King Abdullah University of
Science and Technology
KAUST

CS 247 – Scientific Visualization
Lecture 6: Scalar Fields, Pt. 2 [preview]

Markus Hadwiger, KAUST

Reading Assignment #3 (until Feb 14)

Read (required):

- Data Visualization book, finish Chapter 3 (read starting with 3.6)
- Data Visualization book, Chapter 5 until 5.3 (inclusive)

Scalar Fields

Contours

Set of points where the scalar field *f(x)* has a given value *c*:

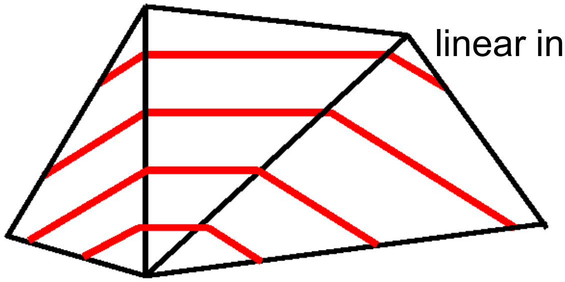
$$S(c) := f^{-1}(c) \qquad S(c) := \{x \in \mathbb{R}^n : f(x) = c\}$$

Common contouring algorithms

- 2D: marching squares, marching triangles
- 3D: marching cubes, marching tetrahedra

Implicit methods

- Point-on-contour test
- Isosurface ray-casting



Contours

Set of points where the scalar field *f(x)* has a given value *c*:

$$S(c) := f^{-1}(c) \qquad S(c) := \{x \in \mathbb{R}^2 : f(x) = c\}$$

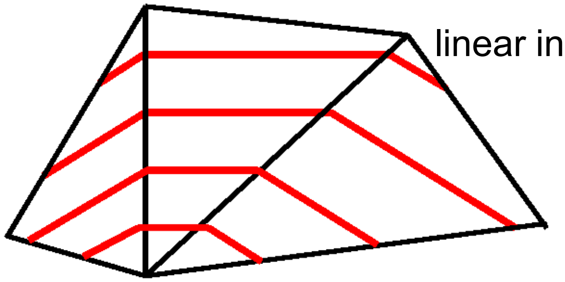
bilinear interpolation

Common contouring algorithms

- 2D: marching squares, marching triangles
- 3D: marching cubes, marching tetrahedra

Implicit methods

- Point-on-contour test
- Isosurface ray-casting



Contours

Set of points where the scalar field *f(x)* has a given value *c*:

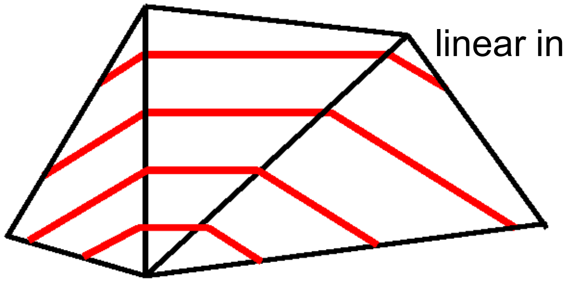
$$S(c) := f^{-1}(c) \qquad S(c) := \{x \in \mathbb{R}^3 : f(x) = c\}$$

Common contouring algorithms

- 2D: marching squares, marching triangles
- 3D: marching cubes, marching tetrahedra

Implicit methods

- Point-on-contour test
- Isosurface ray-casting



What are contours?

Set of points where the scalar field *s* has a given value *c*:

$$S(c) := \{x \in \mathbb{R}^n : f(x) = c\}$$

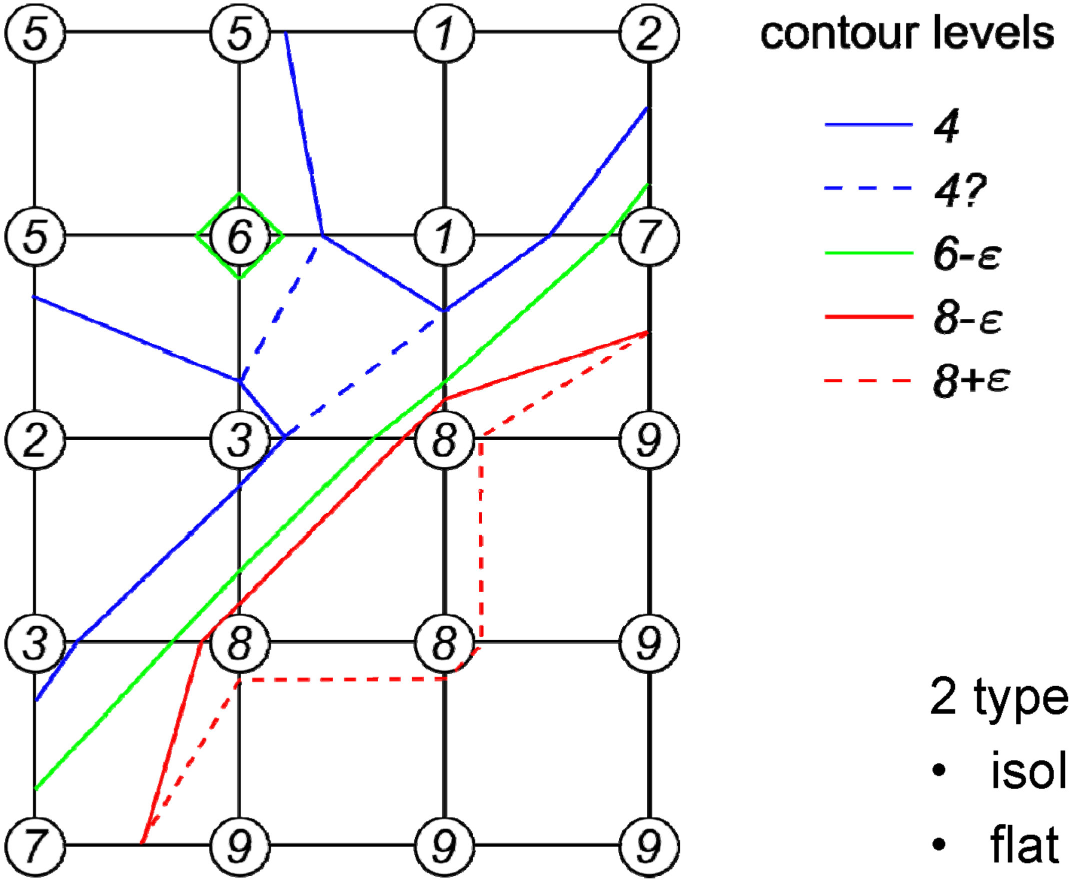
Examples in 2D:

- height contours on maps
- isobars on weather maps

Contouring algorithm:

- find intersection with grid edges
- connect points in each cell

Example



2 types of degeneracies:

- isolated points (c=6)
- flat regions (c=8)

