CS 247 – Scientific Visualization
Lecture 10: Scalar Field Visualization, Pt. 3

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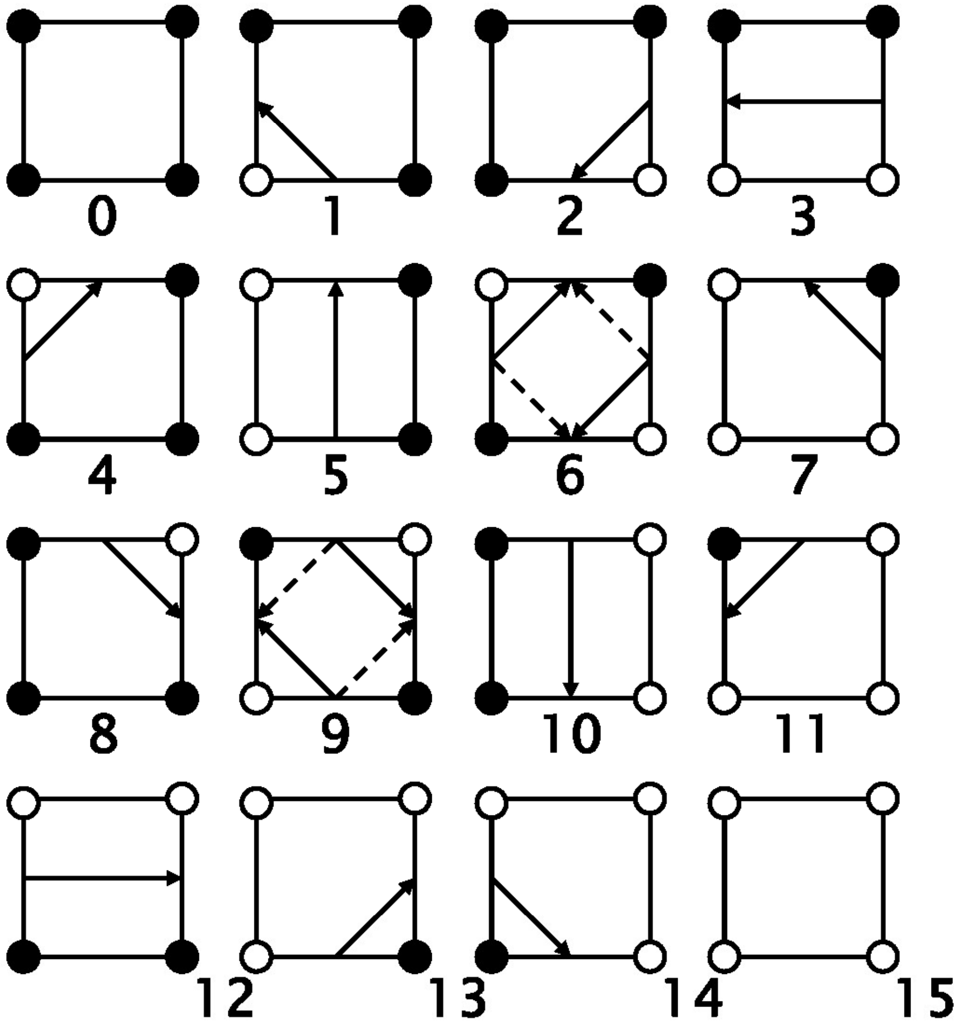
Reading Assignment #6 (until Mar 9)

Read (required):

- Real-Time Volume Graphics, Chapter 2
  (*GPU Programming*)
- Reminder: Real-Time Volume Graphics, Chapter 5.4

Contours in a quadrangle cell



● $f(x_i) \leq c$

○ $f(x_i) > c$

Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Orientability (1-manifold embedded in 2D)

not orientable

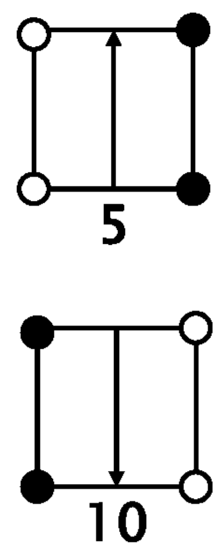
Moebius strip
(only one side!)

Orientability of 1-manifold:

Possible to assign consistent left/right orientation

Iso-contours

- Consistent side for scalar values…
  - greater than iso-value (e.g, *left* side)
  - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices
  (e.g., larger vertex index is “tip” of arrow;
  if (0,1) points “up”, “left” is left, …)



● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

Orientability of 2-manifold:

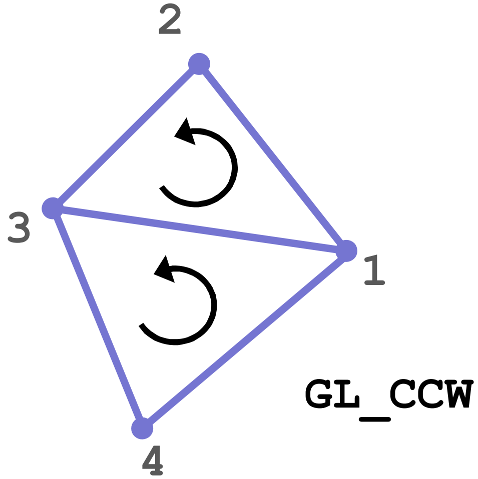
Possible to assign consistent normal vector orientation

not orientable

Moebius strip
(only one side!)

Triangle meshes

- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”



Topological consistency

To avoid degeneracies, use symbolic perturbations:

If level c is found as a node value, set the level to c-ε where ε is a symbolic infinitesimal.

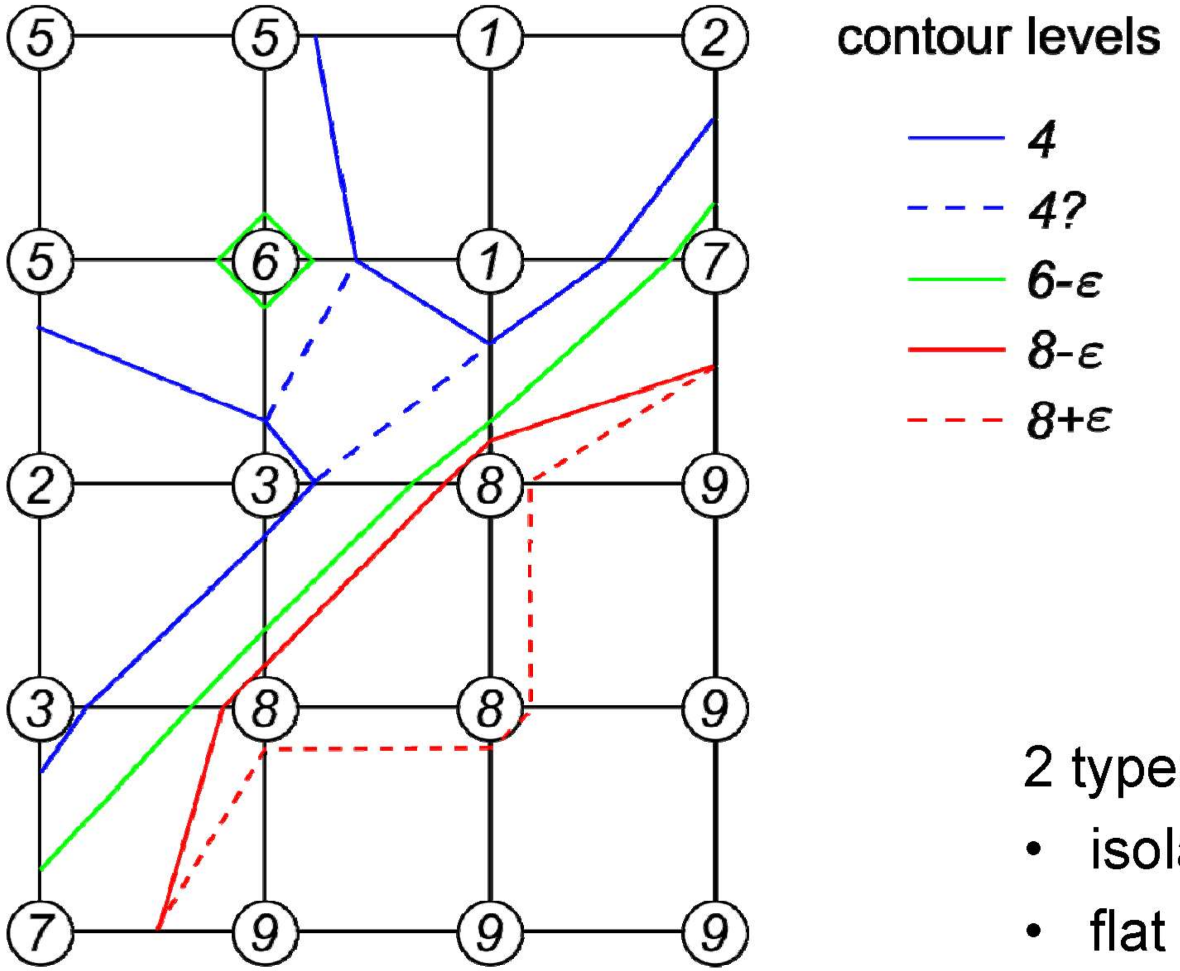
Then:

- contours intersect edges at some (possibly infinitesimal) distance from end points
- flat regions can be visualized by pair of contours at c-ε and c+ε
- contours are topologically consistent, meaning:

Contours are closed, orientable, nonintersecting lines.

(except where the boundary is hit)

Example



2 types of degeneracies:

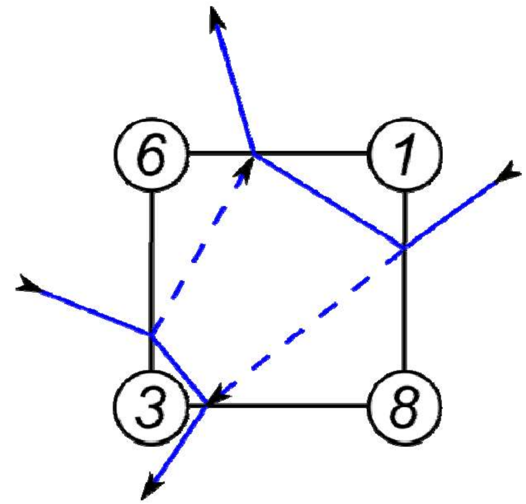
- isolated points (*c*=6)
- flat regions (*c*=8)

Ambiguities of contours

What is the correct contour of c=4?

Two possibilities, both are orientable:

- connect high values ————
- connect low values - - - - - - - - - -



Answer: correctness depends on interior values of $f(x)$.

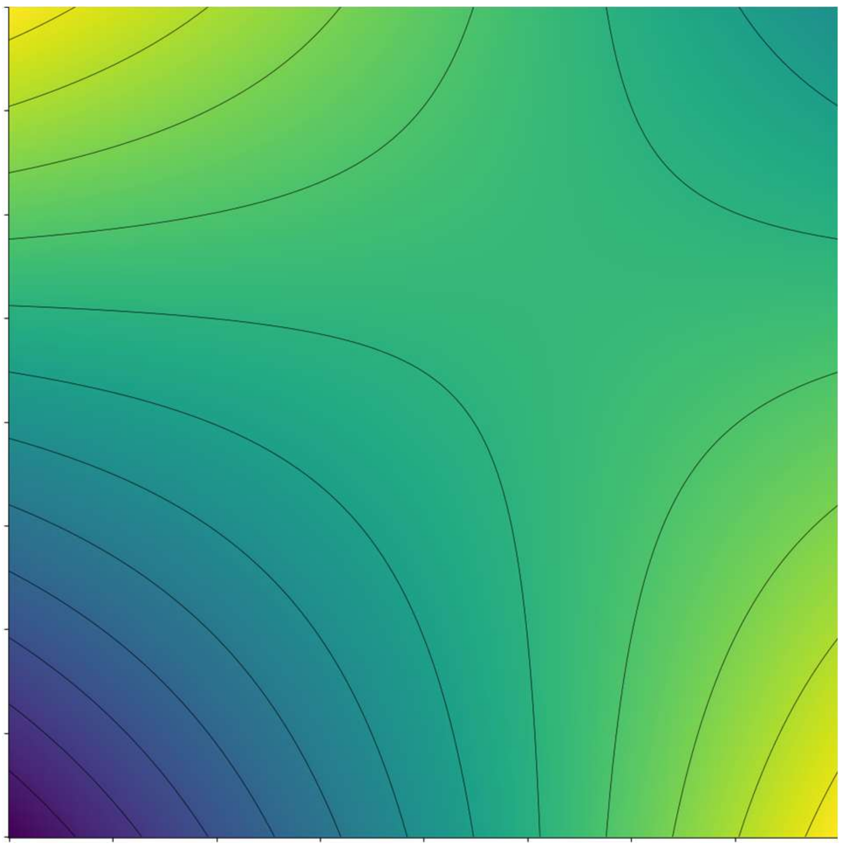
But: different interpolation schemes are possible.

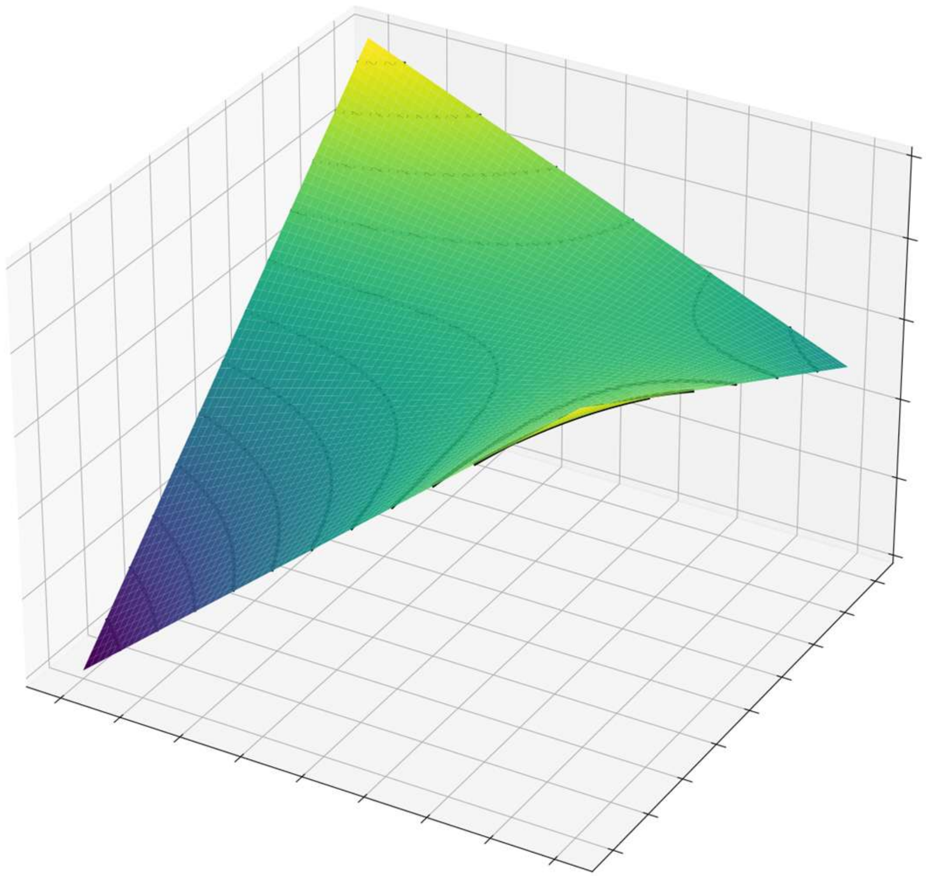
Better question: What is the correct contour with respect to bilinear interpolation?

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples

Example: 1.0 at top-left and bottom-right, 0.0 at bottom-left, 0.5 at top-right





Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= (1-\alpha_1)(1-\alpha_2)v_{00} + \alpha_1(1-\alpha_2)v_{10} + (1-\alpha_1)\alpha_2 v_{01} + \alpha_1\alpha_2 v_{11}$$

$$= v_{00} + \alpha_1(v_{10} - v_{00}) + \alpha_2(v_{01} - v_{00}) + \alpha_1\alpha_2(v_{00} + v_{11} - v_{10} - v_{01})$$

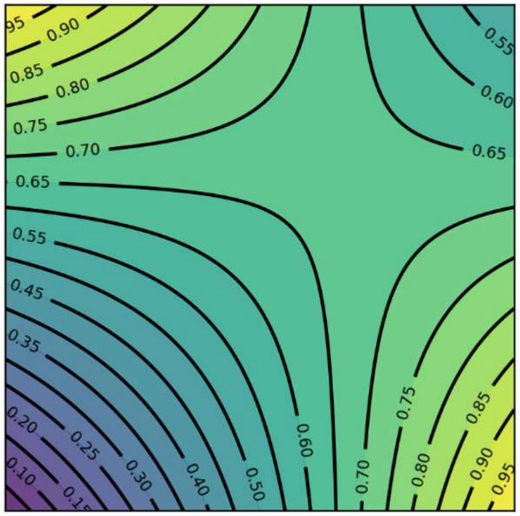
Bi-Linear Interpolation: Contours

Find one specific iso-contour (can of course do this for any/all isovalues):

$$f(\alpha_1, \alpha_2) = c$$

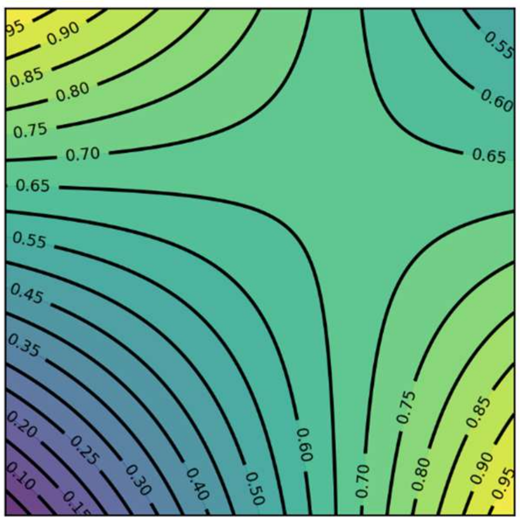
Find all (α_1, α_2) where:

$$v_{00} + \alpha_1 (v_{10} - v_{00}) + \alpha_2 (v_{01} - v_{00}) + \alpha_1 \alpha_2 (v_{00} + v_{11} - v_{10} - v_{01}) = c$$

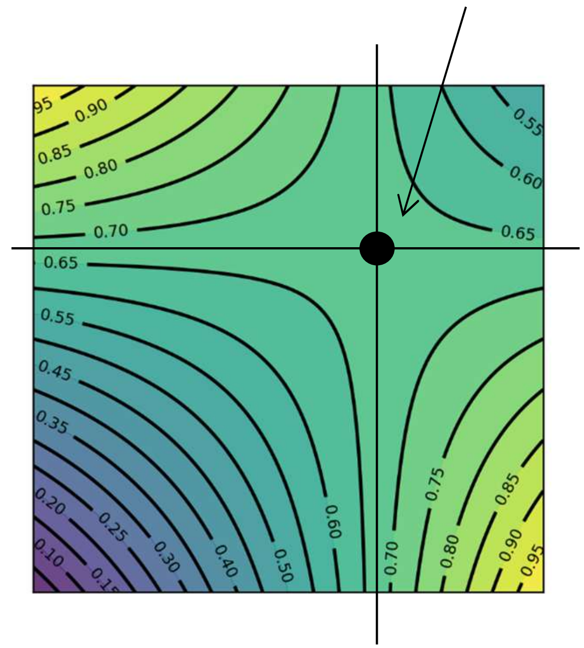


Bi-Linear Interpolation: Critical Points

Critical points are where the gradient vanishes (i.e., is the zero vector)



critical point
(saddle point)



here, the critical
value is 2/3=0.666…

“Asymptotic decider”: resolve ambiguous configurations (6 and 9) by comparing specific iso-value with critical value (scalar value at critical point)

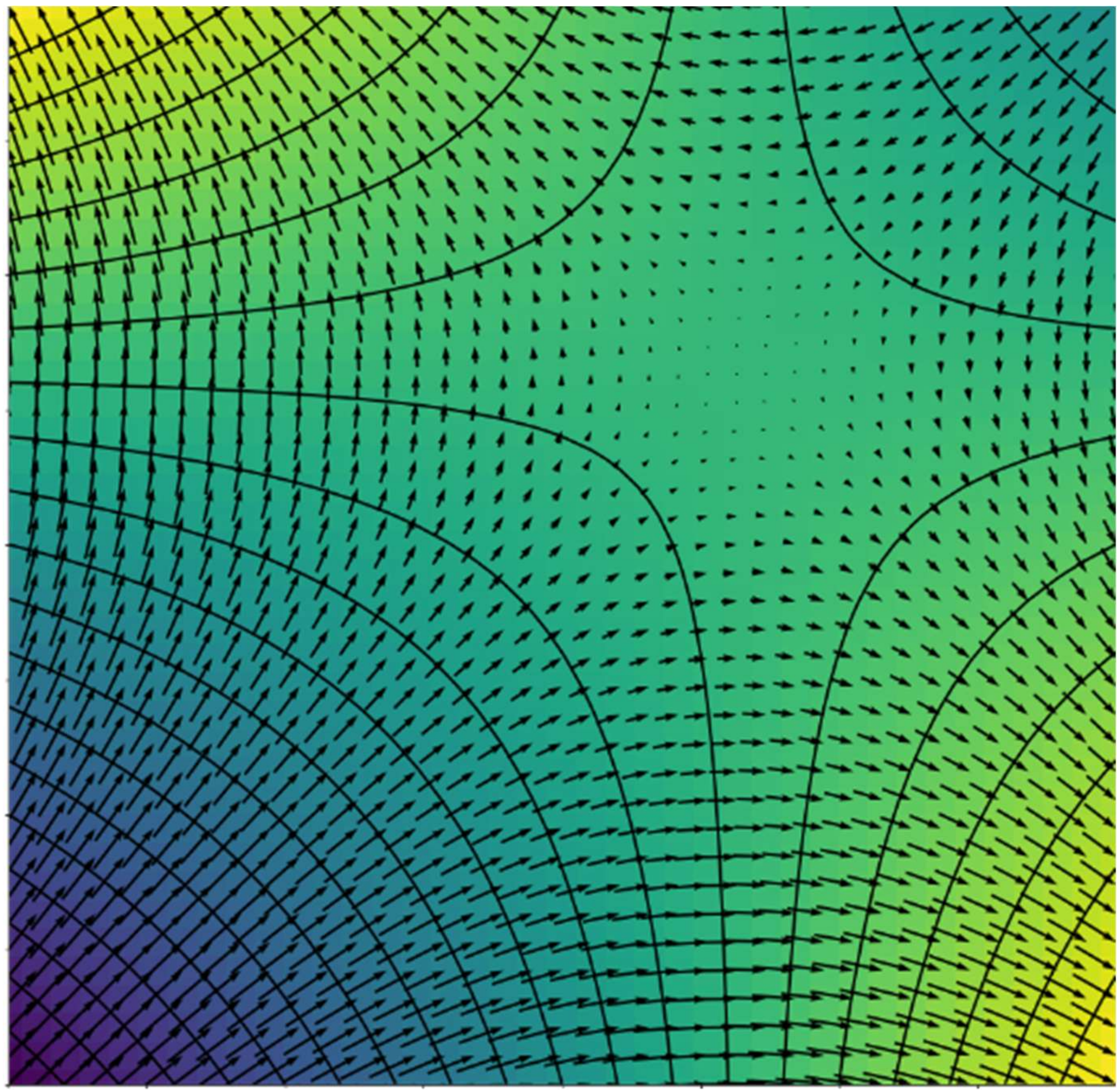
Bi-Linear Interpolation: Critical Points

Compute gradient

Note that isolines are farther apart where gradient is smaller

Note the horizontal and vertical lines where gradient becomes vertical/horizontal

Note the critical point



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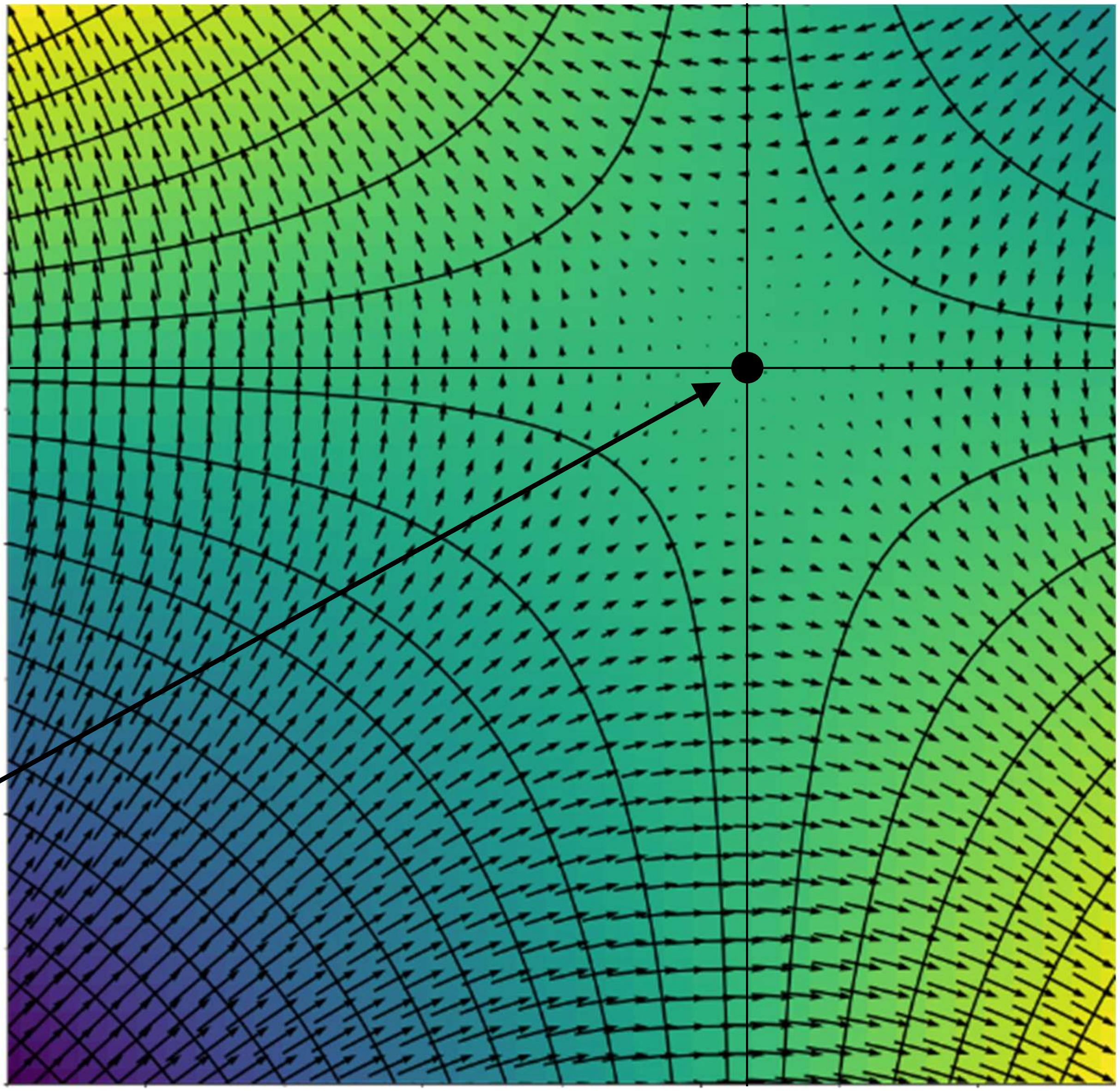

Interlude: Implicit Function Theorem

When can I write an implicit function in $\mathbb{R}^{n+m}$ such that it is the graph of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ *at least locally?*

That is: is this implicitly described function an *n*-manifold embedded in $\mathbb{R}^{n+m}$? (with local coordinates in $\mathbb{R}^n$)

$$G(f) := \{(x, f(x)) | x \in \mathbb{R}^n\} \subset \mathbb{R}^n \times \mathbb{R}^m \simeq \mathbb{R}^{n+m}$$

Theorem: if *m* x *m* Jacobian matrix is invertible

(easier for scalar field: check if gradient of f is non-zero)

See **`https://en.wikipedia.org/wiki/Implicit_function_theorem`**

General result: *constant rank theorem*

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama