King Abdullah University of Science and Technology
KAUST

CS 380 - GPU and GPGPU Programming
Lecture 25: GPU Texturing, Pt. 2

Markus Hadwiger, KAUST

Reading Assignment #10 (until Nov 11)

Read (required):

- Interpolation for Polygon Texture Mapping and Shading, Paul Heckbert and Henry Moreton

  `https://www.ri.cmu.edu/publications/interpolation-for-polygon-texture-mapping-and-shading/`

- Homogeneous Coordinates

  `https://en.wikipedia.org/wiki/Homogeneous_coordinates`

Next Lectures

Lecture 26: Tue, Nov 5 (make-up lecture; 14:30 – 15:45)

Lecture 27: Thu, Nov 7: Vulkan tutorial #2

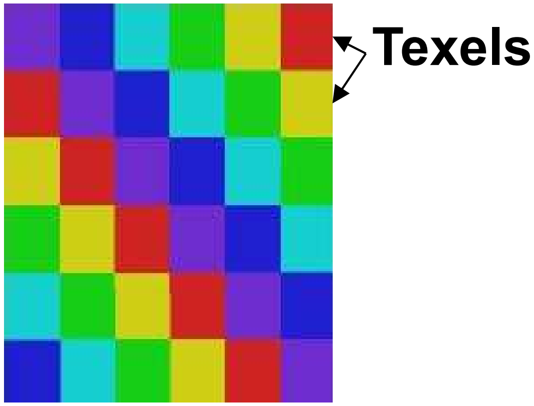
Lecture 28: Mon, Nov 11: 10:00-11:30 (on Zoom)

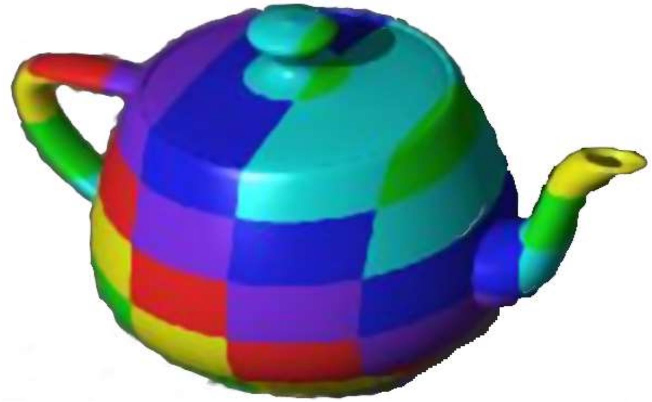
Lecture 29: Thu, Nov 14: 10:00-11:30 (on Zoom)

Lecture 30: Mon, Nov 18: Quiz #3

GPU Texturing

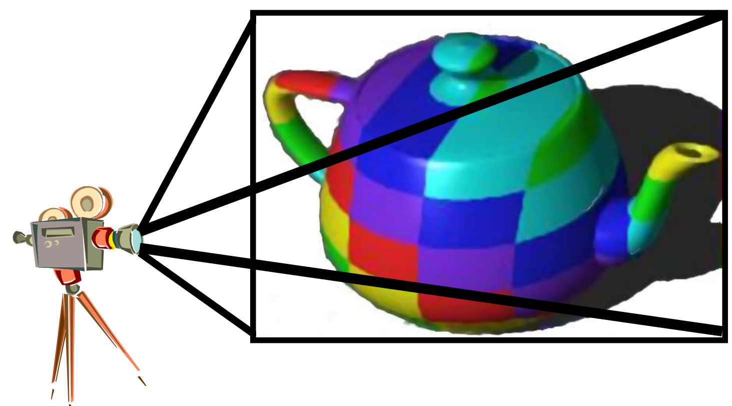
Texturing: General Approach





Texture space *(u,v)* **Object space** (x_O,y_O,z_O) **Image Space** (x_I,y_I)