Contours in a quadrangle cell

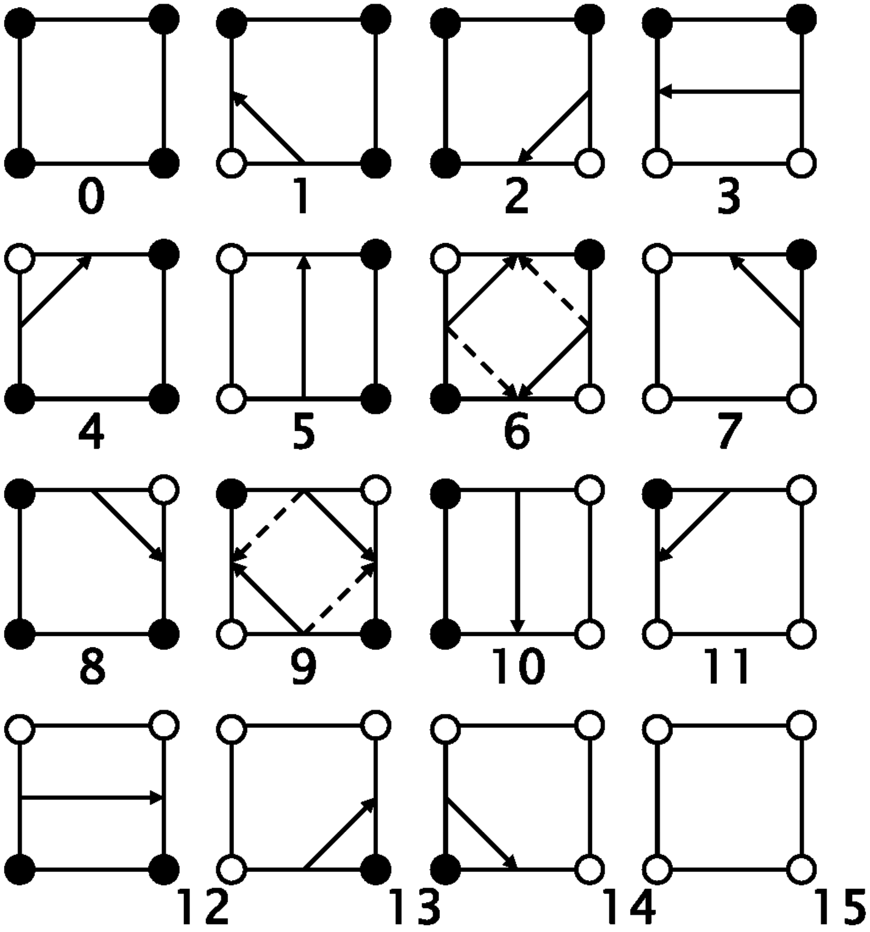
Basic contouring algorithms:

- cell-by-cell algorithms: simple structure, but generate disconnected segments, require post-processing
- contour propagation methods: more complicated, but generate connected contours

"Marching squares" algorithm (systematic cell-by-cell):

- process nodes in ccw order, denoted here as x_0, x_1, x_2, x_3
- compute at each node $\mathbf{x}_i$ the reduced field $\tilde{f}(x_i) = f(x_i) - (c - \varepsilon)$ (which is forced to be nonzero)
- take its sign as the i^{th} bit of a 4-bit integer
- use this as an index for lookup table containing the connectivity information:

Contours in a quadrangle cell



● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

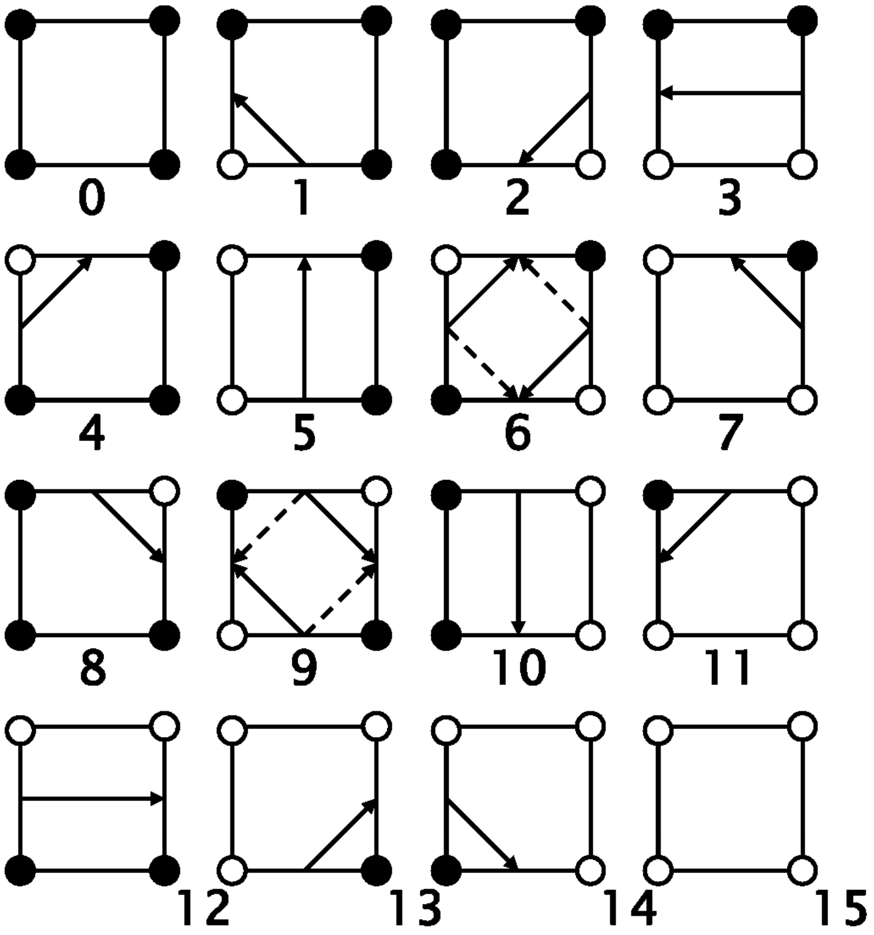
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



- ● $f(x_i) < c$
- ○ $f(x_i) \geq c$

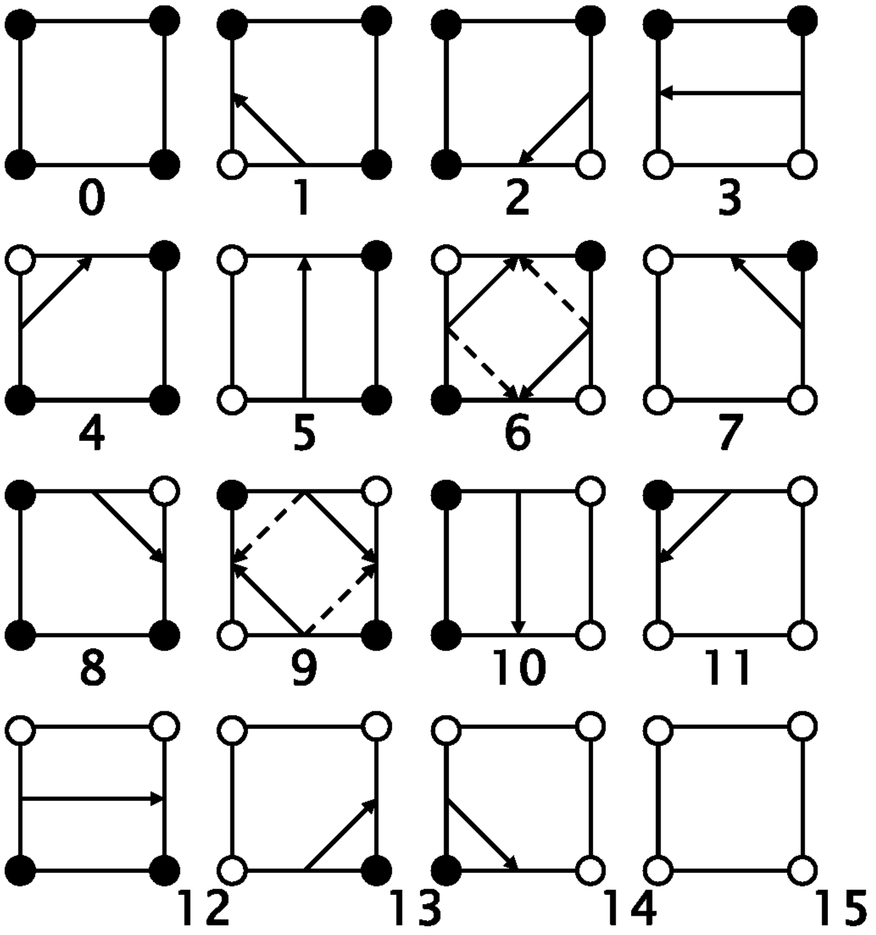
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



● $f(x_i) \leq c$

○ $f(x_i) > c$

Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

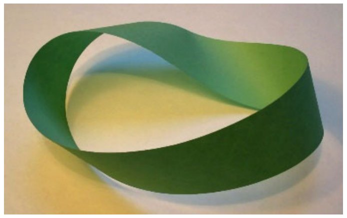
This allows to have a fixed table of 16 cases.

Orientability (1-manifold embedded in 2D)

Orientability of 1-manifold:

Possible to assign consistent left/right orientation

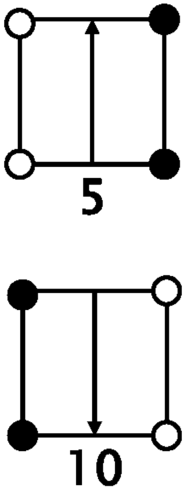
not orientable



Moebius strip
(only one side!)

Iso-contours

- Consistent side for scalar values…
  - greater than iso-value (e.g, *left* side)
  - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices
  (e.g., larger vertex index is “tip” of arrow;
  if (0,1) points “up”, “left” is left, …)



● $\tilde{f}(x_i) < 0$

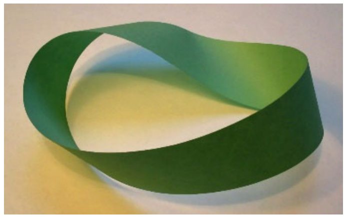
○ $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

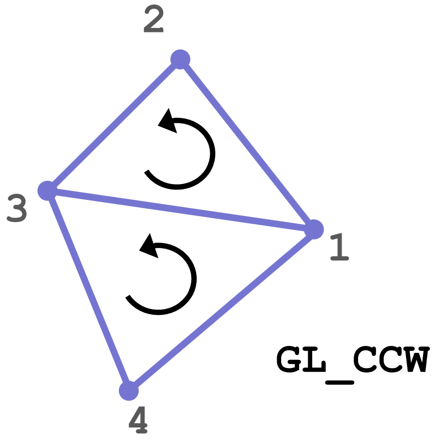
not orientable



Moebius strip
(only one side!)

Triangle meshes

- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”



Topological consistency

To avoid degeneracies, use symbolic perturbations:

If level *c* is found as a node value, set the level to $c-\varepsilon$ where ε is a symbolic infinitesimal.

Then:

- contours intersect edges at some (possibly infinitesimal) distance from end points
- flat regions can be visualized by pair of contours at $c-\varepsilon$ and $c+\varepsilon$
- contours are topologically consistent, meaning:

Contours are closed, orientable, nonintersecting lines.

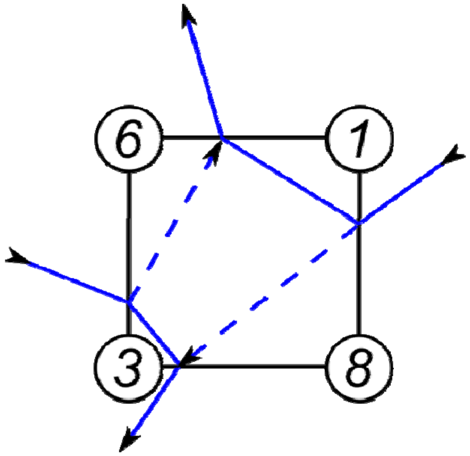
(except where the
boundary is hit)

Ambiguities of contours

What is the correct contour of $c=4$?

Two possibilities, both are orientable:

- connect high values ————
- connect low values - - - - - - - - - -



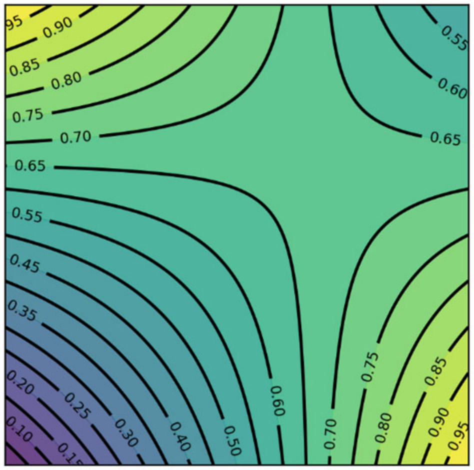
Answer: correctness depends on interior values of $f(x)$.

But: different interpolation schemes are possible.

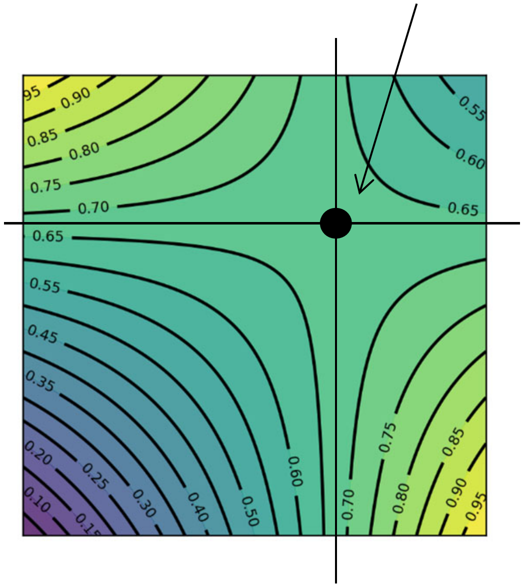
Better question: What is the correct contour with respect to bilinear interpolation?

Bi-Linear Interpolation: Critical Points

Critical points are where the gradient vanishes (i.e., is the zero vector)



critical point
(saddle point)



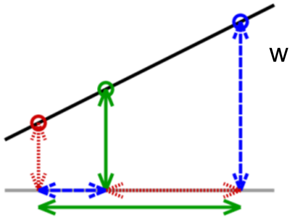
here, the critical
value is 2/3=0.666…

“Asymptotic decider”: resolve ambiguous configurations (6 and 9) by
comparing specific iso-value with critical value (scalar value at critical point)

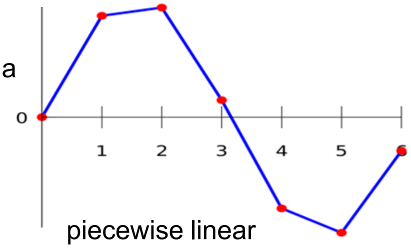
Linear Interpolation / Convex Combinations

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$



wikipedia



piecewise linear

Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad f(\alpha) = v_1 + \alpha(v_2 - v_1)$$

$$\alpha_1 + \alpha_2 = 1 \qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

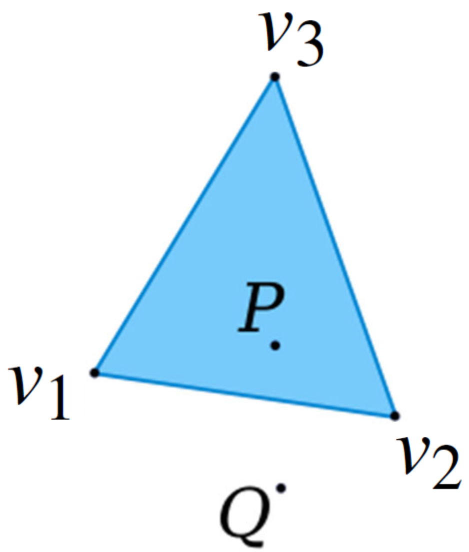
Compare to line parameterization with parameter t:

$$v(t) = v_1 + t(v_2 - v_1)$$

Linear Interpolation / Convex Combinations

Linear combination (n-dim. space):

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$



Affine combination: Restrict to $(n-1)$-dim. subspace:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

Convex combination: $\alpha_i \geq 0$

(restrict to simplex in subspace)

Linear Interpolation / Convex Combinations

The weights α_i are the n normalized ***barycentric*** coordinates

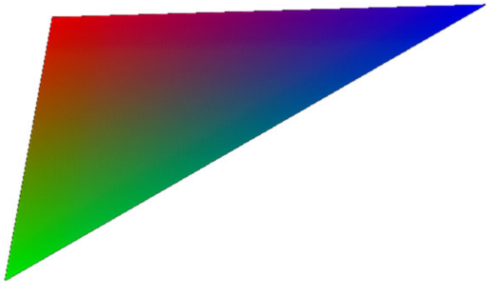
→ linear attribute interpolation in simplex

