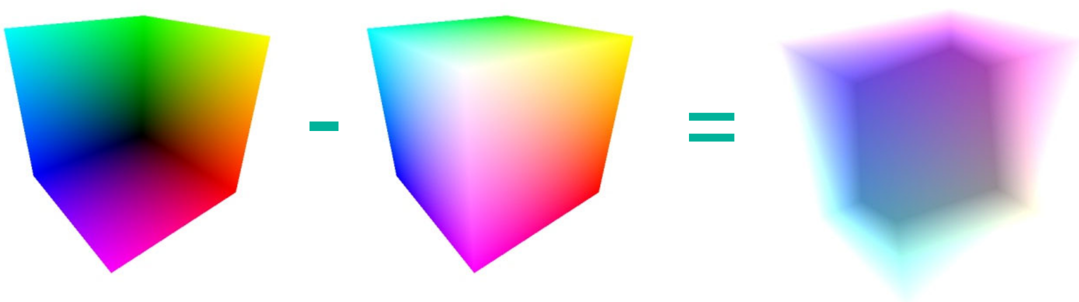
- Ray is given by camera position and volume entry position
- Exit criterion needed

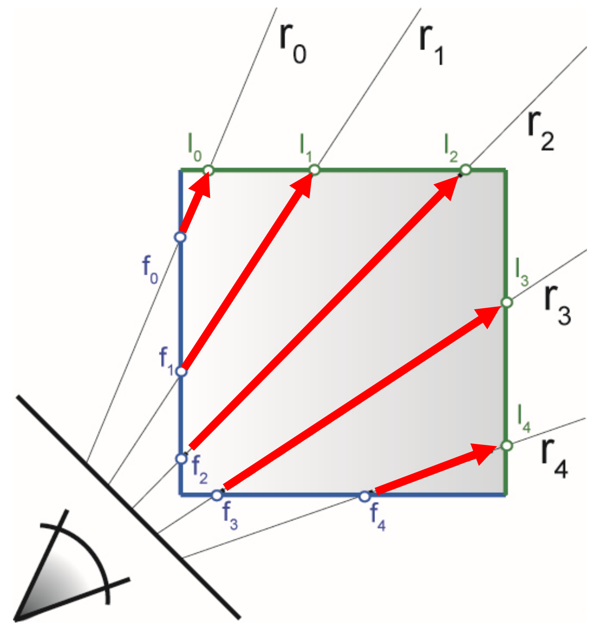
- Pro: simple and self-contained
- Con: full computational load per-pixel/fragment



Rasterization-Based Ray Setup

- Fragment == ray
- Need ray start pos, direction vector
- Rasterize bounding box

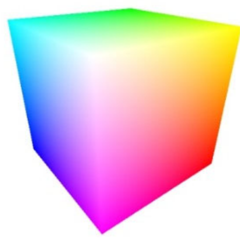


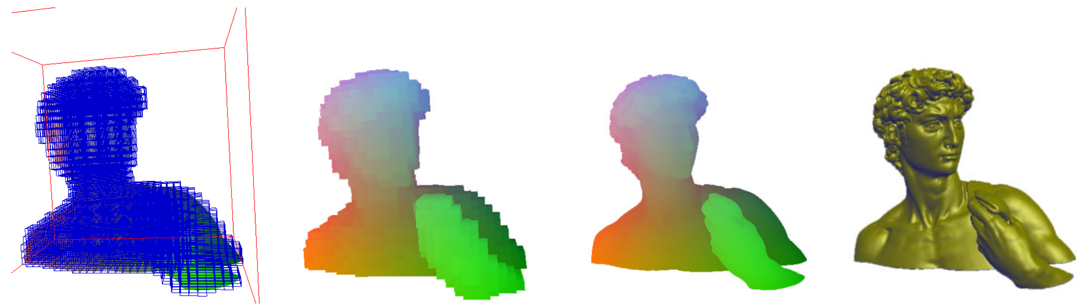


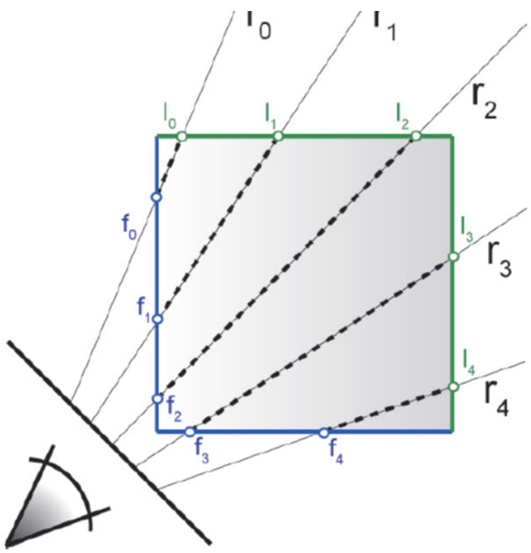
- Identical for orthogonal and perspective projection!

Object-Order Empty Space Skipping

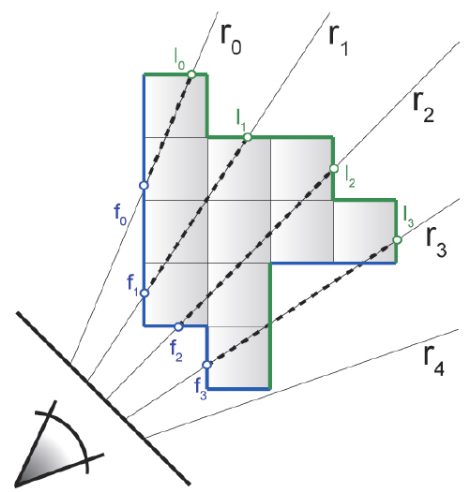
Modify initial rasterization step







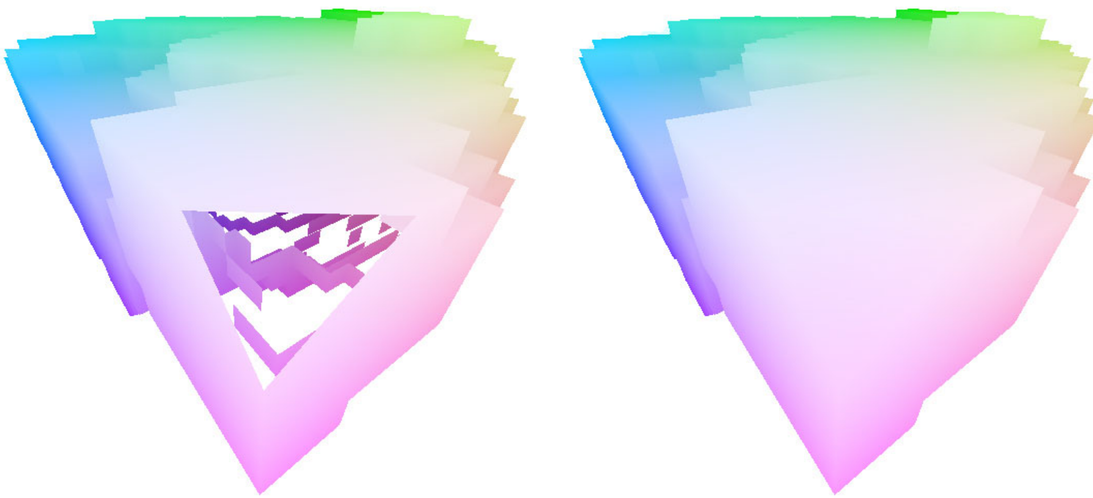
rasterize bounding box

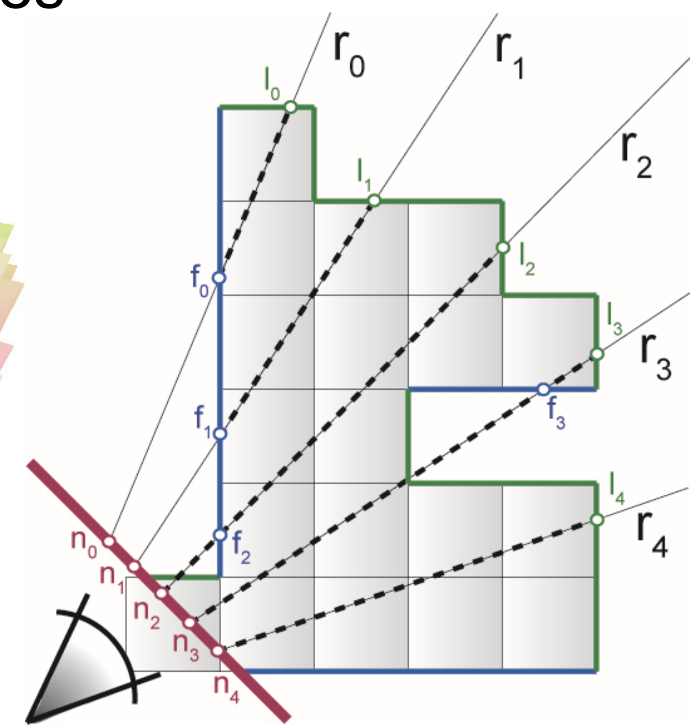


rasterize “tight" bounding geometry

Moving Into The Volume

Near clipping plane clips into front faces





Fill in holes with near clipping plane

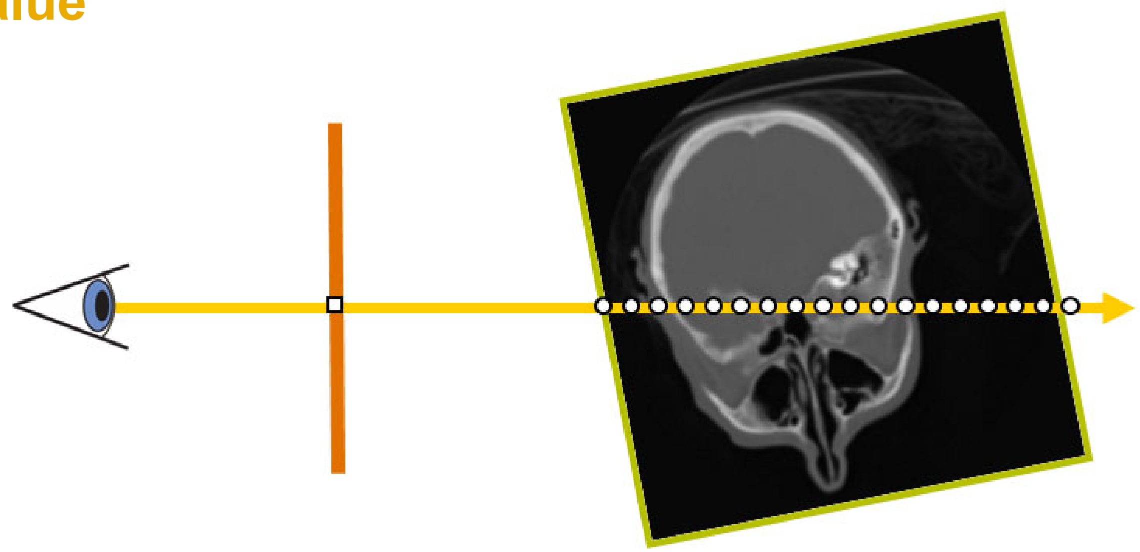
Can use depth buffer [Scharsach et al., 2006]

Implementation

Ray setup

Loop over ray

- **Resample scalar value**
- **Classification**
- Shading
- Compositing



Classification – Transfer Functions

During Classification the user defines the ***"look"*** of the data.

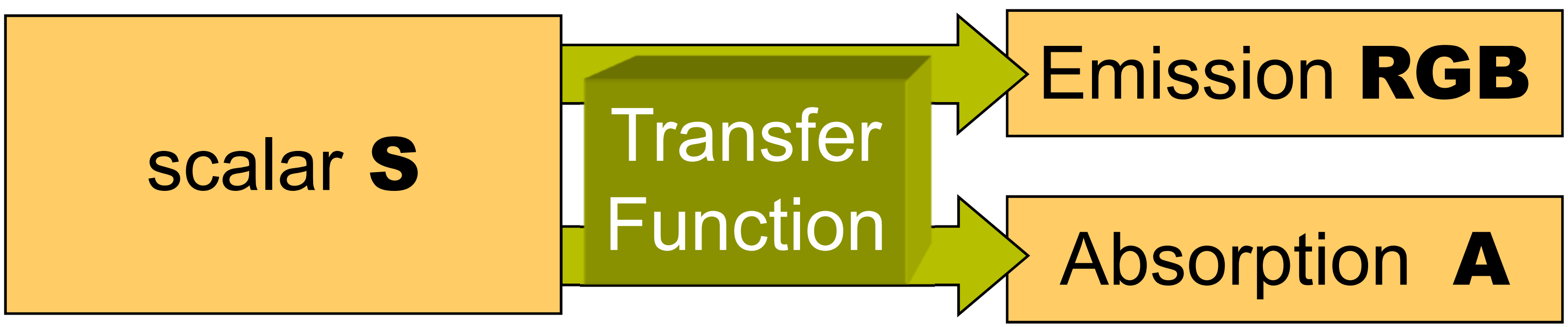
- Which parts are transparent?
- Which parts have what color?

Classification – Transfer Functions

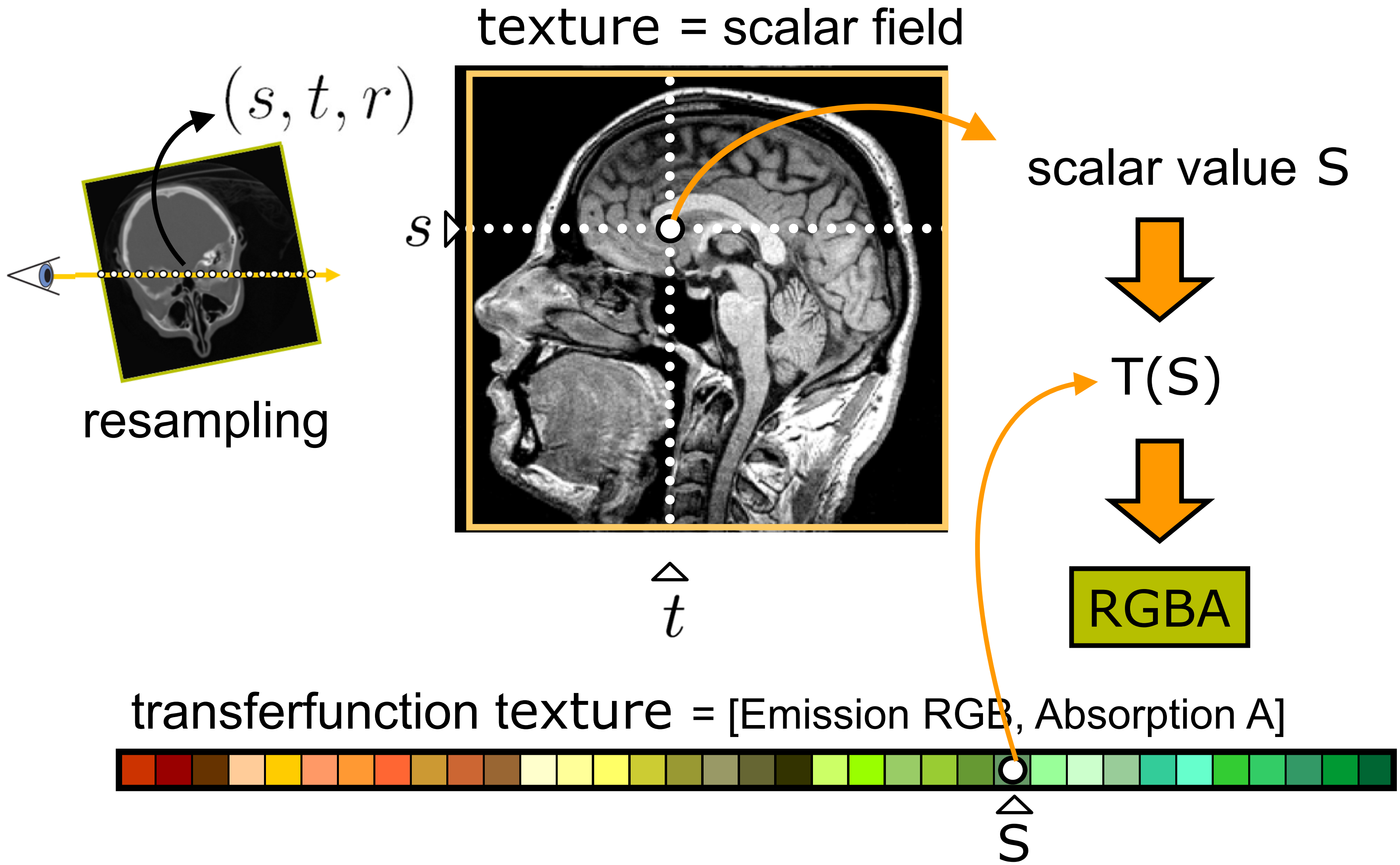
During Classification the user defines the ***“look“*** of the data.

- Which parts are transparent?
- Which parts have what color?

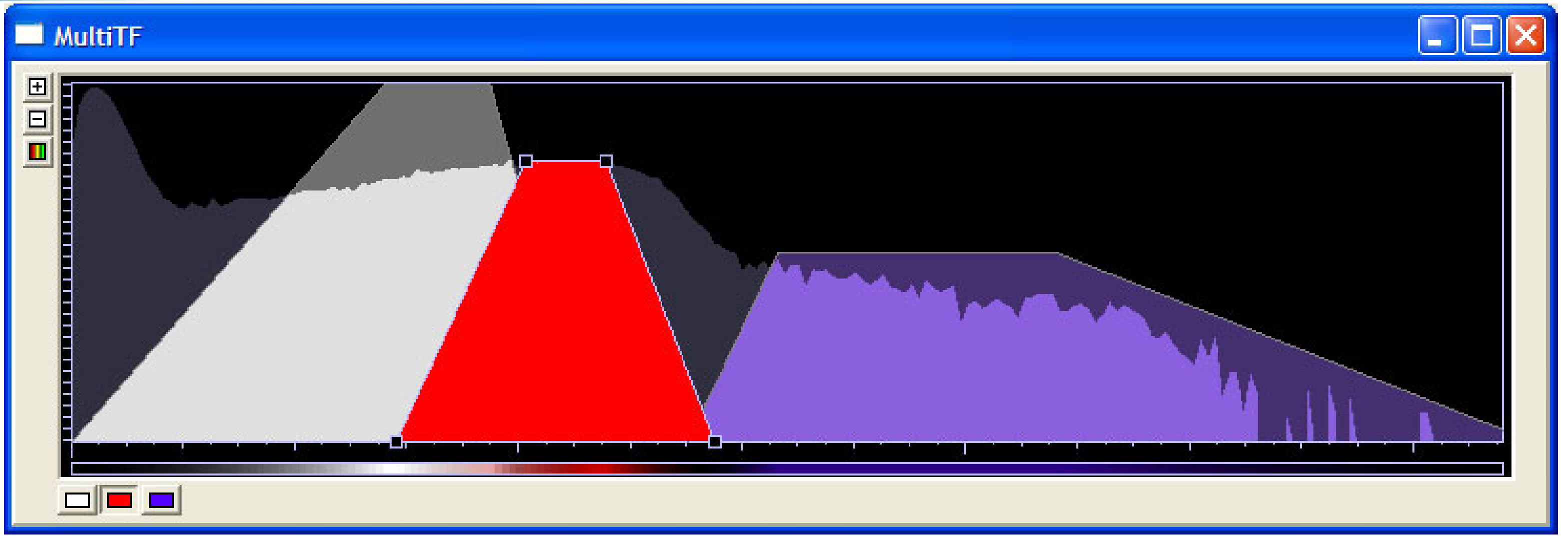
The user defines a ***transfer function.***

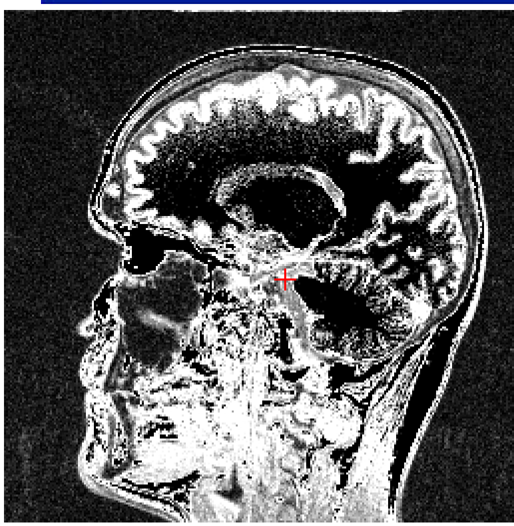


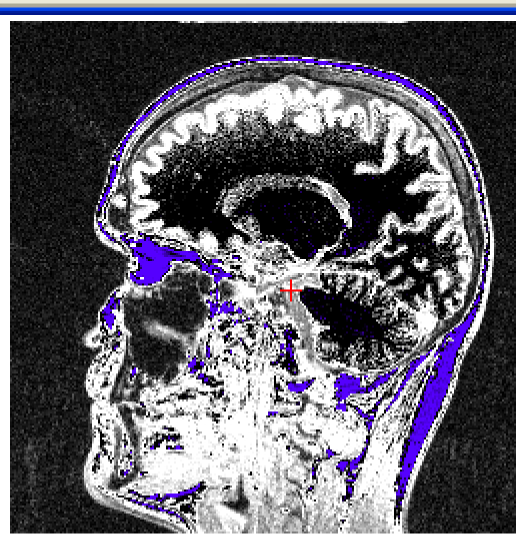
1D Transfer Functions

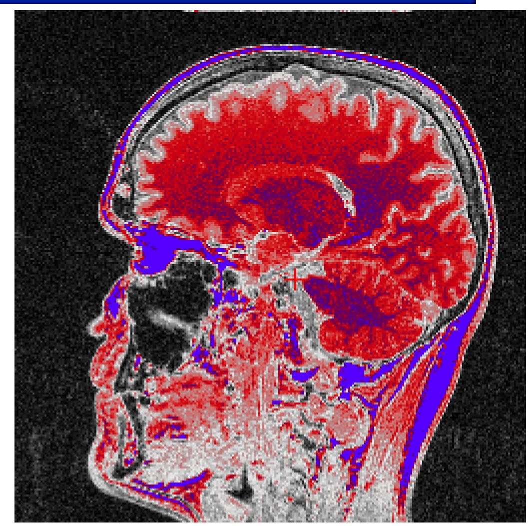


1D Transfer Functions









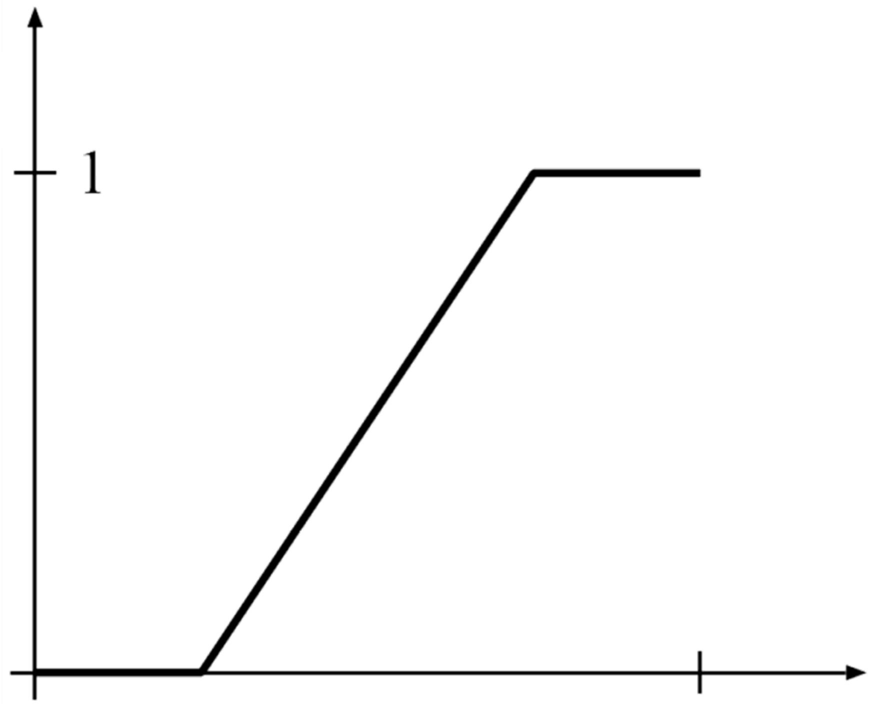
Applying Transfer Function: Cg Example

```
// Cg fragment program for post-classification
// using 3D textures
float4 main (float3 texUV : TEXCOORD0,
             uniform sampler3D volume_texture,
             uniform sampler1D transfer_function) :
  COLOR
{
   float index = tex3D(volume_texture, texUV);
   float4 result = tex1D(transfer_function, index);
   return result;
}
```

Windowing Transfer Function

Map input scalar range to output intensity range

- Select scalar range of interest
- Adjust contrast

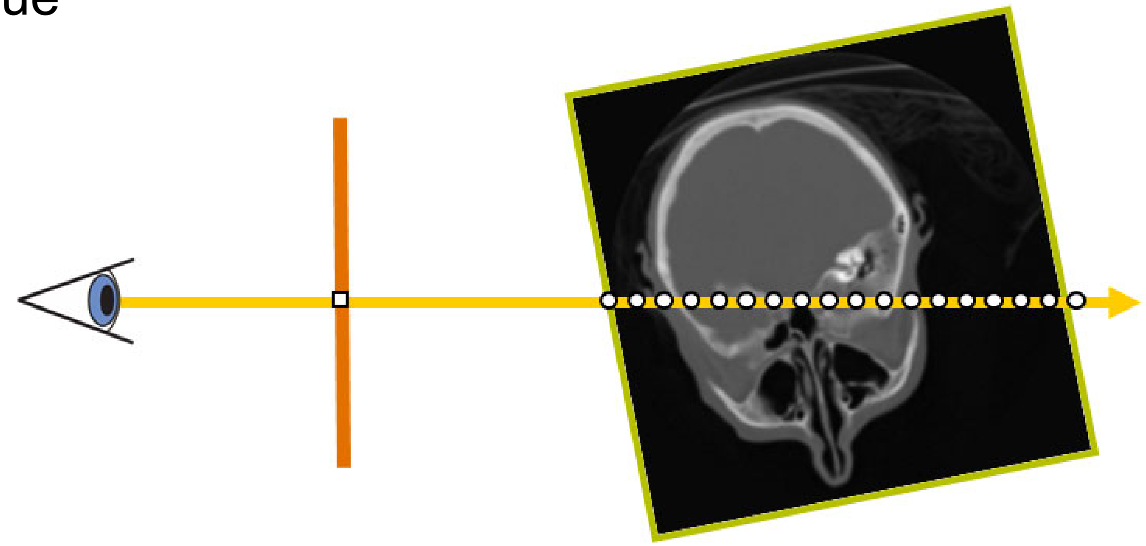


Implementation

Ray setup

Loop over ray

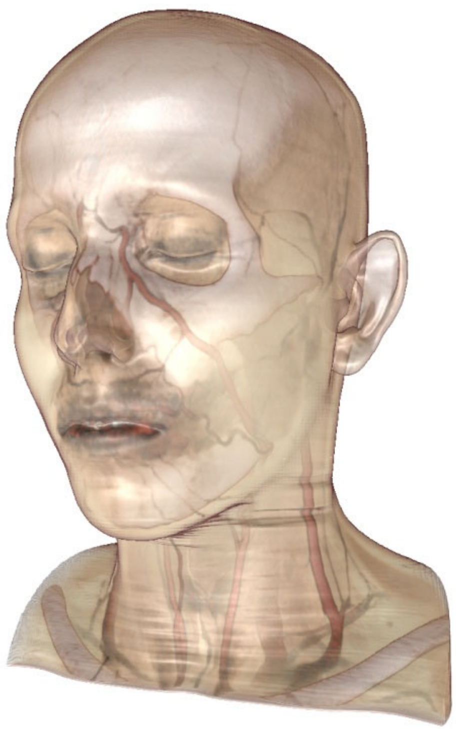
- Resample scalar value
- Classification
- **Shading**
- Compositing

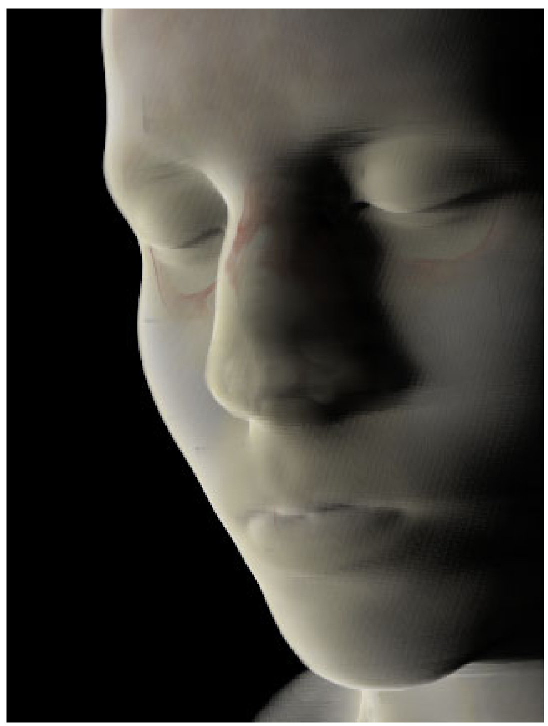


Volume Shading

Local illumination vs. global illumination

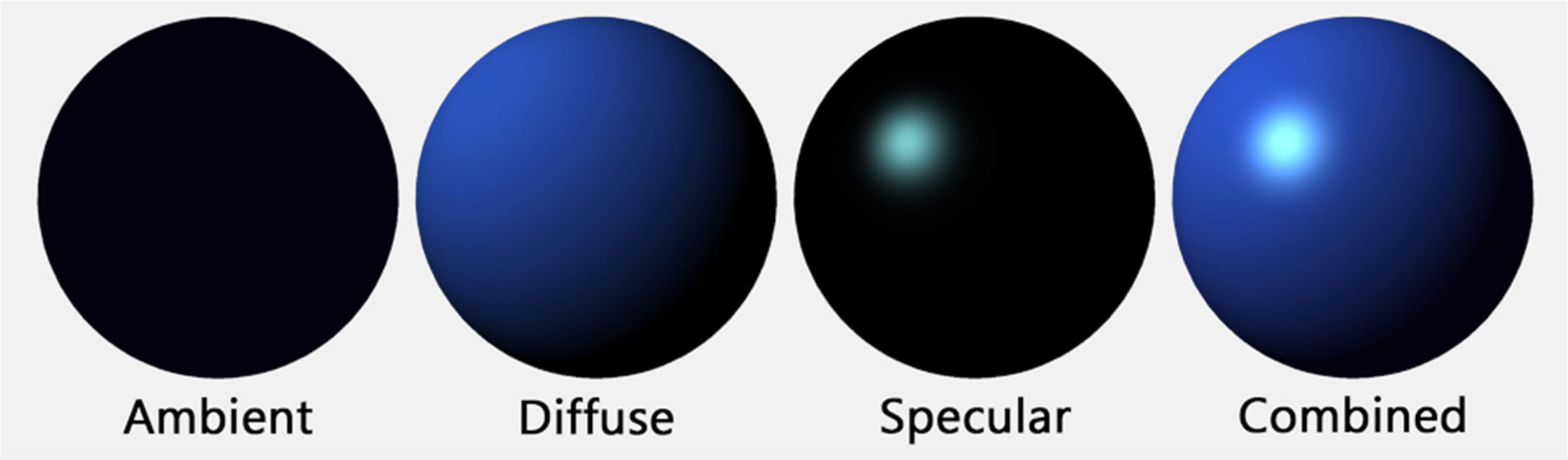
- Gradient-based or gradient-less
- Shadows, (multiple) scattering, …





Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$

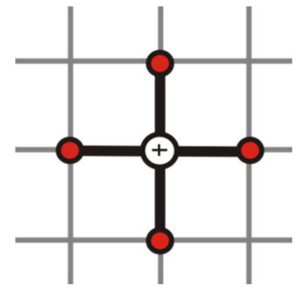


On-the-fly Gradient Estimation

$$\nabla f(x,y,z) \approx \frac{1}{2h}\begin{pmatrix} f(x+h,y,z) - f(x-h,y,z) \\ f(x,y+h,z) - f(x,y-h,z) \\ f(x,y,z+h) - f(x,y,z-h) \end{pmatrix}$$

```
float3 sample1, sample2;
// six texture samples for the gradient
sample1.x = tex3D(texture,uvw-half3(DELTA,0.0,0.0)).x;
sample2.x = tex3D(texture,uvw+half3(DELTA,0.0,0.0)).x;
sample1.y = tex3D(texture,uvw-half3(0.0,DELTA,0.0)).x;
sample2.y = tex3D(texture,uvw+half3(0.0,DELTA,0.0)).x;
sample1.z = tex3D(texture,uvw-half3(0.0,0.0,DELTA)).x;
sample2.z = tex3D(texture,uvw+half3(0.0,0.0,DELTA)).x;
// central difference and normalization
float3 N = normalize(sample2-sample1);
```

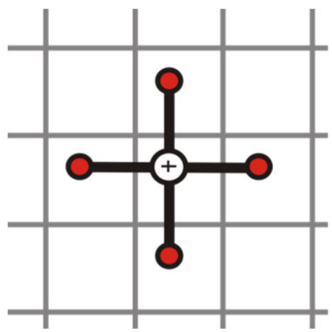
On-The-Fly Gradients



Reduce texture memory consumption!

Central differences before and after linear interpolation of values at grid points yield the same results

Caveat: texture filter precision



Filter kernel methods are expensive, but:

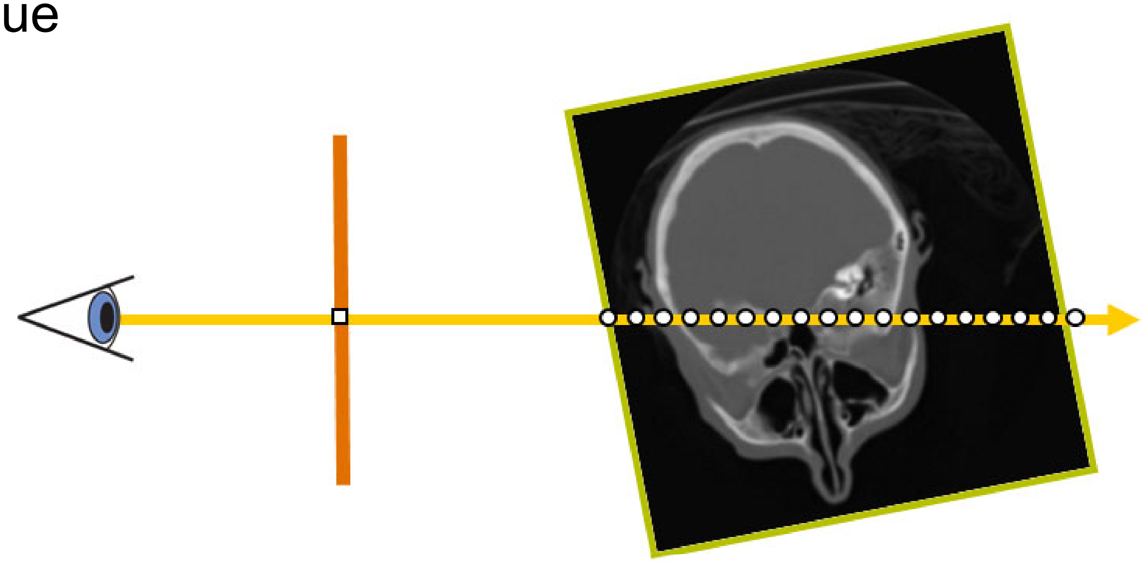
Tri-cubic B-spline kernels can be used in real-time (e.g., GPU Gems 2 Chapter “Fast Third-Order Filtering”)

Implementation

Ray setup

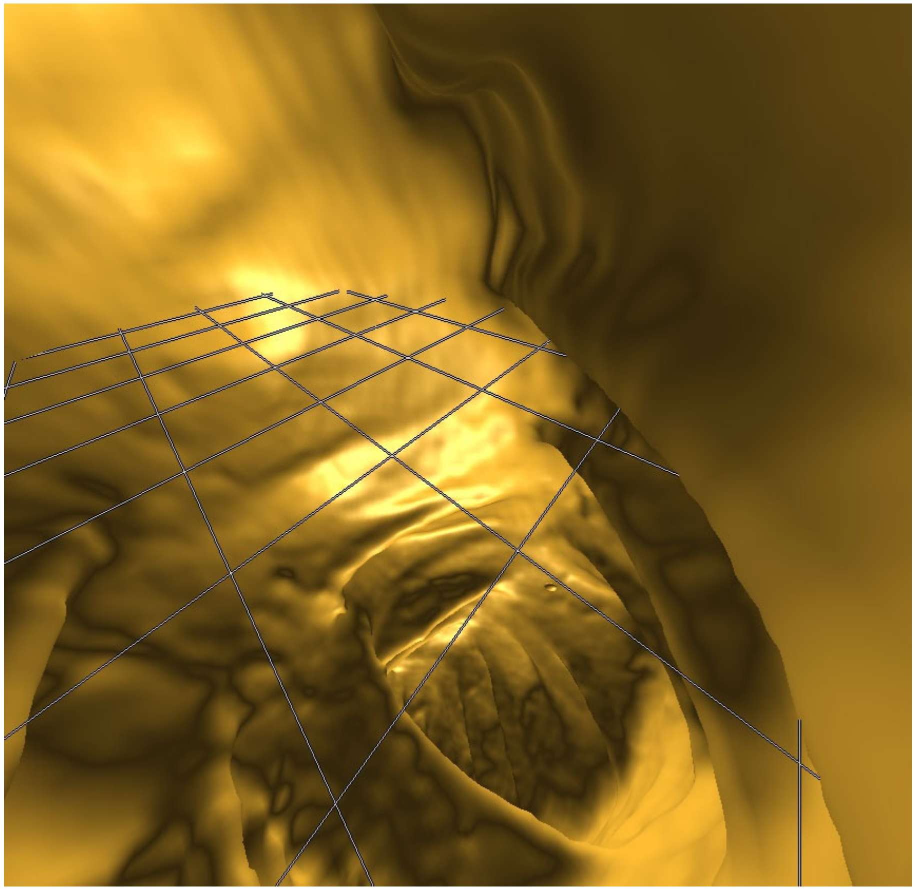
Loop over ray

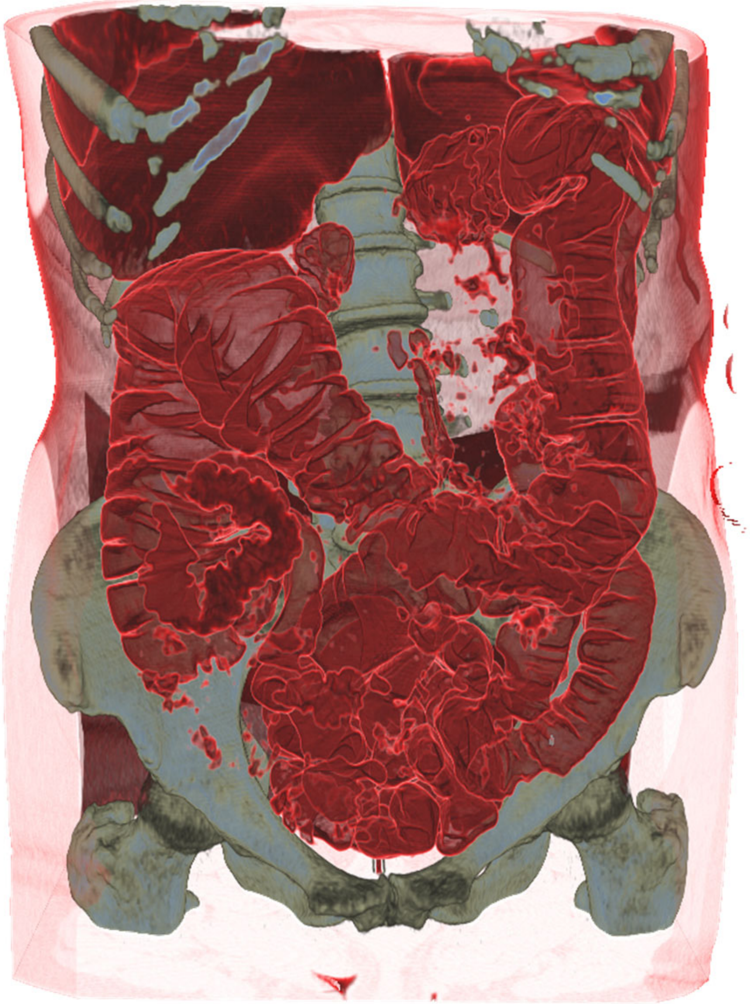
- Resample scalar value
- Classification
- Shading
- **Compositing**



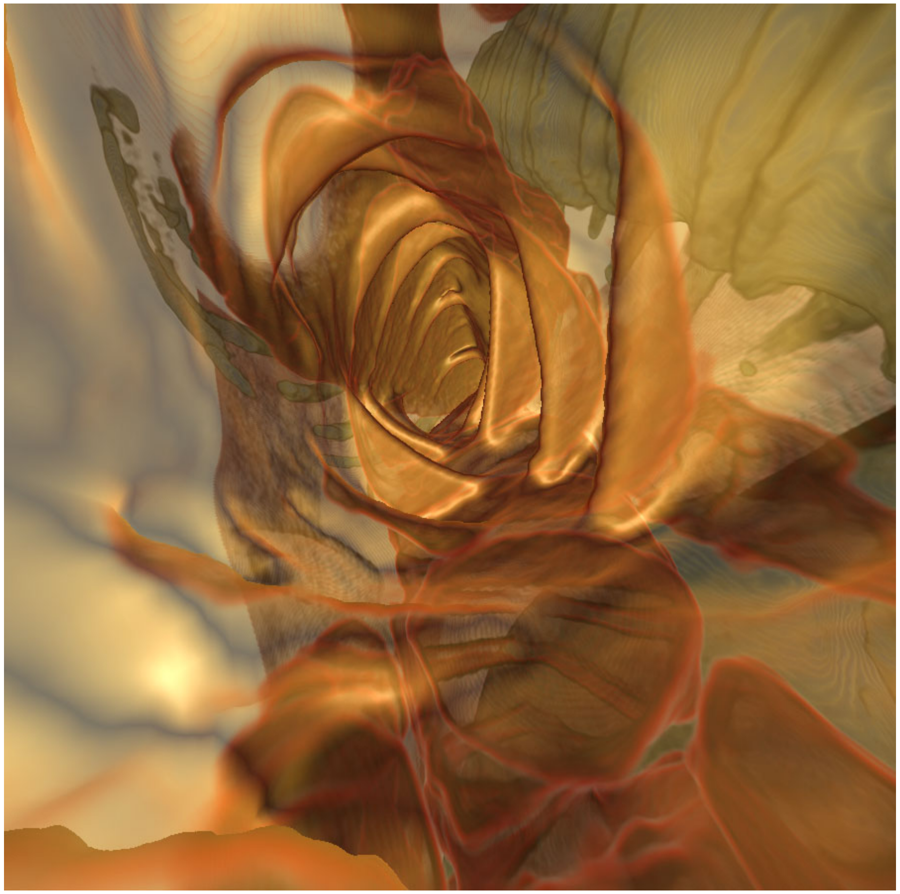
$$
\begin{aligned}
C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\
A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i
\end{aligned}
$$

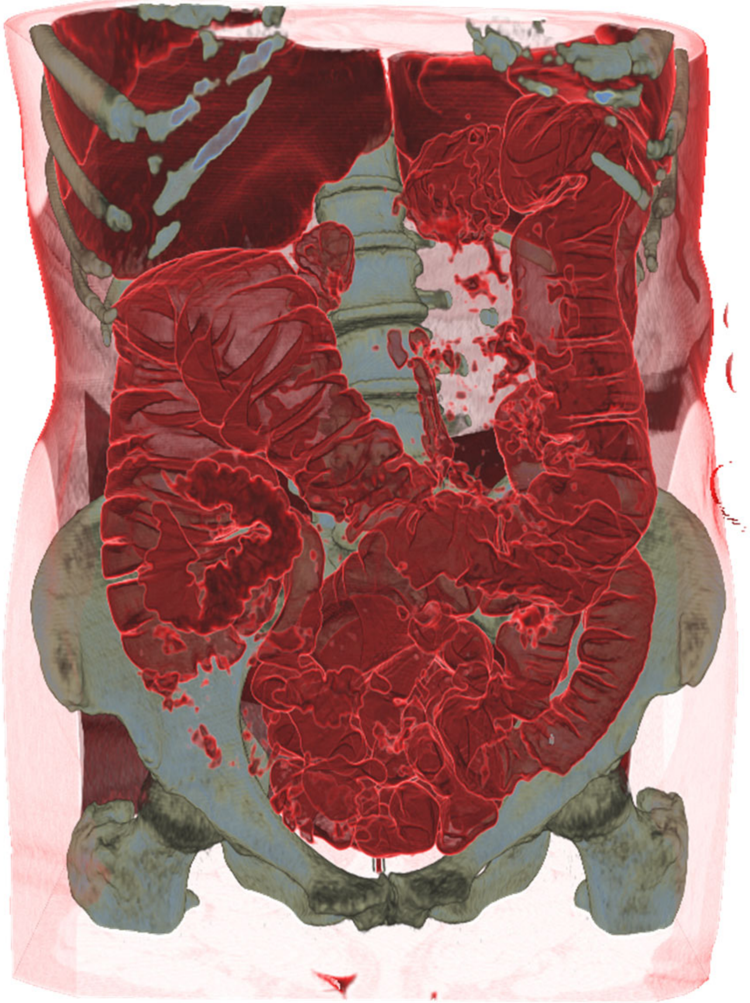
Compositing





Compositing





Fragment Shader

- Rasterize front faces of volume bounding box
- Texcoords are volume position in [0,1]
- Subtract camera position
- Repeatedly check for exit of bounding box

```
// Cg fragment shader code for single-pass ray casting
float4 main(VS_OUTPUT IN, float4 TexCoord0 : TEXCOORD0,
            uniform sampler3D SamplerDataVolume,
            uniform sampler1D SamplerTransferFunction,
            uniform float3 camera,
            uniform float stepsize,
            uniform float3 volExtentMin,
            uniform float3 volExtentMax
            ) : COLOR
{
    float4 value;
    float scalar;
    // Initialize accumulated color and opacity
    float4 dst = float4(0,0,0,0);
    // Determine volume entry position
    float3 position = TexCoord0.xyz;
    // Compute ray direction
    float3 direction = TexCoord0.xyz - camera;
    direction = normalize(direction);
    // Loop for ray traversal
    for (int i = 0; i < 200; i++)  // Some large number
    {
        // Data access to scalar value in 3D volume texture
        value = tex3D(SamplerDataVolume, position);
        scalar = value.a;
        // Apply transfer function
        float4 src = tex1D(SamplerTransferFunction, scalar);
        // Front-to-back compositing
        dst = (1.0-dst.a) * src + dst;
        // Advance ray position along ray direction
        position = position + direction * stepsize;
        // Ray termination: Test if outside volume ...
        float3 temp1 = sign(position - volExtentMin);
        float3 temp2 = sign(volExtentMax - position);
        float inside = dot(temp1, temp2);
        // ... and exit loop
        if (inside < 3.0)
            break;
    }
    return dst;
}
```

CUDA Kernel

- Image-based ray setup
  - Ray start image
  - Direction image
- Ray-cast loop
  - Sample volume
  - Accumulate color and opacity
- Terminate
- Store output

```
__global__
void RayCastCUDAKernel( float *d_output_buffer, float *d_startpos_buffer, float *d_direction_buffer )
{
    // output pixel coordinates
    dword screencoord_x = __umul24( blockIdx.x, blockDim.x ) + threadIdx.x;
    dword screencoord_y = __umul24( blockIdx.y, blockDim.y ) + threadIdx.y;

    // target pixel (RGBA-tuple) index
    dword screencoord_indx = ( __umul24( screencoord_y, cu_screensize.x ) + screencoord_x ) * 4;

    // get direction vector and ray start
    float4 dir_vec  = d_direction_buffer[ screencoord_indx ];
    float4 startpos = d_startpos_buffer[ screencoord_indx ];

    // ray-casting loop
    float4 color     = make_float4( 0.0f );
    float poscount   = 0.0f;
    for ( int i = 0; i < 8192; i++ ) {

        // next sample position in volume space
        float3 samplepos = dir_vec * poscount + startpos;
        poscount += cu_sampling_distance;

        // fetch density
        float tex_density = tex3D( cu_volume_texture, samplepos.x, samplepos.y, samplepos.z );

        // apply transfer function
        float4 col_classified = tex1D( cu_transfer_function_texture, tex_density );

        // compute (1-previous.a)*tf.a
        float prev_alpha = -color.w * col_classified.w + col_classified.w;

        // composite color and alpha
        color.xyz = prev_alpha * col_classified.xyz + color.xyz;
        color.w  += prev_alpha;

        // break if ray terminates (behind exit position or alpha threshold reached)
        if ( ( poscount > dir_vec.w ) || ( color.w > 0.98f ) ) {
            break;
        }
    }

    // store output color and opacity
    d_output_buffer[ screencoord_indx ] = __saturatef( color );
}
```

VolVis: Opacity Correction

Opacity Correction

Simple compositing only works as far as the opacity values are correct… and they depend on the sample distance!

$$T_i = e^{-\int_{s_i}^{s_i+\Delta t} \kappa(t)\,dt} \approx e^{-\kappa(s_i)\Delta t} = e^{-\kappa_i \Delta t}$$

$$A_i = 1 - e^{-\kappa_i \Delta t} \qquad \tilde{T}_i = T_i^{\left(\frac{\Delta \tilde{t}}{\Delta t}\right)}$$

$$\boxed{\tilde{A}_i = 1 - (1 - A_i)^{\left(\frac{\Delta \tilde{t}}{\Delta t}\right)}}$$

opacity correction formula

Beware that usually this is done *for each different scalar value* (every transfer function entry), not actually at spatial positions/intervals *i*

Associated Colors

Associated (or “opacity-weighted” colors) are often used in compositing equations

Every color is *pre-multiplied* by its corresponding opacity

$$\begin{bmatrix} R \\ G \\ B \\ A \end{bmatrix} \Rightarrow \begin{pmatrix} R*A \\ G*A \\ B*A \\ A \end{pmatrix}$$

Our compositing equations assume associated colors!

Important: After opacity-correction, all associated colors must be updated! (or combined/multiplied correctly on-the-fly!)

Associated Colors in Volume Rendering

Standard emission-absorption optical model

- Only one kind of particle: the same particles that absorb light, emit light
- Aha! Therefore lower absorption means lower emission as well

Light observed from (in front of) segment *i* (without any light behind it):

$$C_i = \frac{q_i}{\kappa_i}\left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i A_i$$

$$q_i := \hat{C}_i \kappa_i$$

$$A_i := 1 - e^{-\kappa_i \Delta t}$$

$$\lim_{\kappa_i \to 0} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \lim_{\kappa_i \to 0} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = 0$$

$$\lim_{\kappa_i \to \infty} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \lim_{\kappa_i \to \infty} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i$$

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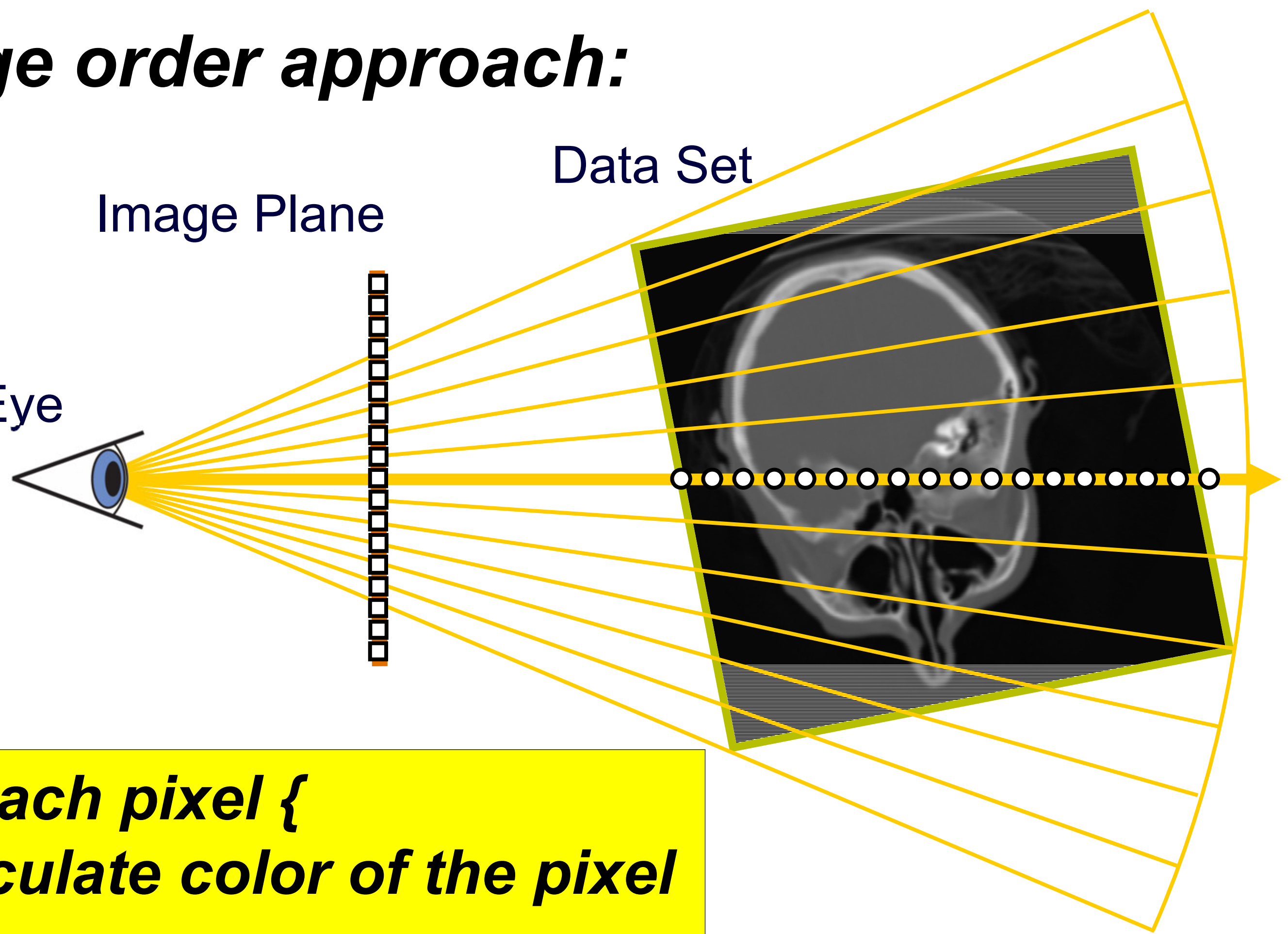
VolVis: Image vs. Object Order

Direct Volume Rendering: Image Order

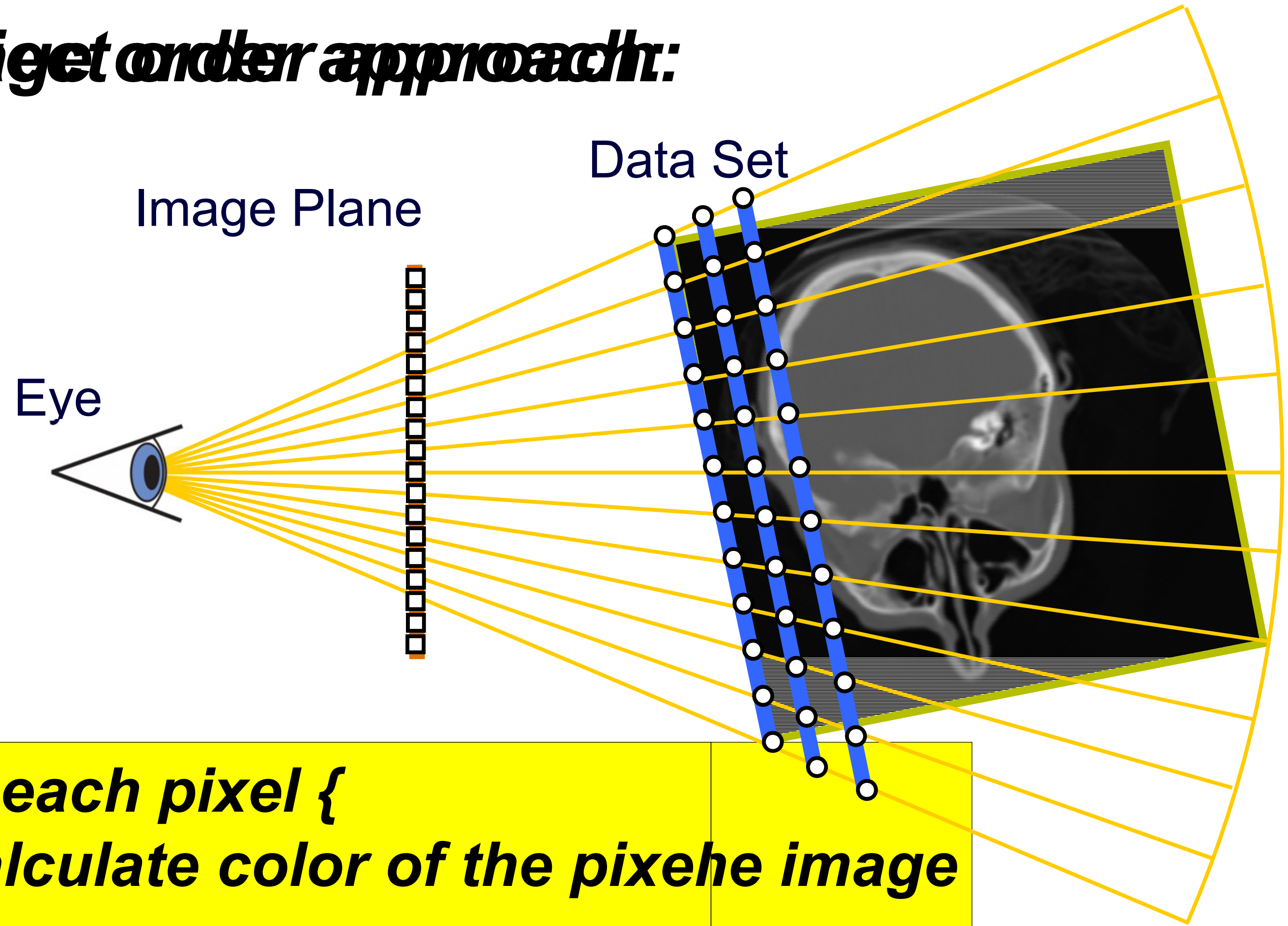
Image order approach:



For each pixel {
calculate color of the pixel
}

Direct Volume Rendering: Object Order

Object order approach:



For each pixel {
calculate color of the pixel
}

he image

Basic Volume Rendering Summary

Volume rendering integral
for *Emission Absorption* model

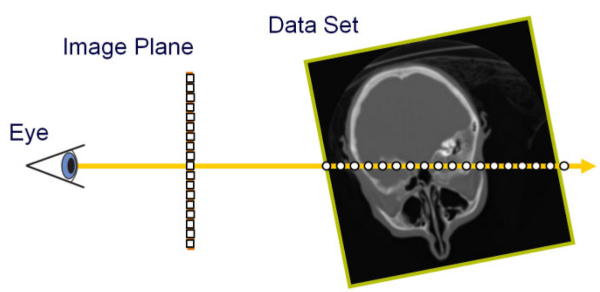
$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

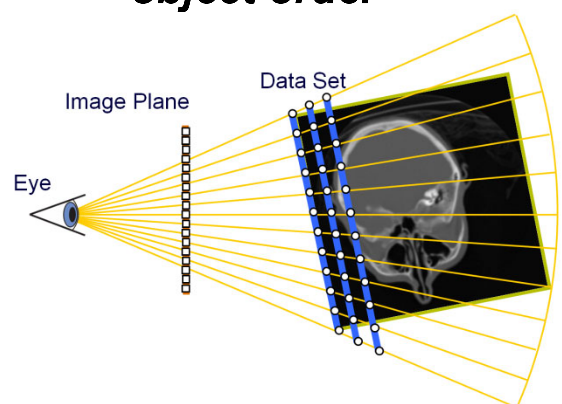
Numerical solutions: ***back-to-front*** ***vs.*** ***front-to-back compositing***

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

Approaches: ***image order*** ***vs.*** ***object order***





Thank you.

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