Parametrization → **Rendering (Projection etc.)** →



Texture Mapping

2D (3D) Texture Space

↓ Texture Transformation

2D Object Parameters

↓ Parameterization

3D Object Space

↓ Model Transformation

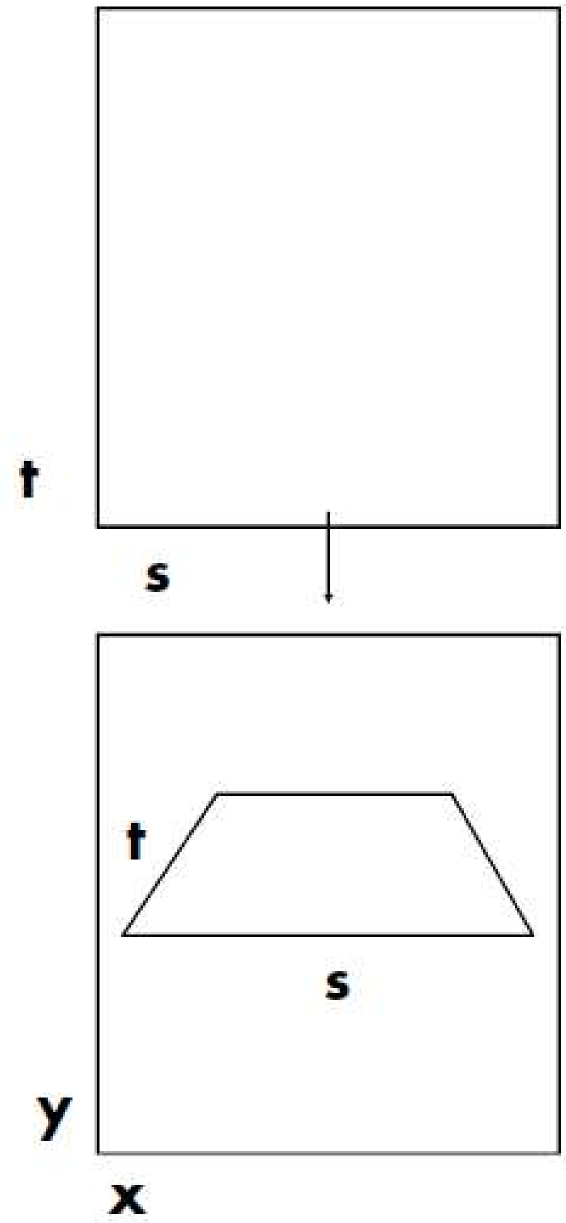
3D World Space

↓ Viewing Transformation

3D Camera Space

↓ Projection

2D Image Space



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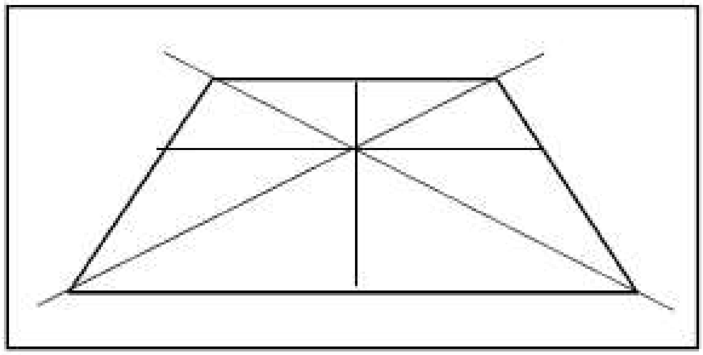
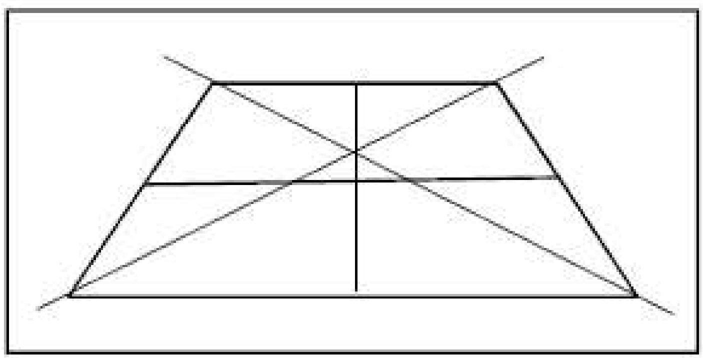
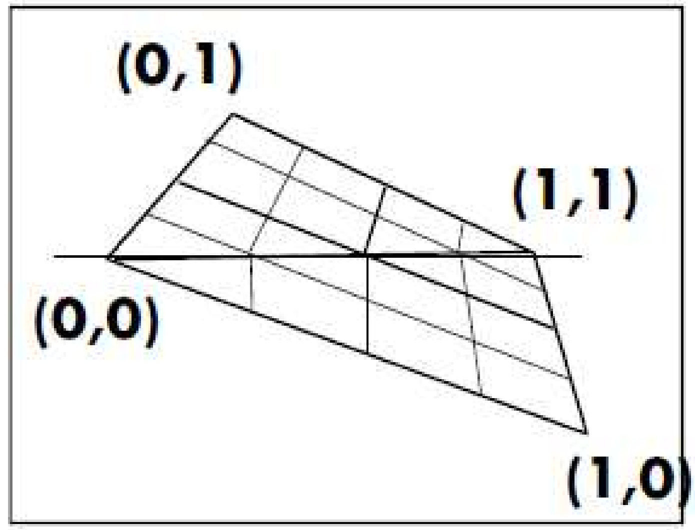
Linear Perspective



Correct Linear Perspective



Incorrect Perspective

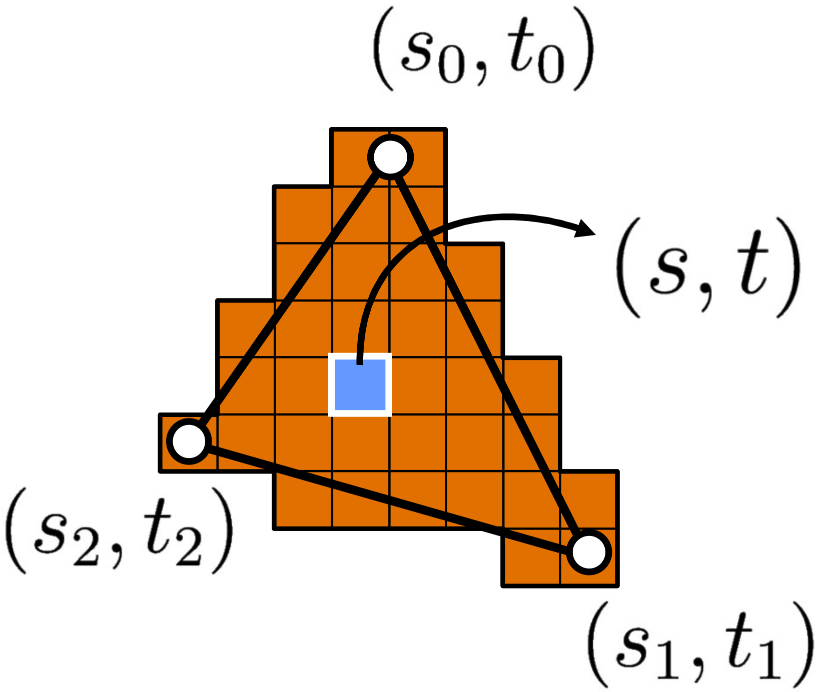


Linear Interpolation, *Bad*

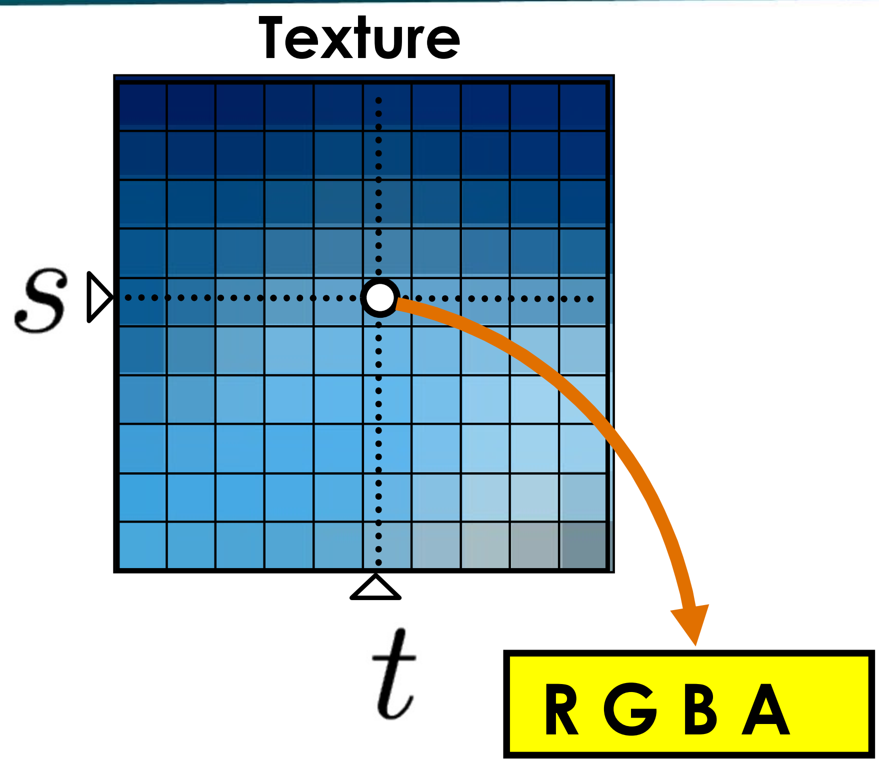
Perspective Interpolation, *Good*

Kurt Akeley, Pat Hanrahan

2D Texture Mapping

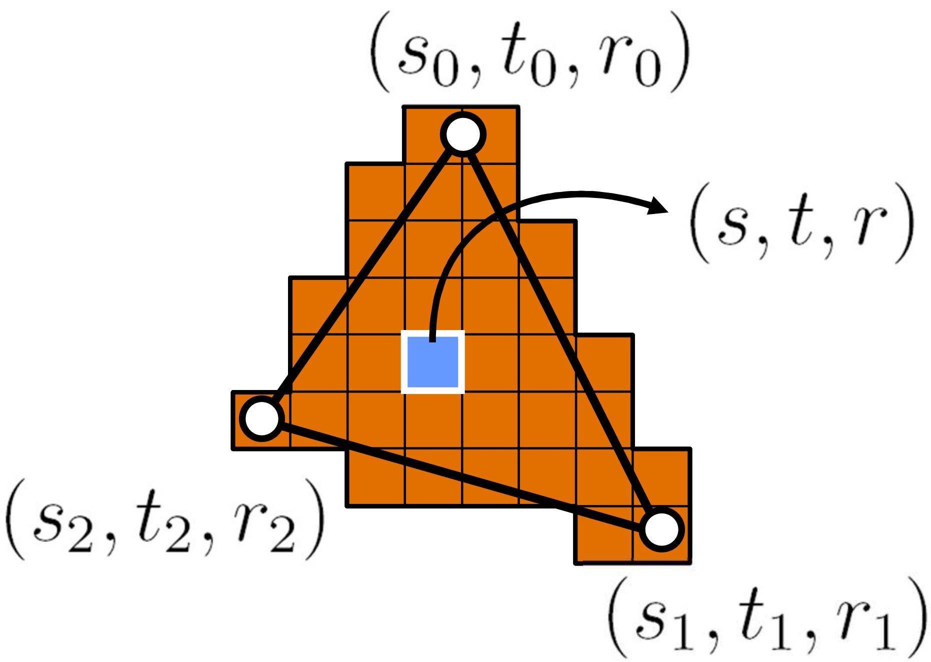


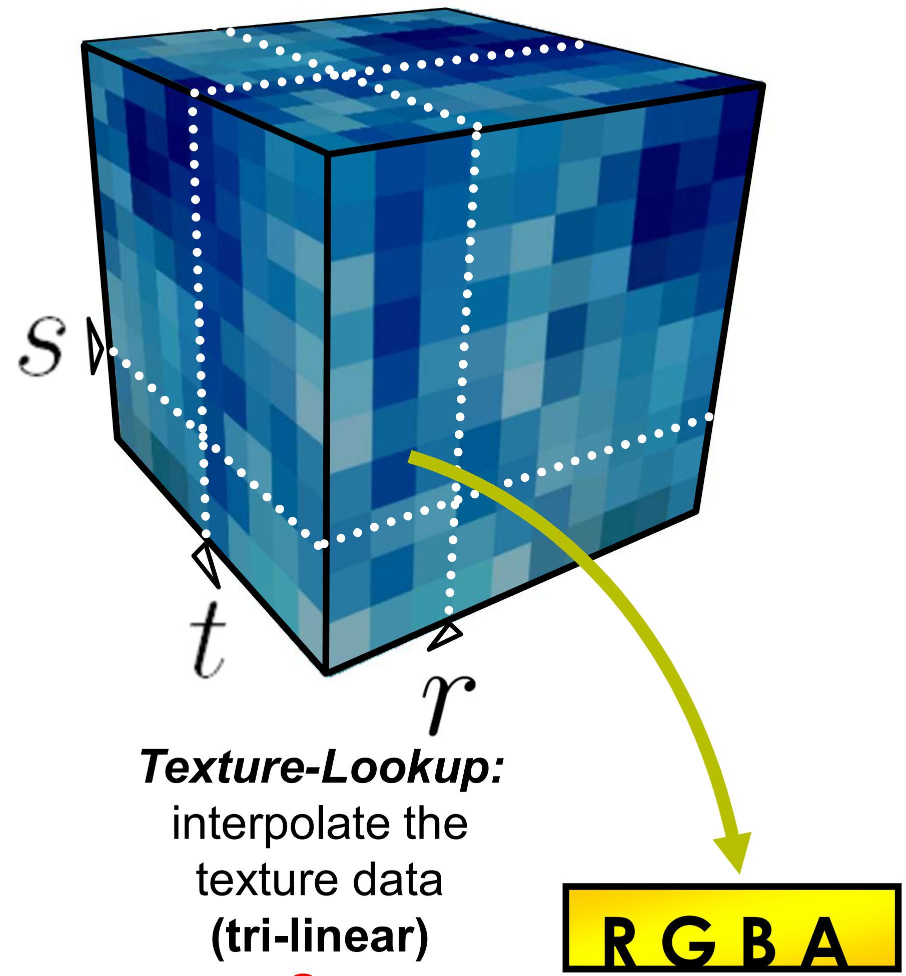
For each fragment:
interpolate the
texture coordinates
(barycentric)
Or:
Use arbitrary, computed coordinates



Texture-Lookup:
interpolate the
texture data
(bi-linear)
Or:
Nearest-neighbor for "array lookup"

3D Texture Mapping





For each fragment:
interpolate the
texture coordinates
(barycentric)
Or:
Use arbitrary, computed coordinates

Texture-Lookup:
interpolate the
texture data
(tri-linear)
Or:
Nearest-neighbor for "array lookup"

Interpolation #1

Interpolation Type + Purpose #1:

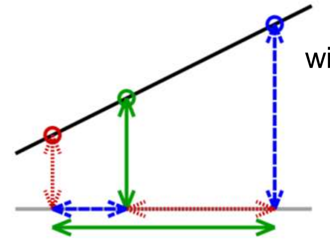
Interpolation of Texture Coordinates

(Linear / Rational-Linear Interpolation)

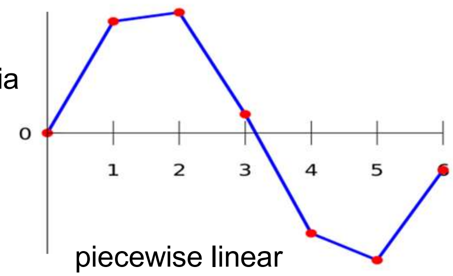
Linear Interpolation / Convex Combinations

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$



wikipedia



Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad\qquad f(\alpha) = v_1 + \alpha(v_2 - v_1)$$

$$\alpha_1 + \alpha_2 = 1 \qquad\qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

Compare to line parameterization with parameter t: $v(t) = v_1 + t(v_2 - v_1)$

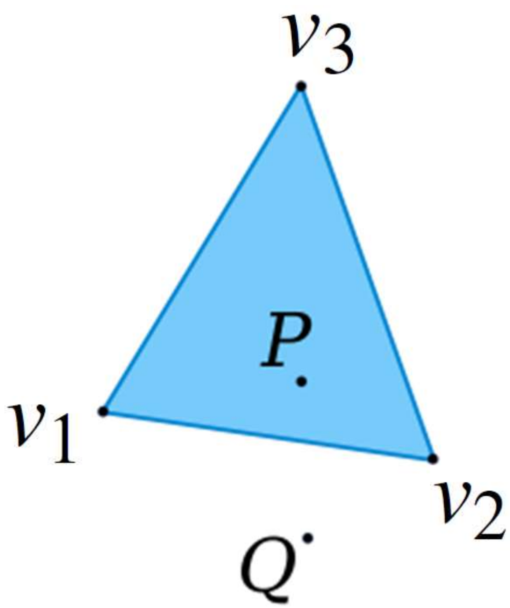
Linear Interpolation / Convex Combinations

Linear combination (n-dim. space):

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

Affine combination: Restrict to $(n-1)$-dim. subspace:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



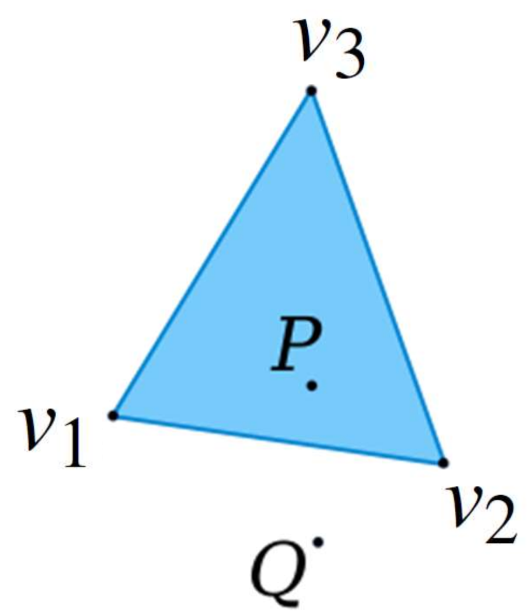
Convex combination: $\alpha_i \geq 0$

(restrict to simplex in subspace)

Linear Interpolation / Convex Combinations

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



Re-parameterize to get affine coordinates:

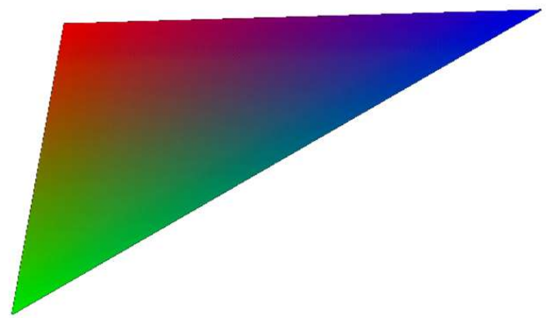
$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 =$$
$$\tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1$$
$$\tilde{\alpha}_1 = \alpha_2$$
$$\tilde{\alpha}_2 = \alpha_3$$

Linear Interpolation / Convex Combinations

The weights α_i are the (normalized) barycentric coordinates

→ linear attribute interpolation in simplex

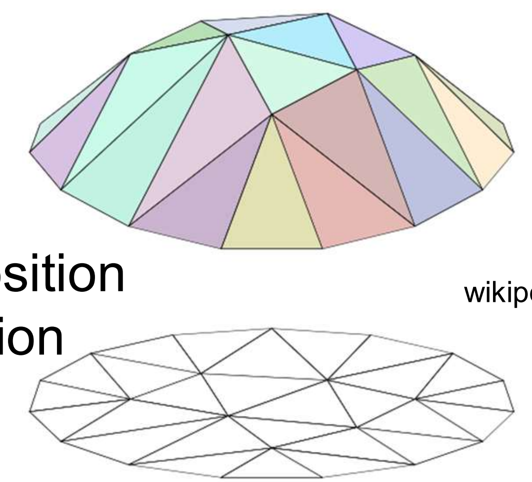
attribute interpolation



$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

$$\alpha_i \geq 0$$



spatial position interpolation

wikipedia

Homogeneous Coordinates (1)

Projective geometry

- (Real) projective spaces RP^n:

  Real projective line RP^1, real projective plane RP^2, ...

- A point in RP^n is a line through the origin (i.e., all the scalar multiples of the same vector) in an (n+1)-dimensional (real) vector space

Homogeneous coordinates of 2D projective point in RP^2

- Coordinates differing only by a non-zero factor λ map to the same point

  $(\lambda x, \lambda y, \lambda)$ dividing out the λ gives $(x, y, 1)$, corresponding to (x,y) in R^2

- Coordinates with last component = 0 map to "points at infinity"

  $(\lambda x, \lambda y, 0)$ division by last component not allowed; but again this is the same point if it only differs by a scalar factor, e.g., this is the same point as $(x, y, 0)$

Homogeneous Coordinates (2)

Examples of usage

- Translation (with translation vector $\vec{b}$)
- Affine transformations (linear transformation + translation)

$$\vec{y} = A\vec{x} + \vec{b}.$$

- With homogeneous coordinates:

$$\begin{bmatrix} \vec{y} \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & A & & \vec{b} \\ 0 & \dots & 0 & 1 \end{array}\right] \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix}$$

- Setting the last coordinate = 1 and the last row of the matrix to [0, ..., 0, 1] results in translation of the point $\vec{x}$ (via addition of translation vector $\vec{b}$)
- The matrix above is a linear map, but because it is one dimension higher, it does not have to move the origin in the (n+1)-dimensional space for translation

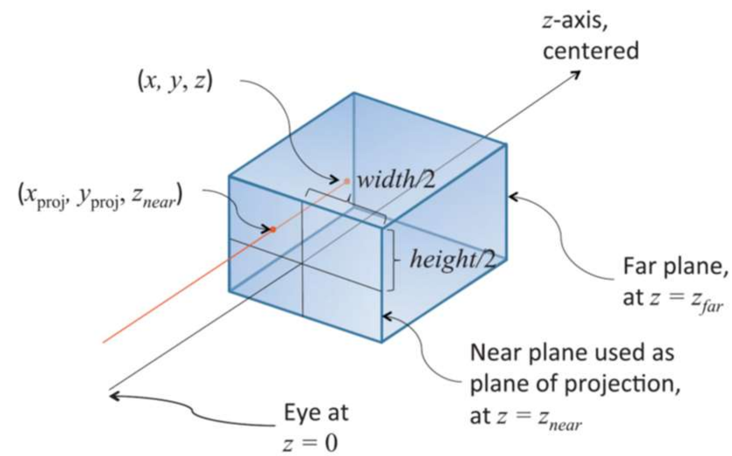
Homogeneous Coordinates (3)

Examples of usage

- Projection (e.g., OpenGL projection matrices)

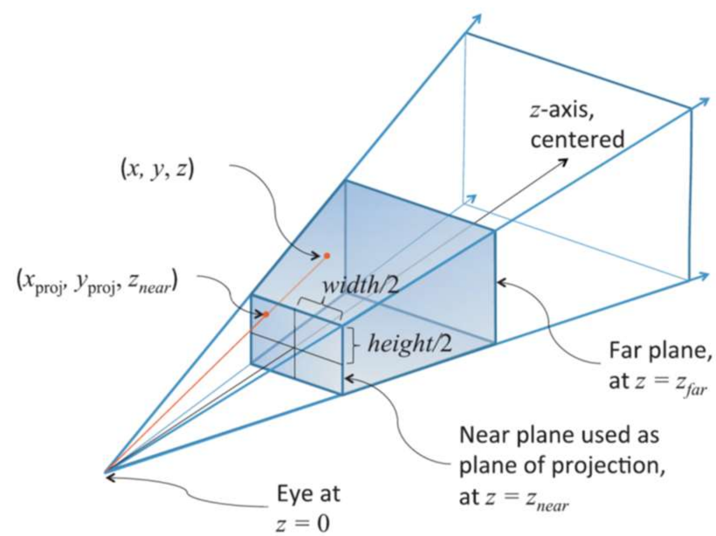
$$\begin{bmatrix} \frac{2}{\text{right}-\text{left}} & 0 & 0 & -\frac{\text{right}+\text{left}}{\text{right}-\text{left}} \\ 0 & \frac{2}{\text{top}-\text{bottom}} & 0 & -\frac{\text{top}+\text{bottom}}{\text{top}-\text{bottom}} \\ 0 & 0 & \frac{-2}{\text{far}-\text{near}} & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

orthographic



$$\begin{bmatrix} \frac{z_{near}}{width/2} & 0.0 & \frac{left+right}{width/2} & 0.0 \\ 0.0 & \frac{z_{near}}{height/2} & \frac{top+bottom}{height/2} & 0.0 \\ 0.0 & 0.0 & -\frac{z_{far}+z_{near}}{z_{far}-z_{near}} & \frac{2z_{far}z_{near}}{z_{far}-z_{near}} \\ 0.0 & 0.0 & -1.0 & 0.0 \end{bmatrix}$$

perspective



Texture Mapping

2D (3D) Texture Space

Texture Transformation

2D Object Parameters

Parameterization

3D Object Space

Model Transformation

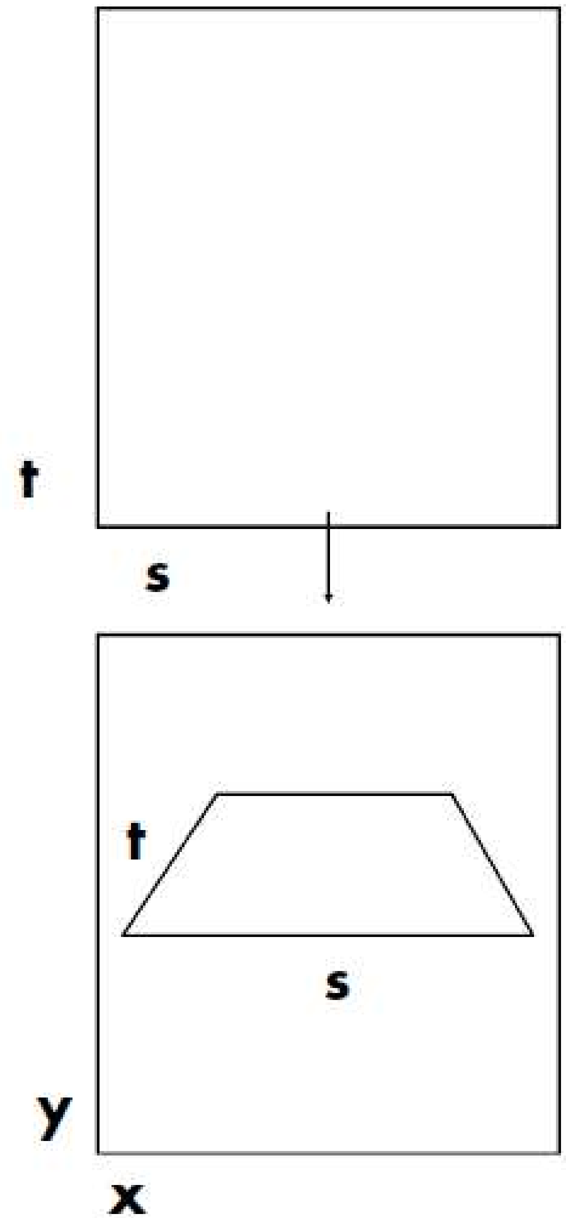
3D World Space

Viewing Transformation

3D Camera Space

Projection

2D Image Space



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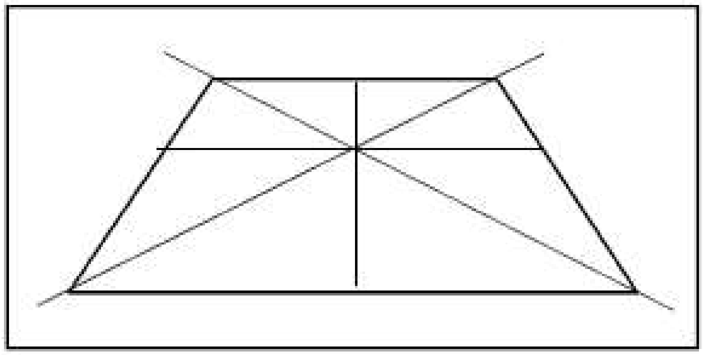
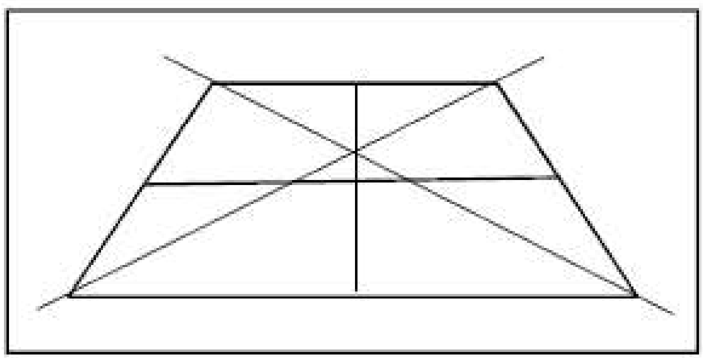
Linear Perspective



Correct Linear Perspective



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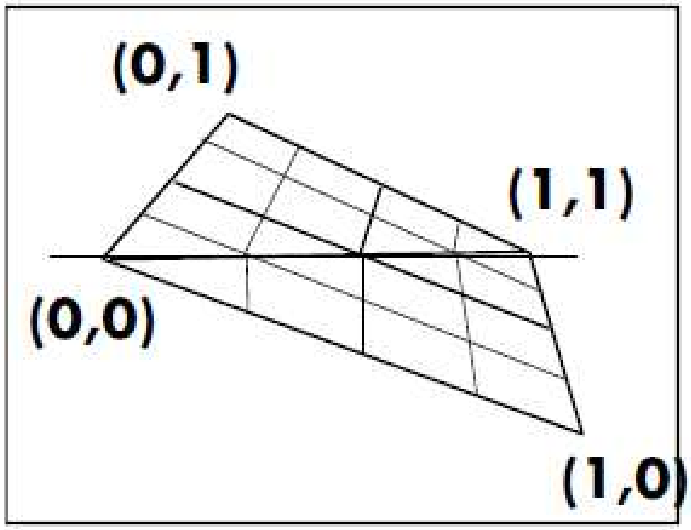

Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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Texture Mapping Polygons

Forward transformation: linear projective map

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} s \\ t \\ r \end{bmatrix}$$

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

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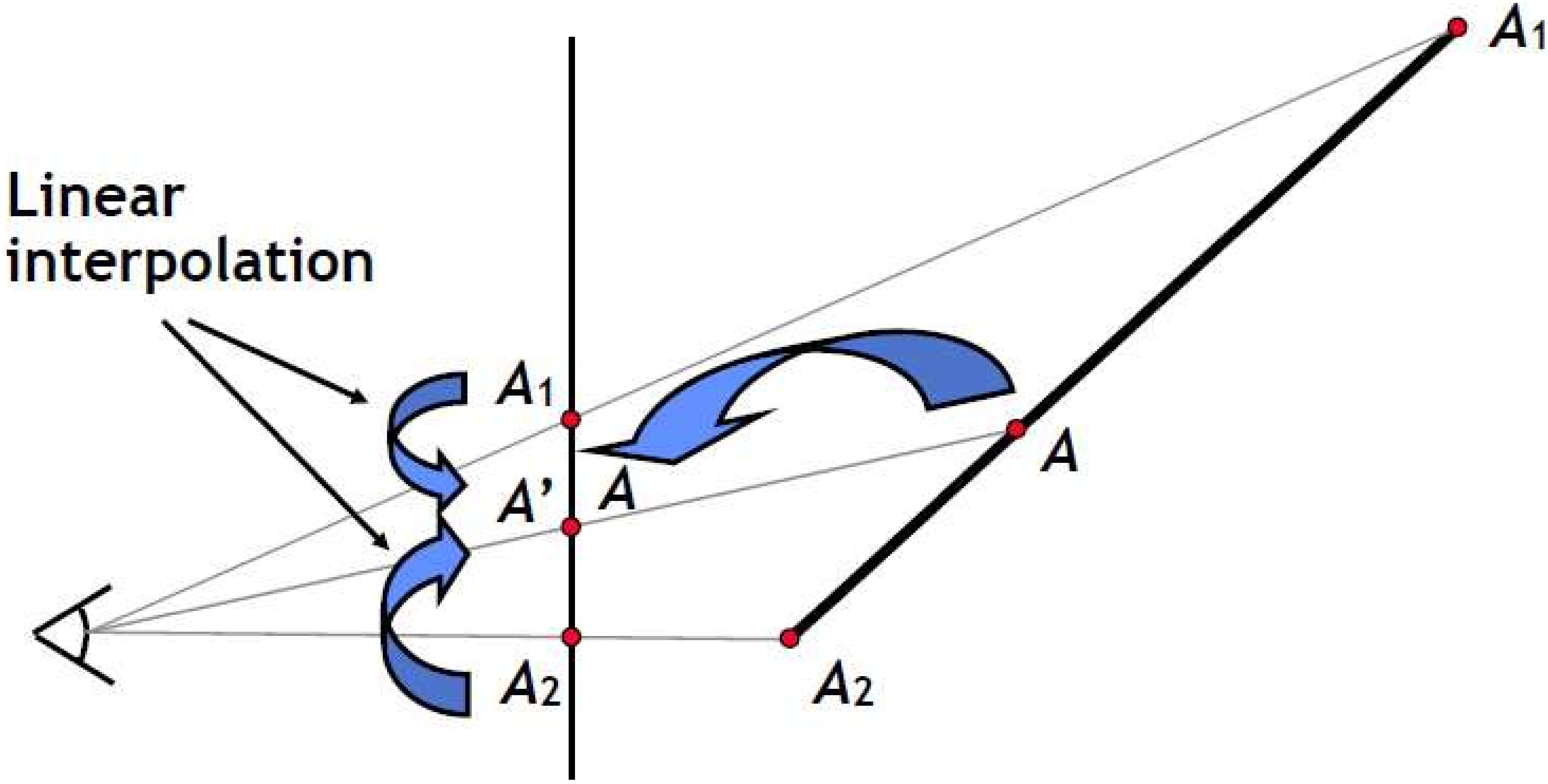
Incorrect attribute interpolation



$\boxed{A' \neq A\,!}$

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Linear interpolation

Compute intermediate attribute value

- Along a line: $A = aA_1 + bA_2$, $a+b=1$
- On a plane: $A = aA_1 + bA_2 + cA_3$, $a+b+c=1$

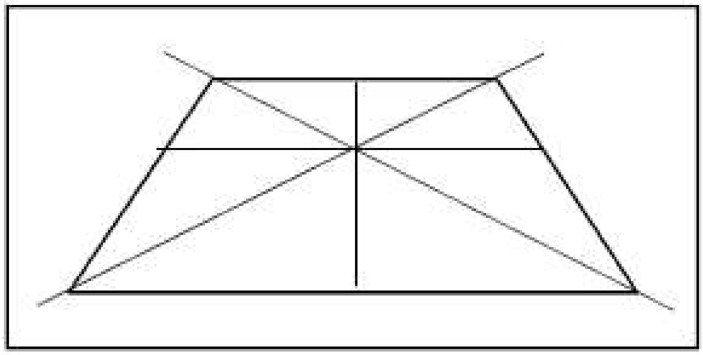
Only projected values interpolate linearly in screen space (straight lines project to straight lines)

- x and y are projected (divided by w)
- Attribute values are not naturally projected

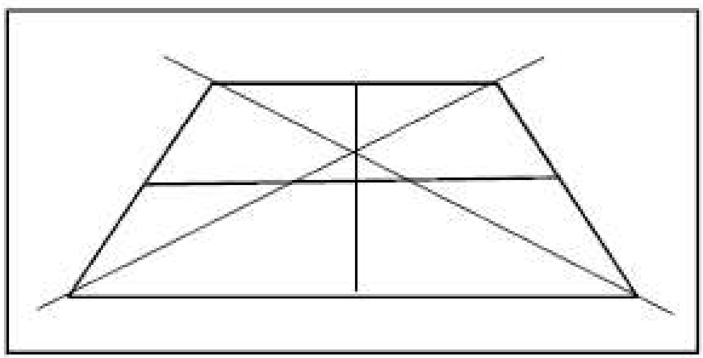
Choice for attribute interpolation in screen space

- Interpolate unprojected values
  - Cheap and easy to do, but gives wrong values
  - Sometimes OK for color, but
  - Never acceptable for texture coordinates
- Do it right

Linear Perspective



Correct Linear Perspective



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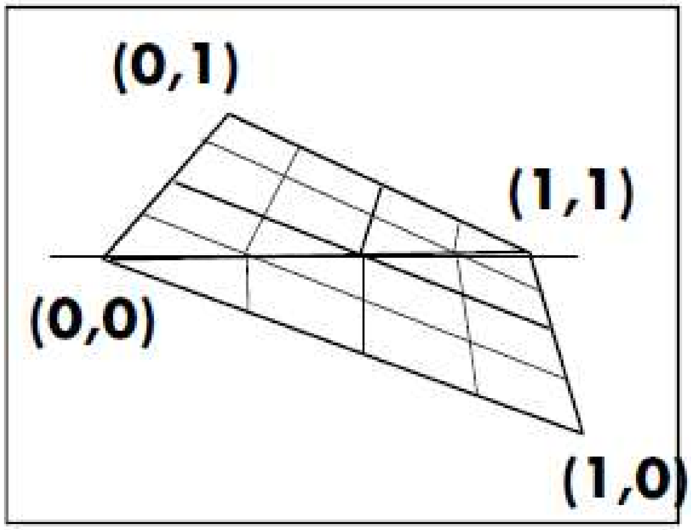

Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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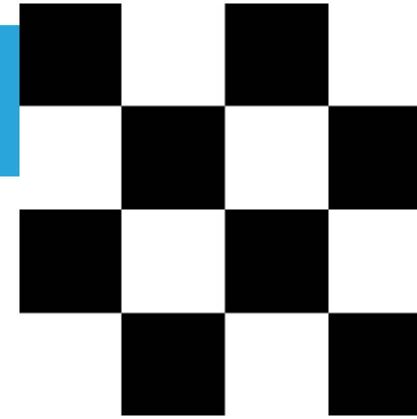
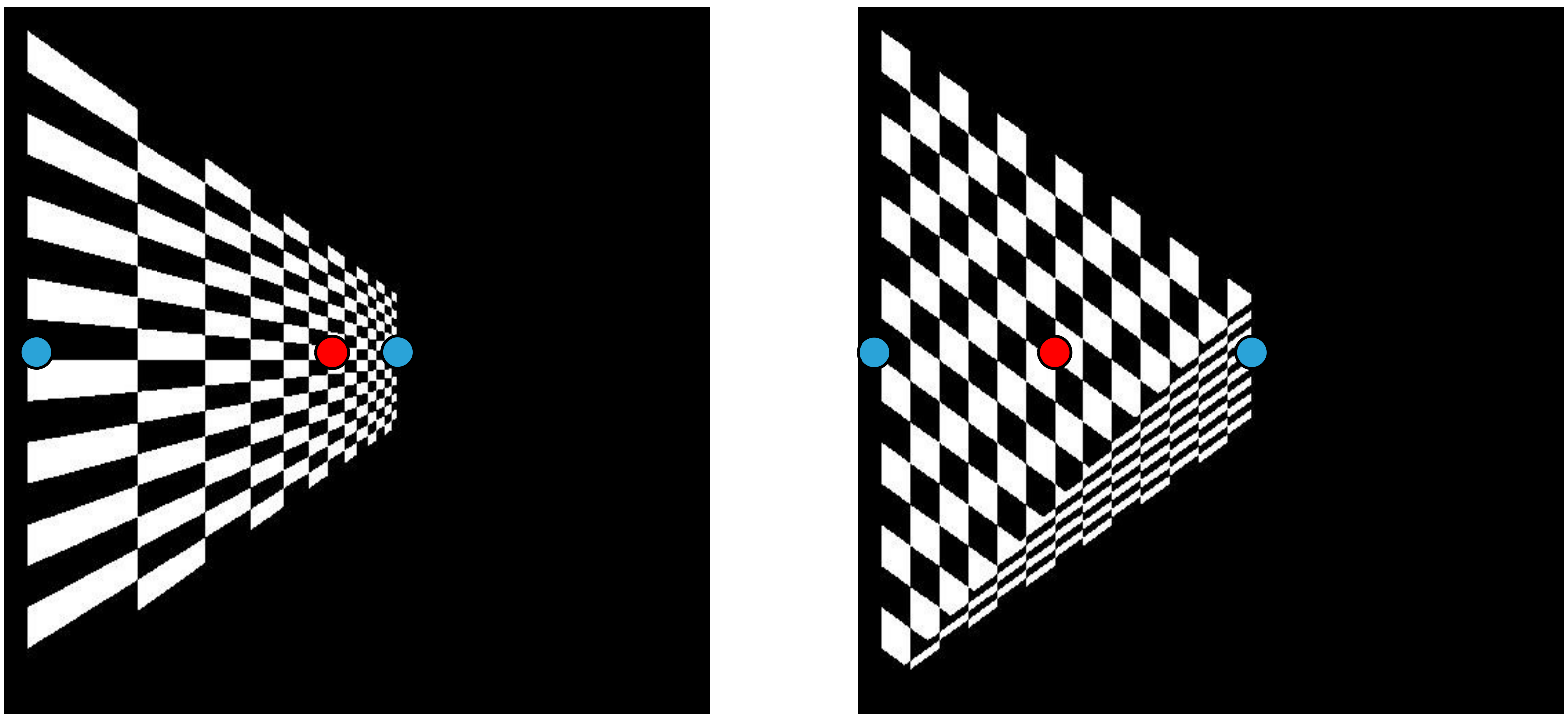
Perspective Texture Mapping



linear interpolation in object space

$$\frac{ax_1 + bx_2}{aw_1 + bw_2} \neq a\frac{x_1}{w_1} + b\frac{x_2}{w_2}$$

linear interpolation in screen space



$a = b = 0.5$

Early Perspective Texture Mapping in Games

Ultima Underworld (Looking Glass, 1992)

Early Perspective Texture Mapping in Games

DOOM (id Software, 1993)

Early Perspective Texture Mapping in Games

Quake (id Software, 1996)

Perspective-correct linear interpolation

Only projected values interpolate correctly, so project *A*

- Linearly interpolate A_1/w_1 and A_2/w_2

Also interpolate $1/w_1$ and $1/w_2$

- These also interpolate linearly in screen space

Divide interpolants at each sample point to recover *A*

- $(A/w) \,/\, (1/w) = A$
- Division is expensive (more than add or multiply), so
  - Recover *w* for the sample point (reciprocate), and
  - Multiply each projected attribute by *w*

Barycentric triangle parameterization:

$$A = \frac{aA_1/w_1 + bA_2/w_2 + cA_3/w_3}{a/w_1 + b/w_2 + c/w_3} \qquad a + b + c = 1$$

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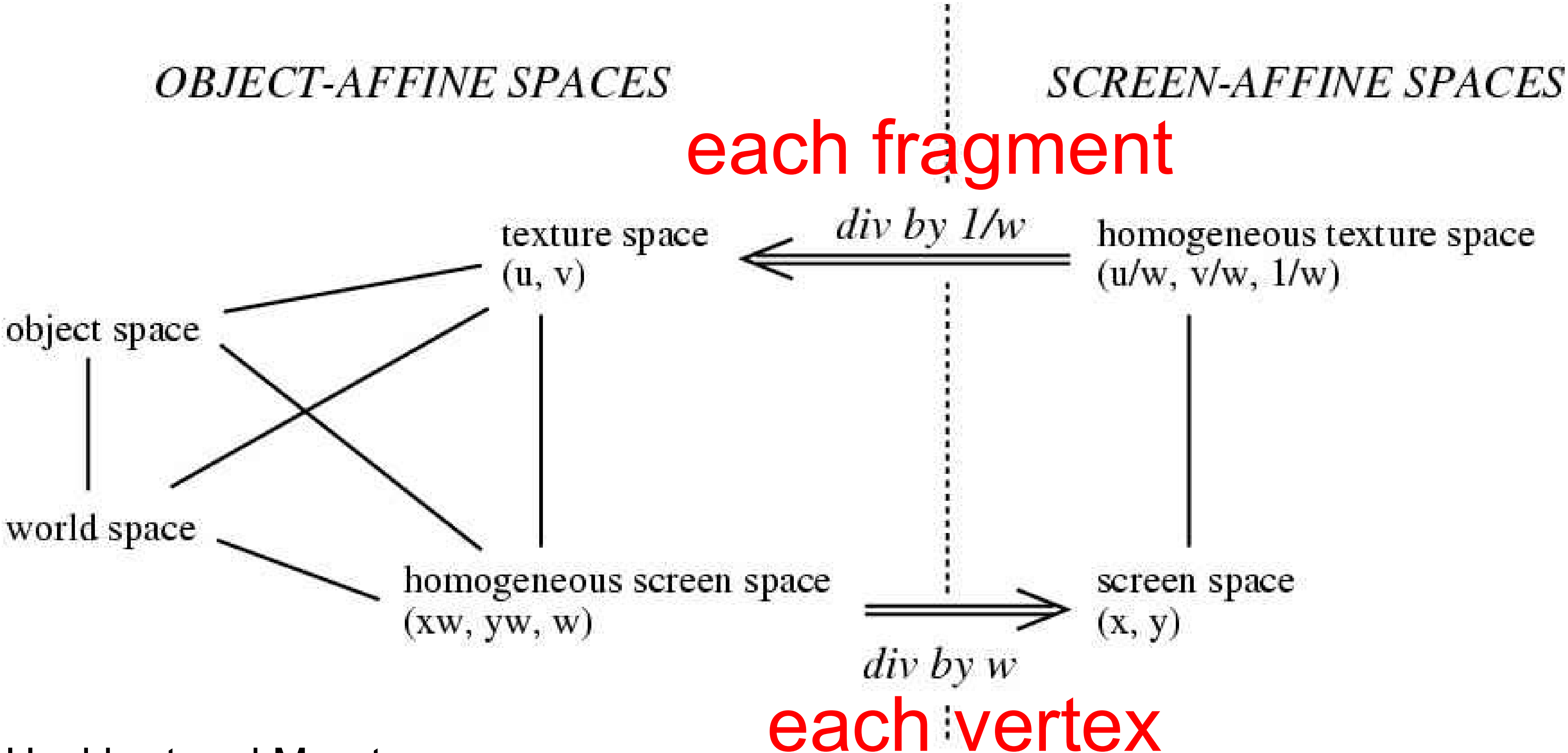
Perspective Texture Mapping

- Solution: interpolate (s/w, t/w, 1/w)
- (s/w) / (1/w) = s etc. at every fragment



Heckbert and Moreton

Perspective-Correct Interpolation Recipe

$$r_i(x,y) = \frac{r_i(x,y)/w(x,y)}{1/w(x,y)}$$

(1) Associate a record containing the n parameters of interest $(r_1, r_2, \cdots, r_n)$ with each vertex of the polygon.

(2) For each vertex, transform object space coordinates to homogeneous screen space using 4×4 object to screen matrix, yielding the values (xw, yw, zw, w).

(3) Clip the polygon against plane equations for each of the six sides of the viewing frustum, linearly interpolating all the parameters when new vertices are created.

(4) At each vertex, divide the homogeneous screen coordinates, the parameters r_i, and the number 1 by w to construct the variable list $(x, y, z, s_1, s_2, \cdots, s_{n+1})$, where $s_i = r_i/w$ for $i \leq n$, $s_{n+1} = 1/w$.

(5) Scan convert in screen space by linear interpolation of all parameters, at each pixel computing $r_i = s_i/s_{n+1}$ for each of the n parameters; use these values for shading.

Heckbert and Moreton

Thank you.