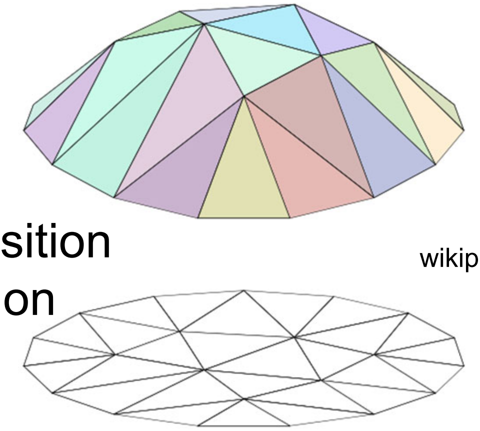
$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

$$\alpha_i \geq 0$$

attribute interpolation





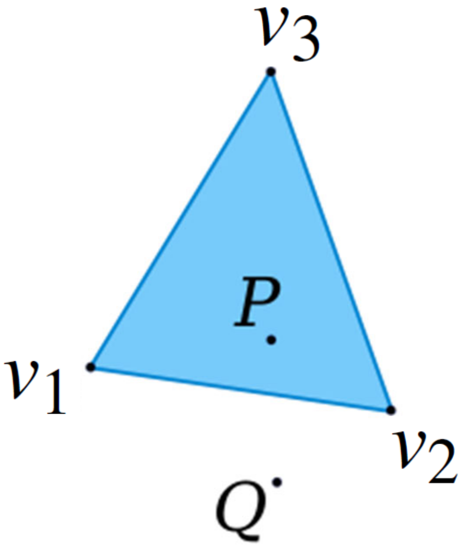
spatial position interpolation

wikipedia

Linear Interpolation / Convex Combinations

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

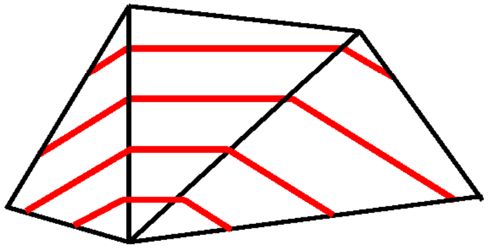
$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



Can re-parameterize to get $(n-1)$ ***affine*** coordinates:

$$\begin{aligned} \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 &= \\ \tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1 & \\ \tilde{\alpha}_1 &= \alpha_2 \\ \tilde{\alpha}_2 &= \alpha_3 \end{aligned}$$

Contours in triangle/tetrahedral cells



Linear interpolation of cells implies piece-wise linear contours.

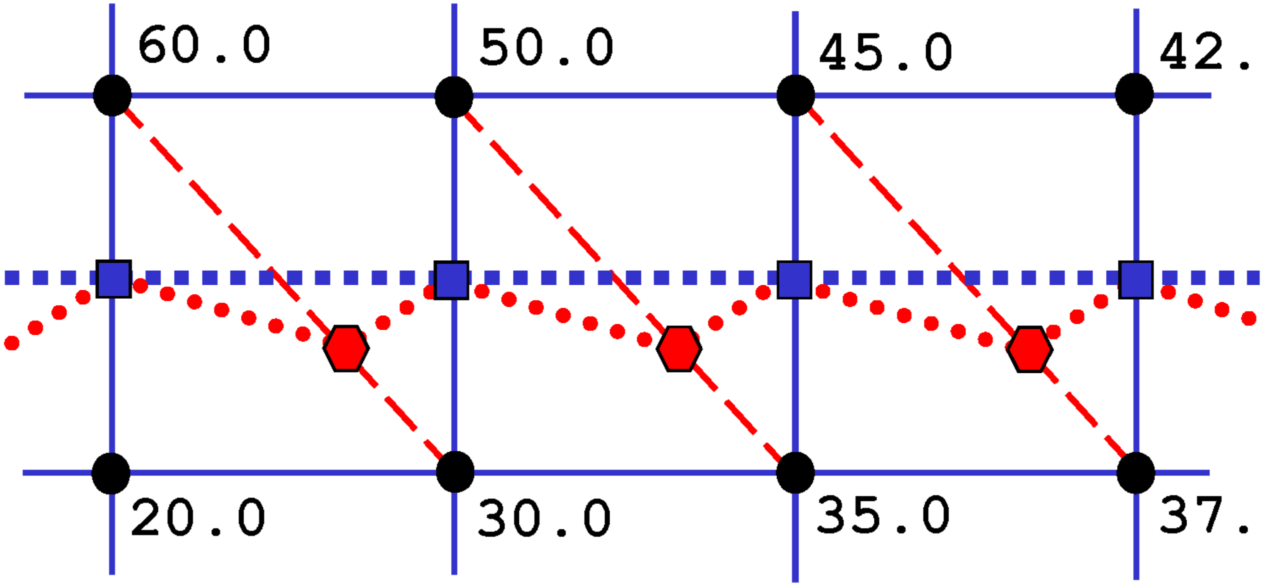
Contours are unambiguous, making "marching triangles" even simpler than "marching squares".

Question: Why not split quadrangles into two triangles (and hexahedra into five or six tetrahedra) and use marching triangles (tetrahedra)?

Answer: This can introduce periodic artifacts!

Contours in triangle/tetrahedral cells

Illustrative example: Find contour at level c=40.0 !



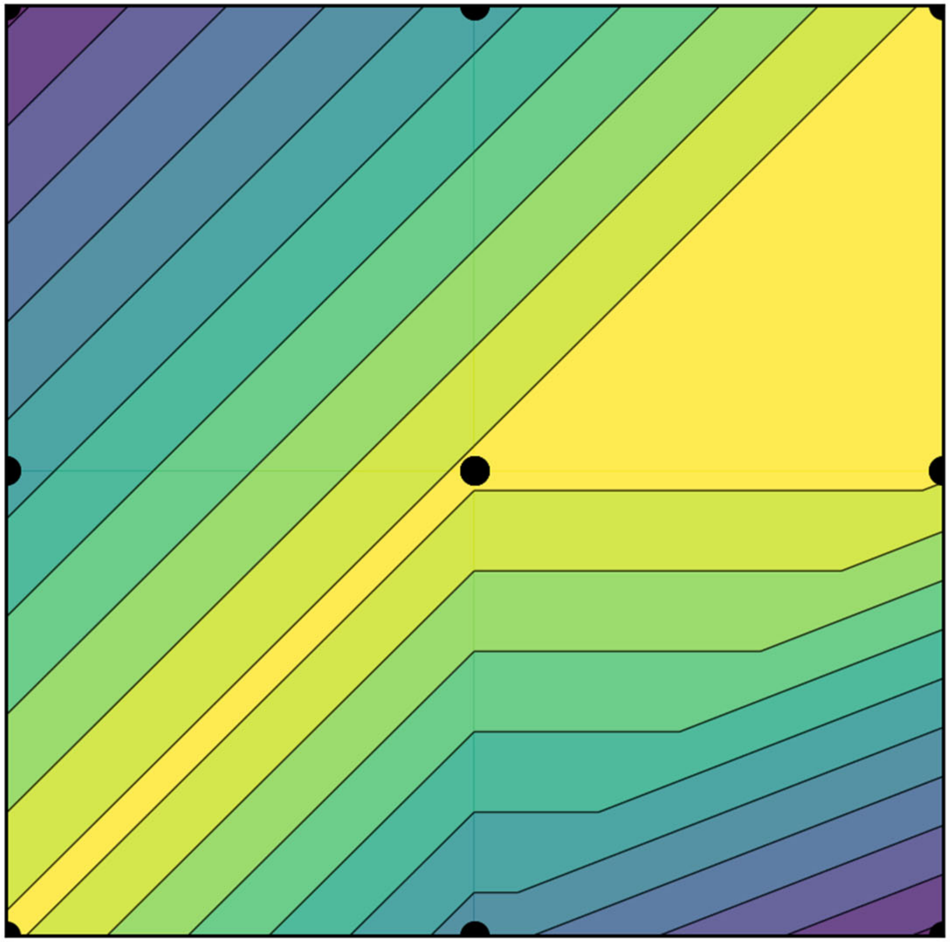
—— original quad grid, yielding vertices ■ and contour ■■■■■■

– – – triangulated grid, yielding vertices ⬢ and contour ••••••

Bi-Linear Interpolation: Comparisons

linear

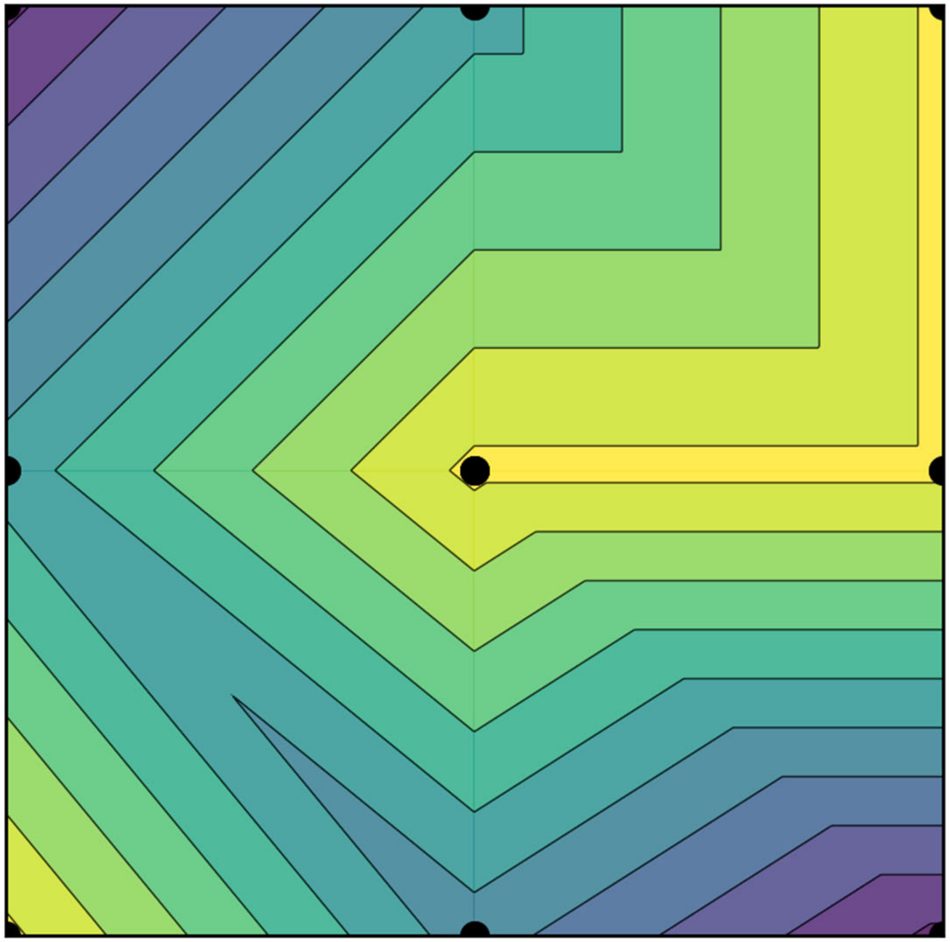
(2 triangles per quad;
diagonal:
bottom-left,
top-right)



Bi-Linear Interpolation: Comparisons

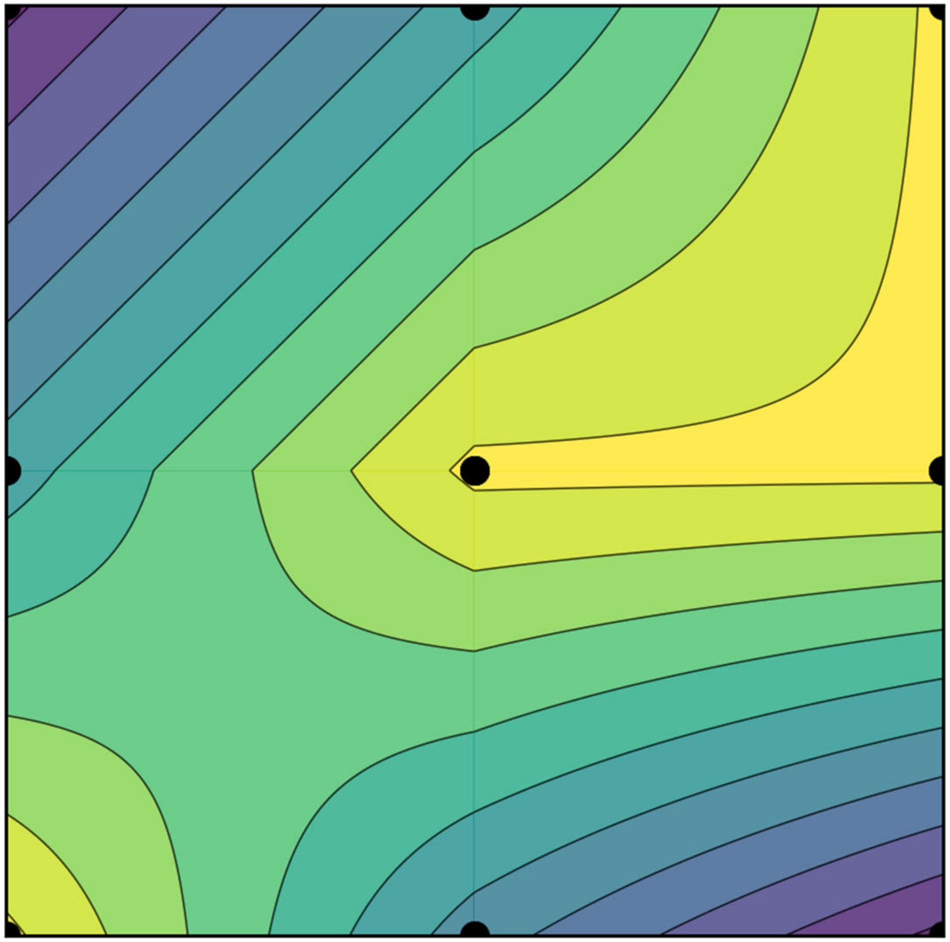
linear

(2 triangles per quad;
diagonal:
top-left,
bottom-right)



Bi-Linear Interpolation: Comparisons

bi-linear



From 2D to 3D (Domain)

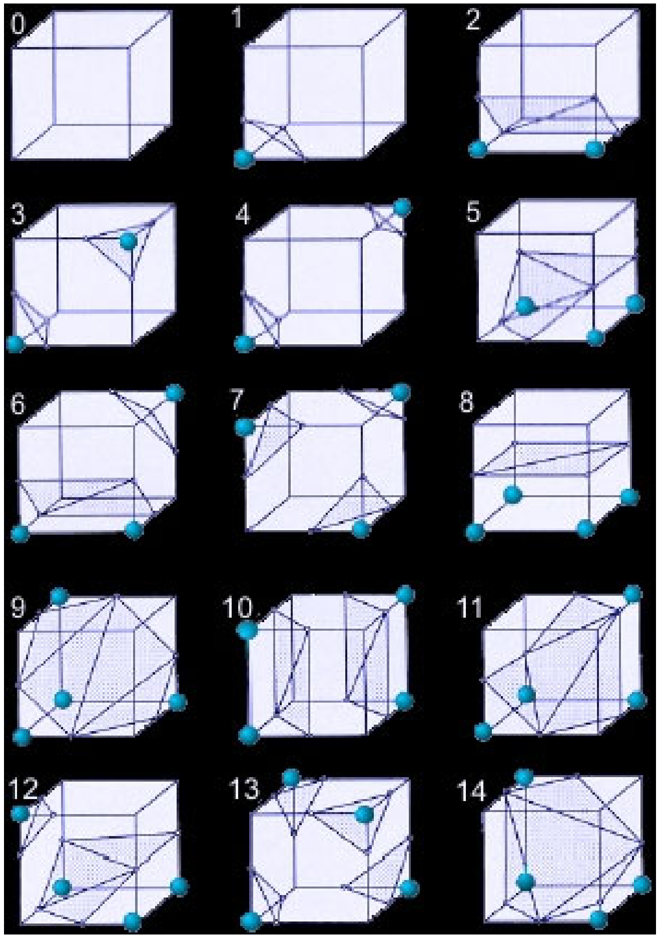
2D - Marching Squares Algorithm:

1. Locate the contour corresponding to a user-specified iso value
2. Create lines

3D - Marching Cubes Algorithm:

1. Locate the surface corresponding to a user-specified iso value
2. Create triangles
3. Calculate normals to the surface at each vertex
4. Draw shaded triangles

Marching Cubes



- For each cell, we have 8 vertices with 2 possible states each (inside or outside).
- This gives us 2^8 possible patterns = 256 cases.
- Enumerate cases to create a LUT
- Use symmetries to reduce problem from 256 to 15 cases.

Explanations

- Data Visualization book, 5.3.2
- Marching Cubes: A high resolution 3D surface construction algorithm, Lorensen & Cline, ACM SIGGRAPH 1987

The marching cubes algorithm

Contours of 3D scalar fields are known as isosurfaces.

Before 1987, isosurfaces were computed as

- contours on planar slices, followed by
- "contour stitching".

The marching cubes algorithm computes contours directly in 3D.

- Pieces of the isosurfaces are generated on a cell-by-cell basis.
- Similar to marching squares, a 8-bit number is computed from the 8 signs of $\tilde{f}(x_i)$ on the corners of a hexahedral cell.
- The isosurface piece is looked up in a table with 256 entries.

The marching cubes algorithm

How to build up the table of 256 cases?

Lorensen and Cline (1987) exploited 3 types of symmetries:

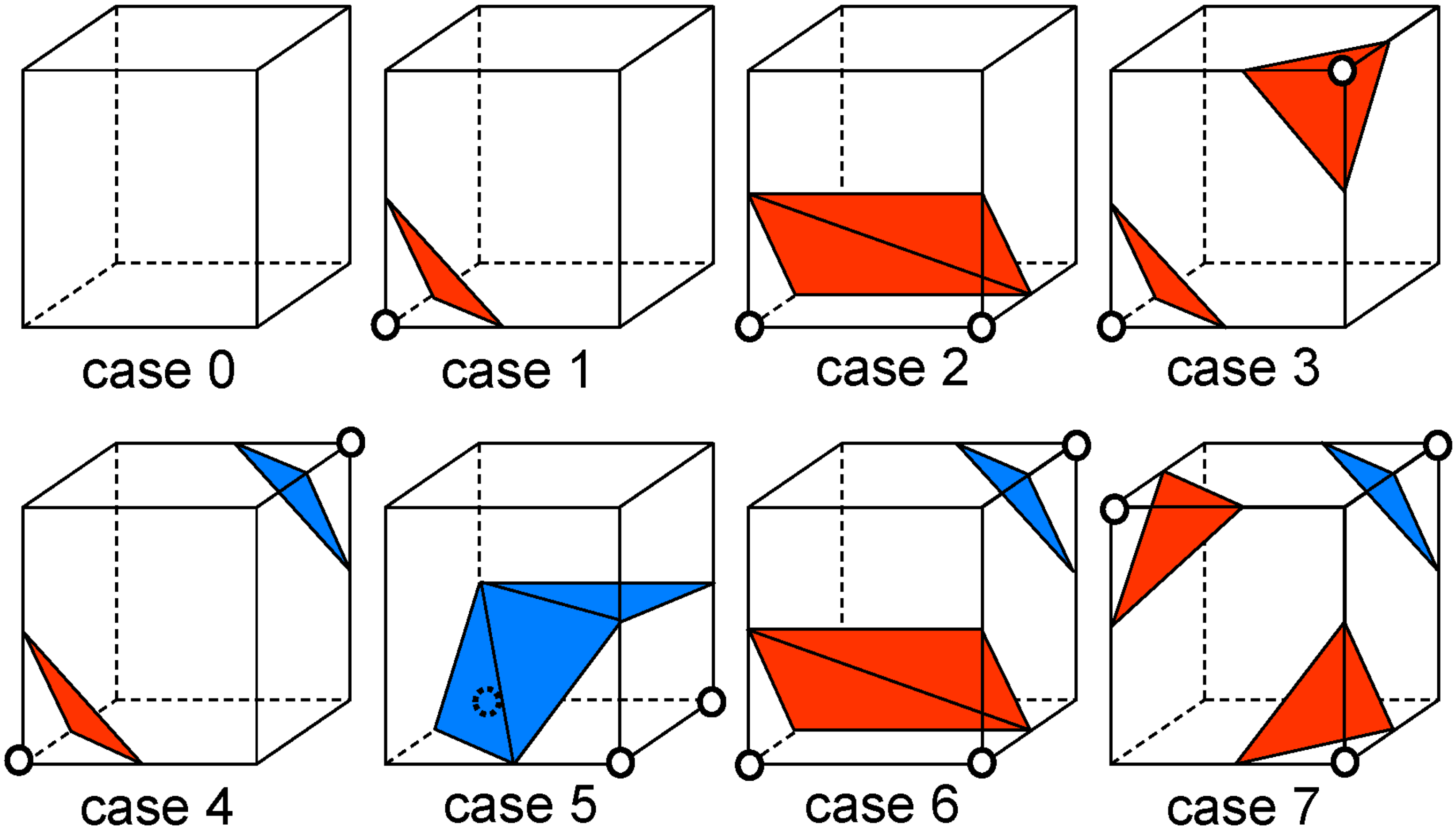
- rotational symmetries of the cube
- reflective symmetries of the cube
- sign changes of $\tilde{f}(x_i)$

They published a reduced set of 14*) cases shown on the next slides where

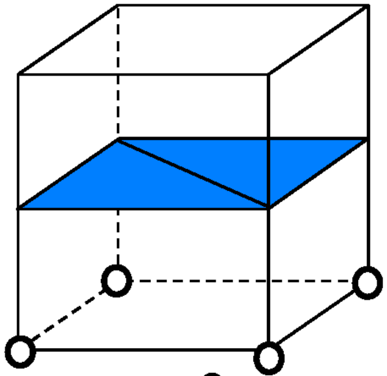
- white circles indicate positive signs of $\tilde{f}(x_i)$
- the positive side of the isosurface is drawn in red, the negative side in blue.

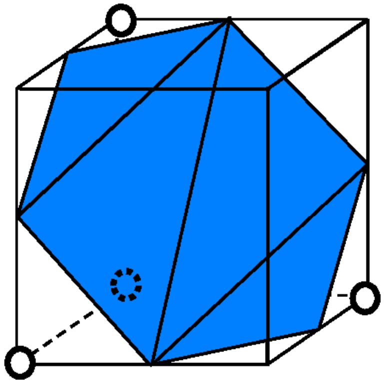
*) plus an unnecessary "case 14" which is a symmetric image of case 11.

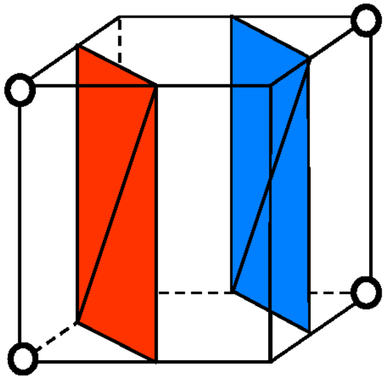
The marching cubes algorithm

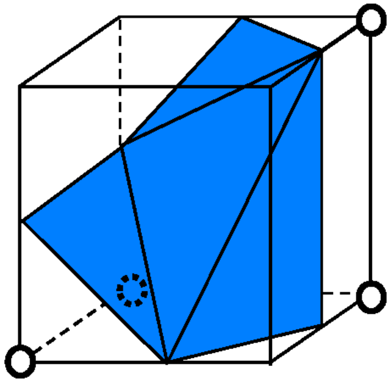


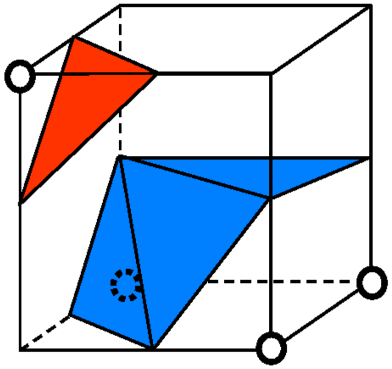
The marching cubes algorithm

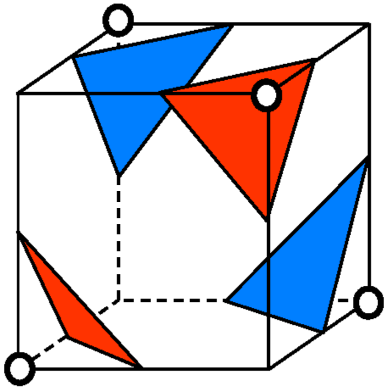

case 8


case 9


case 10


case 11


case 12


case 13

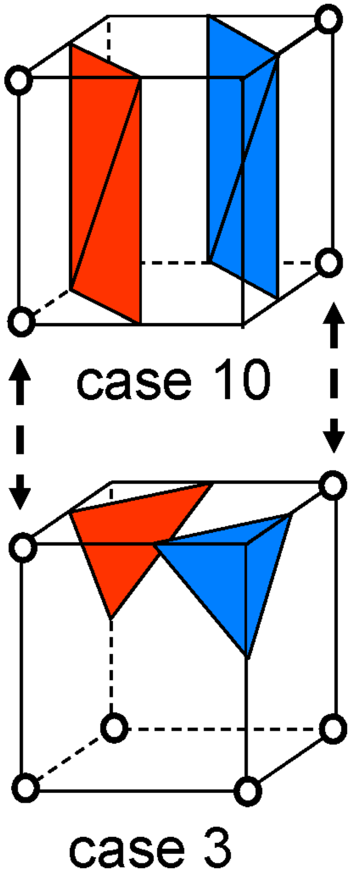
Do the pieces fit together?

- The correct isosurfaces of the trilinear interpolant would fit (trilinear reduces to bilinear on the cell interfaces)
- but the marching cubes polygons don't necessarily fit.

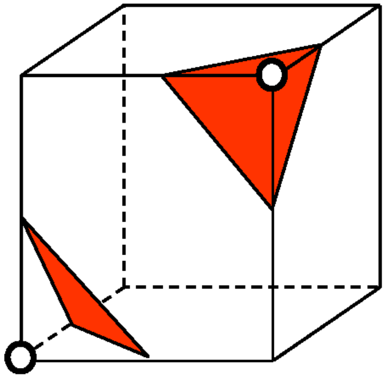
Example

- case 10, on top of
- case 3 (rotated, signs changed)

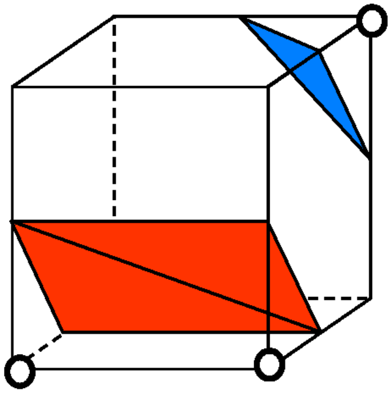
have matching signs at nodes but polygons don't fit.



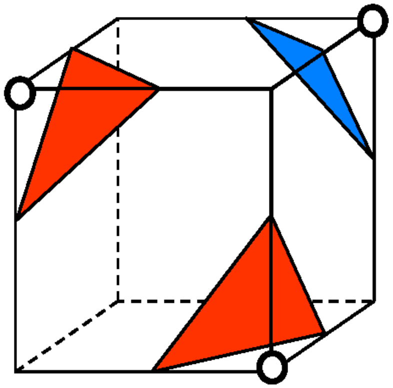
The marching cubes algorithm



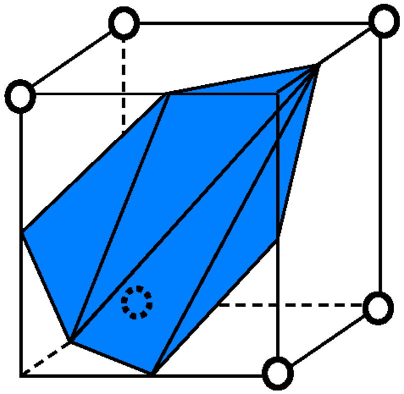
case 3



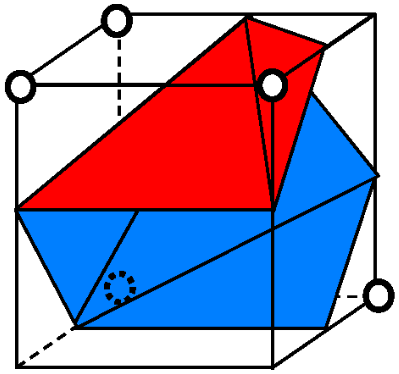
case 6



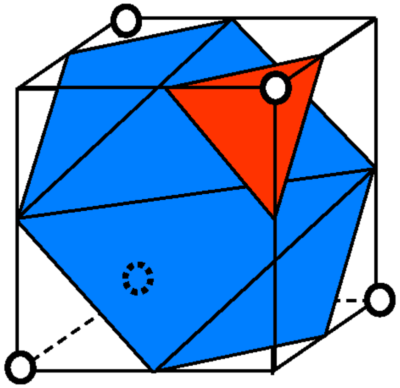
case 7



case 3c



case 6c



case 7c

The marching cubes algorithm

Summary of marching cubes algorithm:

Pre-processing steps:

- build a table of the 28 cases
- derive a table of the 256 cases, containing info on
  - intersected cell edges, e.g. for case 3/256 (see case 2/28): (0,2), (0,4), (1,3), (1,5)
  - triangles based on these points, e.g. for case 3/256: (0,2,1), (1,3,2).

The marching cubes algorithm

Loop over cells:

- find sign of $\tilde{f}(x_i)$ for the 8 corner nodes, giving 8-bit integer
- use as index into (256 case) table
- find intersection points on edges listed in table, using linear interpolation
- generate triangles according to table

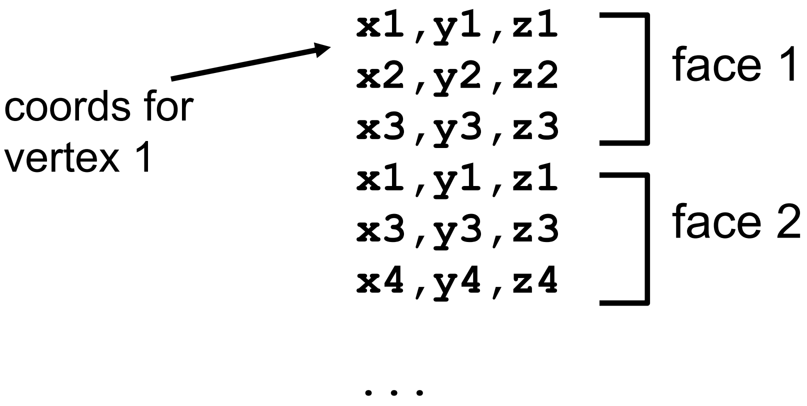
Post-processing steps:

- connect triangles (share vertices)
- compute normal vectors
  - by averaging triangle normals (problem: thin triangles!)
  - by estimating the gradient of the field $f(x_i)$ (better)

Triangle Mesh Data Structure (1)

Store list of vertices; vertices shared by triangles are replicated

Render, e.g., with OpenGL immediate mode, …



coords for vertex 1

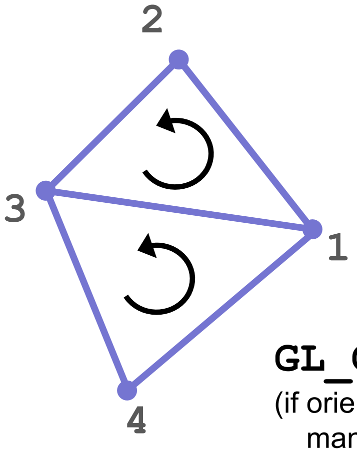
```
x1,y1,z1
x2,y2,z2
x3,y3,z3
x1,y1,z1
x3,y3,z3
x4,y4,z4

...
```

face 1

face 2

```
struct face
  float verts[3][3]
  DataType val;
```



GL_CCW
(if orientable manifold)

Redundant, large storage size, cannot modify shared vertices easily

Store data values per face, or separately

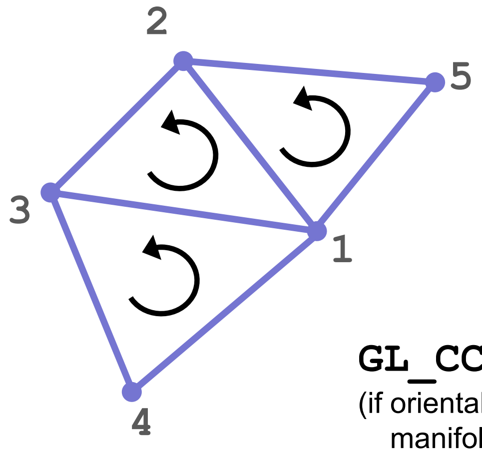
Triangle Mesh Data Structure (2)

Indexed face set: store list of vertices; store triangles as indexes

Render using separate vertex and index arrays / buffers

coords for vertex 1 →

vertex list	face list
`x1,y1,(z1)`	`1,2,3`
`x2,y2,(z2)`	`1,3,4`
`x3,y3,(z3)`	`2,1,5`
`x4,y4,(z4)`	
	`...`
`...`	



`GL_CCW`

(if orientable manifold)

Less redundancy, more efficient in terms of memory

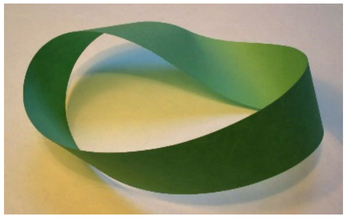
Easy to change vertex positions; still have to do (global) search for shared edges (local information)

Orientability (2-manifold embedded in 3D)

Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

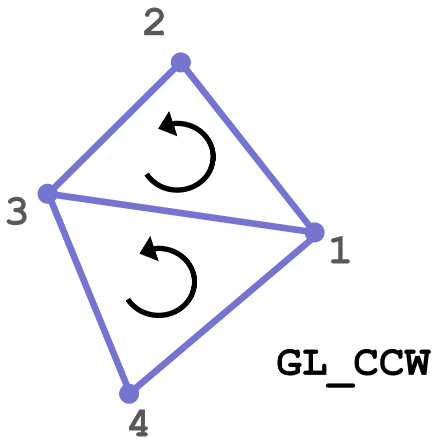
not orientable



Moebius strip
(only one side!)

Triangle meshes

- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”

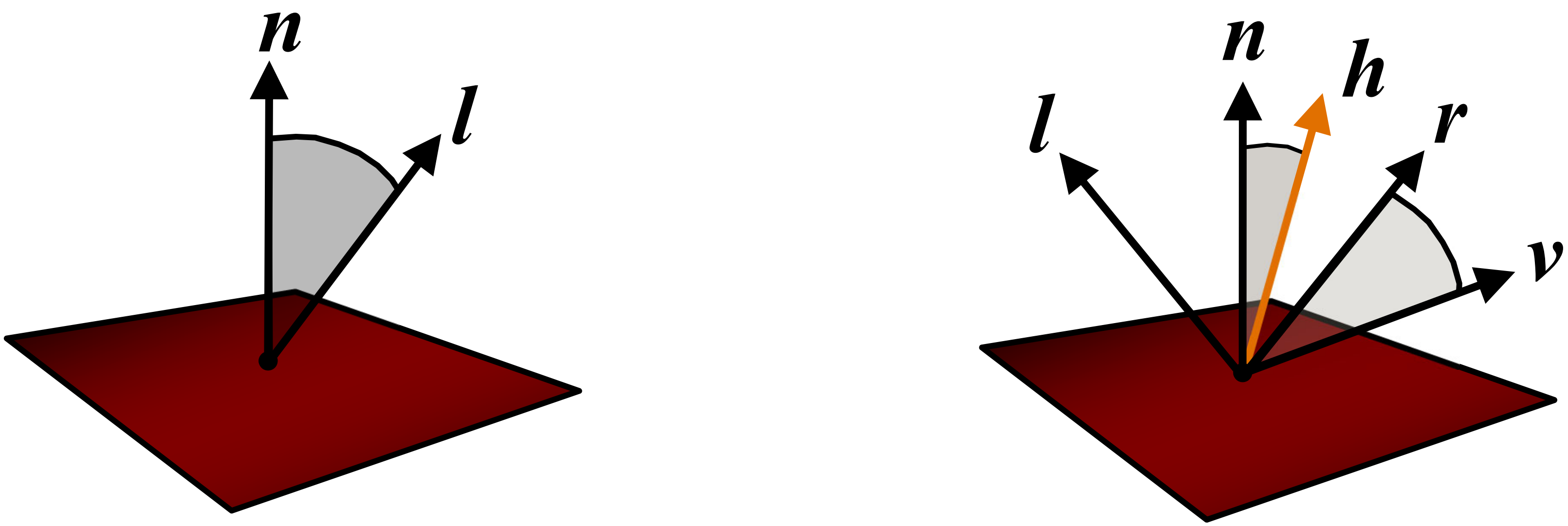


Local Shading Equations

Standard volume shading adapts surface shading

Most commonly Blinn/Phong model

But what about the "surface" normal vector?



diffuse reflection

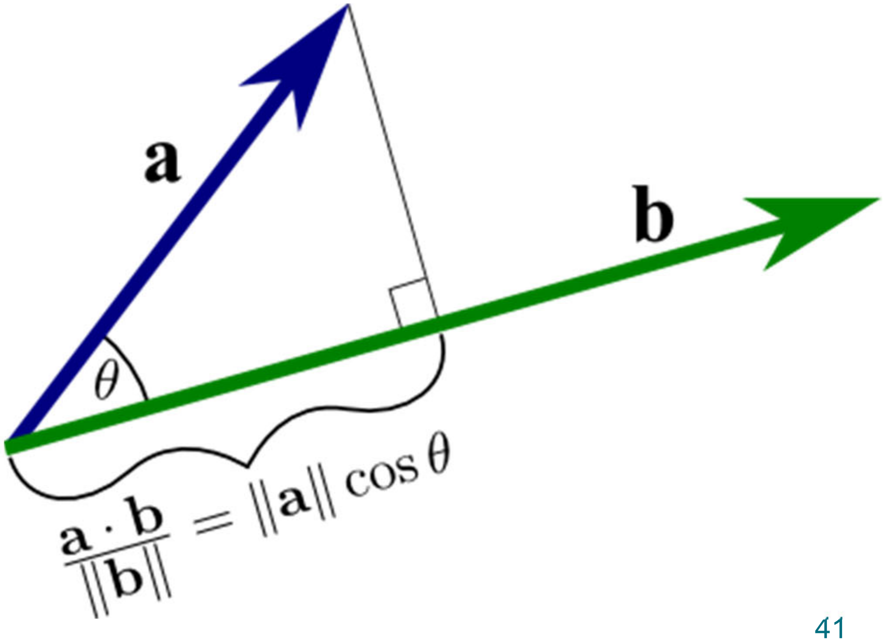
specular reflection

The Dot Product (Scalar / Inner Product)

Cosine of angle between two vectors times their lengths

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i \qquad \mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

(standard inner product in Cartesian coordinates)



Many uses:

- Project vector onto another vector,
  project into basis,
  project into tangent plane,
  ...

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama