King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 23: Vector / Flow Visualization, Pt. 2 [preview]

Markus Hadwiger, KAUST

Reading Assignment #12 (until Apr 18)

Read (required):

- Data Visualization book
  - Chapter 6 (Vector Visualization)
    - Beginning (before 6.1)
    - Chapters 6.2, 6.3, 6.5
- More general vector field basics (the book is not very precise on the basics)

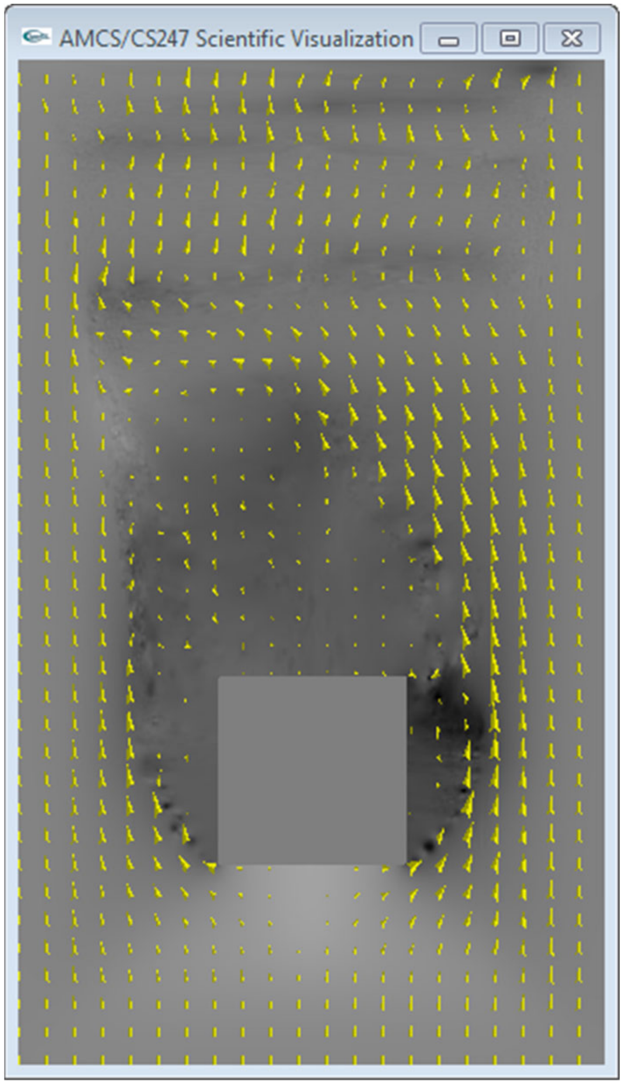
  `https://en.wikipedia.org/wiki/Vector_field`

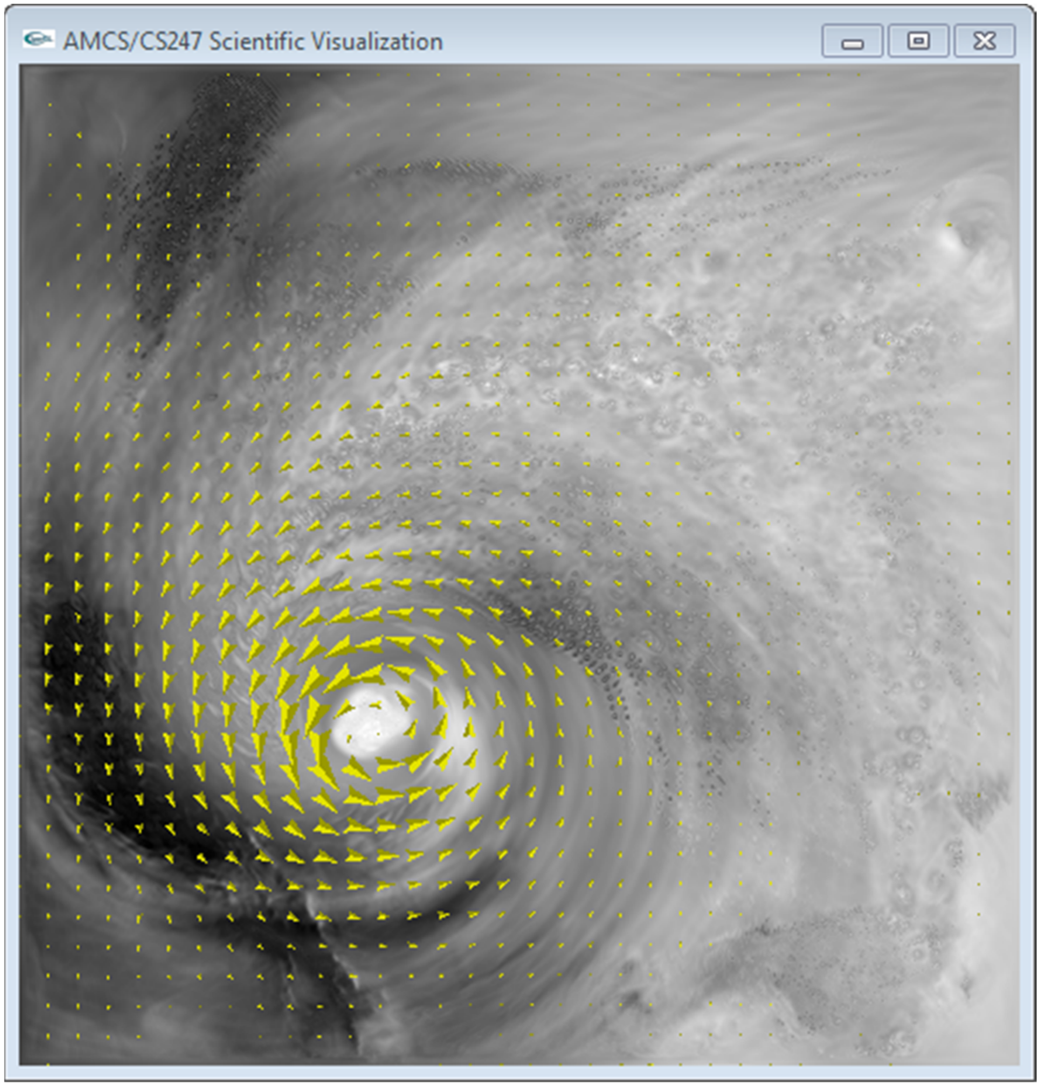
Read (optional):

- Paper:
  Bruno Jobard and Wilfrid Lefer
  Creating Evenly-Spaced Streamlines of Arbitrary Density,

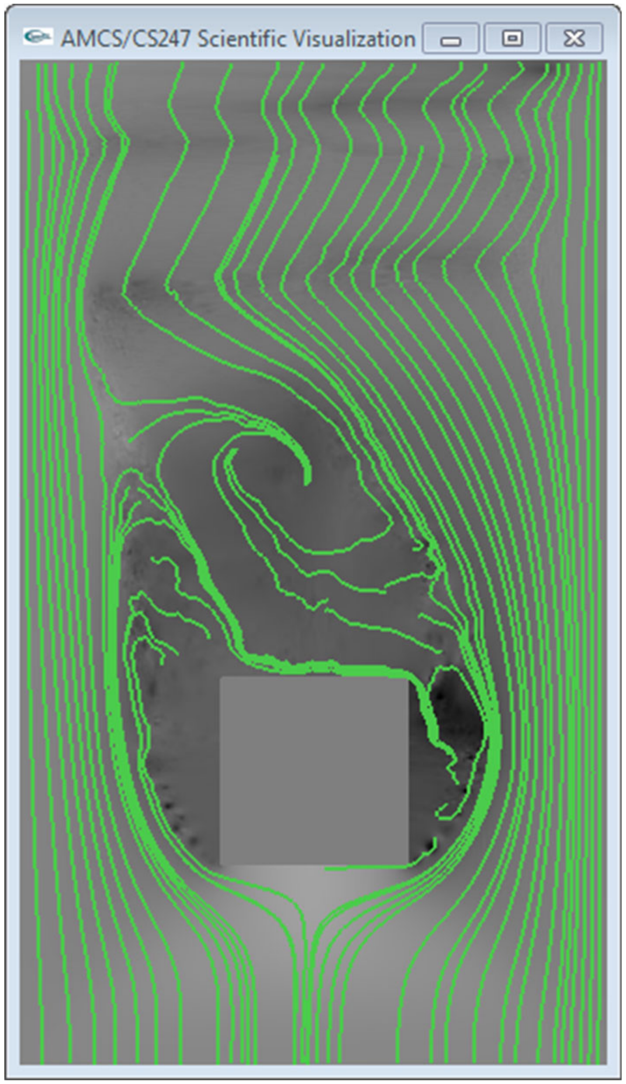
  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.9498`

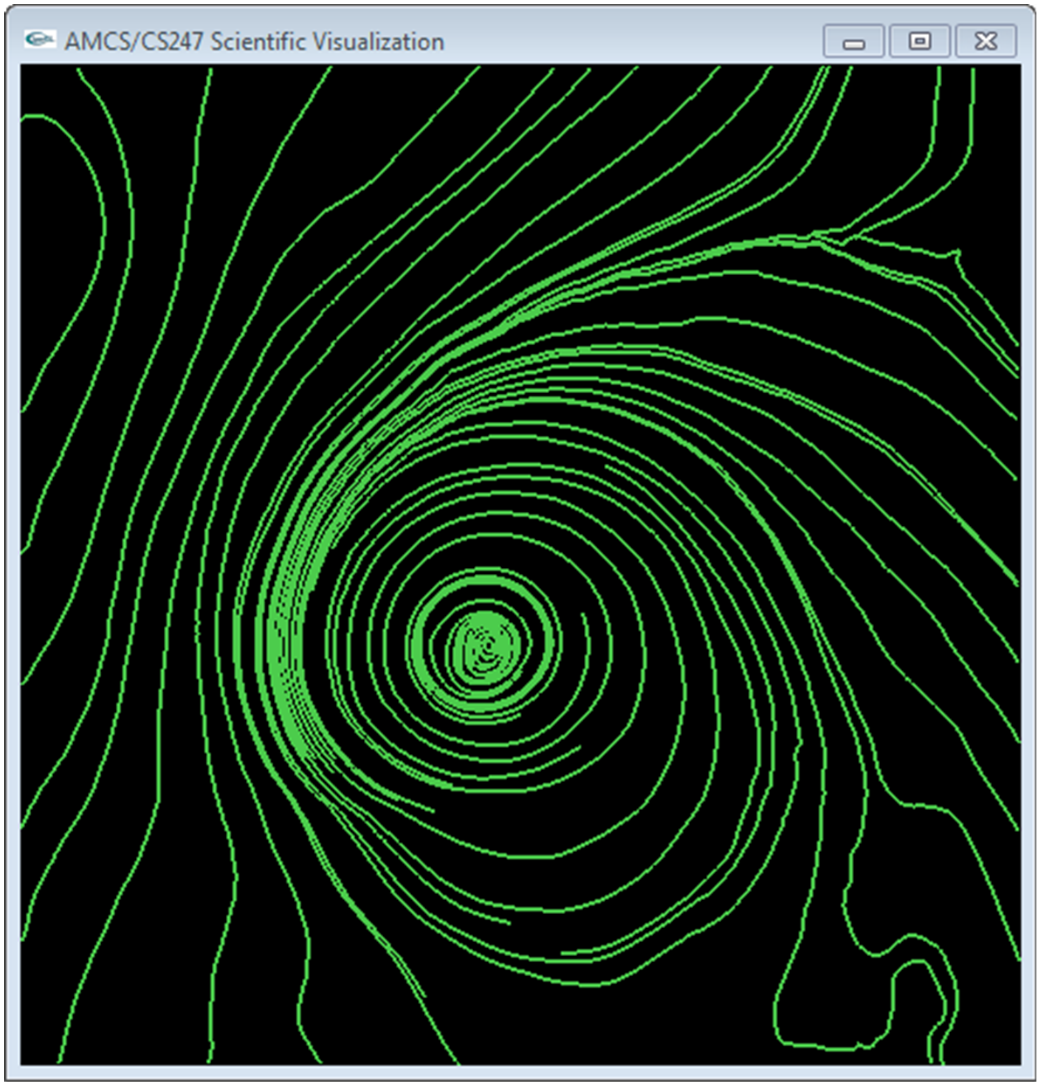
Programming Assignment #5: Flow Vis 1



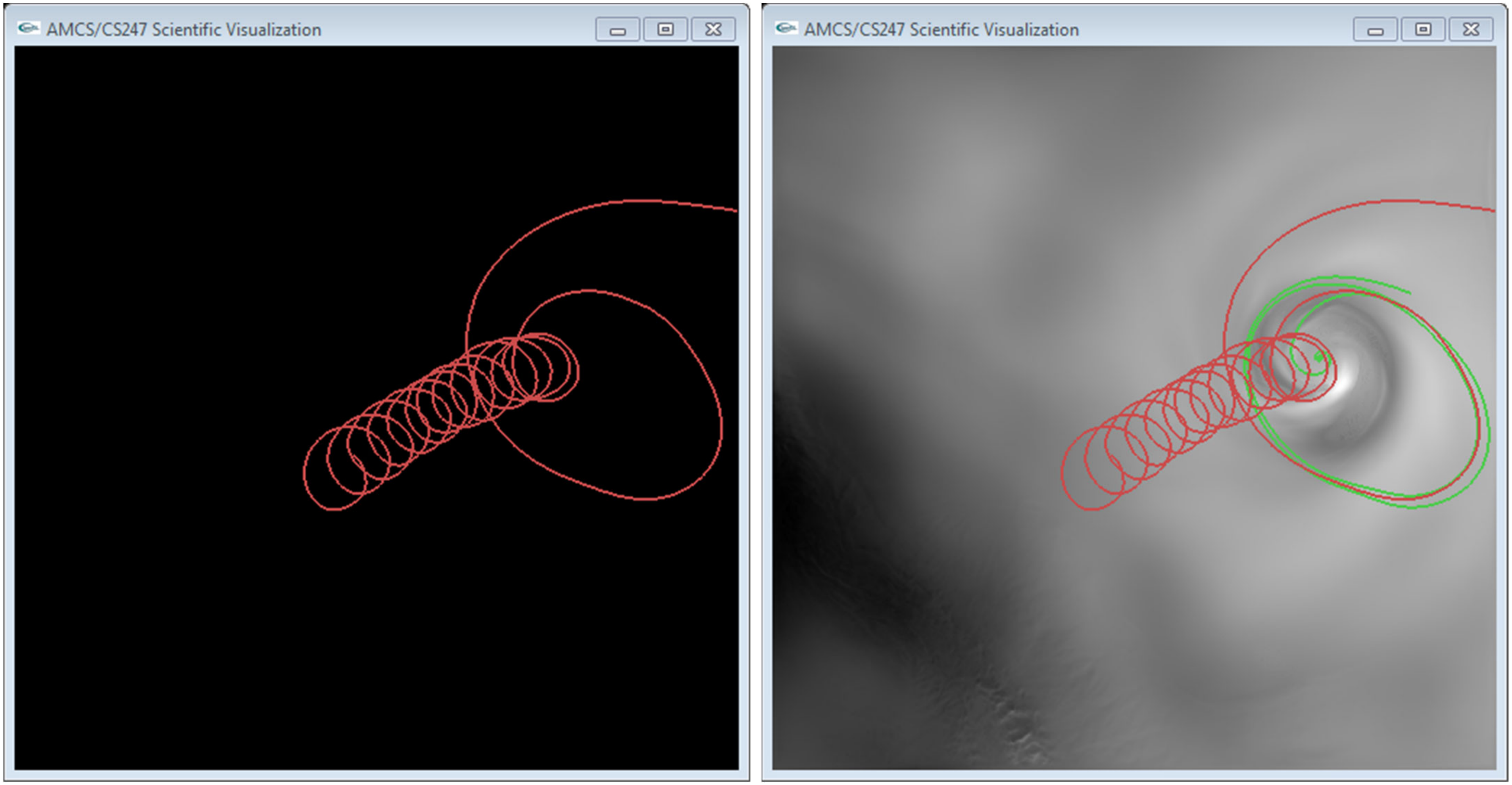


Programming Assignment #5: Flow Vis 1





Programming Assignment #5: Flow Vis 1



Online Demos and Info

Numerical ODE integration methods (Euler vs. Runge Kutta, etc.)

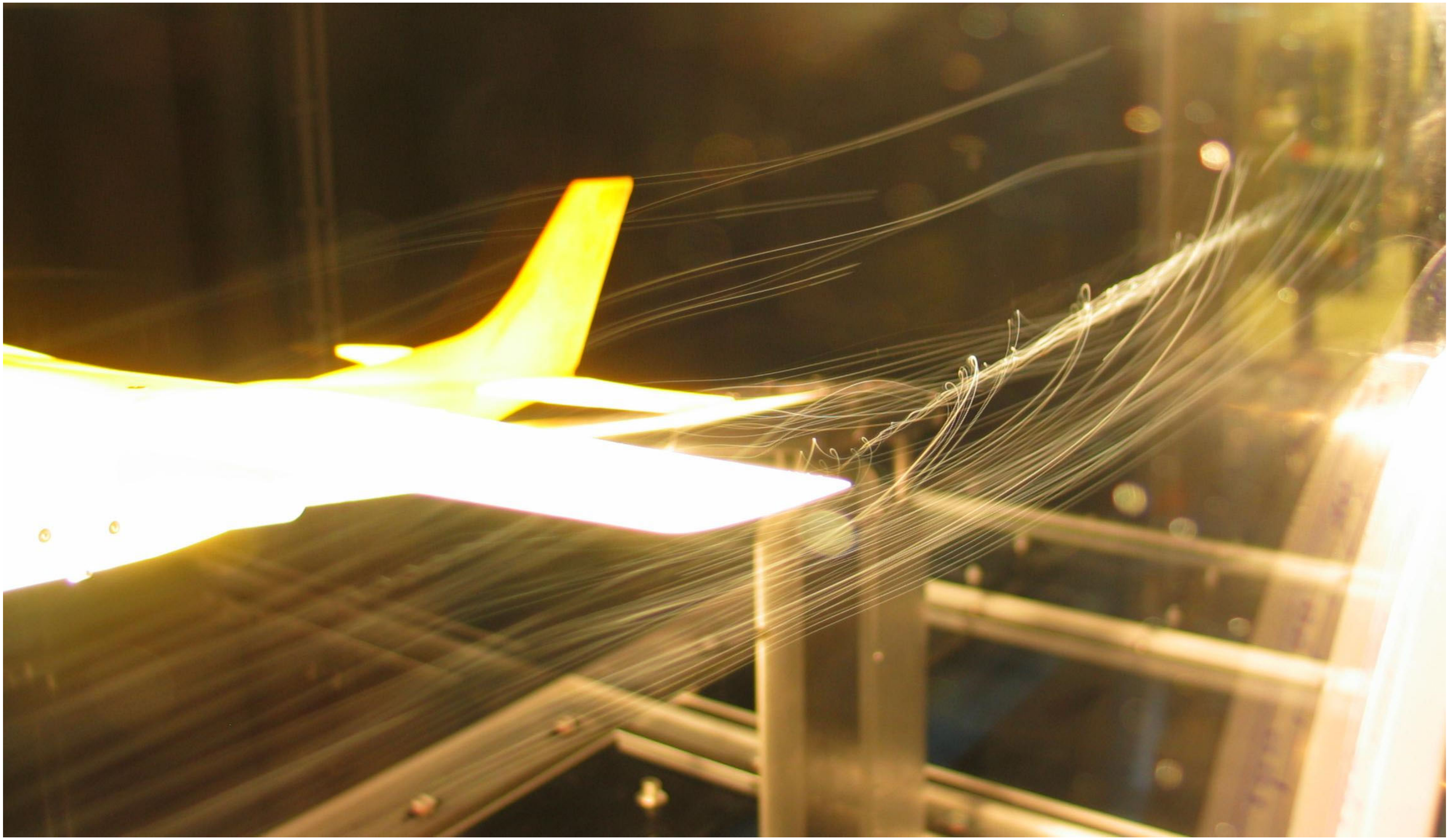
https://demonstrations.wolfram.com/NumericalMethodsForDifferentialEquations/

Flow visualization concepts

https://www3.nd.edu/~cwang11/flowvis.html

Vector Fields: Motivation

Smoke angel
A C-17 Globemaster III from the 14th Airlift Squadron, Charleston Air Force Base, S.C. flies off after releasing flares over the Atlantic Ocean near Charleston, S.C., during a training mission on Tuesday, May 16, 2006. The "smoke angel" is caused by the vortex from the engines.
(U.S. Air Force photo/Tech. Sgt. Russell E. Cooley IV)



A wind tunnel model of a Cessna 182 showing a wingtip vortex.
Tested in the RPI (Rensselaer Polytechnic Institute) Subsonic Wind Tunnel.
By Ben FrantzDale (2007).

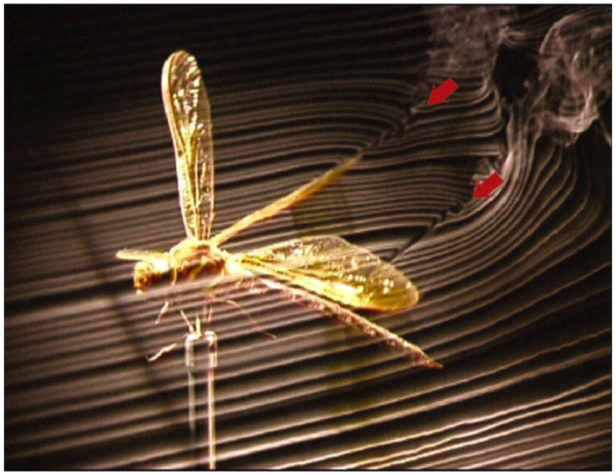

http://autospeed.com/cms/A_108677/article.html

wool tufts


http://autospeed.com/cms/A_108677/article.html

smoke injection

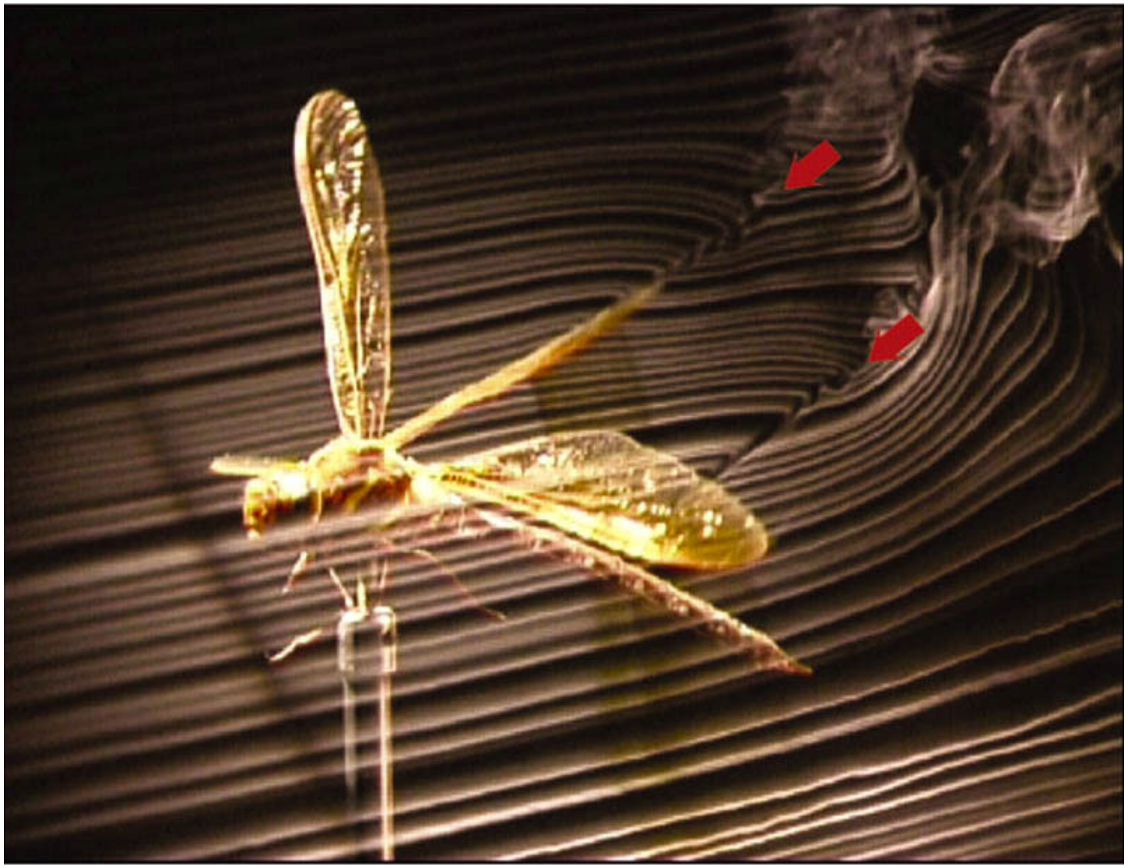
http://autospeed.com/cms/A_108677/article.html smoke nozzles



[NASA, J. Exp. Biol.]

http://autospeed.com/cms/A_108677/article.html smoke nozzles

Smoke injection

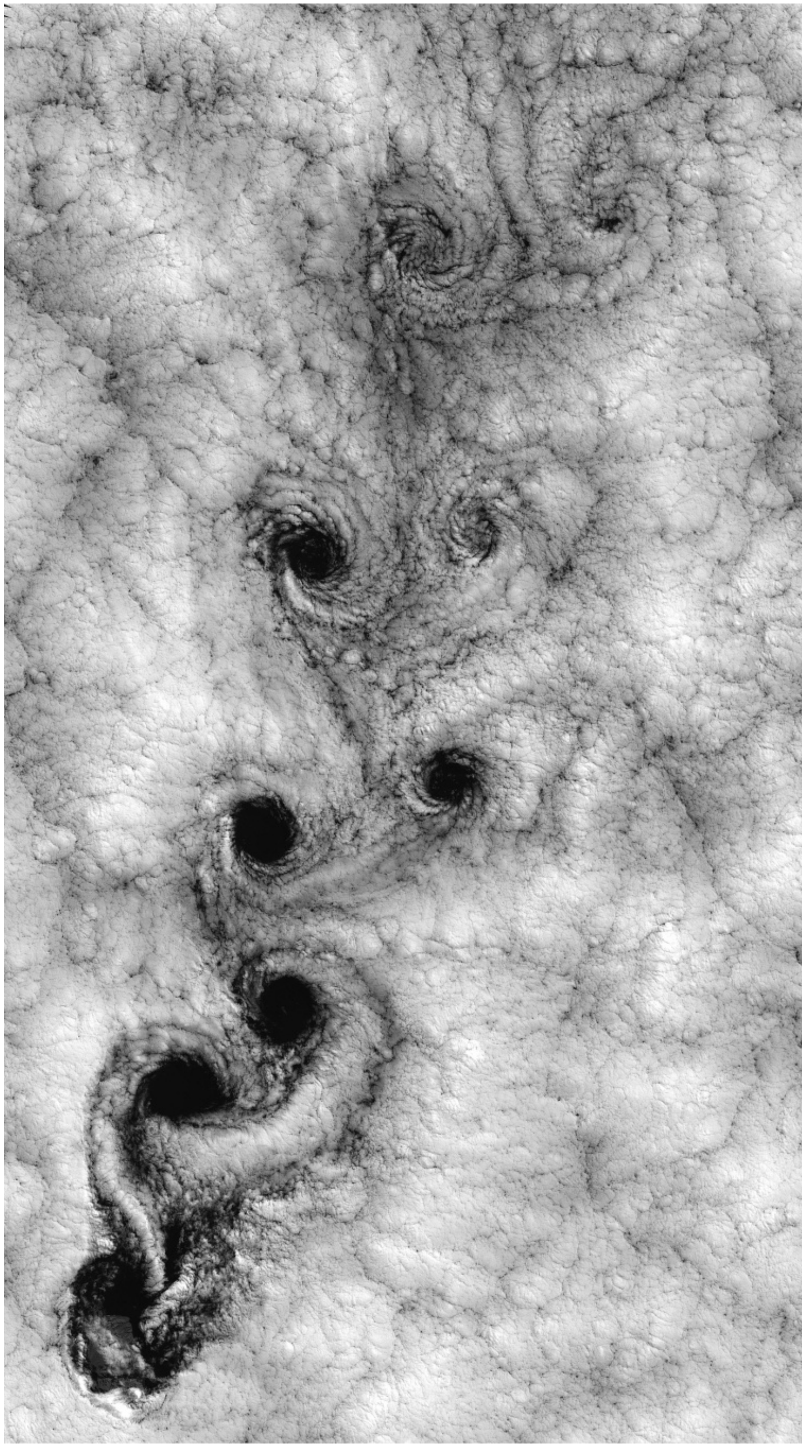


A. L. R. Thomas, G. K. Taylor, R. B. Srygley, R. L. Nudds, and R. J. Bomphrey. Dragonfly flight: free-flight and tethered flow visualizations reveal a diverse array of unsteady lift-generating mechanisms, controlled primarily via angle of attack. J Exp Biol, 207(24):4299–4323, 2004.

http://de.wikipedia.org/wiki/Bild:Airplane_vortex_edit.jpg

Smoke injection

http://www-me.ccny.cuny.edu/research/aerolab/facilities/images/wt2.jpg



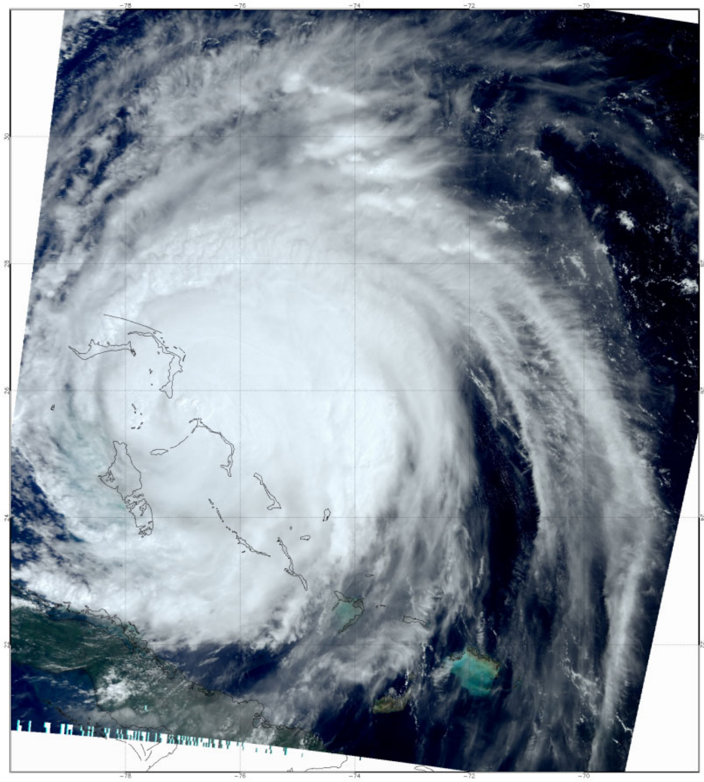
Clouds (satellite image)

Juan Fernandez Islands

http://de.wikipedia.org/wiki/Bild:Vortex-street-1.jpg

Clouds (satellite image)

http://daac.gsfc.nasa.gov/gallery/frances/

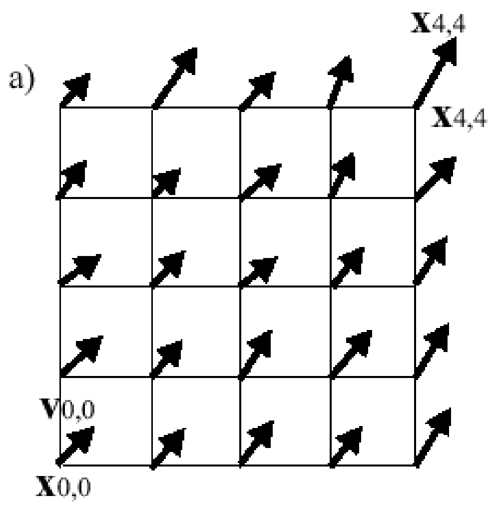


- **Vortex/ Vortex core lines**
  - There is no exact definition of vortices
  - capturing some swirling behavior

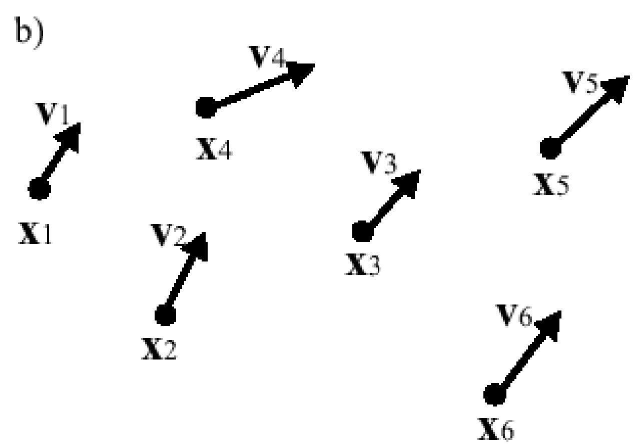
Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)



vectors given at grid points



vectors given at particle positions

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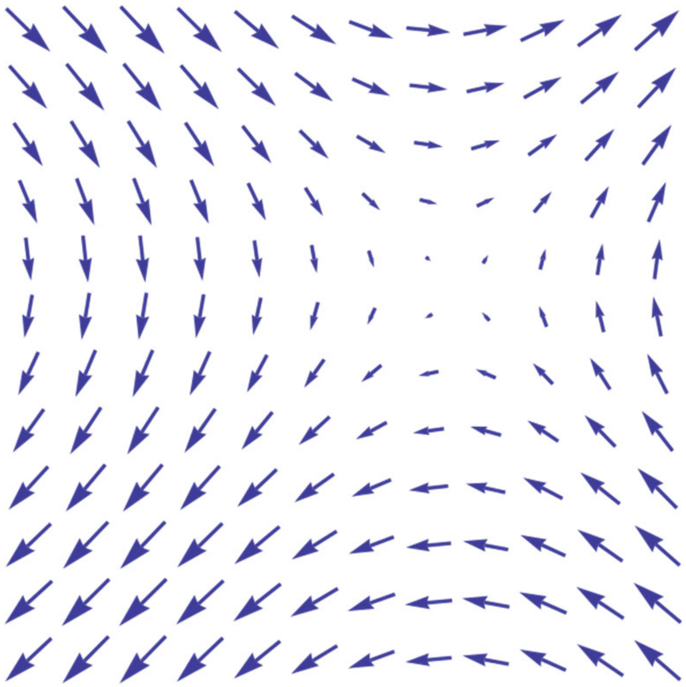
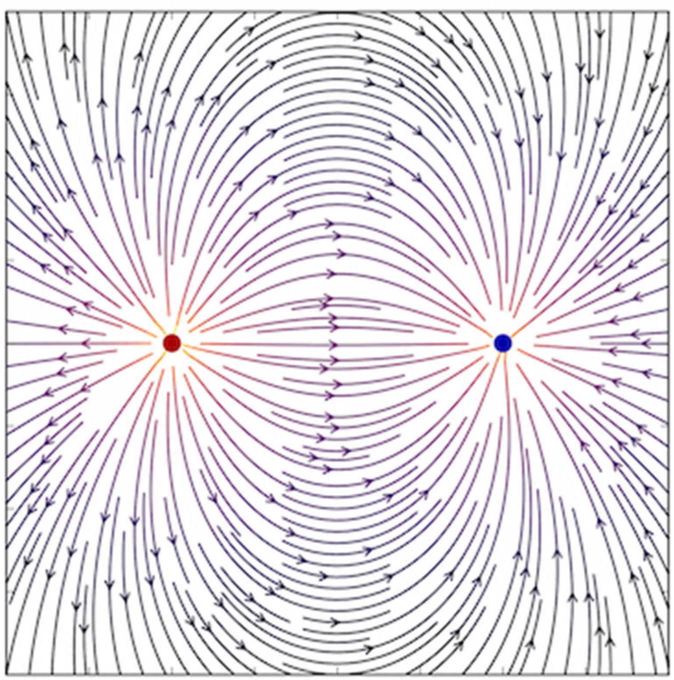
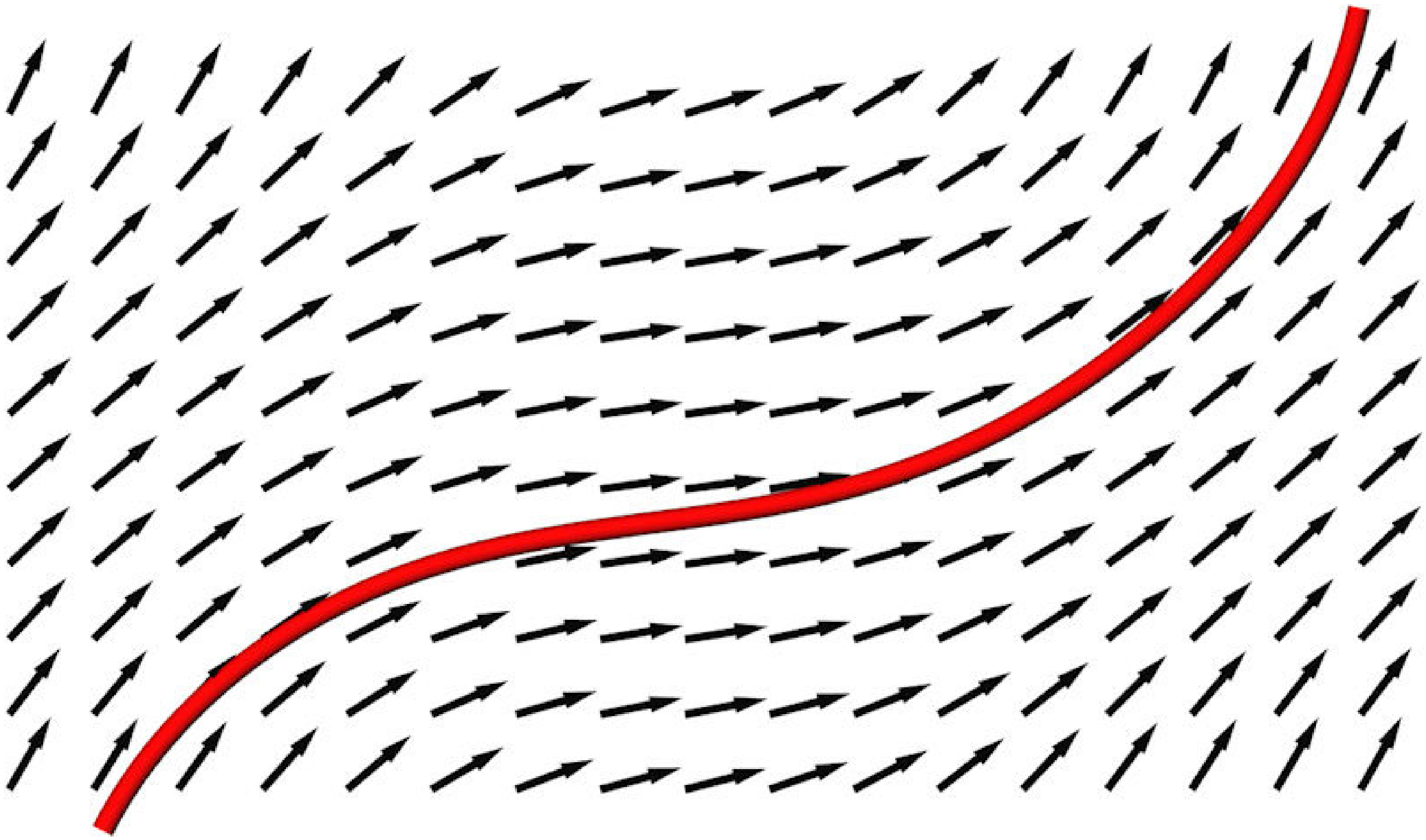




images from wikipedia

Integral Curves / Stream Objects

Integrating velocity over time yields spatial motion



Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
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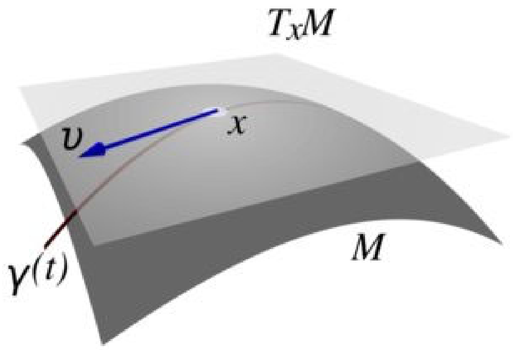
Each vector in a vector field
  lives in the **tangent space**
  of the manifold at that point:

Each vector is a **tangent vector**



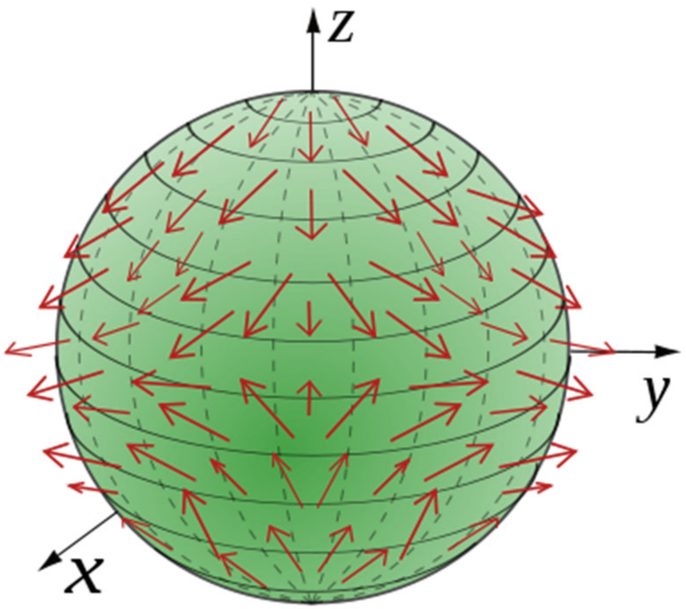


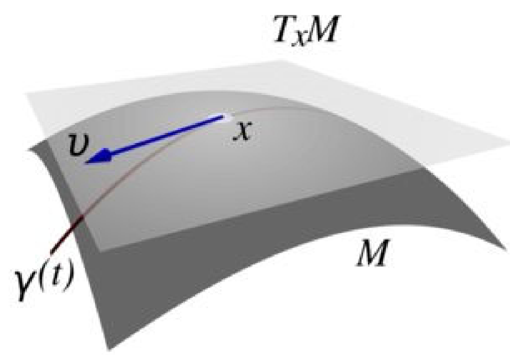
image from wikipedia

Vector Fields

Vector fields on general manifolds M (not just Euclidean space)

Tangent space at a point $x \in M$:

$$T_x M$$



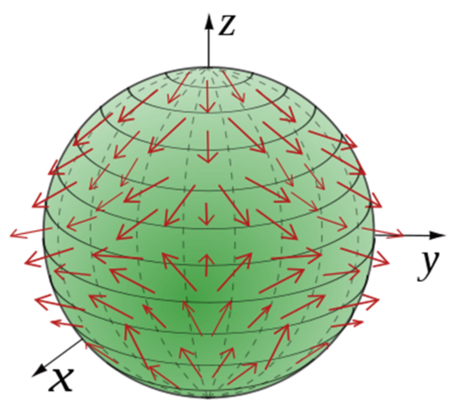
Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \to M$$

Vector field: *Section* of tangent bundle

$$s : M \to TM, \quad x \mapsto s(x).$$

$$\pi(s(x)) = x$$



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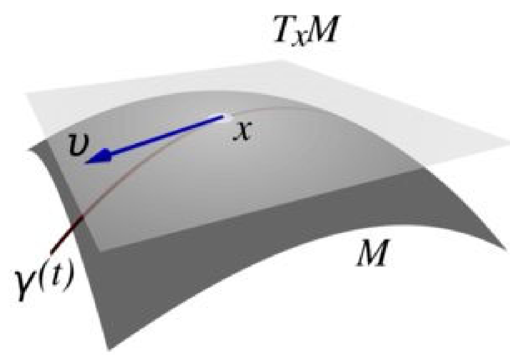

Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \rightarrow M$$

Vector field: *Section* of tangent bundle

$$\mathbf{v} : M \rightarrow TM, \quad x \mapsto \mathbf{v}(x).$$

$$\mathbf{v}(x) \in T_x M$$

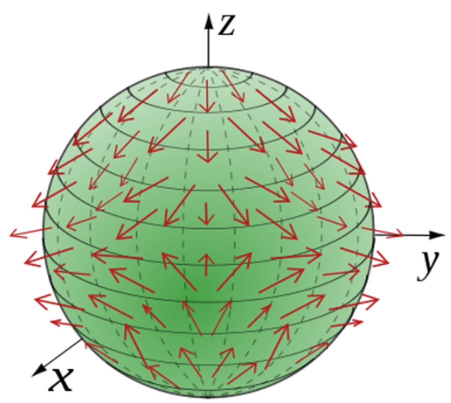

image from wikipedia

Interlude: Coordinate Charts

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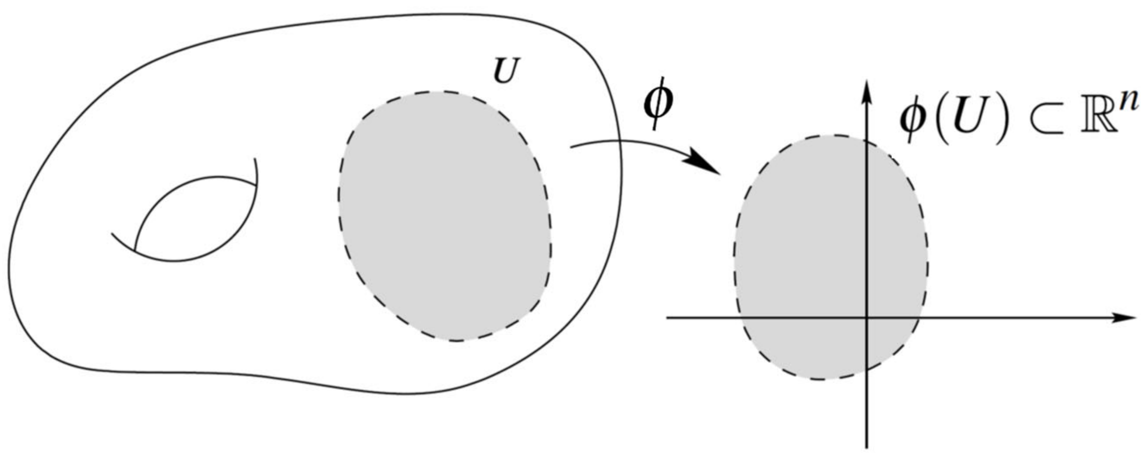
Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$



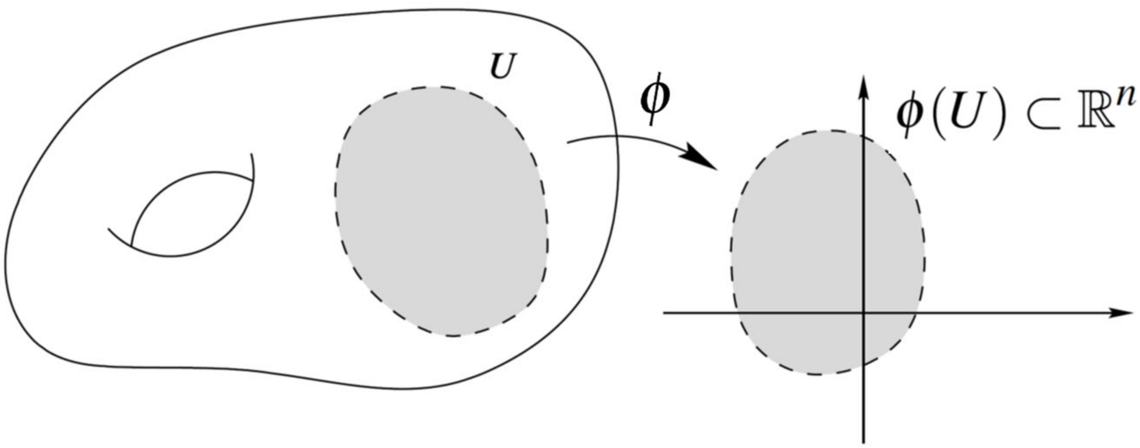
Interlude: Coordinate Charts

Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Coordinate functions

$$x^i : U \subset M \to \mathbb{R},$$
$$x \mapsto x^i(x).$$



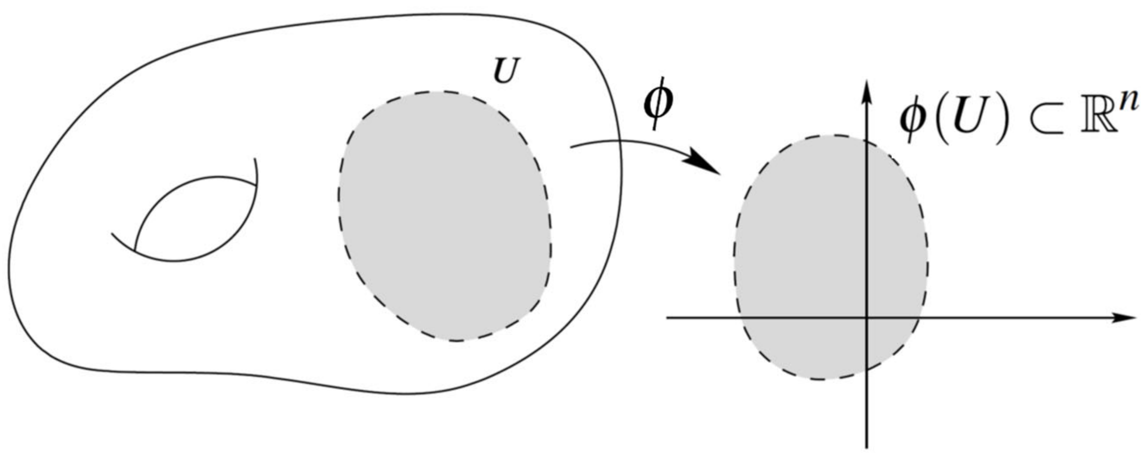
Interlude: Coordinate Charts

Coordinate charts

$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Atlas

$$\{(U_\alpha, \phi_\alpha)\}_{\alpha \in I}$$



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$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n, \qquad x \mapsto (x^1(x), x^2(x), \ldots, x^n(x)).$$

Atlas

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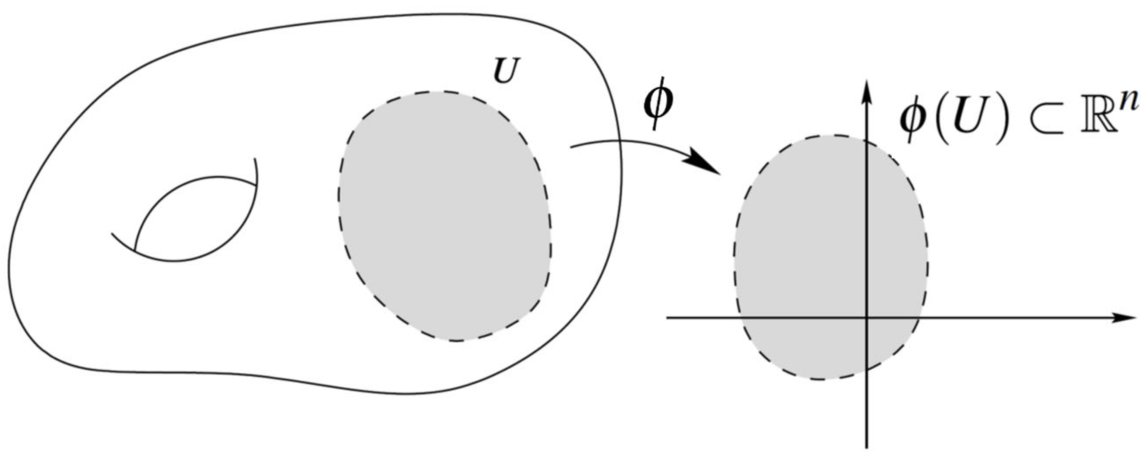

Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\mathbf{v} : U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
$$(x, y) \mapsto \begin{bmatrix} u \\ v \end{bmatrix}.$$

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$$\mathbf{v}: U \subset \mathbb{R}^3 \to \mathbb{R}^3,$$
$$(x, y, z) \mapsto \begin{bmatrix} u \\ v \\ w \end{bmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
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Vector Fields vs. Vectors in Components

$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{pmatrix} v^1(x^1, x^2, \ldots, x^n) \\ v^2(x^1, x^2, \ldots, x^n) \\ \vdots \\ v^n(x^1, x^2, \ldots, x^n) \end{pmatrix}.$$

Vector Fields vs. Vectors in Components

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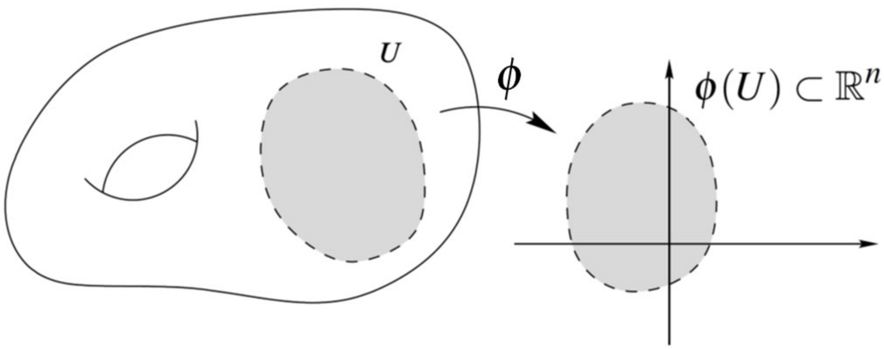

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$$\mathbf{v}\big|_U : \phi(U) \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \quad x \mapsto \mathbf{e}_i(x)$$

$$\{\mathbf{e}_i(x)\}_{i=1}^{n} \quad \text{basis for } T_xM$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\begin{aligned} \mathbf{e}_i : U \subset M &\to TM, \\ x &\mapsto \mathbf{e}_i(x) \end{aligned} \qquad \{\mathbf{e}_i(x)\}_{i=1}^n \quad \text{basis for } T_xM$$

$$\begin{aligned} \mathbf{v} : U \subset M &\to TM, \\ x &\mapsto v^1 \mathbf{e}_1 + v^2 \mathbf{e}_2 + \ldots + v^n \mathbf{e}_n. \end{aligned}$$

$$\begin{aligned} \mathbf{v} : U \subset M &\to TM, \\ x &\mapsto v^1(x)\, \mathbf{e}_1(x) + v^2(x)\, \mathbf{e}_2(x) + \ldots + v^n(x)\, \mathbf{e}_n(x). \end{aligned}$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \quad x \mapsto \mathbf{e}_i(x)$$

$$\{\mathbf{e}_i(x)\}_{i=1}^{n} \quad \text{basis for } T_xM$$

Coordinate basis:

$$\mathbf{e}_i := \frac{\partial}{\partial x^i}$$

$$\mathbf{v} : U \subset M \to TM,$$
$$x \mapsto v^1 \mathbf{e}_1 + v^2 \mathbf{e}_2 + \ldots + v^n \mathbf{e}_n.$$

$$\mathbf{v} : U \subset M \to TM,$$
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Examples of Coordinate Curves and Bases

Coordinate functions, coordinate curves, bases

- Coordinate functions are real-valued (“scalar”) functions on the domain
- On each coordinate curve, *one* coordinate changes, *all others stay constant*
- Basis: n linearly independent vectors at each point of domain

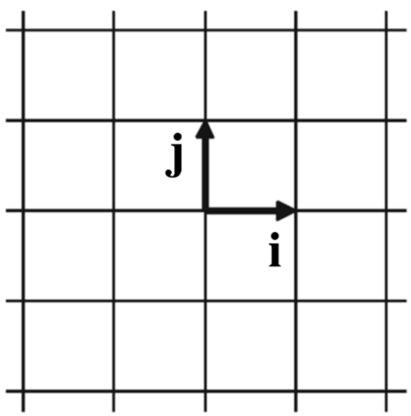
Cartesian coordinates

$$x^1 = x$$

$$x^2 = y$$

$$\mathbf{e}_1 = \frac{\partial}{\partial x} = \mathbf{i}$$

$$\mathbf{e}_2 = \frac{\partial}{\partial y} = \mathbf{j}$$



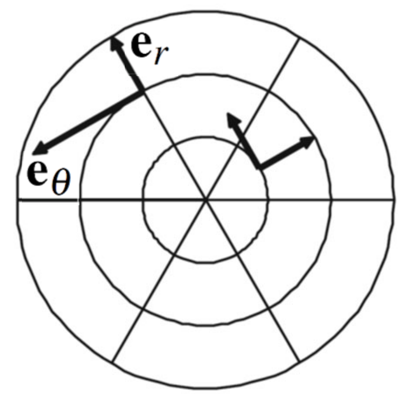
polar coordinates

$$x^1 = r$$

$$x^2 = \theta$$

$$\mathbf{e}_1 = \frac{\partial}{\partial r} = \mathbf{e}_r$$

$$\mathbf{e}_2 = \frac{\partial}{\partial \theta} = \mathbf{e}_\theta$$

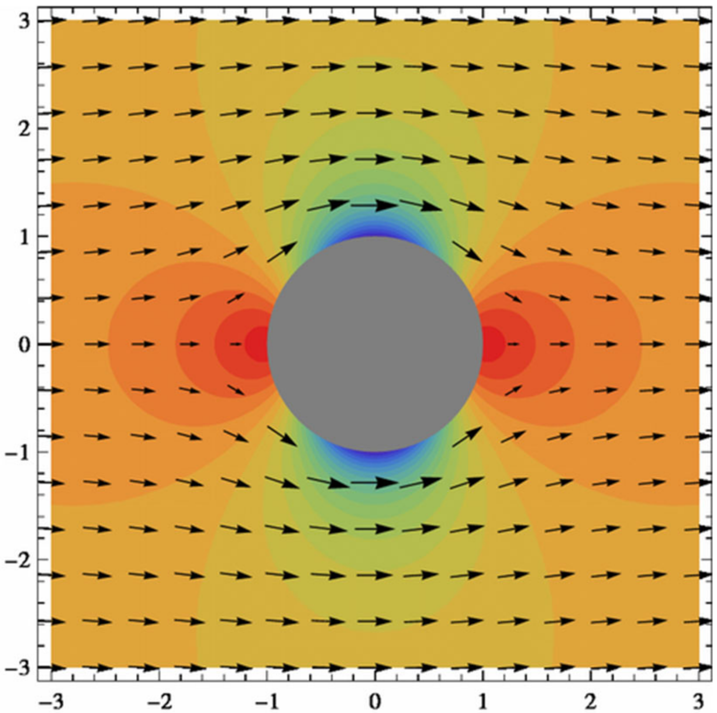


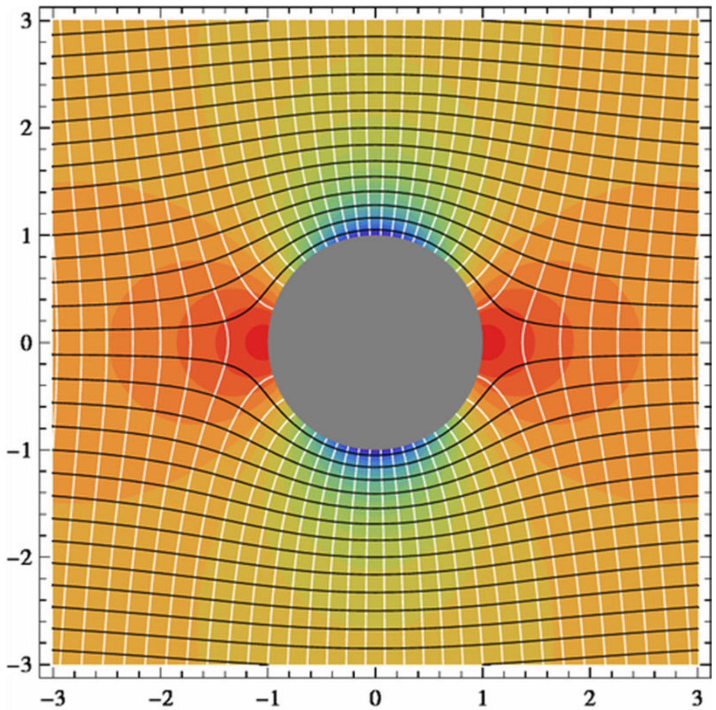
Flow Field Example (1)

Potential flow around a circular cylinder

`https://en.wikipedia.org/wiki/Potential_flow_around_a_circular_cylinder`

Inviscid, incompressible flow that is irrotational (curl-free) and can be modeled as the gradient of a scalar function called the (scalar) velocity potential





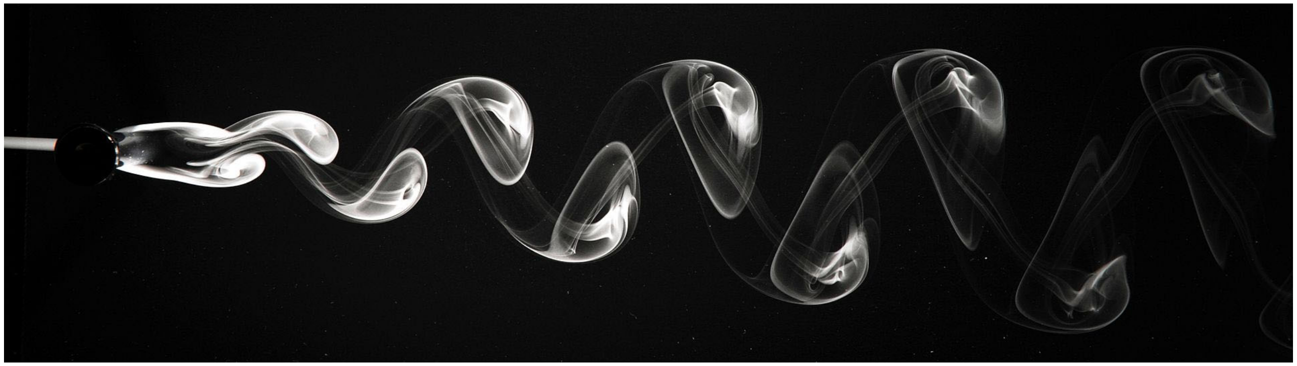
images from wikipedia

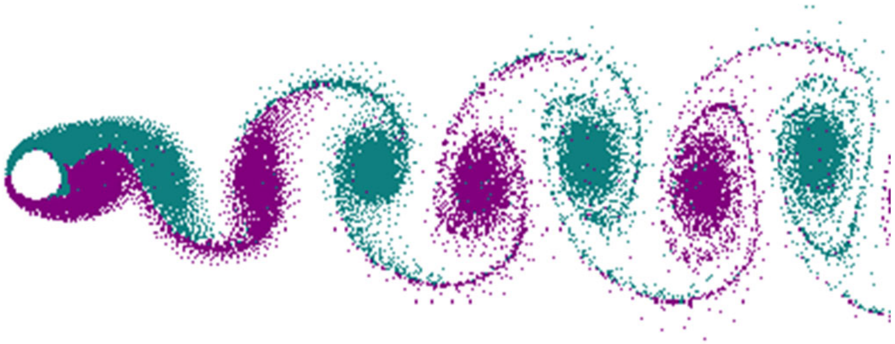
Flow Field Example (2)

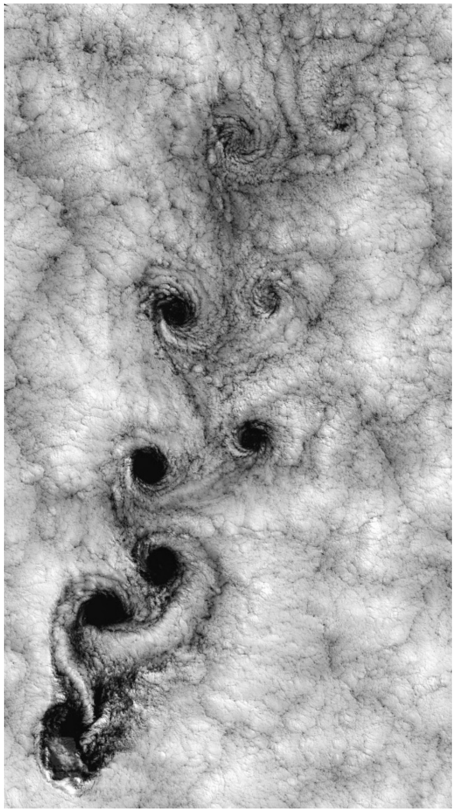
Depending on Reynolds number, turbulence will develop

Example: von Kármán vortex street: vortex shedding

`https://en.wikipedia.org/wiki/Karman_vortex_street`







images from wikipedia

Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
  - Example: laminar flows

$$\mathbf{v}: \mathbb{R}^n \to \mathbb{R}^n, \quad x \mapsto \mathbf{v}(x).$$

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x}, t)$
  - Example: turbulent flows

$$\mathbf{v}: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n, \quad x \mapsto \mathbf{v}(x, t).$$

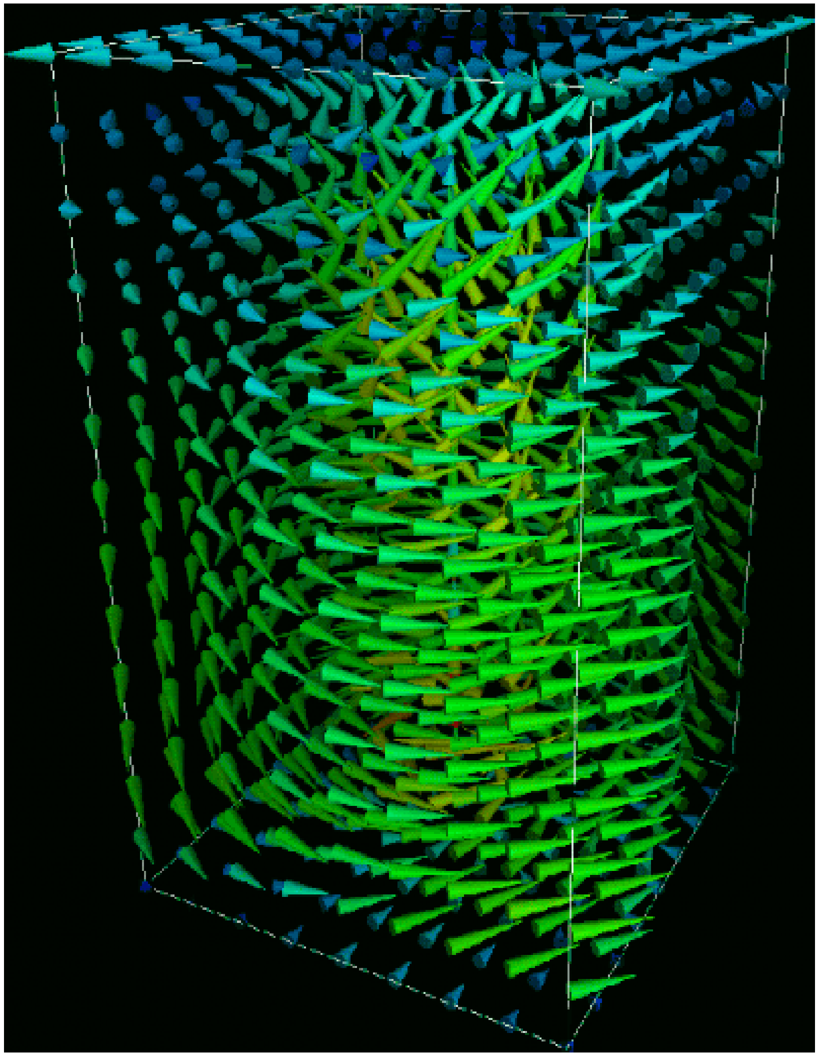
(here just for Euclidean domain; analogous on general manifolds)

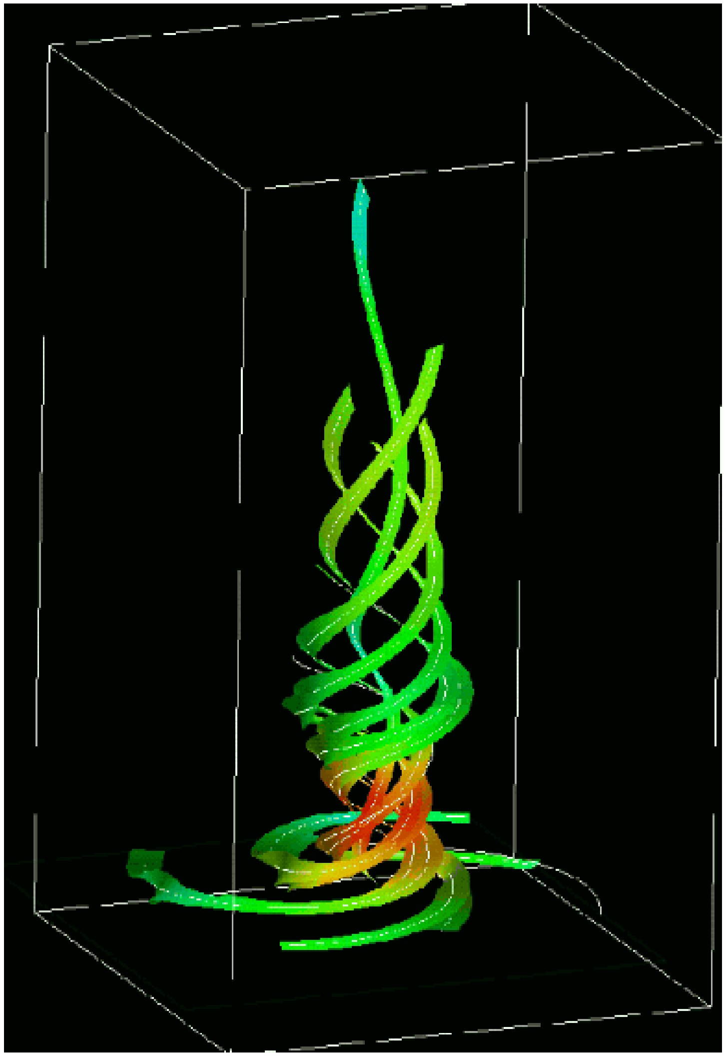
Direct vs. Indirect Flow Visualization

- Direct flow visualization
  - Overview of current flow state
  - Visualization of vectors: arrow plots (“hedgehog” plots)

- Indirect flow visualization
  - Use intermediate representation: vector field integration over time
  - Visualization of temporal evolution
  - Integral curves: streamlines, pathlines, streaklines, timelines
  - Integral surfaces: streamsurfaces, pathsurfaces, streaksurfaces

Direct vs. Indirect Flow Visualization

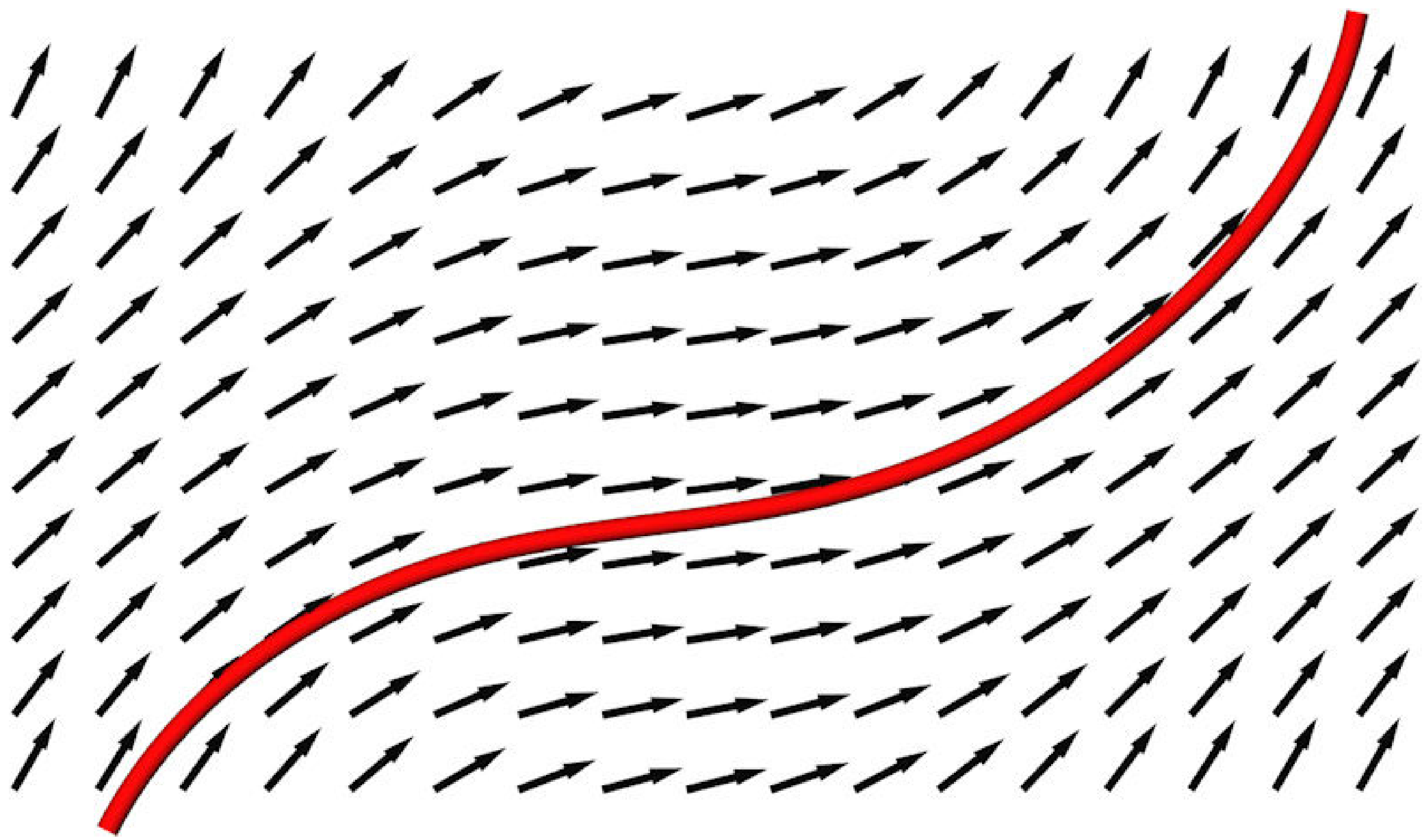




Integral Curves: Intro

Integral Curves / Stream Objects

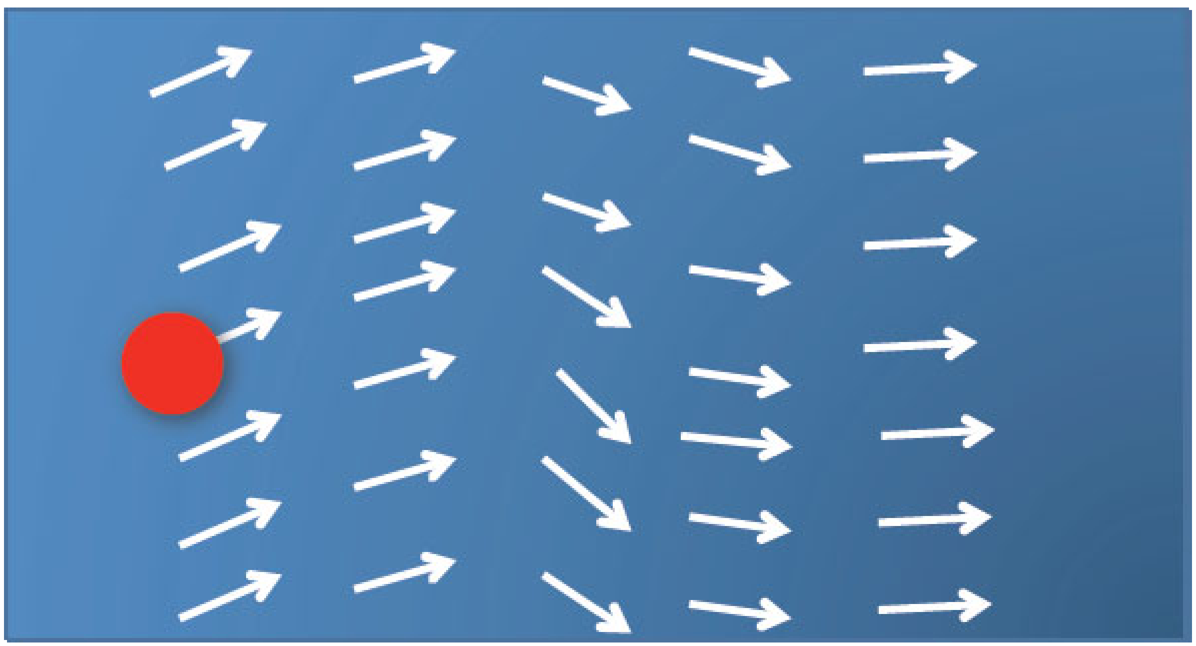
Integrating velocity over time yields spatial motion



Particle Trajectories

Courtesy Jens Krüger

Particle Trajectories



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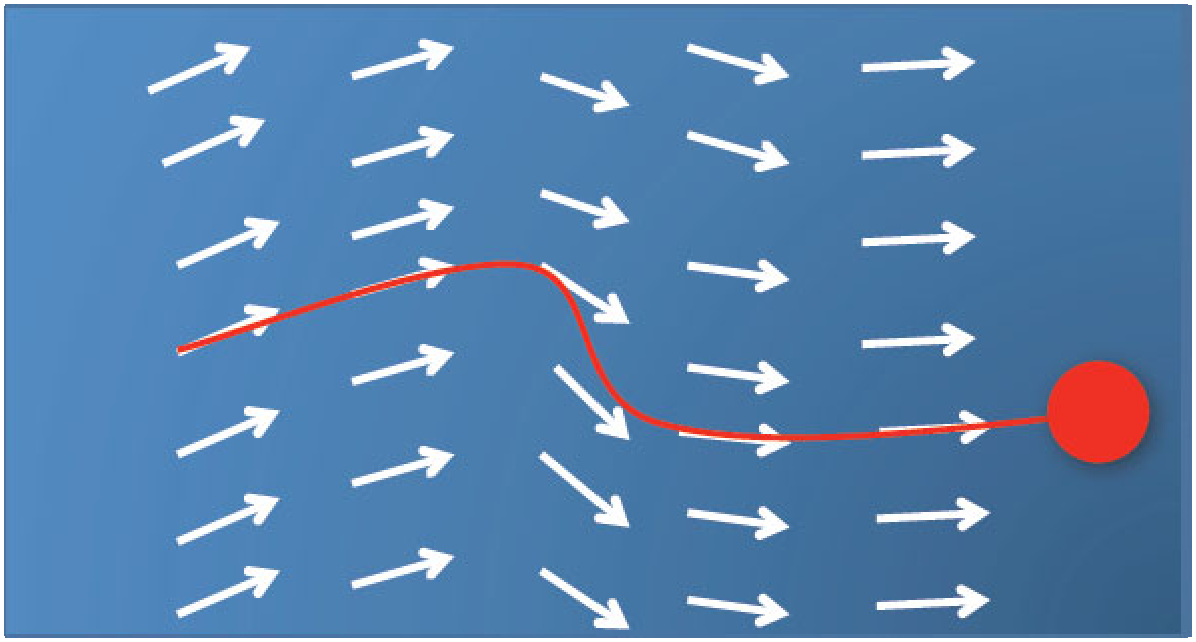
Particle Trajectories



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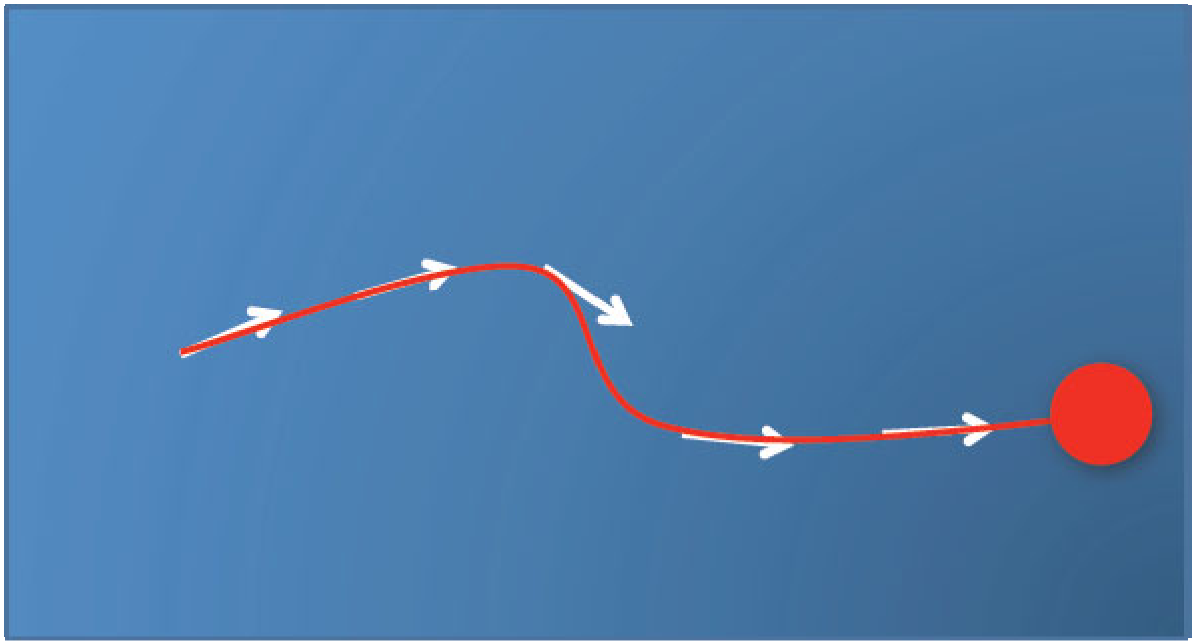
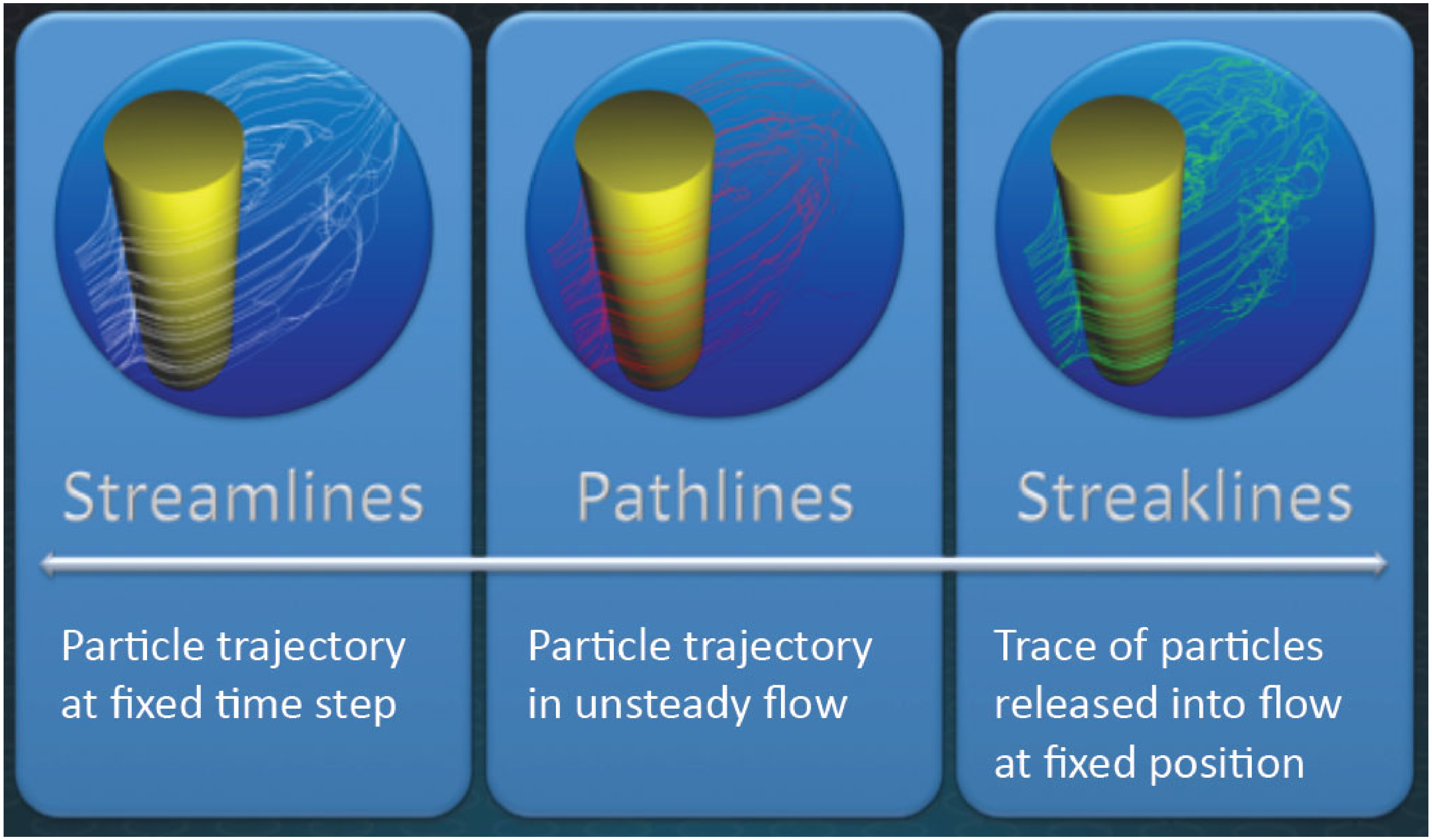


Courtesy Jens Krüger

Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

- Describes motion of a massless particle over time

Streakline

- Location of all particles released at a *fixed position* over time

Timeline

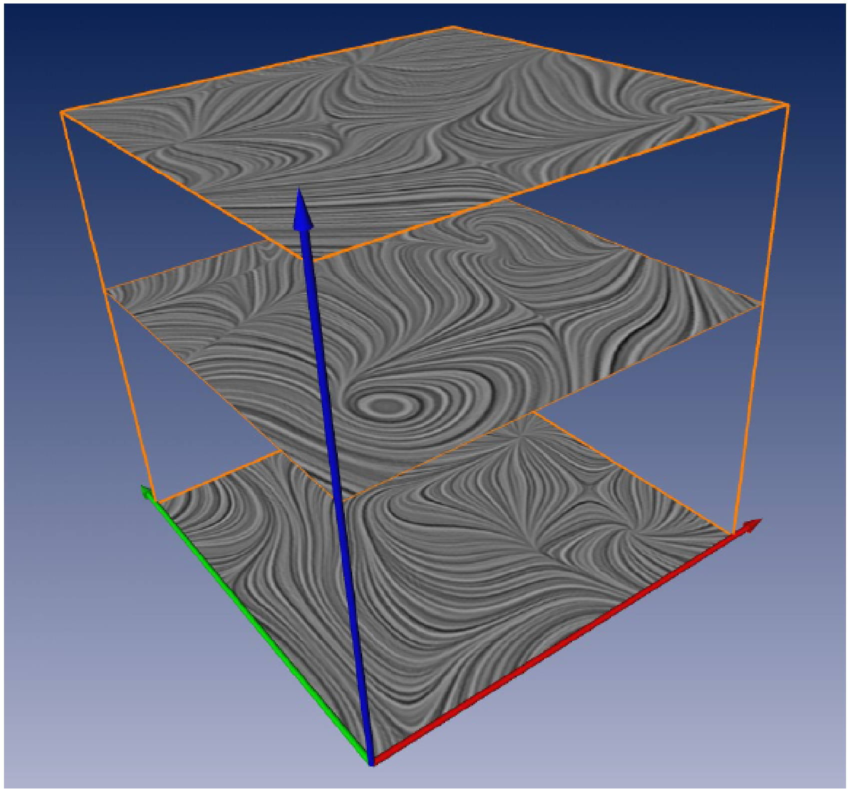
- Location of all particles released along a line at a *fixed time*

Integral Curves

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

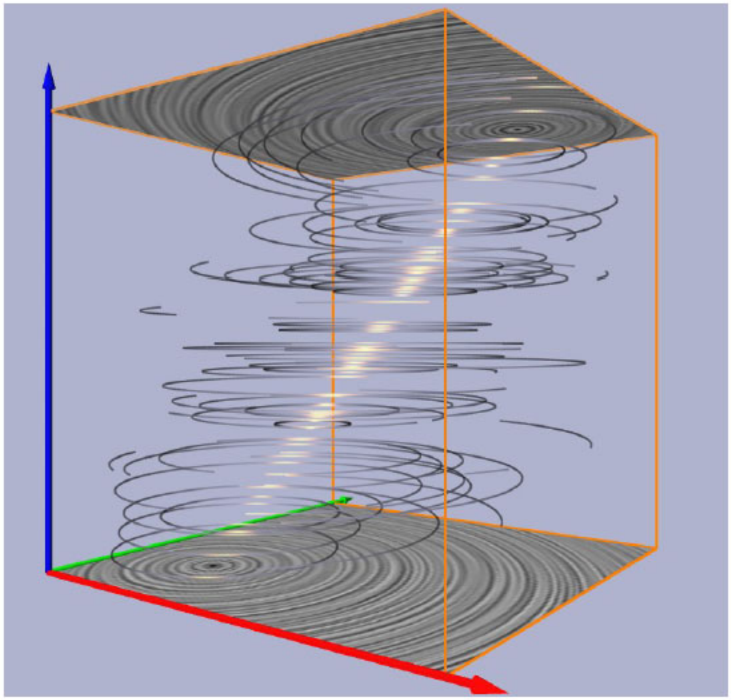


Tino Weinkauf

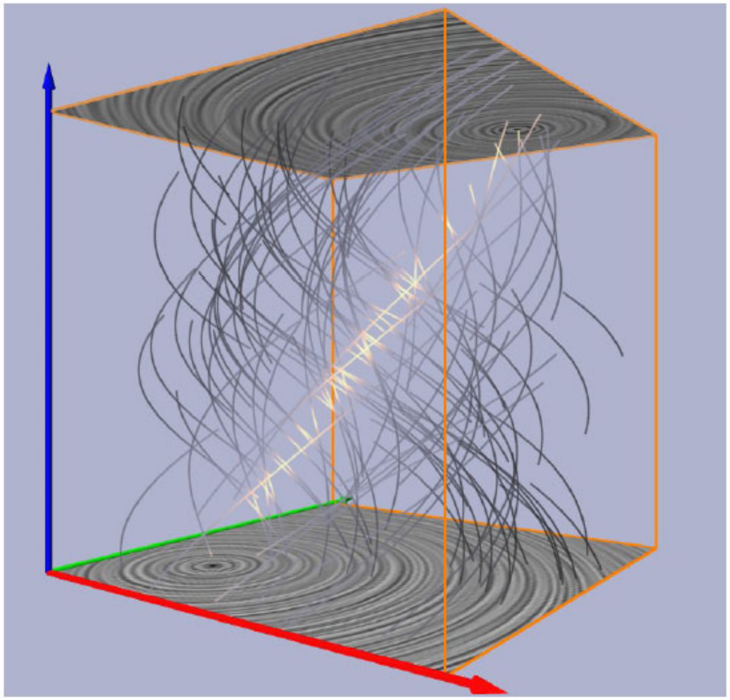
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Vector fields

A static vector field $\mathbf{v}(\mathbf{x})$ is a vector-valued function of space.

A time-dependent vector field $\mathbf{v}(\mathbf{x}, t)$ depends also on time.

In the case of velocity fields, the terms steady and unsteady flow are used.

The dimensions of **x** and **v** are equal, often 2 or 3, and we denote components by *x,y,z* and *u,v,w*:

$$\mathbf{x} = (x, y, z), \ \mathbf{v} = (u, v, w)$$

Sometimes a vector field is defined on a surface $\mathbf{x}(i, j)$. The vector field is then a function of parameters and time:

$$\mathbf{v}(i, j, t)$$

Vector fields as ODEs

For simplicity, the vector field is now interpreted as a velocity field.

Then the field $\mathbf{v}(\mathbf{x},t)$ describes the connection between location and velocity of a (massless) particle.

It can equivalently be expressed as an ordinary differential equation

$$\dot{\mathbf{x}}(t)=\mathbf{v}(\mathbf{x}(t),t)$$

This ODE, together with an initial condition

$$\mathbf{x}(t_0)=\mathbf{x}_0 ,$$

is a so-called initial value problem (IVP).

Its solution is the integral curve (or trajectory)

$$\mathbf{x}(t)=\mathbf{x}_0+\int_{t_0}^{t}\mathbf{v}(\mathbf{x}(\tau),\tau)\,d\tau$$

The integral curve is a pathline, describing the path of a massless particle which was released at time t_0 at position x_0.

Remark: $t < t_0$ is allowed.

For static fields, the ODE is autonomous:

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t))$$

and its integral curves

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau))\, d\tau$$

are called field lines, or (in the case of velocity fields) streamlines.

In static vector fields, pathlines and streamlines are identical.

In time-dependent vector fields, instantaneous streamlines can be computed from a "snapshot" at a fixed time T (which is a static vector field)

$$\mathbf{v}_T(\mathbf{x}) = \mathbf{v}(\mathbf{x}, T)$$

In practice, time-dependent fields are often given as a dataset per time step. Each dataset is then a snapshot.

Streamline integration

Outline of algorithm for numerical streamline integration (with obvious extension to pathlines):

Inputs:

- static vector field $\mathbf{v}(\mathbf{x})$
- seed points with time of release $(\mathbf{x}_0, t_0)$
- control parameters:
  - step size (temporal, spatial, or in local coordinates)
  - step count limit, time limit, etc.
  - order of integration scheme

Output:

- streamlines as "polylines", with possible attributes (interpolated field values, time, speed, arc length, etc.)

Preprocessing:

- set up search structure for point location
- for each seed point:
  - global point location: Given a point **x**,
    find the cell containing **x** and the local coordinates (ξ, η, ζ)
    or ir the grid is structured:
    find the computational space coordinates $(i+\xi,\ j+\eta,\ k+\zeta)$
  - If **x** is not found in a cell, remove seed point

Integration loop, for each seed point **x**:

- interpolate ***v*** trilinearly to local coordinates (ξ, η, ζ)
- do an integration step, producing a new point **x'**
- incremental point location: For position **x'** find cell and local coordinates (ξ', η', ζ') making use of information (coordinates, local coordinates, cell) of old point **x**

Termination criteria:

- grid boundary reached
- step count limit reached
- optional: velocity close to zero
- optional: time limit reached
- optional: arc length limit reached

Integration step: widely used integration methods:

- Euler (used only in special speed-optimized techniques, e.g. GPU-based texture advection)

$$\mathbf{x}_{new} = \mathbf{x} + \mathbf{v}(\mathbf{x}, t) \cdot \Delta t$$

- Runge-Kutta, 2nd or 4th order

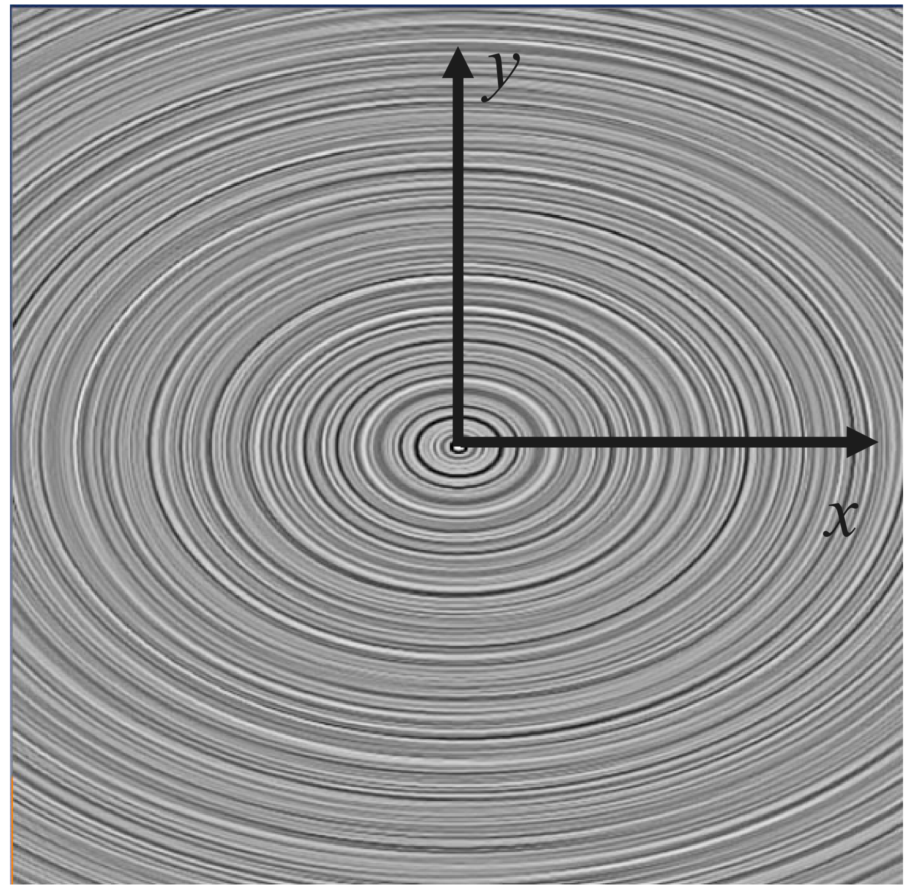
Higher order than 4th?

- often too slow for visualization
- study (Yeung/Pope 1987) shows that, when using standard trilinear interpolation, interpolation errors dominate integration errors.

- **Numerical integration of stream lines:**

- approximate streamline by polygon $\mathbf{x}_i$

- **Testing example:**
  - $\mathbf{v}(x,y) = (-y, x/2)^T$
  - exact solution: ellipses
  - starting integration from (0,-1)



Bonus Slides:
Vectors as Derivative Operators

Vectors as Derivative Operators

A vector applied to a (real) function on the manifold gives the *directional derivative* in that direction

- From this viewpoint, the vector is a derivative operator (actually, a *derivation*)
- Can be used as *definition* of a vector (must fulfill props. of a derivation; esp. Leibniz rule)

$$
\begin{aligned}
f : M &\to \mathbb{R}, \qquad\qquad \mathbf{v} f \\
x &\mapsto f(x).
\end{aligned}
$$

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Kronecker delta
(“identity matrix”)

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Kronecker delta ("identity matrix")

For vector field: obtain directional derivative at each point

$$\mathbf{v}f : M \to \mathbb{R},\quad x \mapsto \mathbf{v}(x)\,f = df(\mathbf{v}(x)).$$

(remember that this just *looks* scary (maybe) ...)

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama