CS 247 – Scientific Visualization
Lecture 24: Vector / Flow Visualization, Pt. 3

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Reading Assignment #13 (until May 4)

Read (required):

- Data Visualization book
  - Chapter 6.1 (Divergence and Vorticity)
  - Chapter 6.6 (Texture-Based Vector Visualization)
- Diffeomorphisms / smooth deformations
  `https://en.wikipedia.org/wiki/Diffeomorphism`
- Learn how convolution (the convolution of two functions) works:
  `https://en.wikipedia.org/wiki/Convolution`
- B. Cabral, C. Leedom:
  Imaging Vector Fields Using Line Integral Convolution, SIGGRAPH 1993
  `http://dx.doi.org/10.1145/166117.166151`

Vector / Flow Visualization

Online Demos and Info

Numerical ODE integration methods (Euler vs. Runge Kutta, etc.)

https://demonstrations.wolfram.com/NumericalMethodsForDifferentialEquations/

Flow visualization concepts

https://www3.nd.edu/~cwang11/flowvis.html

Vector Fields: Motivation

http://de.wikipedia.org/wiki/Bild:Airplane_vortex_edit.jpg

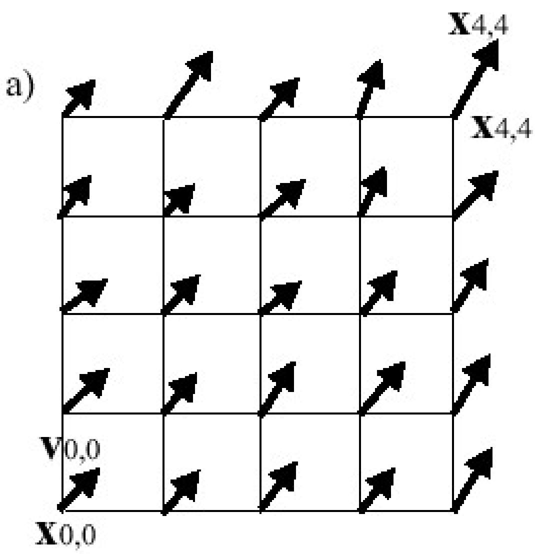
- **Vortex/ Vortex core lines**
  - There is no exact definition of vortices
  - capturing some swirling behavior

Vector Fields

Each vector is usually thought of as a velocity vector

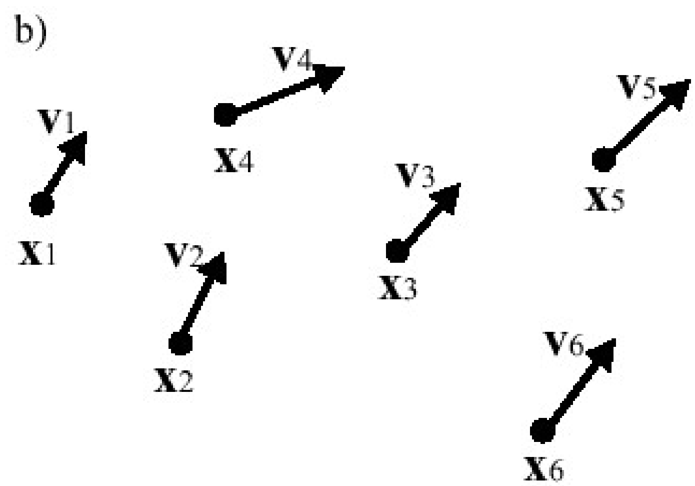
- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

Eulerian specification:



vectors given at grid points
(grid points do not move)

Lagrangian specification:

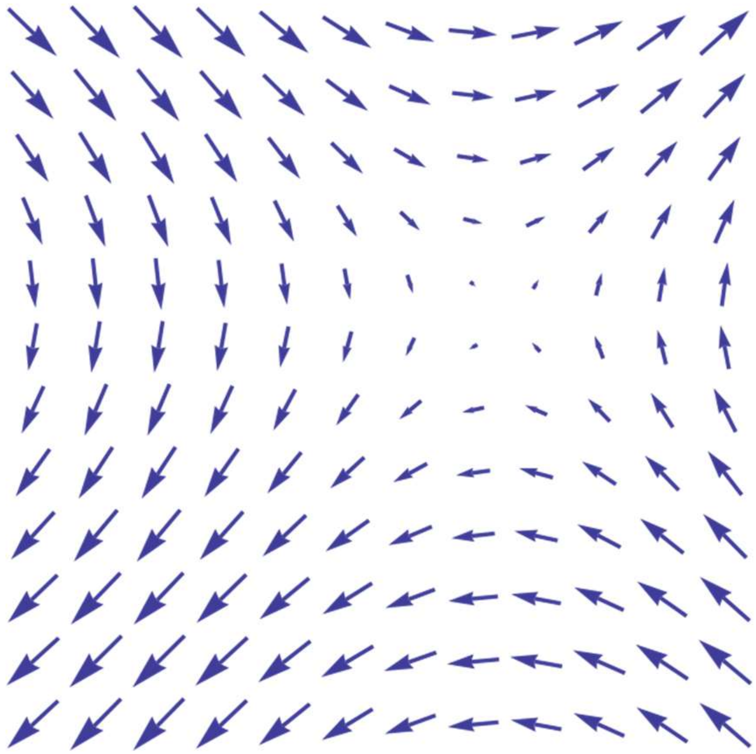


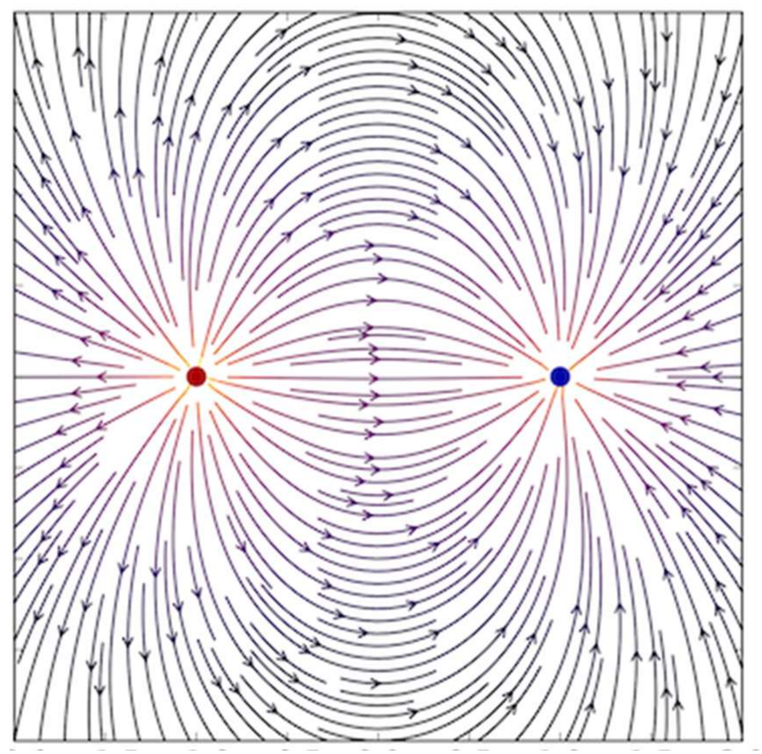
vectors given at particle positions
(particle positions do move)

Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

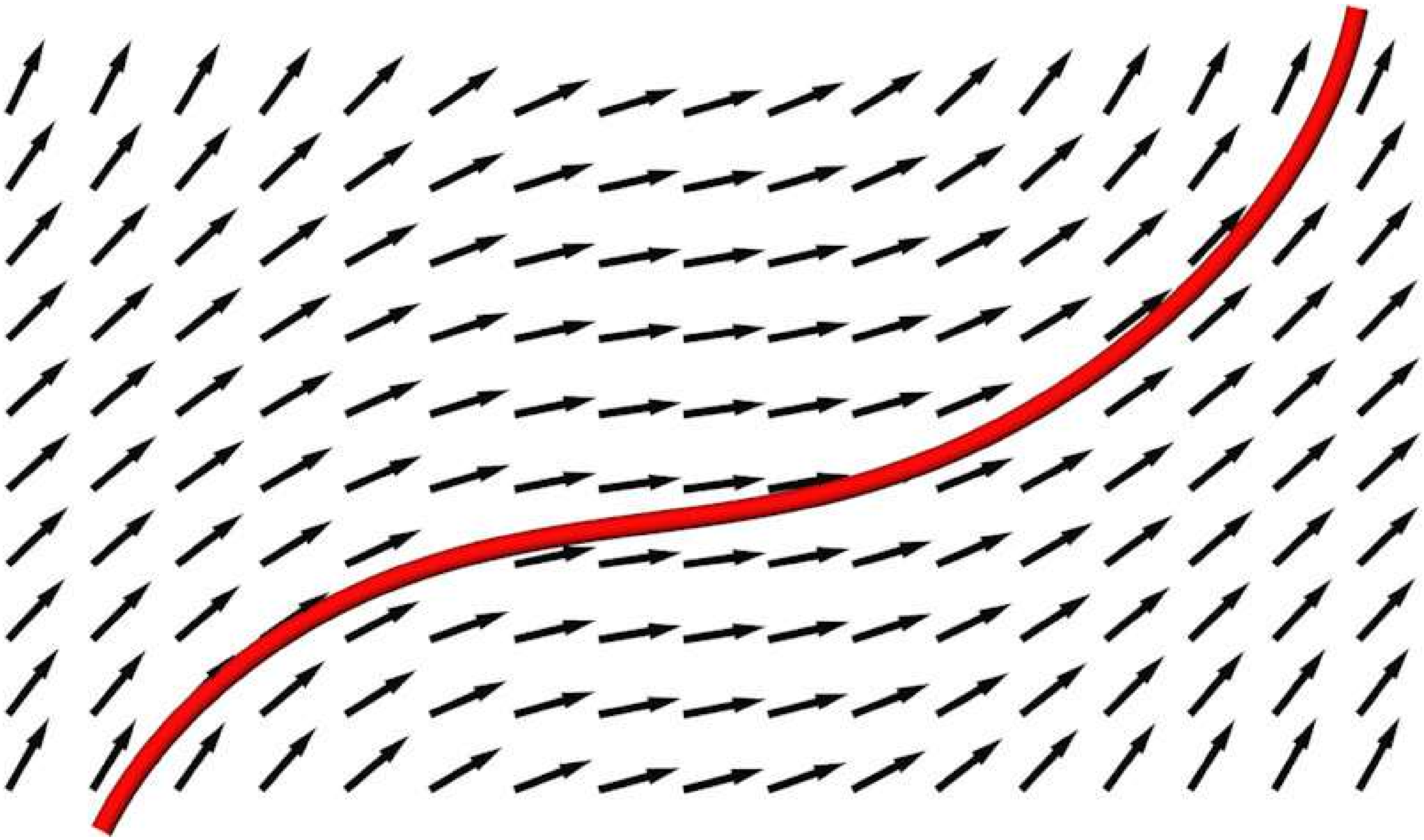




images from wikipedia

Integral Curves / Stream Objects

Integrating velocity over time yields spatial motion

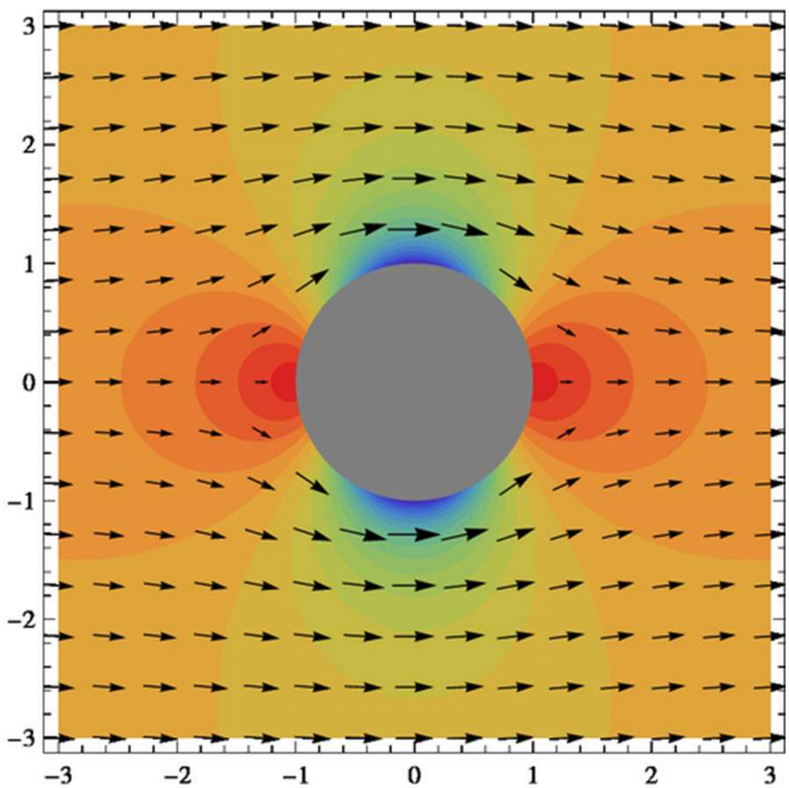


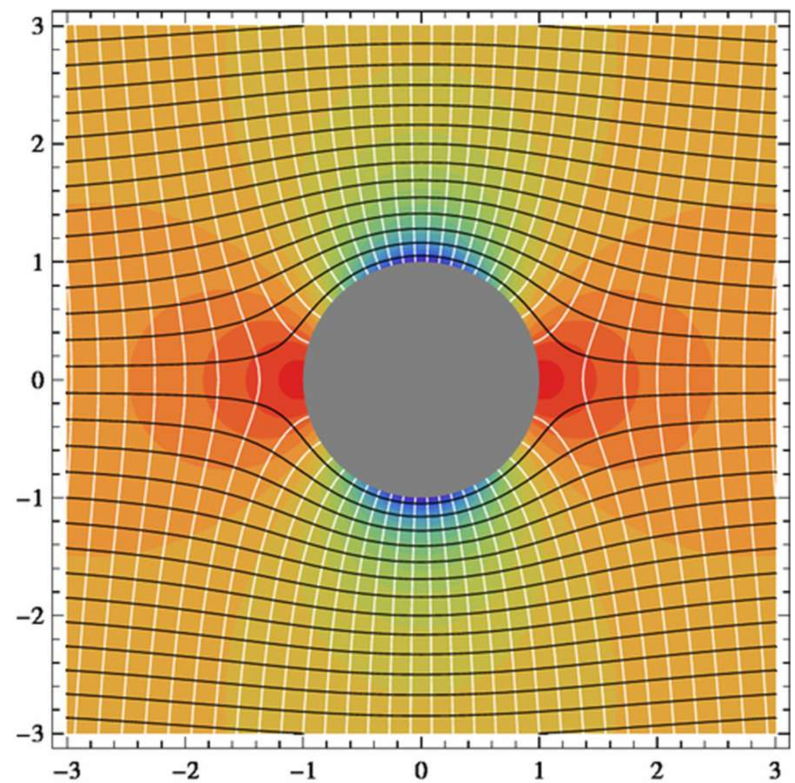
Flow Field Example (1)

Potential flow around a circular cylinder

`https://en.wikipedia.org/wiki/Potential_flow_around_a_circular_cylinder`

Inviscid, incompressible flow that is irrotational (curl-free) and can be modeled as the gradient of a scalar function called the (scalar) velocity potential





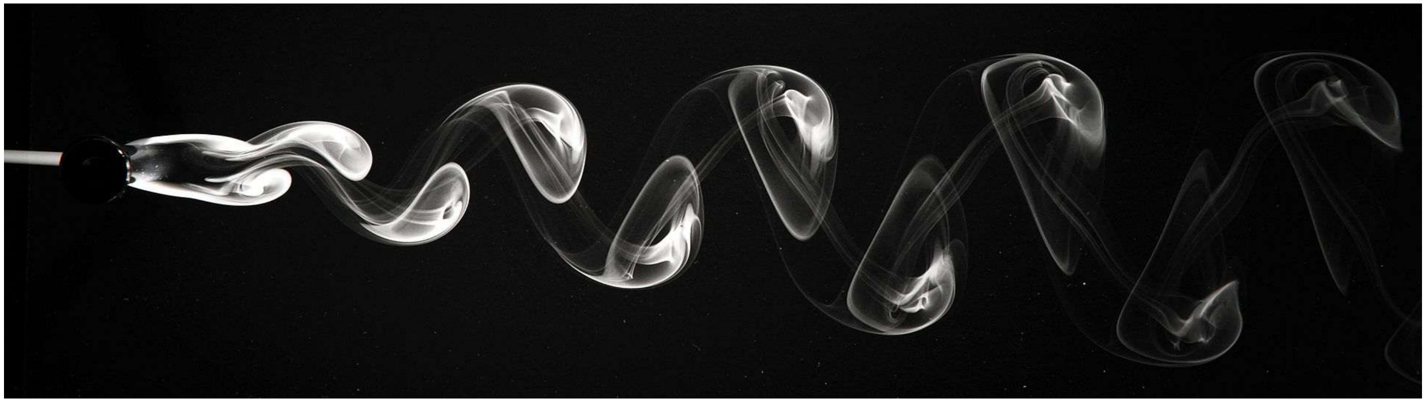
images from wikipedia

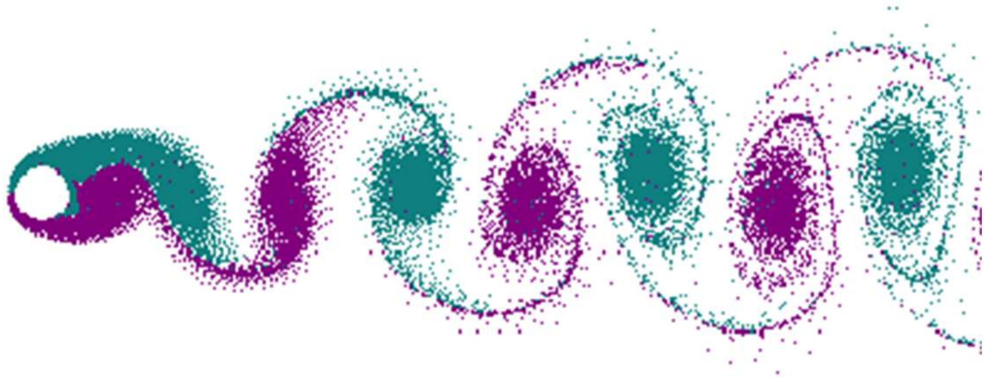
Flow Field Example (2)

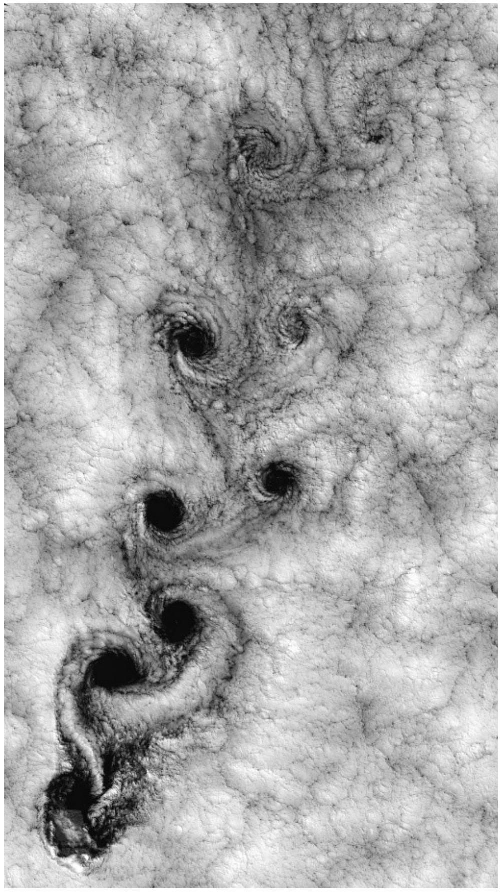
Depending on Reynolds number, turbulence will develop

Example: von Kármán vortex street: vortex shedding

`https://en.wikipedia.org/wiki/Karman_vortex_street`







images from wikipedia

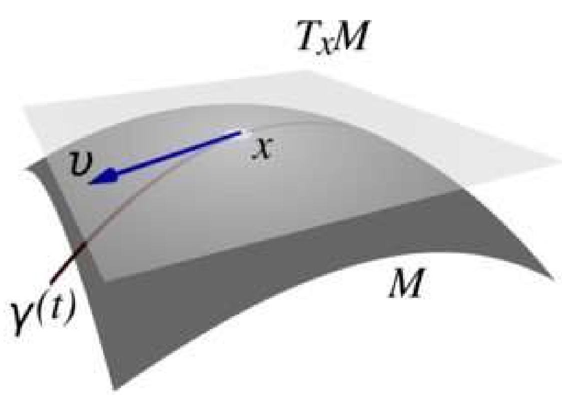
Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

Each vector in a vector field
lives in the **tangent space**
of the manifold at that point:

Each vector is a **tangent vector**



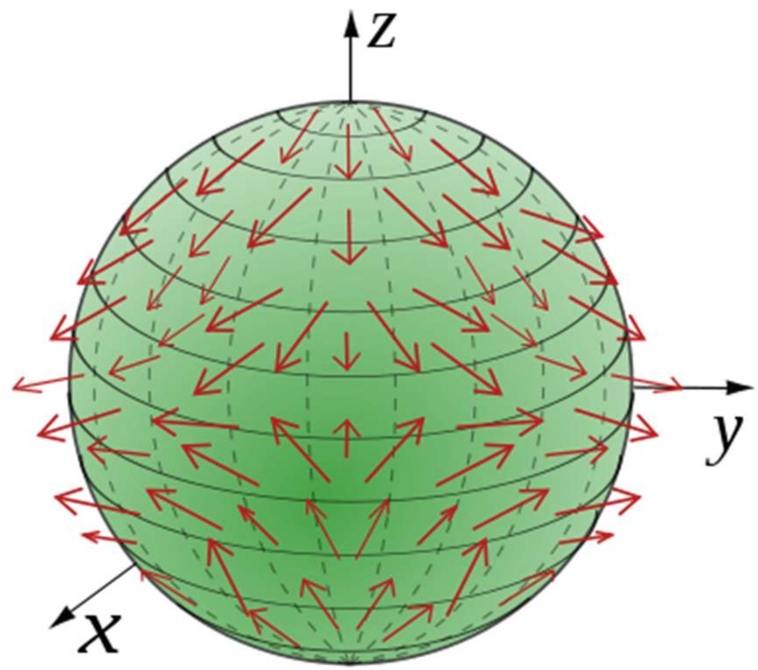


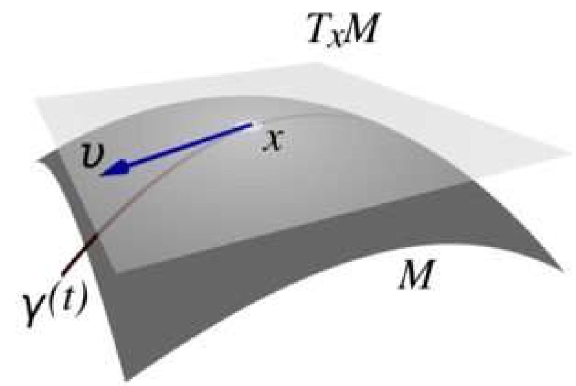
image from wikipedia

Vector Fields

Vector fields on general manifolds M (not just Euclidean space)

Tangent space at a point $x \in M$:

$$T_x M$$



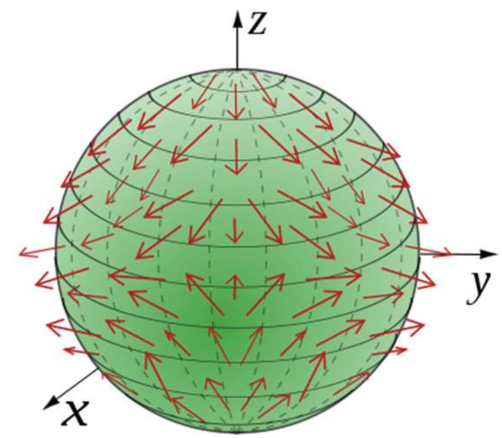
Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \rightarrow M$$

Vector field: *Section* of tangent bundle

$$s : M \rightarrow TM,$$
$$x \mapsto s(x).$$

$$\pi\left(s(x)\right) = x$$



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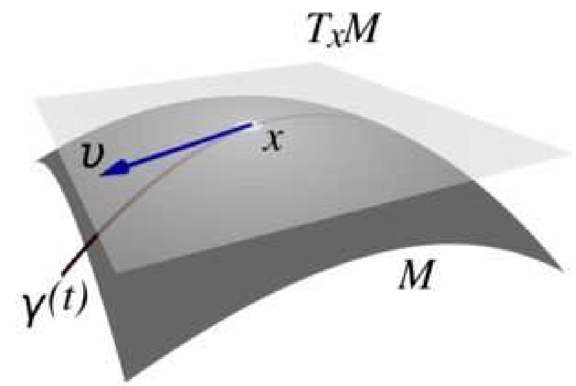


Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \to M$$

Vector field: *Section* of tangent bundle

$$\mathbf{v} : M \to TM, \quad x \mapsto \mathbf{v}(x). \qquad \mathbf{v}(x) \in T_x M$$

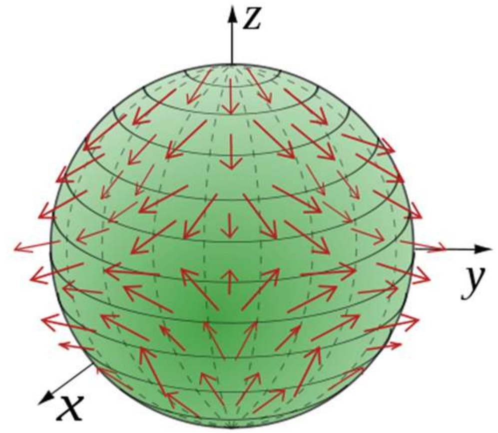


image from wikipedia

Vector fields

A static vector field $\mathbf{v}(\mathbf{x})$ is a vector-valued function of space.

A time-dependent vector field $\mathbf{v}(\mathbf{x},t)$ depends also on time.

In the case of velocity fields, the terms steady and unsteady flow are used.

The dimensions of **x** and **v** are equal, often 2 or 3, and we denote components by *x,y,z* and *u,v,w*:

$$\mathbf{x} = (x,y,z),\ \mathbf{v} = (u,v,w)$$

Sometimes a vector field is defined on a surface $\mathbf{x}(i,j)$. The vector field is then a function of parameters and time:

$$\mathbf{v}(i,j,t)$$

Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
    $$\mathbf{v} : \mathbb{R}^n \to \mathbb{R}^n, \quad x \mapsto \mathbf{v}(x).$$
  - Example: laminar flows

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x}, t)$
    $$\mathbf{v} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n, \quad (x,t) \mapsto \mathbf{v}(x,t).$$
  - Example: turbulent flows

(here just for Euclidean domain; analogous on general manifolds)

Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
  - Example: laminar flows

$$\mathbf{v}:\ M \to \mathbb{R}^n,$$
$$x \mapsto \mathbf{v}(x).$$

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x},t)$
  - Example: turbulent flows

$$\mathbf{v}:\ M \times \mathbb{R} \to \mathbb{R}^n,$$
$$(x,t) \mapsto \mathbf{v}(x,t).$$

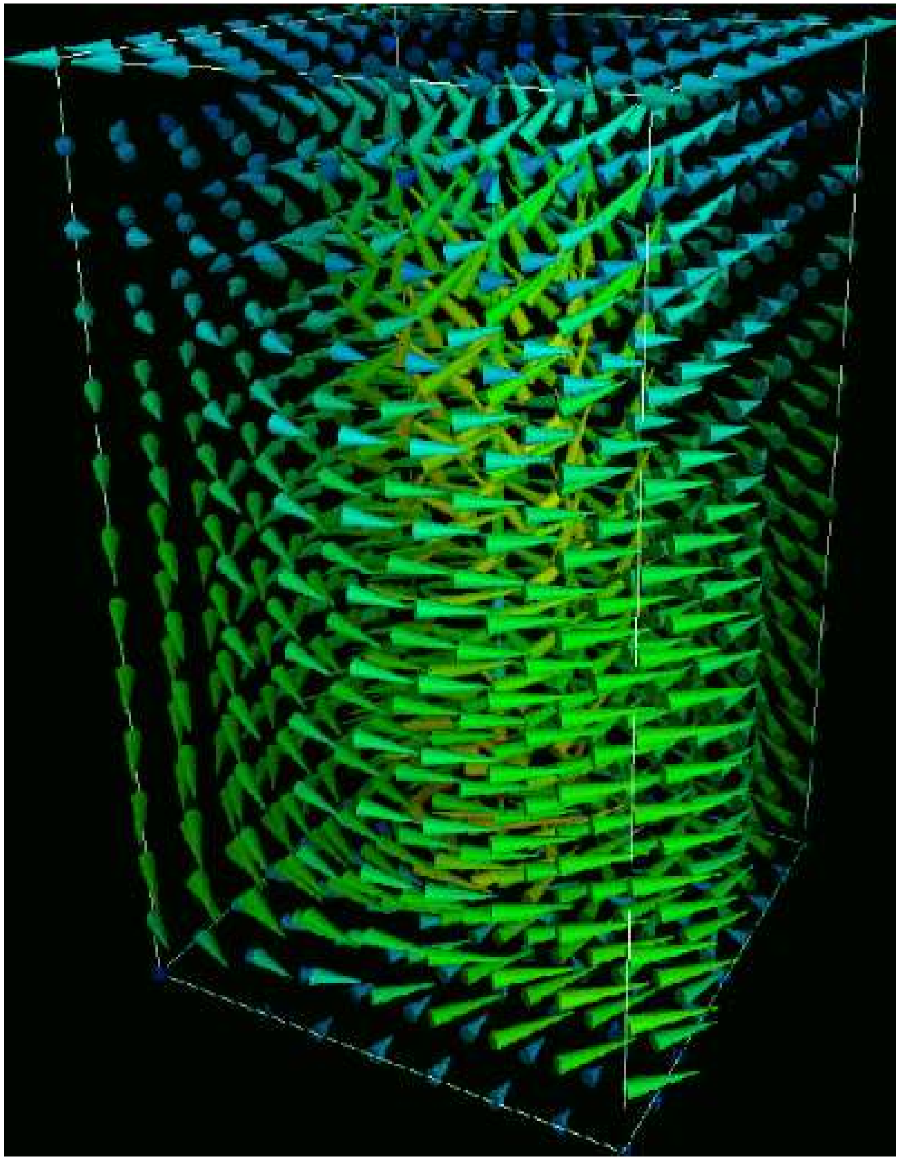
(here now for general manifolds)

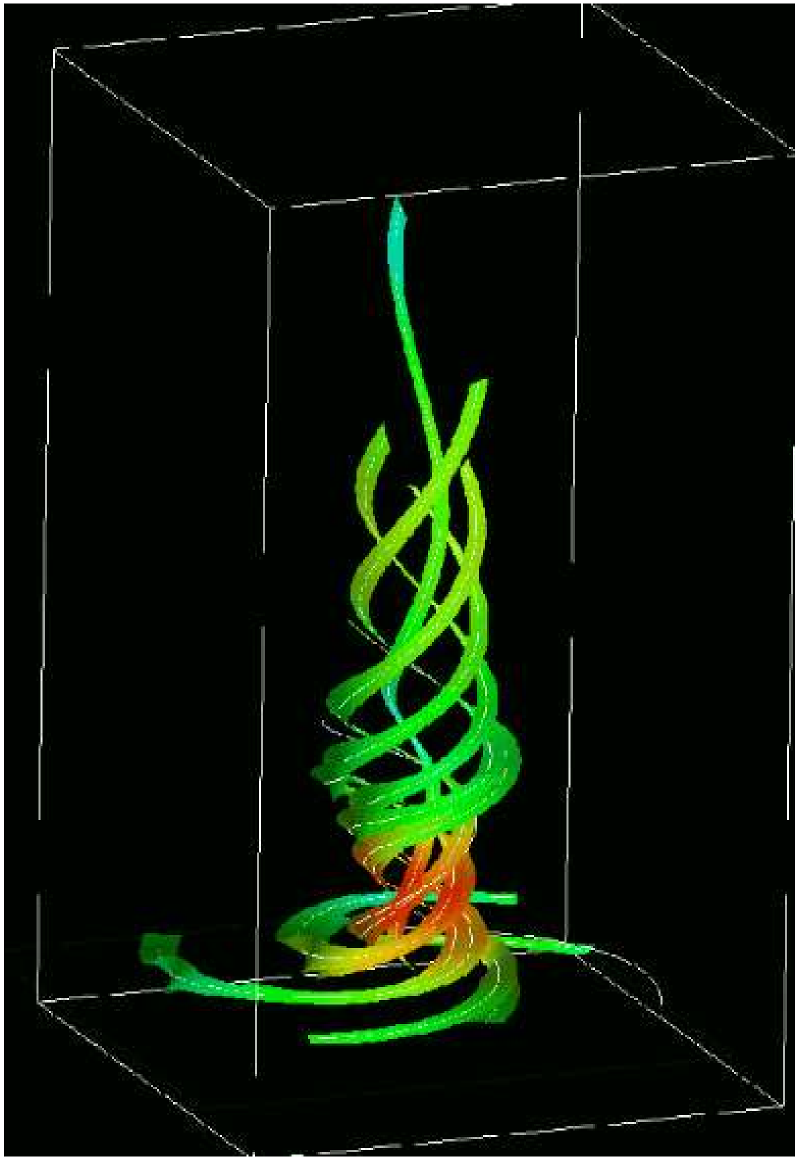
Direct vs. Indirect Flow Visualization

- Direct flow visualization
  - Overview of current flow state
  - Visualization of vectors: arrow plots (“hedgehog” plots)

- Indirect flow visualization
  - Use intermediate representation: vector field integration over time
  - Visualization of temporal evolution
  - Integral curves: streamlines, pathlines, streaklines, timelines
  - Integral surfaces: streamsurfaces, pathsurfaces, streaksurfaces

Direct vs. Indirect Flow Visualization

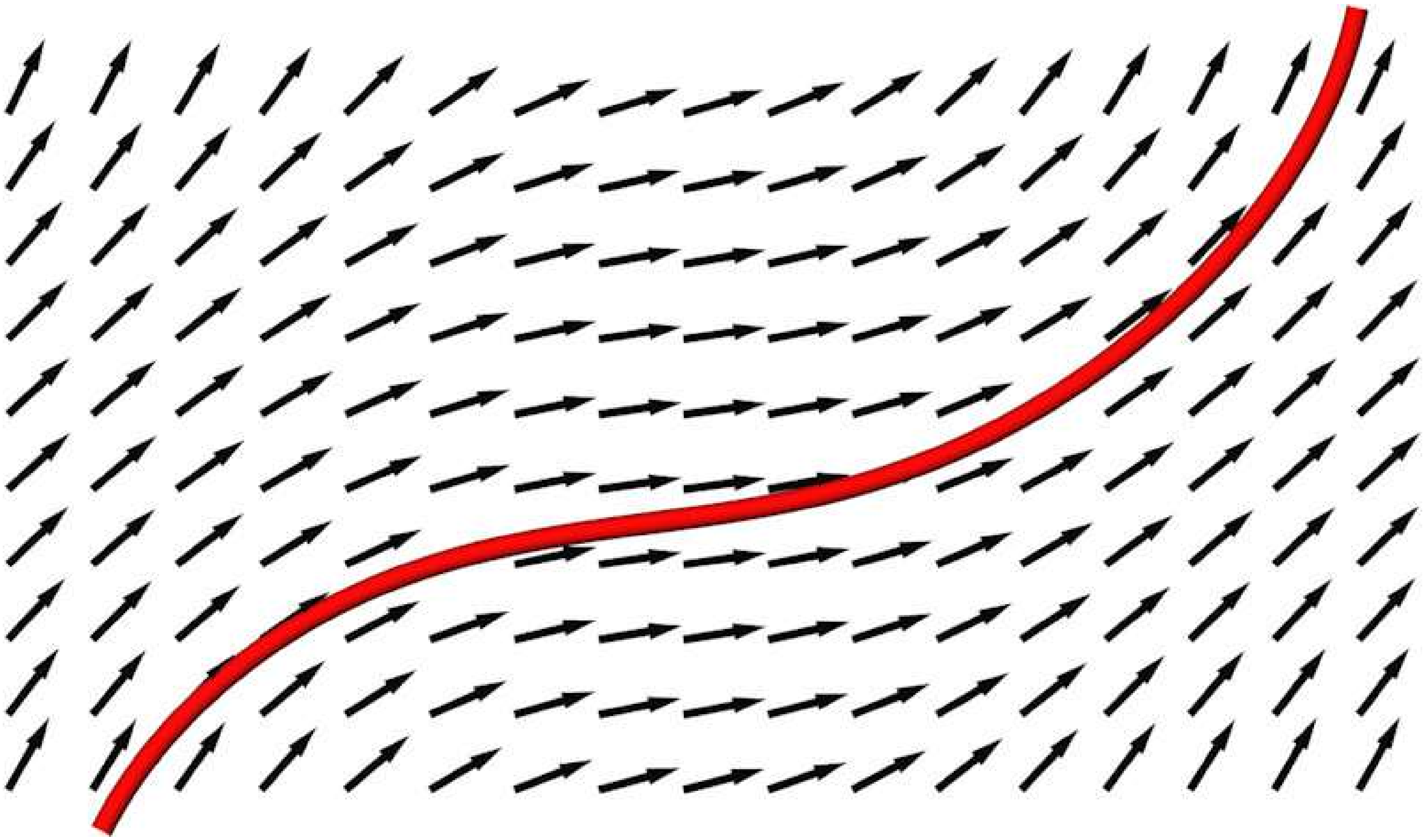




Integral Curves: Intro

Integral Curves / Stream Objects

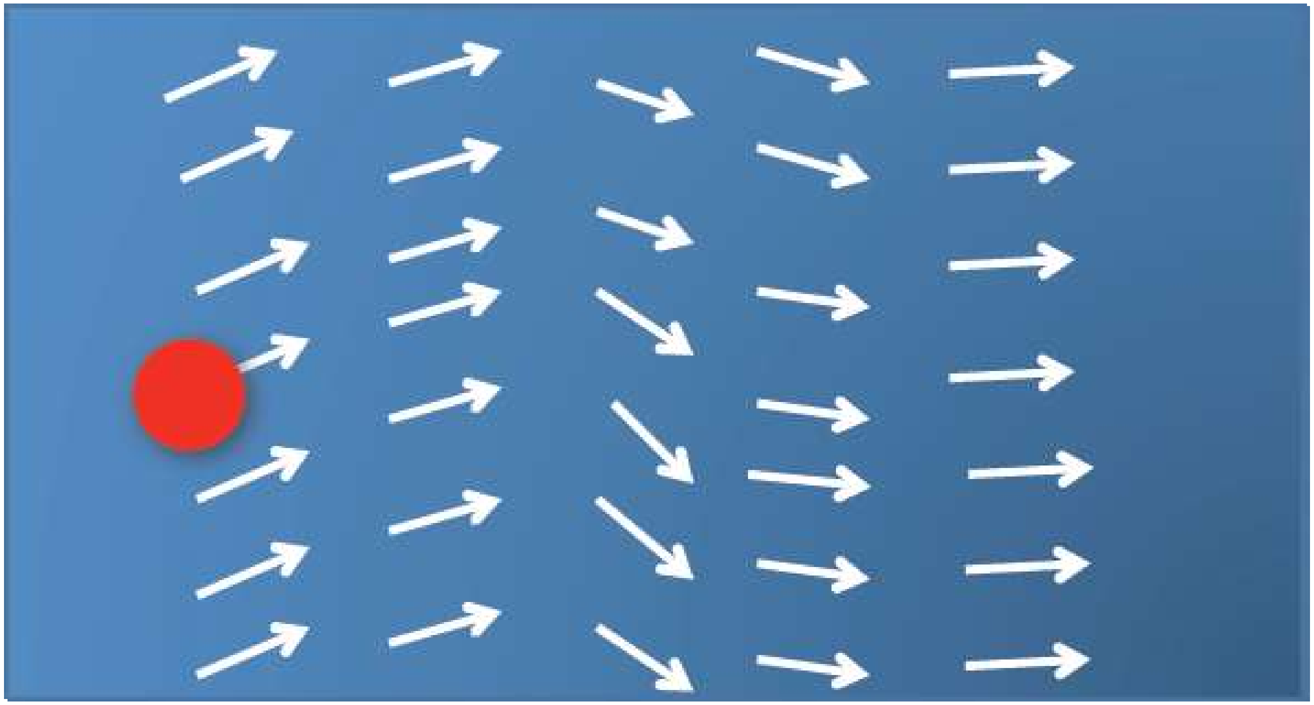
Integrating velocity over time yields spatial motion



Particle Trajectories

Courtesy Jens Krüger

Particle Trajectories



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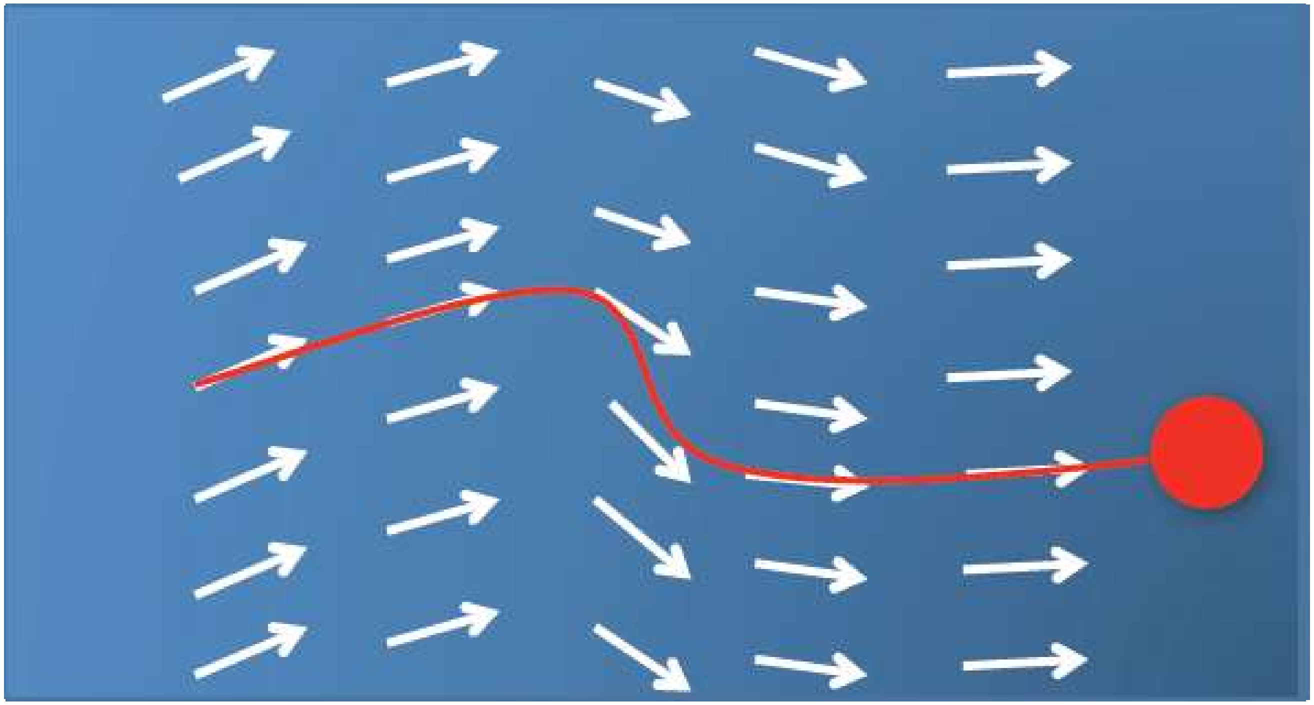

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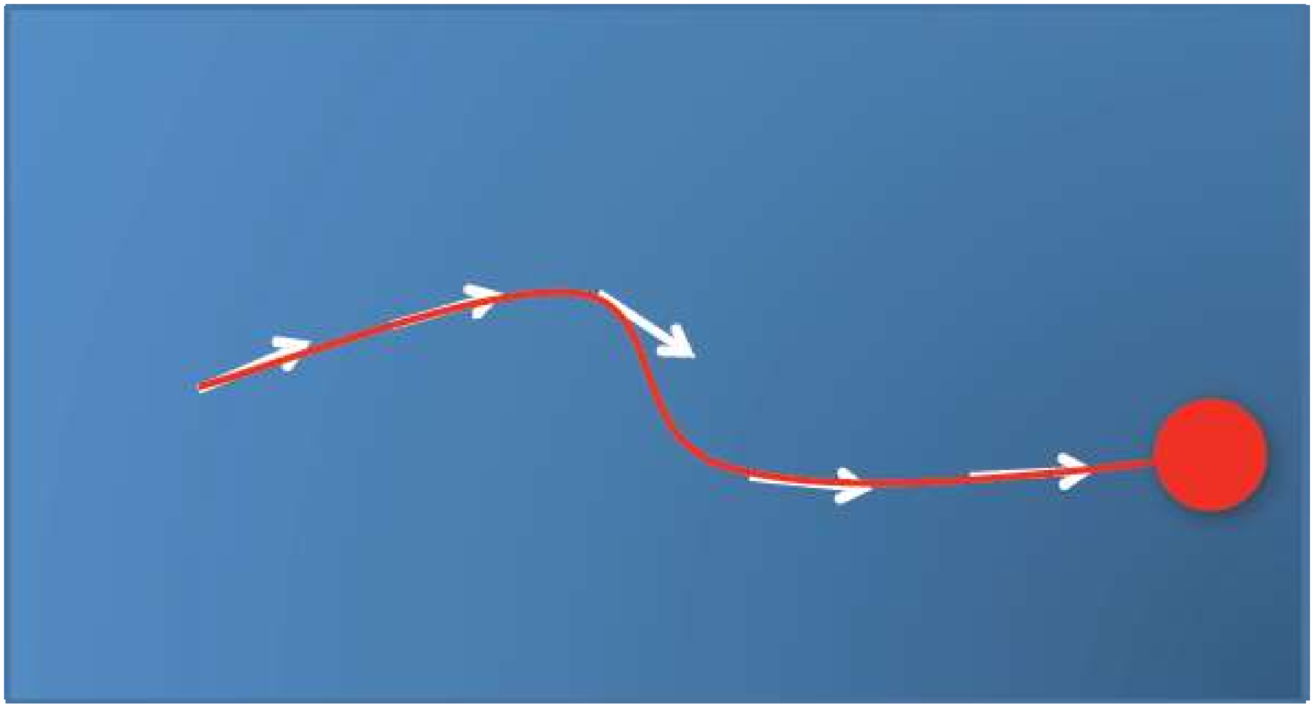
Particle Trajectories



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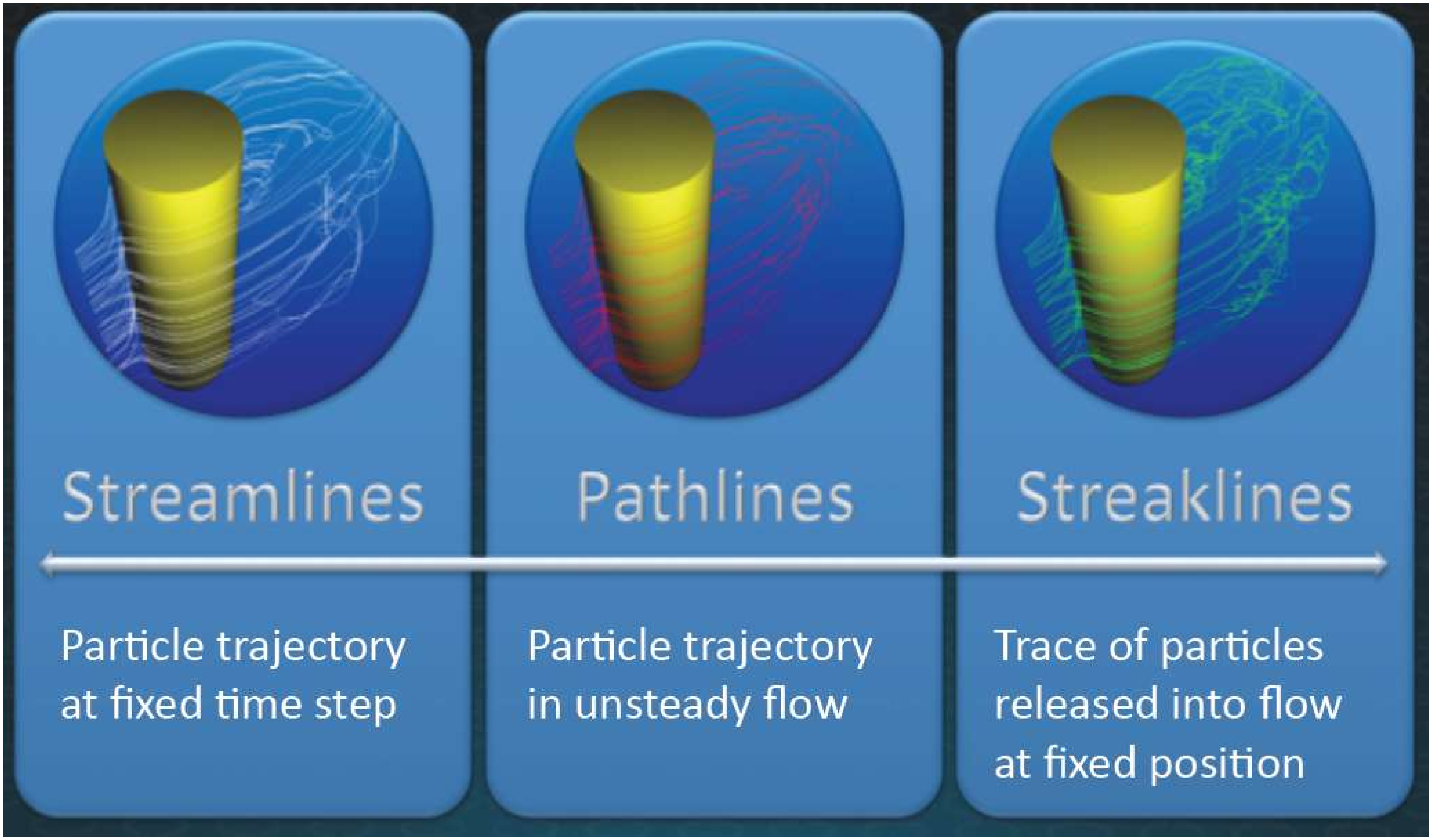
Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

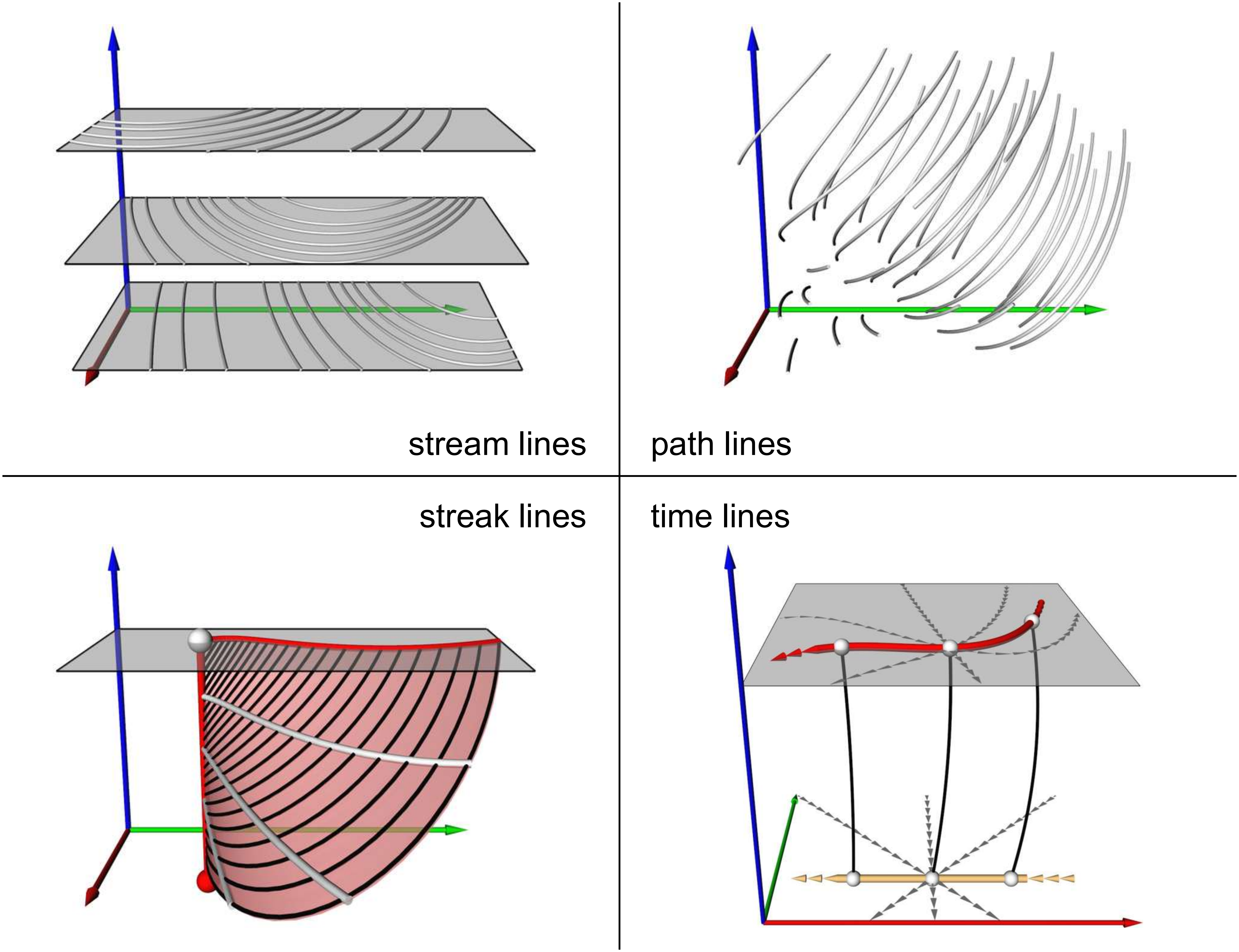
- Describes motion of a massless particle over time

Streakline

- Location of all particles released at a *fixed position* over time

Timeline

- Location of all particles released along a line at a *fixed time*


stream lines
path lines
streak lines
time lines

Comparison of techniques:

(1) Pathlines:

- are physically meaningful
- allow comparison with experiment (observe marked particles)
- are well suited for dynamic visualization (of particles)

(2) Streamlines:

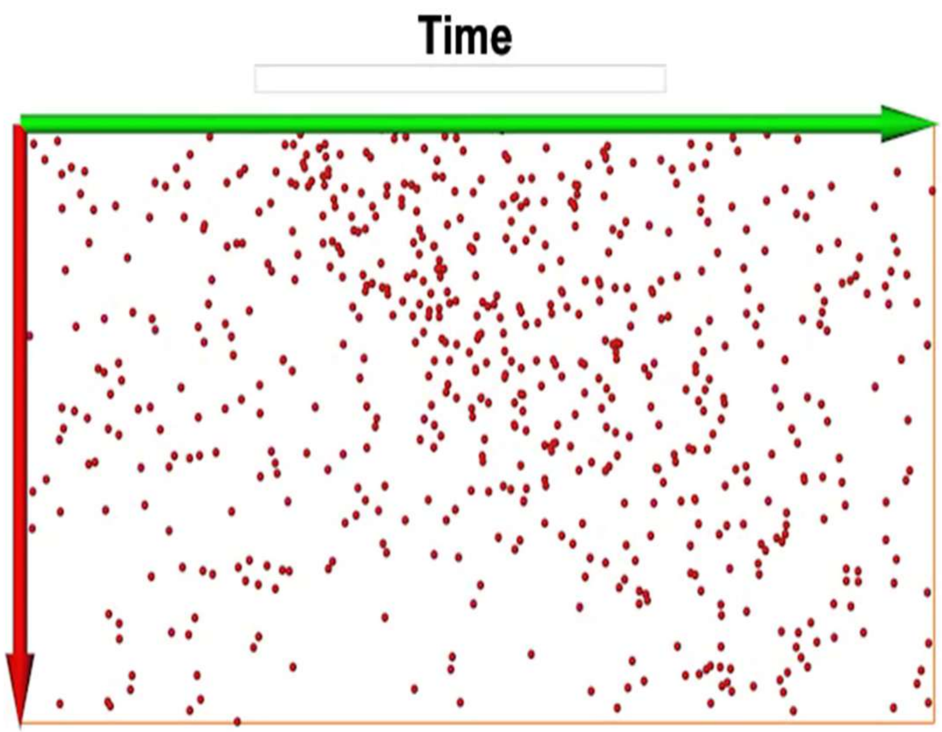
- are only geometrically, not physically meaningful
- are easiest to compute (no temporal interpolation, single IVP)
- are better suited for static visualization (prints)
- don't intersect (under reasonable assumptions)

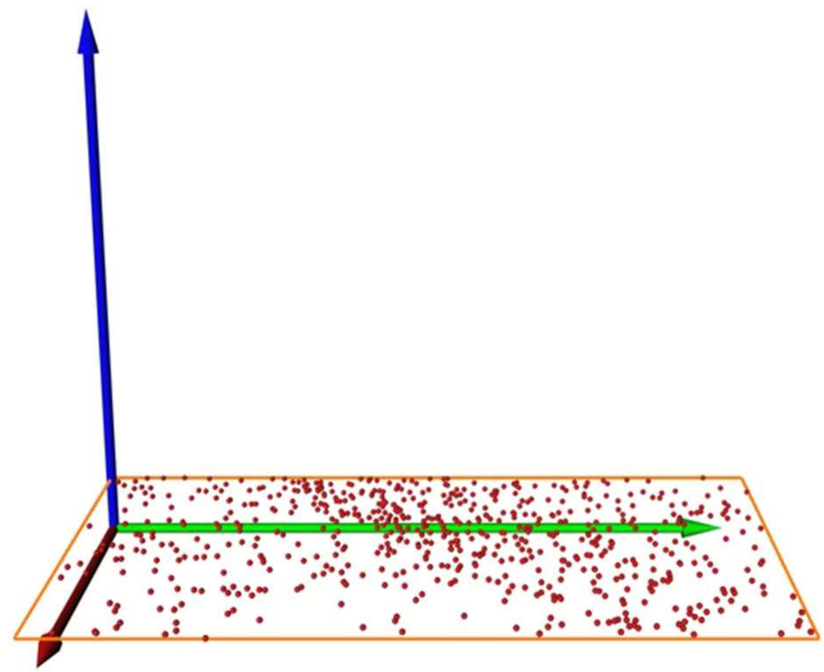
(3) Streaklines:

- are physically meaningful
- allow comparison with experiment (dye injection)
- are well suited for static and dynamic visualization
- good choice for fast moving vortices
- can be approximated by set of disconnected particles

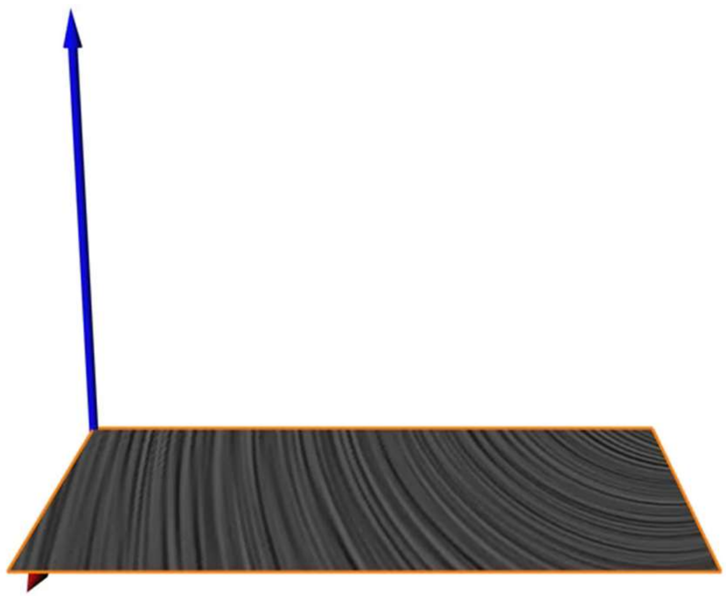
(4) Timelines:

- are physically meaningful
- are well suited for static and dynamic visualization
- can be approximated by set of disconnected particles

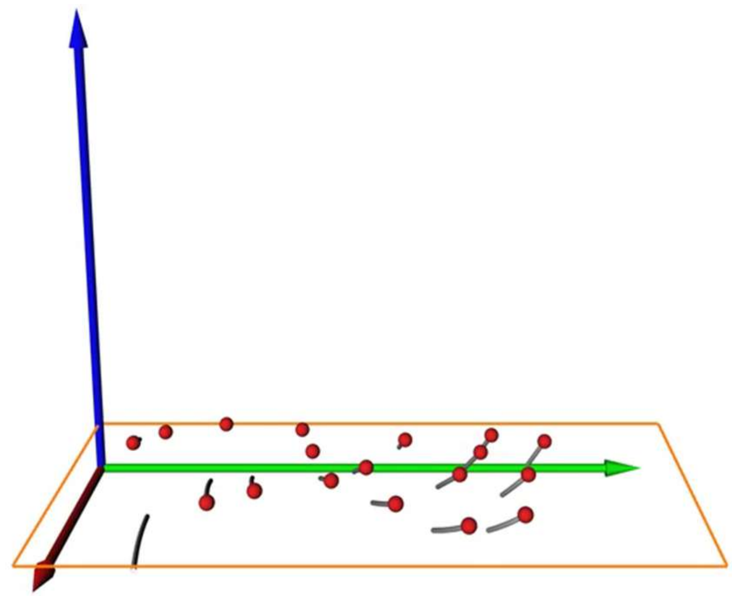




2D time-dependent vector field
particle visualization



stream lines



path lines

curve parallel to the vector field in each point for a **fixed time**

describes motion of a massless particle in an **steady** flow field

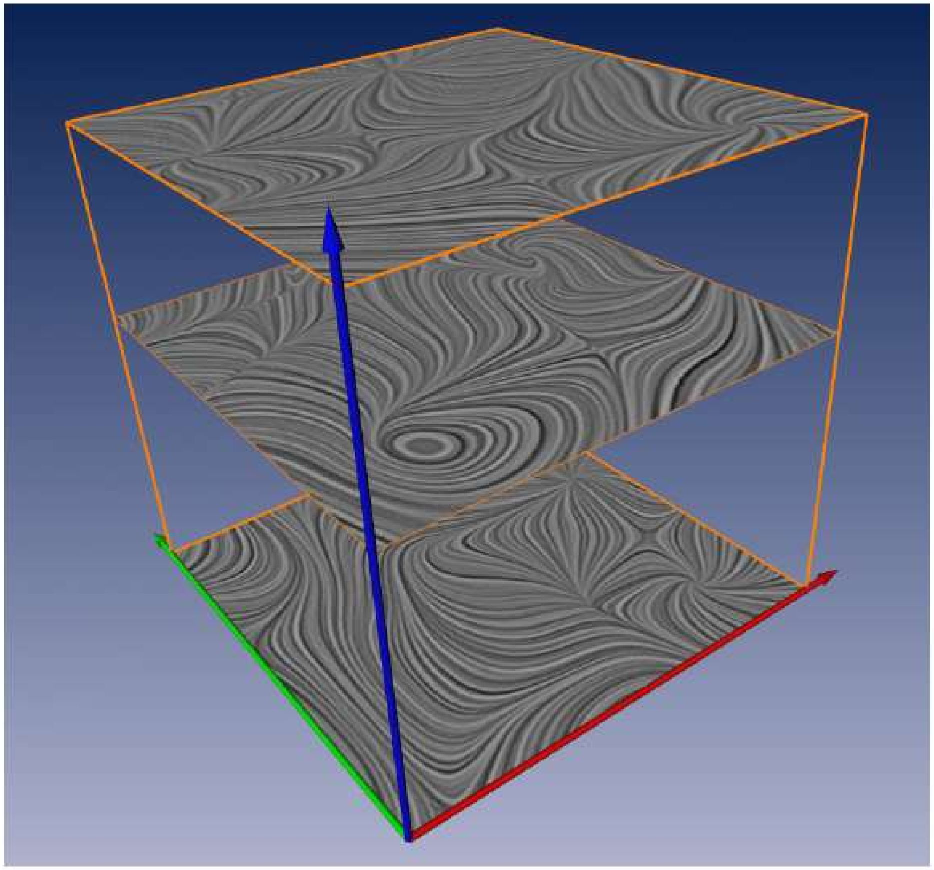
curve parallel to the vector field in each point **over time**

describes motion of a massless particle in an **unsteady** flow field

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

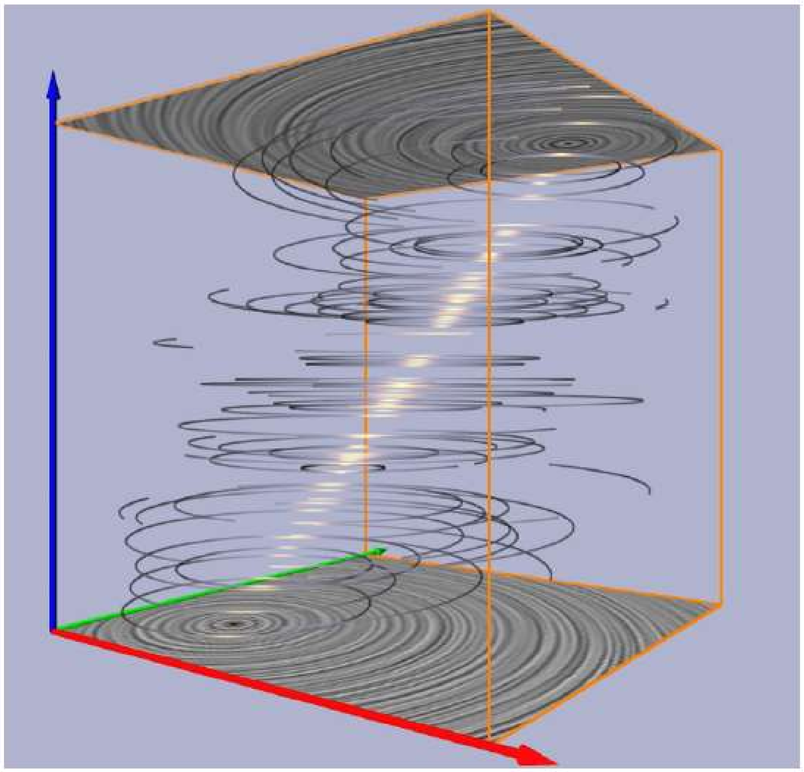


Tino Weinkauf

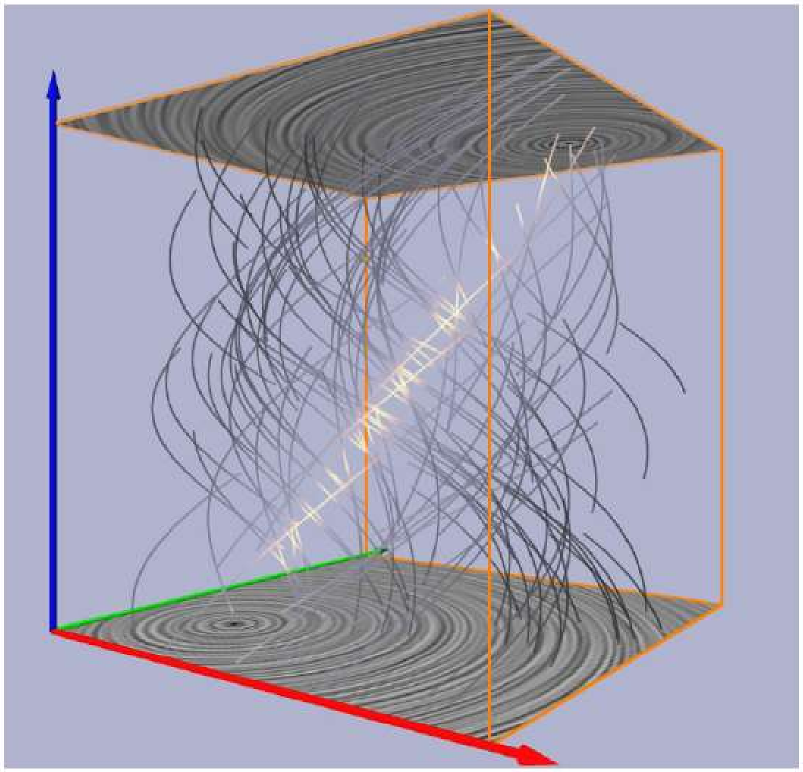
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Vector fields as ODEs

For simplicity, the vector field is now interpreted as a velocity field.

Then the field $\mathbf{v}(\mathbf{x},t)$ describes the connection between location and velocity of a (massless) particle.

It can equivalently be expressed as an ordinary differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t),t)$$

This ODE, together with an initial condition

$$\mathbf{x}(t_0) = \mathbf{x}_0 \ ,$$

is a so-called initial value problem (IVP).

Its solution is the integral curve (or trajectory)

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau),\tau)\,d\tau$$

The integral curve is a pathline, describing the path of a massless particle which was released at time t_0 at position x_0.

Remark: $t < t_0$ is allowed.

For static fields, the ODE is autonomous:

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t))$$

and its integral curves

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau))\,d\tau$$

are called field lines, or (in the case of velocity fields) streamlines.

In static vector fields, pathlines and streamlines are identical.

In time-dependent vector fields, instantaneous streamlines can be computed from a "snapshot" at a fixed time T (which is a static vector field)

$$\mathbf{v}_T(\mathbf{x}) = \mathbf{v}(\mathbf{x}, T)$$

In practice, time-dependent fields are often given as a dataset per time step. Each dataset is then a snapshot.

Streamline integration

Outline of algorithm for numerical streamline integration (with obvious extension to pathlines):

Inputs:

- static vector field $\mathbf{v}(\mathbf{x})$
- seed points with time of release $(\mathbf{x}_0, t_0)$
- control parameters:
  - step size (temporal, spatial, or in local coordinates)
  - step count limit, time limit, etc.
  - order of integration scheme

Output:

- streamlines as "polylines", with possible attributes (interpolated field values, time, speed, arc length, etc.)

Preprocessing:

- set up search structure for point location
- for each seed point:
  - global point location: Given a point **x**,
    find the cell containing **x** and the local coordinates (ξ, η, ζ)
    or ir the grid is structured:
    find the computational space coordinates $(i+\xi,\ j+\eta,\ k+\zeta)$
  - If **x** is not found in a cell, remove seed point

Integration loop, for each seed point **x**:

- interpolate ***v*** trilinearly to local coordinates (ξ, η, ζ)
- do an integration step, producing a new point **x'**
- incremental point location: For position **x'** find cell and local coordinates (ξ', η', ζ') making use of information (coordinates, local coordinates, cell) of old point **x**

Termination criteria:

- grid boundary reached
- step count limit reached
- optional: velocity close to zero
- optional: time limit reached
- optional: arc length limit reached

Streamline integration

Integration step: widely used integration methods:

- Euler (used only in special speed-optimized techniques, e.g. GPU-based texture advection)

$$\mathbf{x}_{new} = \mathbf{x} + \mathbf{v}(\mathbf{x}, t) \cdot \Delta t$$

- Runge-Kutta, 2nd or 4th order

Higher order than 4th?

- often too slow for visualization
- study (Yeung/Pope 1987) shows that, when using standard trilinear interpolation, interpolation errors dominate integration errors.

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama