King Abdullah University of Science and Technology

KAUST

# CS 247 – Scientific Visualization
# Lecture 15: Volume Visualization, Pt. 2

Markus Hadwiger, KAUST

# Reading Assignment #8 (until Mar 26)

Read (required):

- Real-Time Volume Graphics, Chapter 4 (Transfer Functions) until Sec. 4.4 (inclusive)
- Paper:

  Jens Krüger and Rüdiger Westermann,
  *Acceleration Techniques for GPU-based Volume Rendering,*
  IEEE Visualization 2003,
  **`http://dl.acm.org/citation.cfm?id=1081482`**

# Volume Rendering

# Theory, Ctd.

# Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model

**true emission** **true *absorption***

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) \;=\; \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

***Back-to-front compositing***

$$C'_i \;=\; C_i + (1 - A_i) C'_{i-1}$$

***Front-to-back compositing***

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1}) C_i$$

$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1}) A_i$$

here, all colors are *associated* colors!

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

***Physical model:*** emission and absorption, no scattering

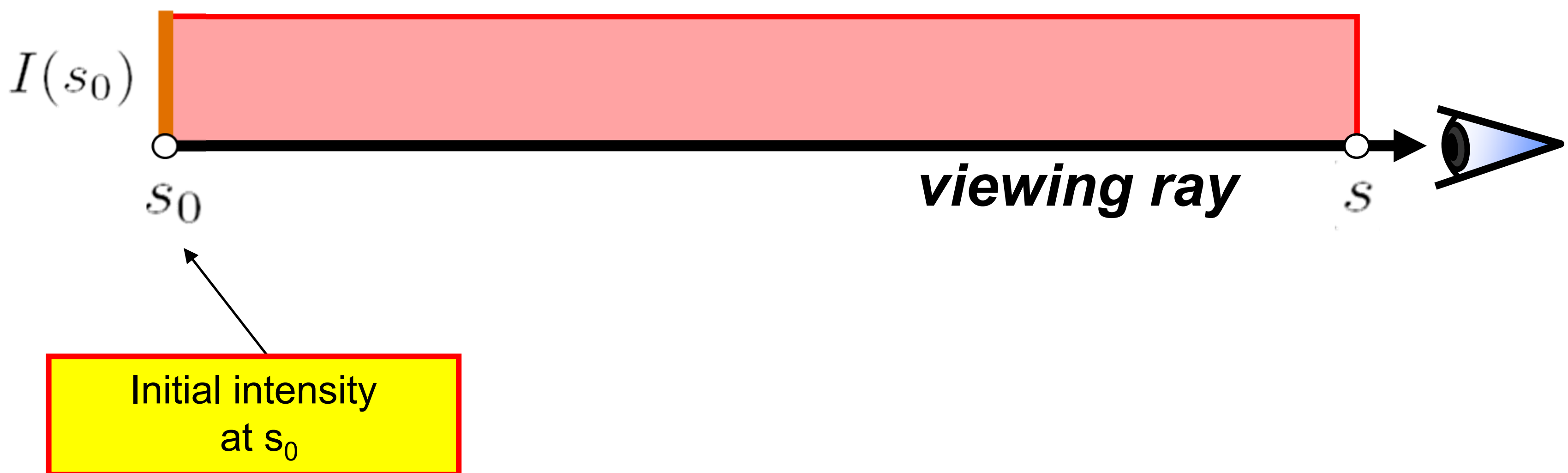

Initial intensity at $s_0$

$$I(s) = \boxed{I(s_0)}$$

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

***Physical model:*** emission and absorption, no scattering



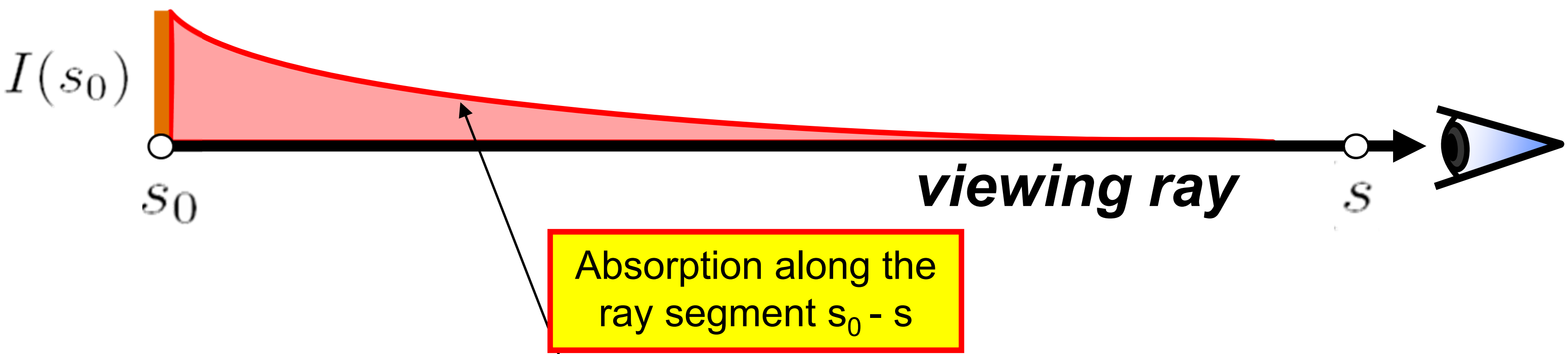

$$I(s) = \boxed{I(s_0)}$$

Without absorption all the initial radiant energy would reach the point ***s.***

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

***Physical model:*** emission and absorption, no scattering

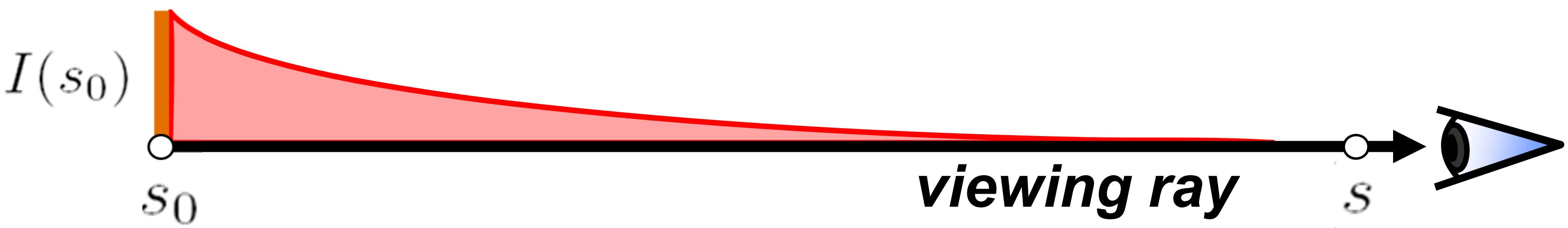



$$I(s) = I(s_0) \boxed{e^{-\tau(s_0,s)}}$$

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

***Physical model:*** emission and absorption, no scattering




**Optical depth $\tau$**

**Absorption $\kappa$**

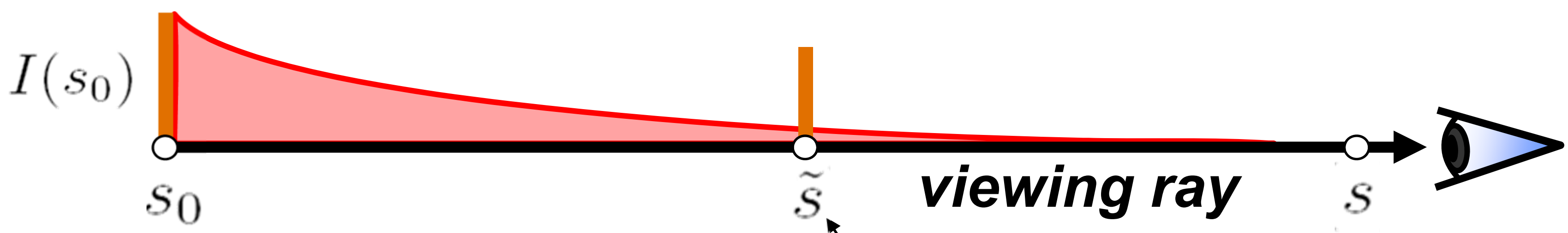
$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)}$$

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

***Physical model:*** emission and absorption, no scattering




One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

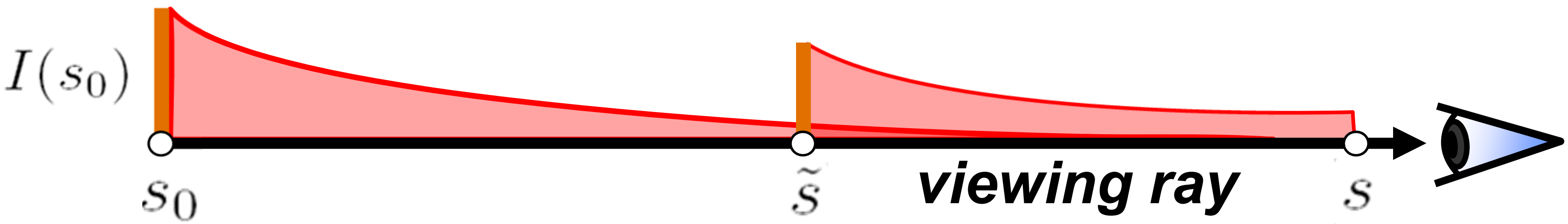
Active emission at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \boxed{q(\tilde{s})}$$

# Volume Rendering Integral

**How do we determine the radiant energy along the ray?**

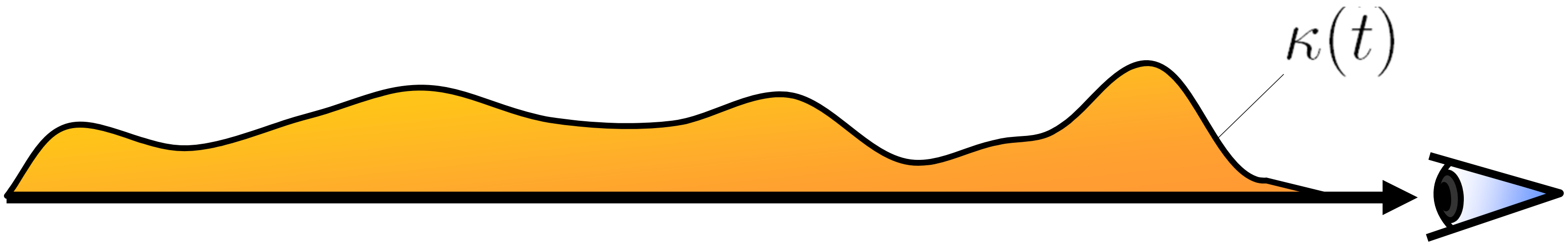
***Physical model:*** emission and absorption, no scattering




***Every*** point $\tilde{s}$ along the viewing ray emits additional radiant energy

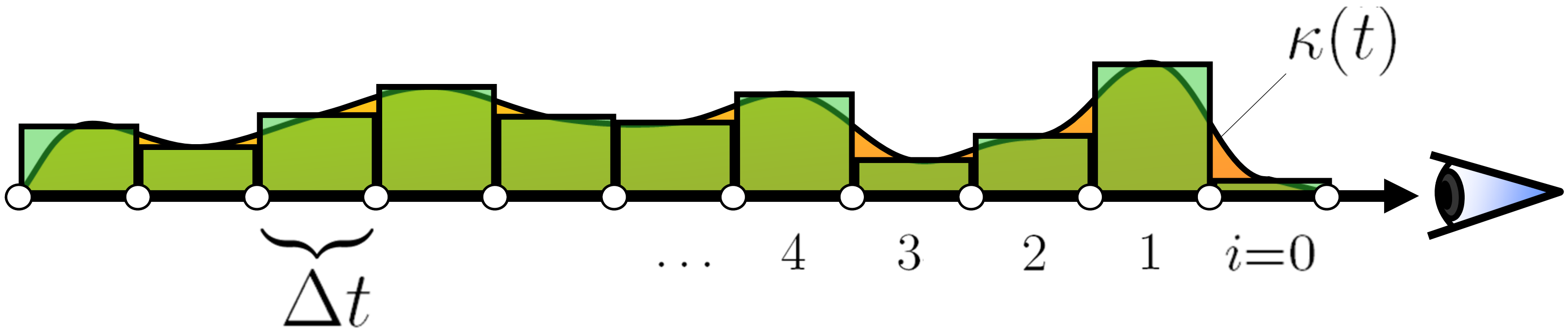
$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} \;+\; \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

# Volume Rendering Integral: Numerical Solution




***Optical depth:*** $\tau(0,t) \quad = \quad \int_0^t \kappa(\hat{t})\, d\hat{t}$

# Volume Rendering Integral: Numerical Solution

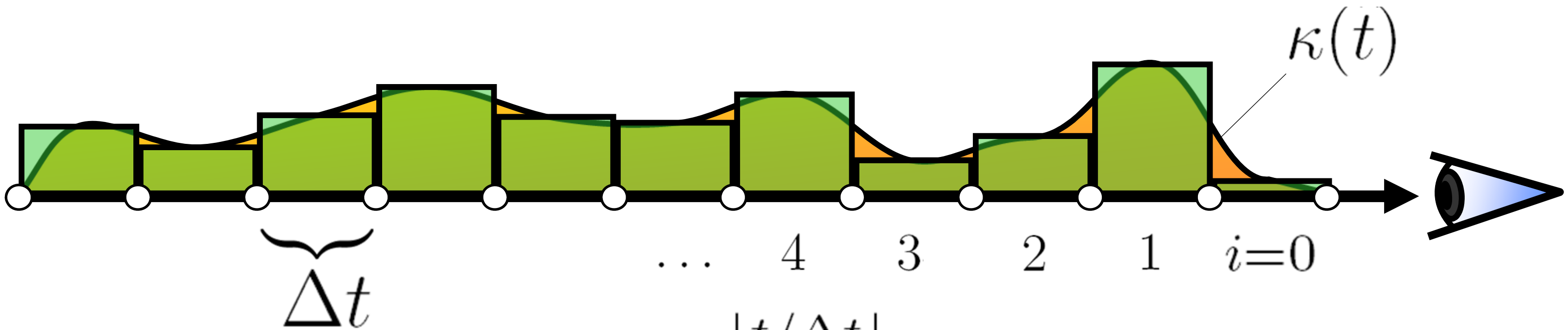



***Optical depth:*** $\tau(0,t) \;=\; \int_0^t \kappa(\hat{t})\,d\hat{t}$

***Approximate Riemann integral by Riemann sum:***

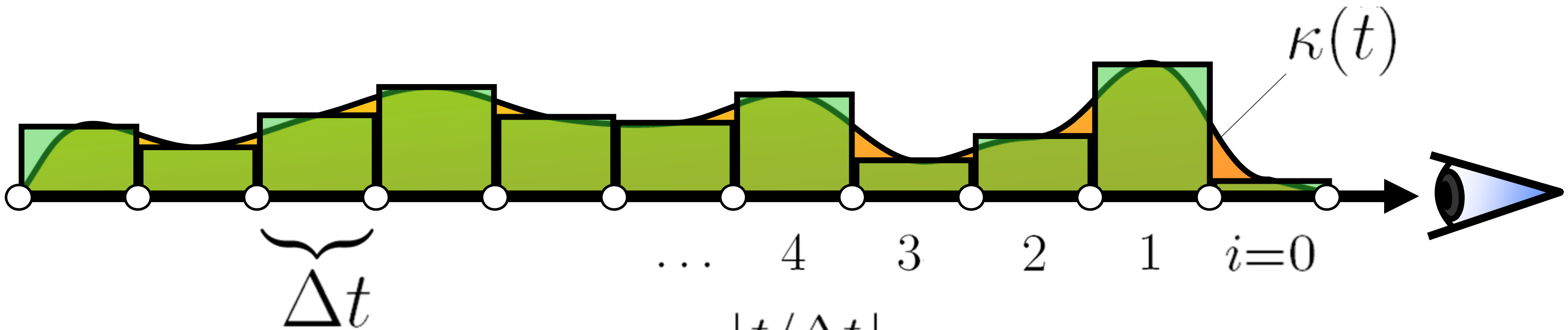
$$\tau(0,t) \;\approx\; \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

# Volume Rendering Integral: Numerical Solution




$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

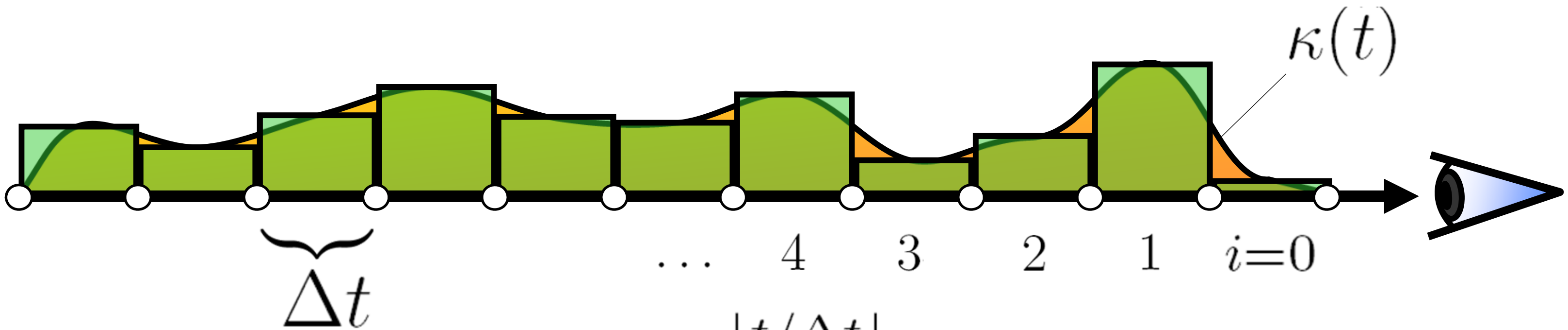
# Volume Rendering Integral: Numerical Solution




$$\tau(0,t) \quad \approx \quad \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} \quad = \quad e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$

# Volume Rendering Integral: Numerical Solution

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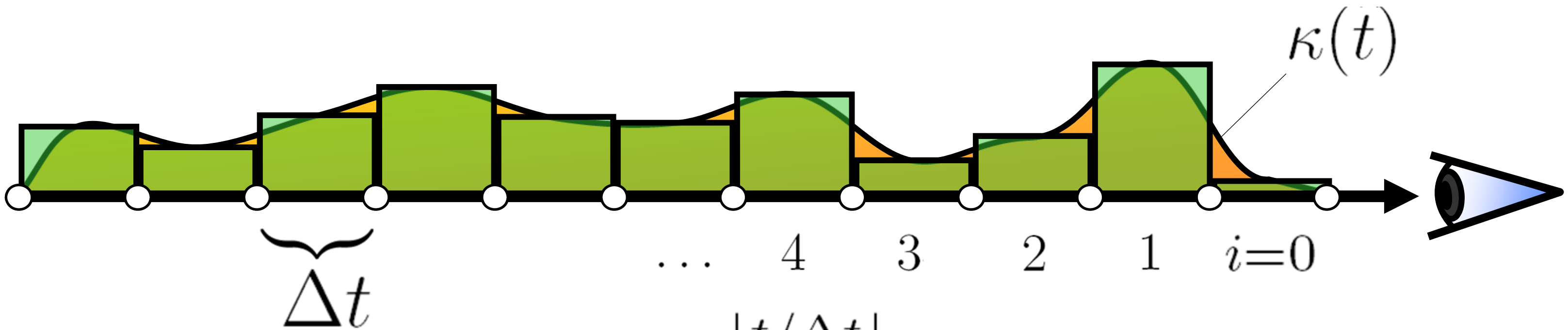


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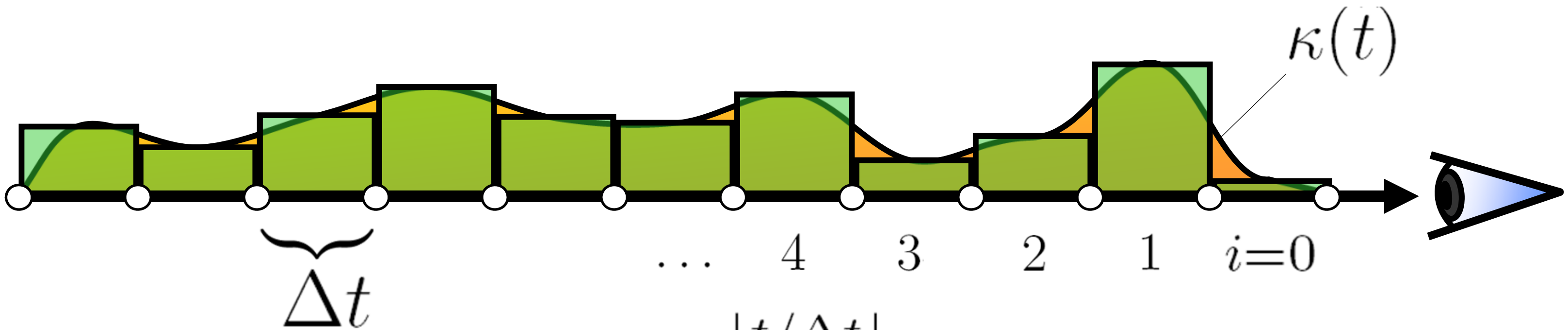
$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

# Volume Rendering Integral: Numerical Solution

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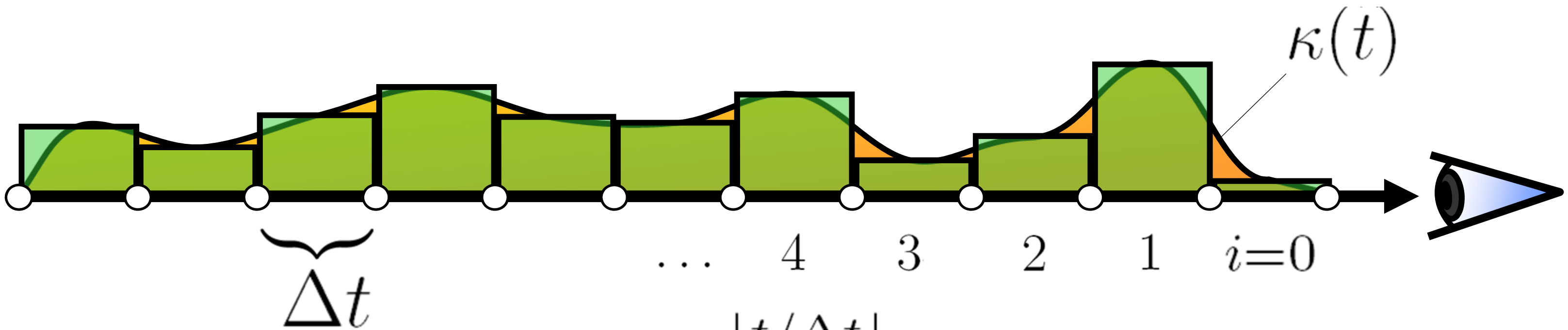
Now we introduce *opacity*:

$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

# Volume Rendering Integral: Numerical Solution

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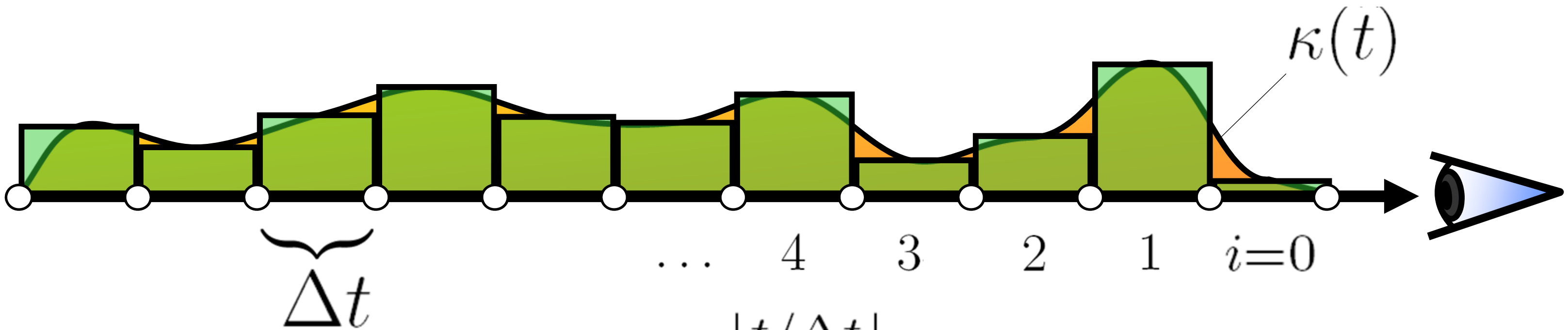


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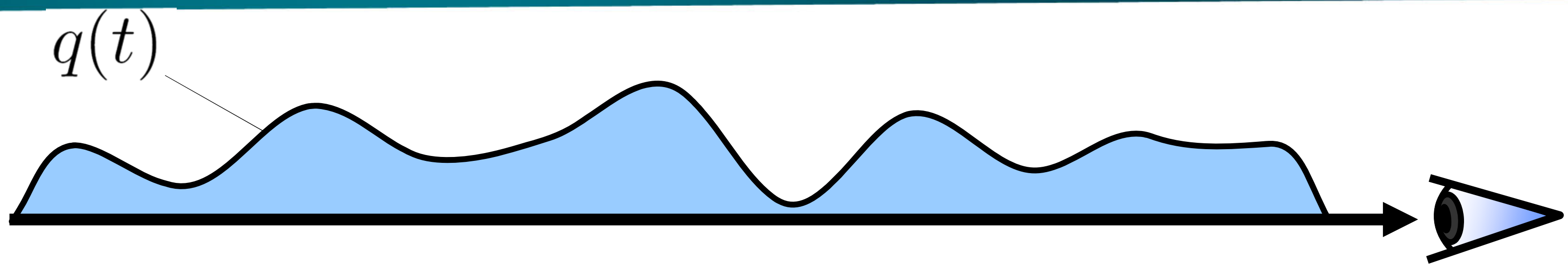
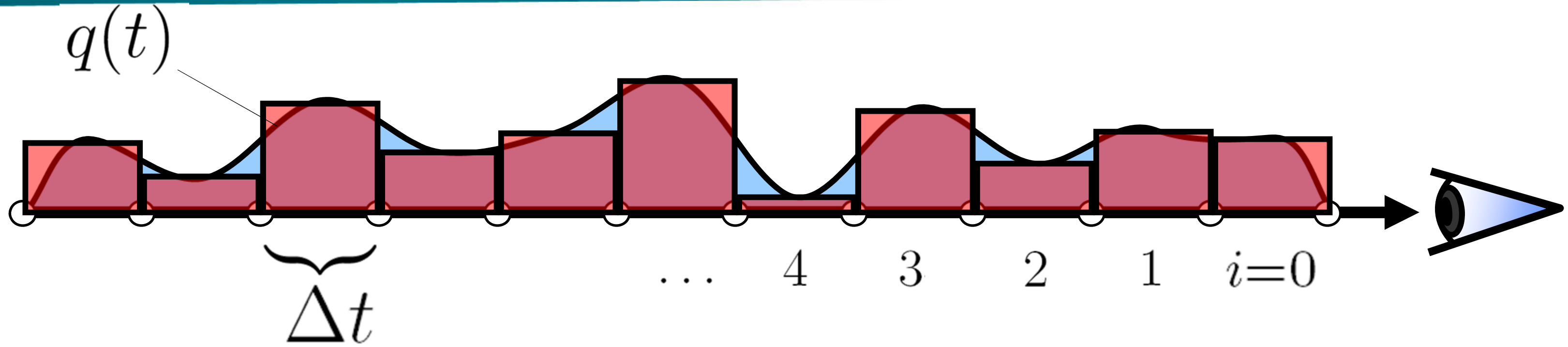


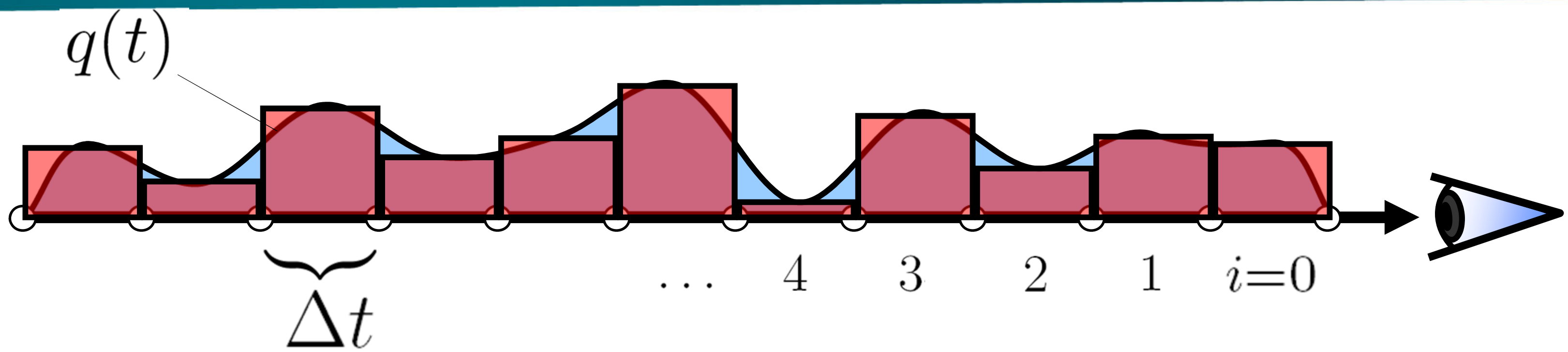
# Volume Rendering Integral: Numerical Solution




$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

# Volume Rendering Integral: Numerical Solution

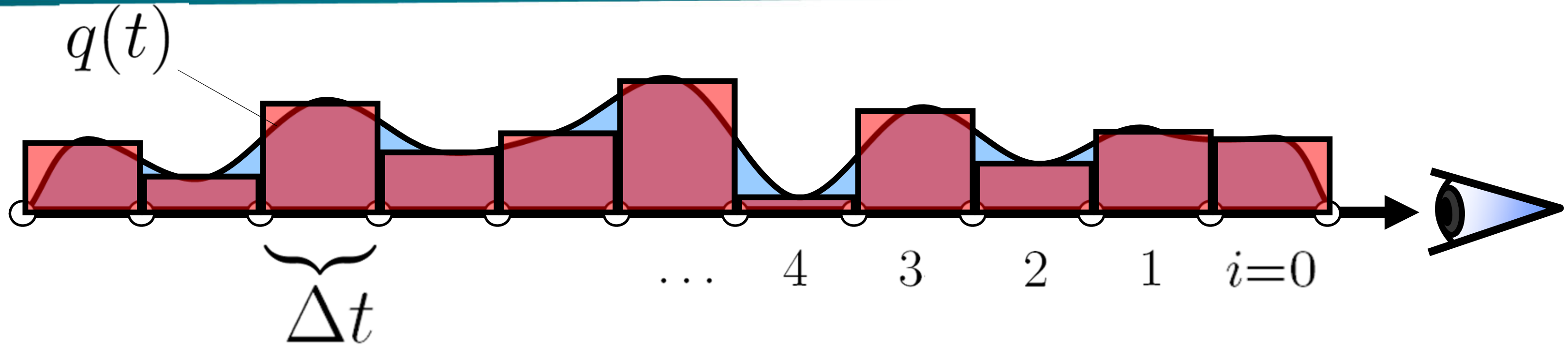



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

# Volume Rendering Integral: Numerical Solution




$$\boxed{e^{-\tilde{\tau}(0,t)}} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\, \Delta t$$

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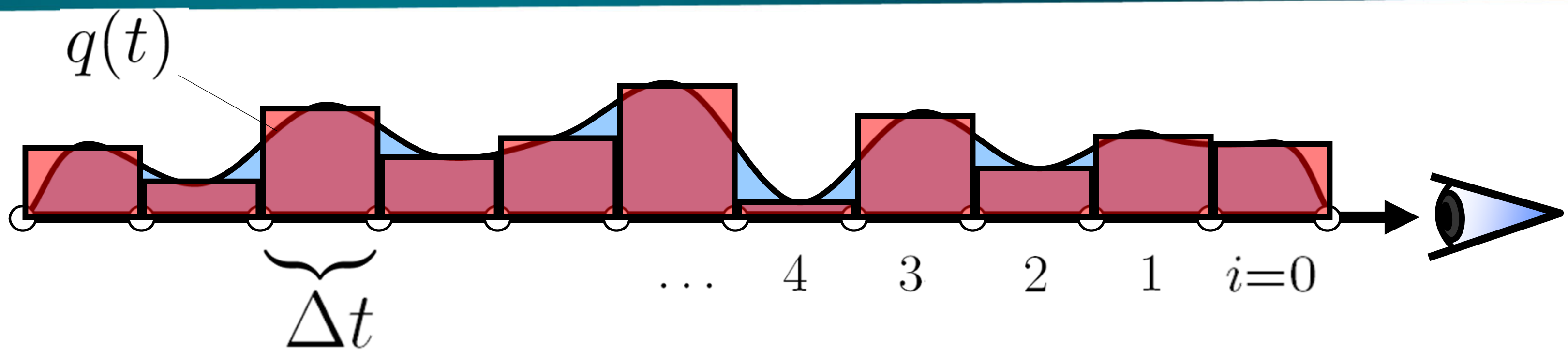
# Volume Rendering Integral: Numerical Solution

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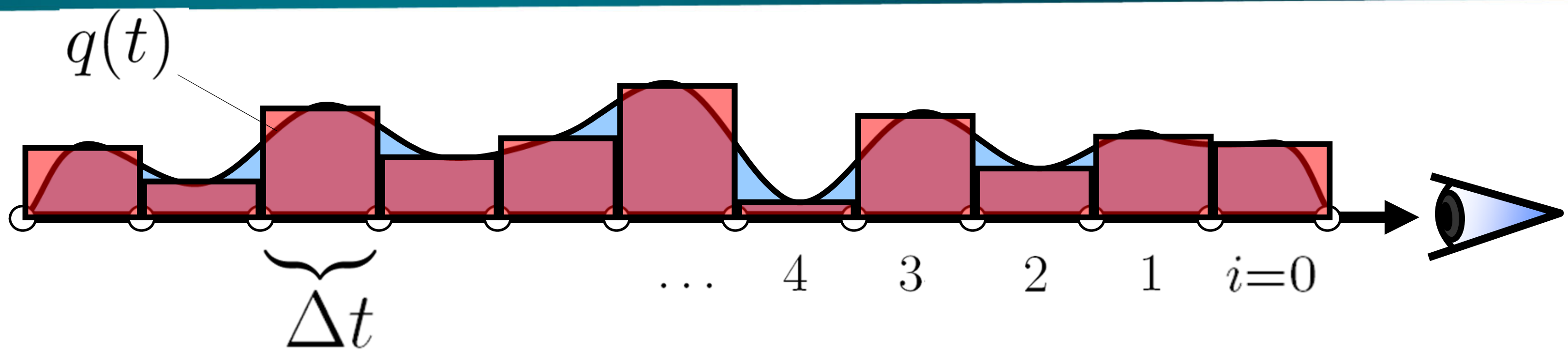
# Volume Rendering Integral: Numerical Solution

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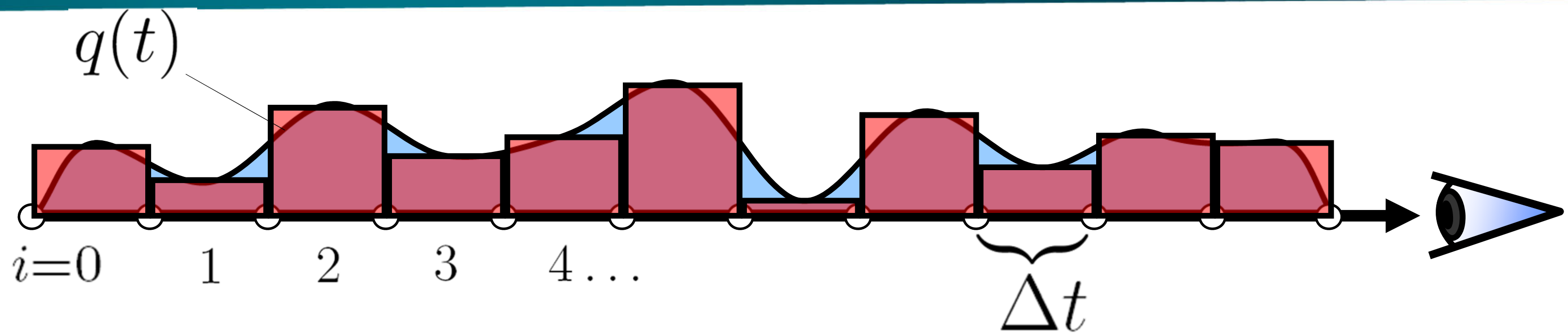
*can be computed iteratively/recursively!*

# Volume Rendering Integral: Numerical Solution




*Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !*

can be computed iteratively/recursively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

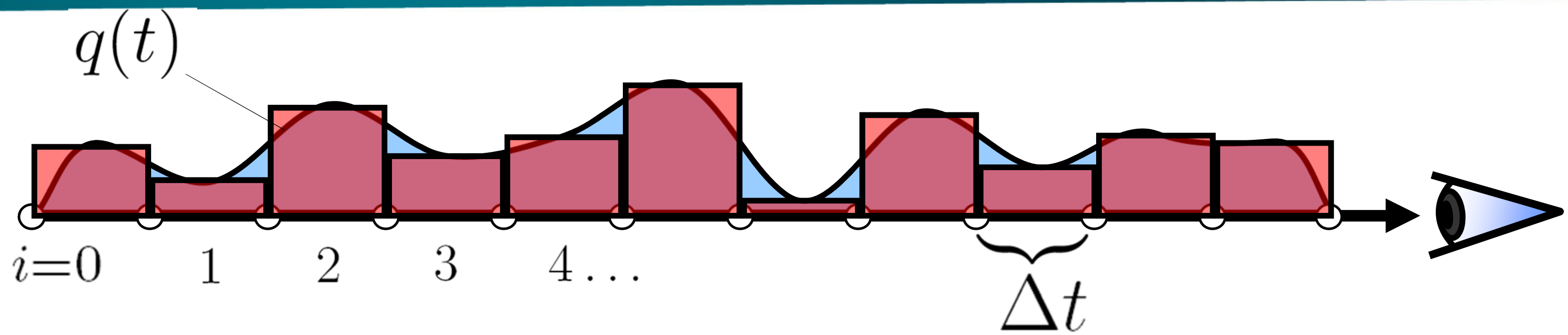
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i*–1**

# Volume Rendering Integral: Numerical Solution




***Back-to-front compositing***

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

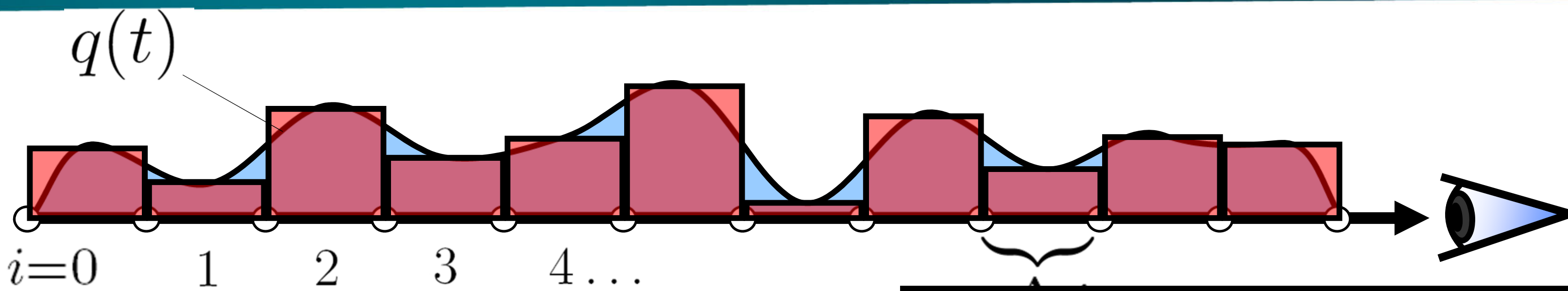
iterate from *i*=0 (back) to *i*=max (front): *i* increases

***Front-to-back compositing***

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

# Volume Rendering Integral: Numerical Solution




***Back-to-front compositing***

$$C'_i \quad = \quad C_i \; + \; (1$$

iterate from *i*=0 (back)

***Early Ray Termination:***
Stop the calculation when

$$A'_i \approx 1$$

***Front-to-back compositing***

$$\begin{aligned} C'_i &= C'_{i+1} + \boxed{(1 - A'_{i+1})} C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

# Volume Rendering Integral Summary

Volume rendering integral
for *Emission Absorption* model

**true emission** **true *absorption***

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) \;=\; \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

***Back-to-front compositing***

$$C'_i \;=\; C_i + (1 - A_i)C'_{i-1}$$

***Front-to-back compositing***

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1})A_i$$

here, all colors are *associated* colors!

# Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama