

CS 247 – Scientific Visualization

Lecture 15: Volume Visualization, Pt. 2

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Reading Assignment #8 (until Mar 26)



Read (required):

- Real-Time Volume Graphics, Chapter 4 (Transfer Functions) until Sec. 4.4 (inclusive)
- Paper:
Jens Krüger and Rüdiger Westermann,
Acceleration Techniques for GPU-based Volume Rendering,
IEEE Visualization 2003,
<http://dl.acm.org/citation.cfm?id=1081482>

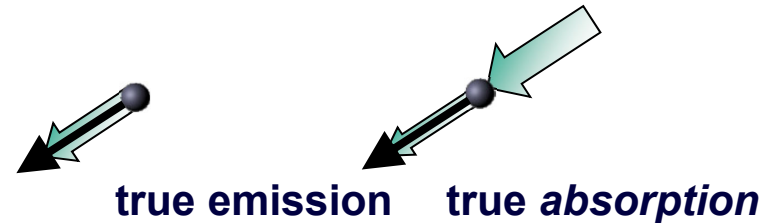
Volume Rendering

Theory, Ctd.

Volume Rendering Integral



Volume rendering integral
for *Emission Absorption* model



$$I(s) = I(s_0) e^{-\tau(s_0, s)} + \int_{s_0}^s q(\tilde{s}) e^{-\tau(\tilde{s}, s)} d\tilde{s}$$

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s) ds.$$

Iterative/recursive numerical solutions:

Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1})C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1})A_i \end{aligned}$$

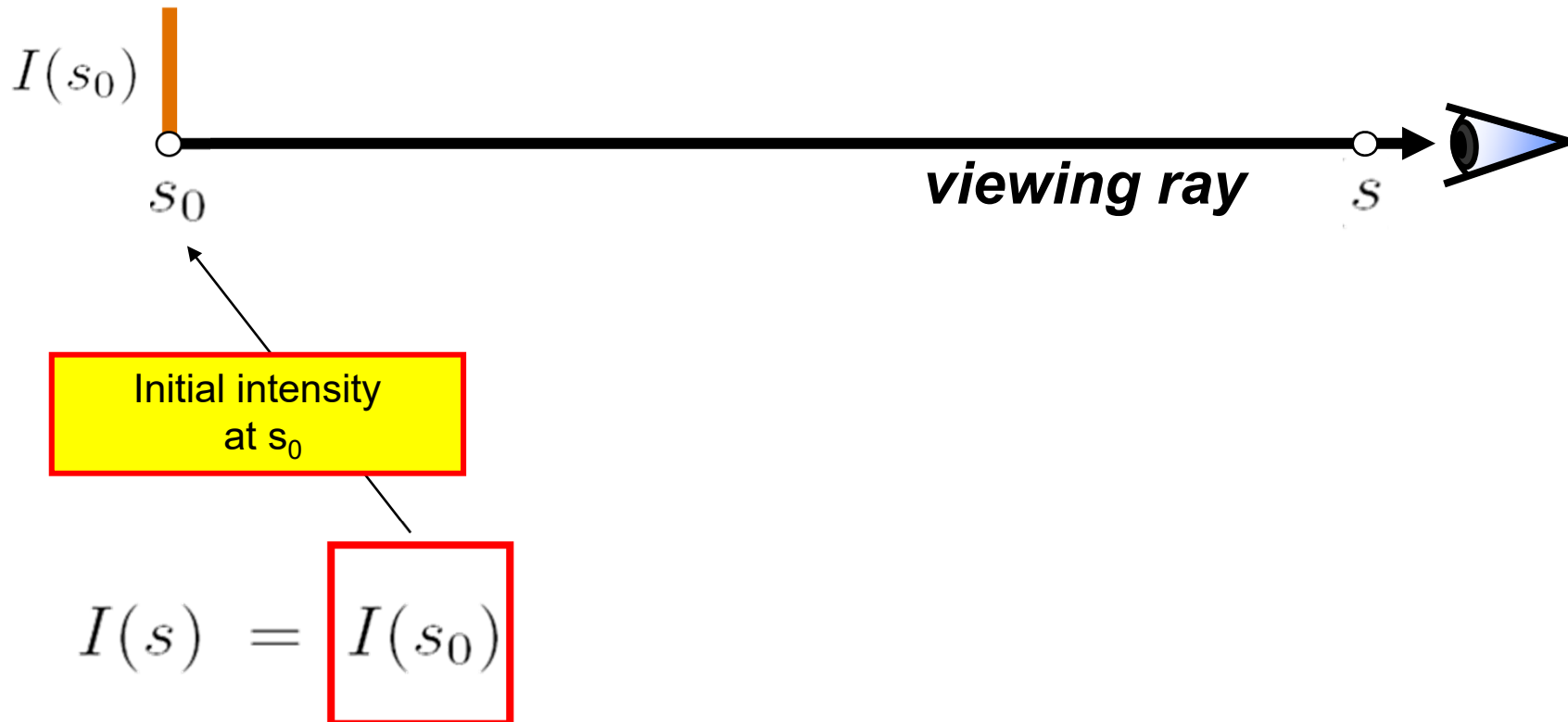
here, all colors are *associated colors*!

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Initial intensity
at s_0

$$I(s) = I(s_0)$$

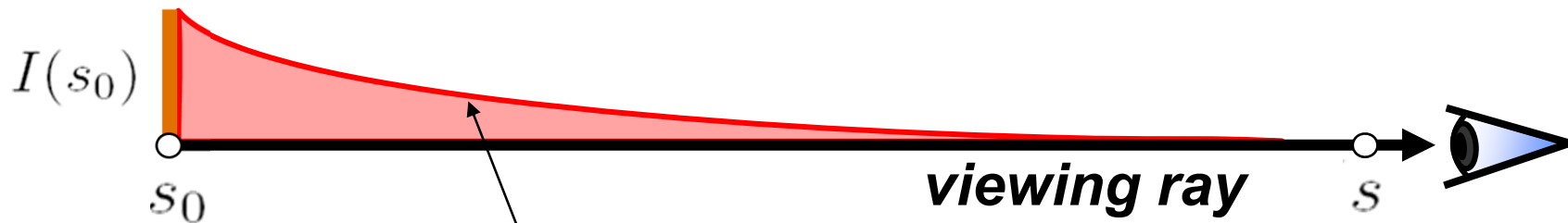
Without absorption all
the initial radiant energy
would reach the point s .

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



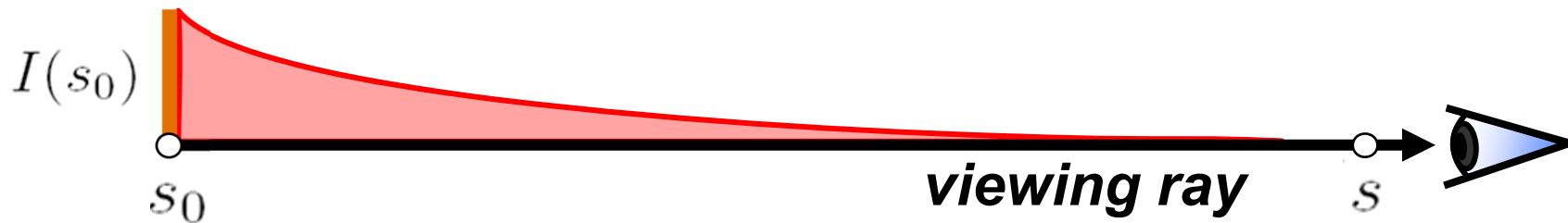
$$I(s) = I(s_0) e^{-\tau(s_0, s)}$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Optical depth τ
Absorption κ

$$I(s) = I(s_0) e^{-\tau(s_0, s)}$$

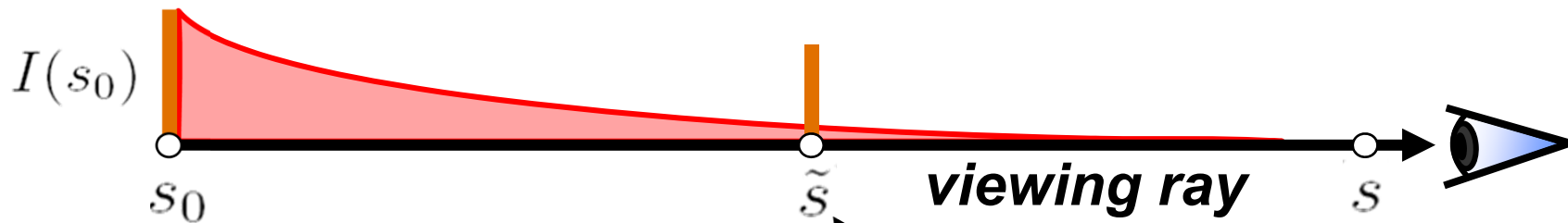
$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s) ds.$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point \tilde{s} along the viewing ray emits additional radiant energy.

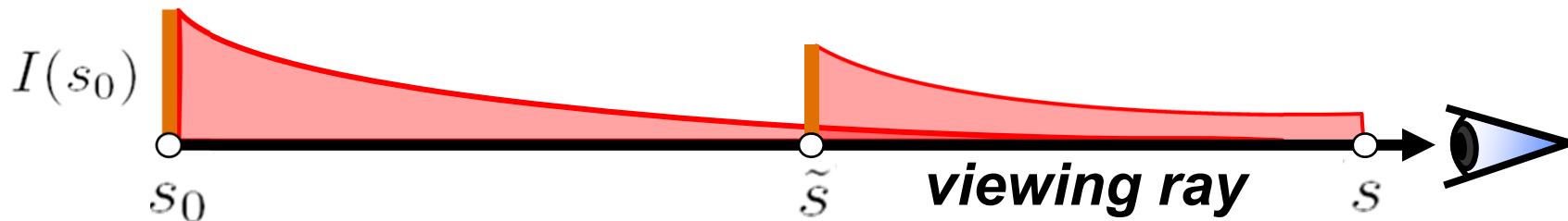
$$I(s) = I(s_0) e^{-\tau(s_0, s)} + q(\tilde{s})$$

Volume Rendering Integral



How do we determine the radiant energy along the ray?

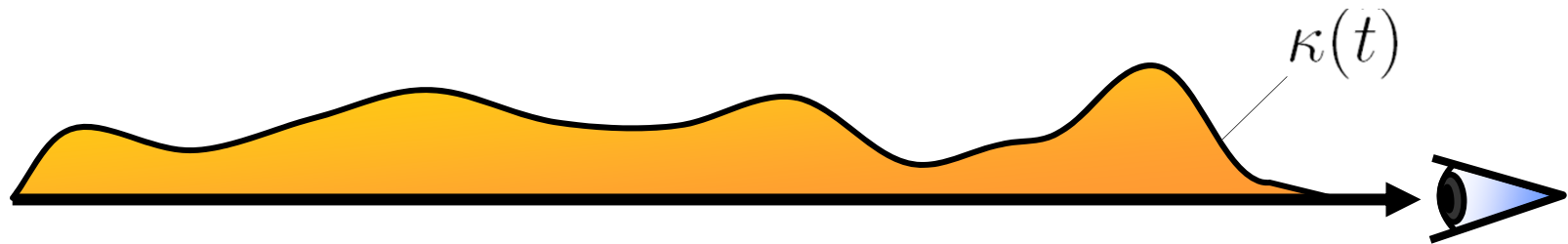
Physical model: emission and absorption, no scattering



Every point \tilde{s} along the viewing ray emits additional radiant energy

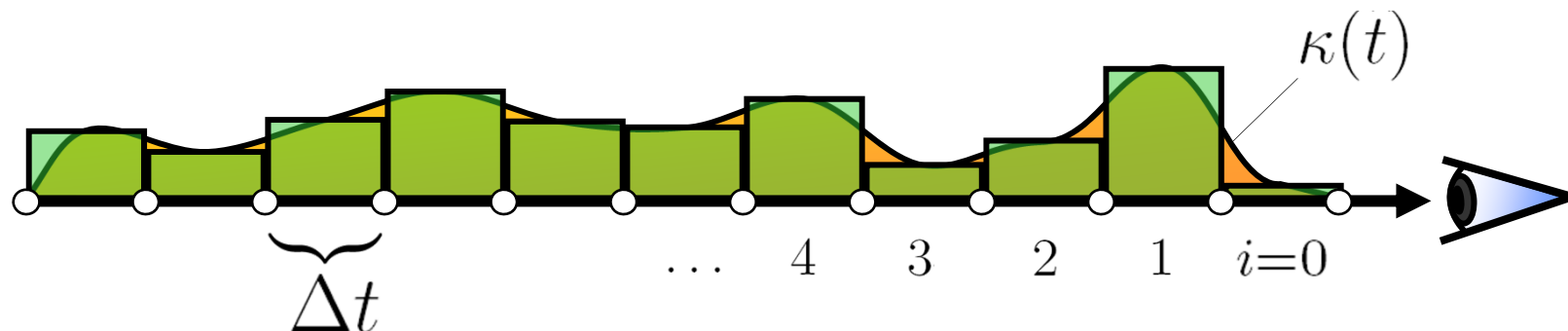
$$I(s) = I(s_0) e^{-\tau(s_0, s)} + \int_{s_0}^s q(\tilde{s}) e^{-\tau(\tilde{s}, s)} d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0, t) = \int_0^t \kappa(\hat{t}) d\hat{t}$

Volume Rendering Integral: Numerical Solution

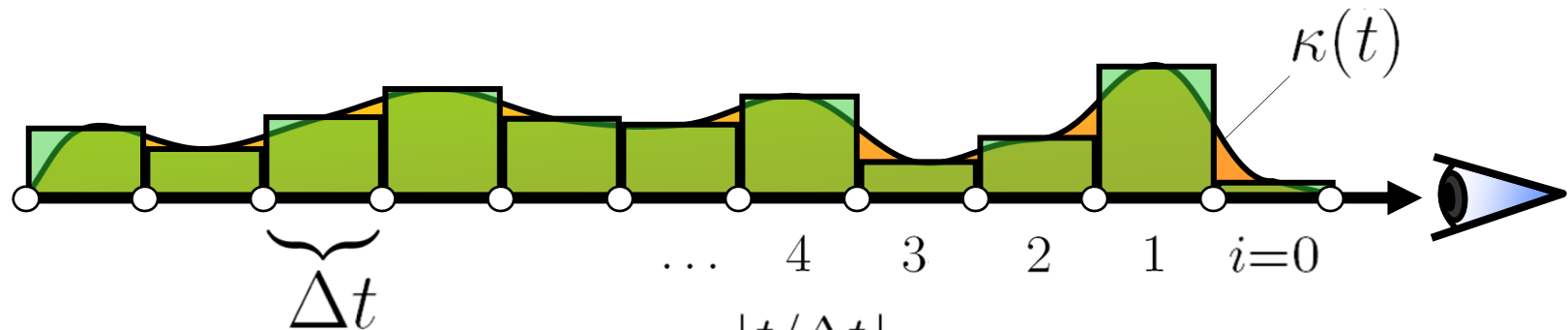


Optical depth: $\tau(0, t) = \int_0^t \kappa(\hat{t}) d\hat{t}$

Approximate Riemann integral by Riemann sum:

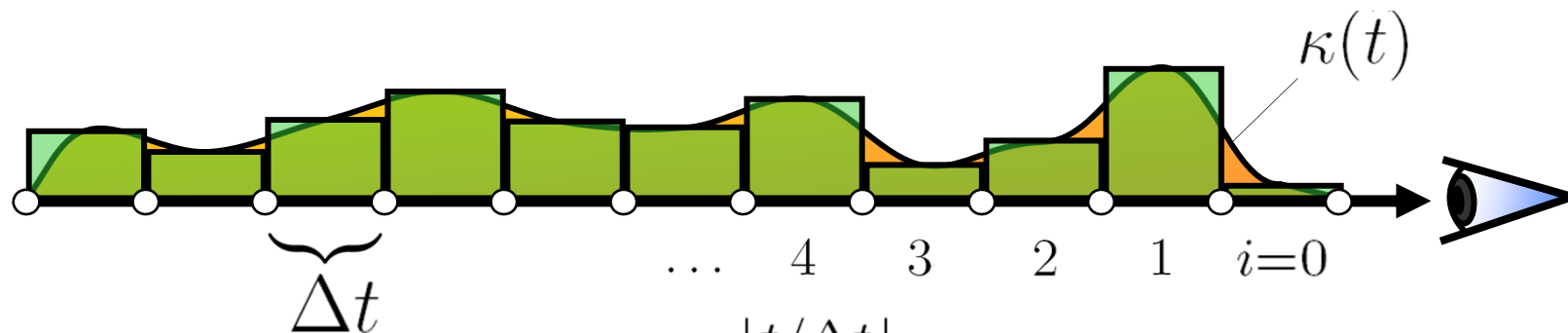
$$\tau(0, t) \approx \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

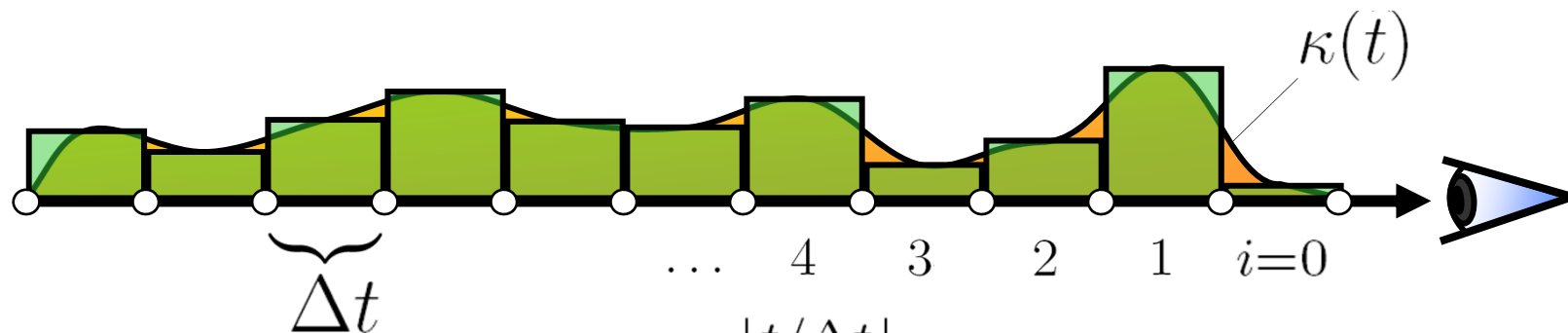
Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

$$e^{-\tilde{\tau}(0, t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t}$$

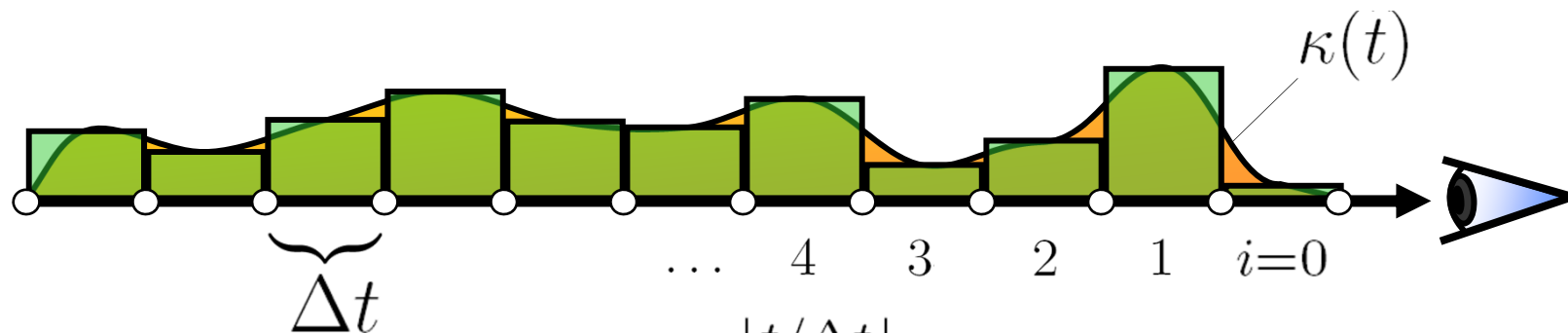
Volume Rendering Integral: Numerical Solution



$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

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Volume Rendering Integral: Numerical Solution



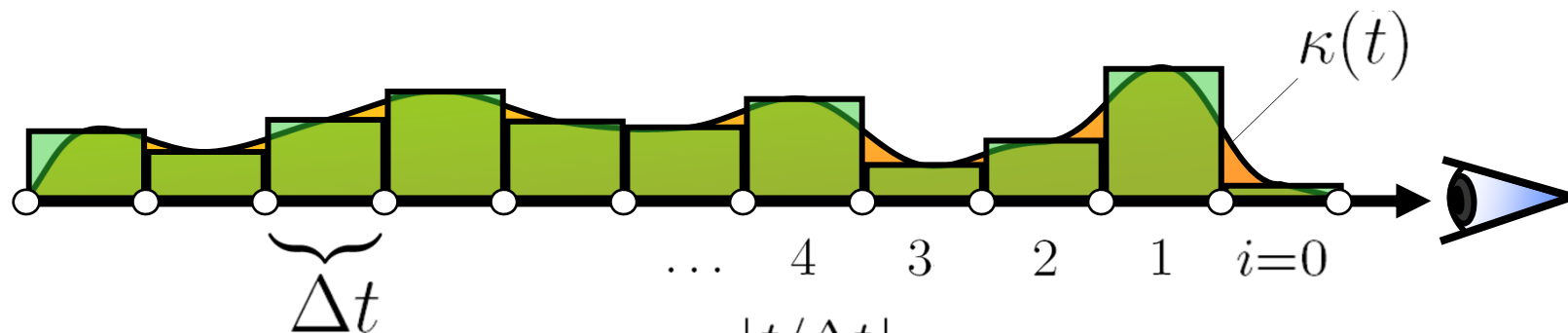
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$$e^{-\tilde{\tau}(0, t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Volume Rendering Integral: Numerical Solution



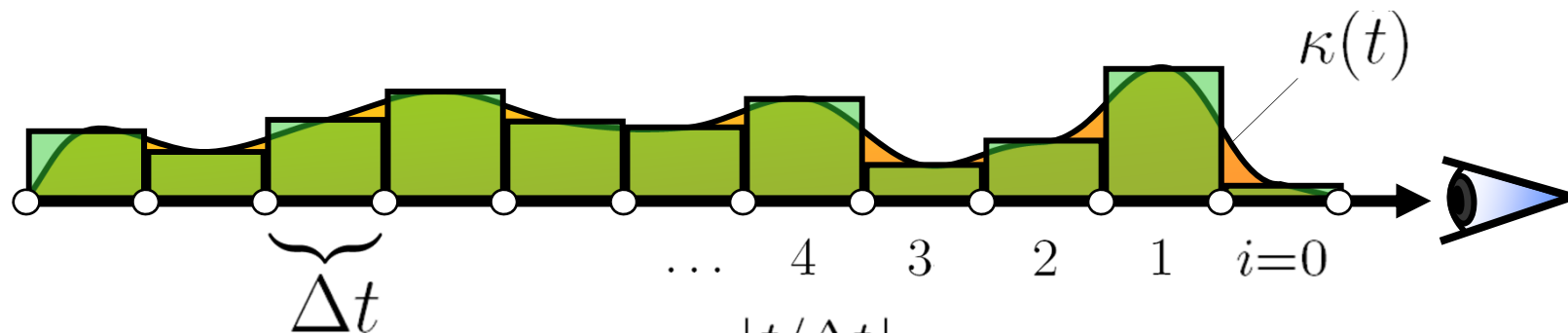
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$$1 - A_i = e^{-\kappa(i \cdot \Delta t) \Delta t}$$

Volume Rendering Integral: Numerical Solution



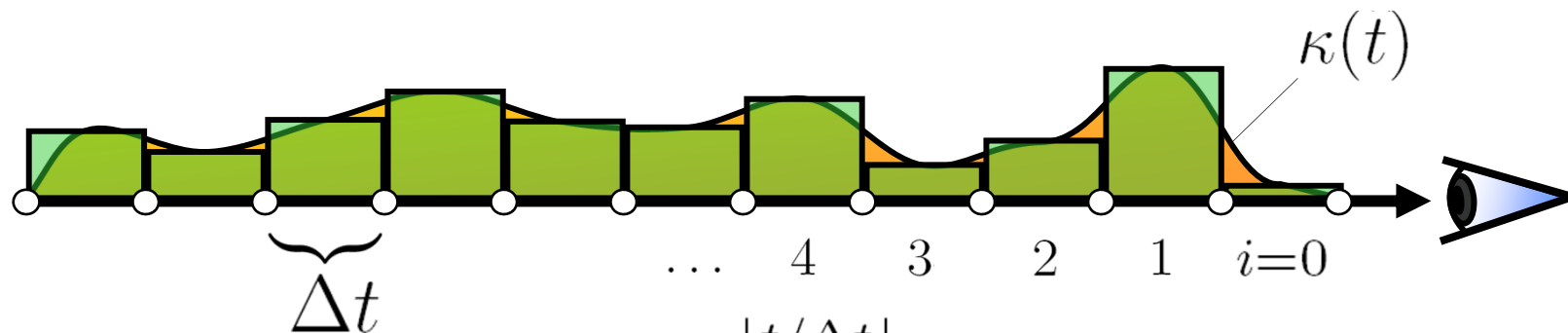
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Volume Rendering Integral: Numerical Solution



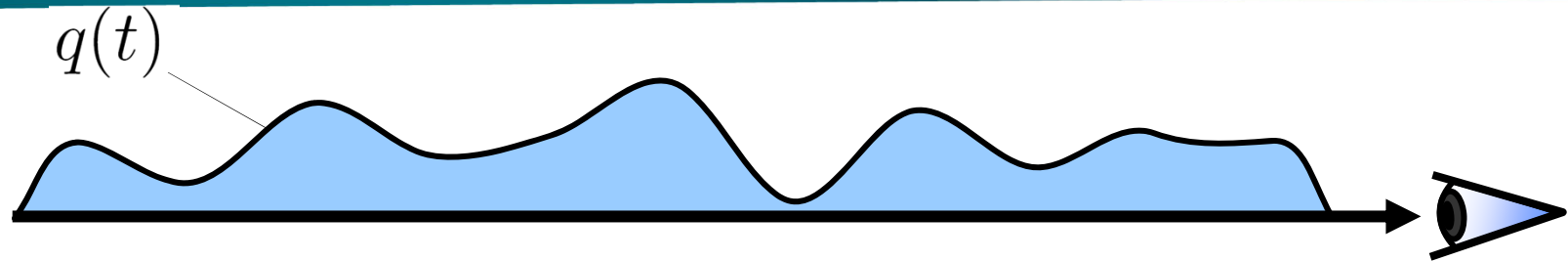
$$\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t) \Delta t$$

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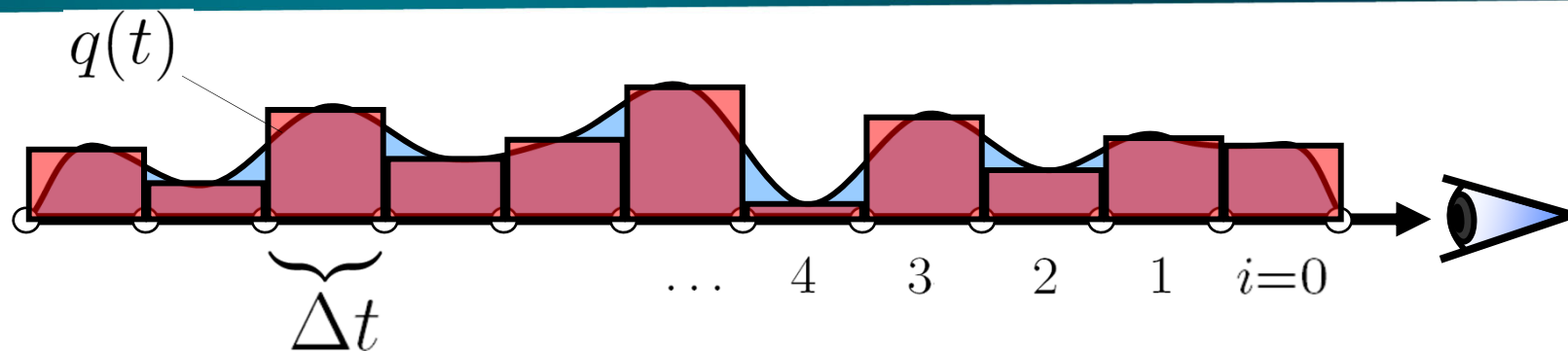
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Volume Rendering Integral: Numerical Solution



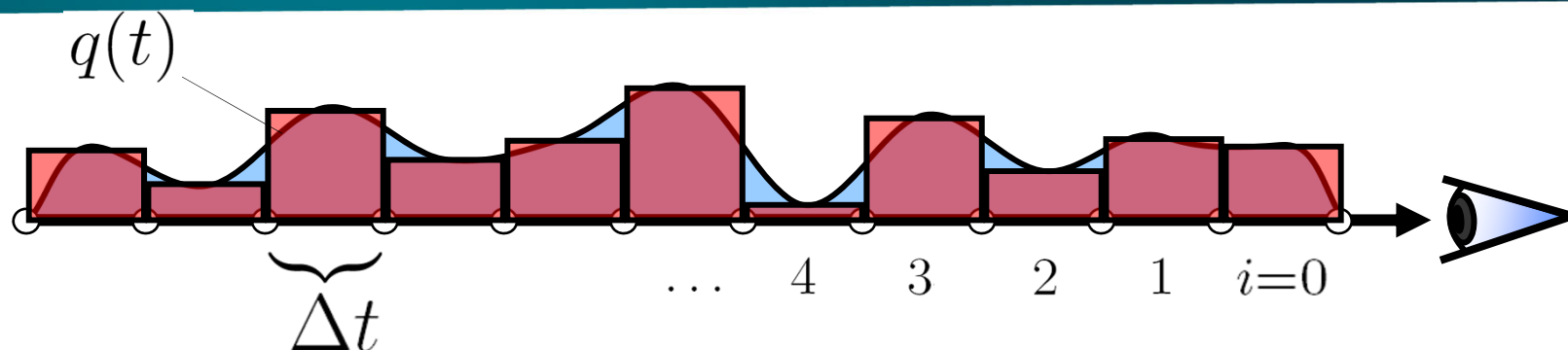
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t) \Delta t$$

Volume Rendering Integral: Numerical Solution

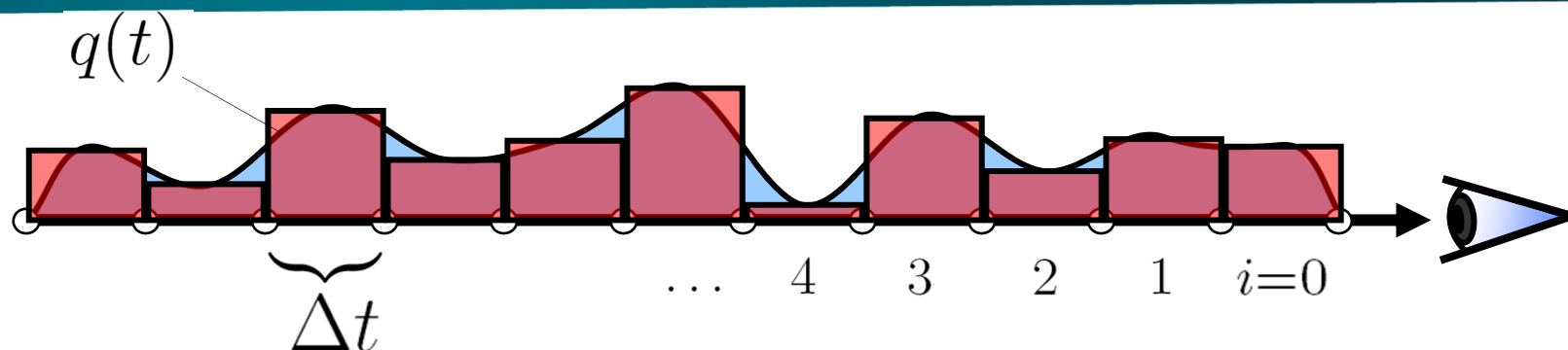


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$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution

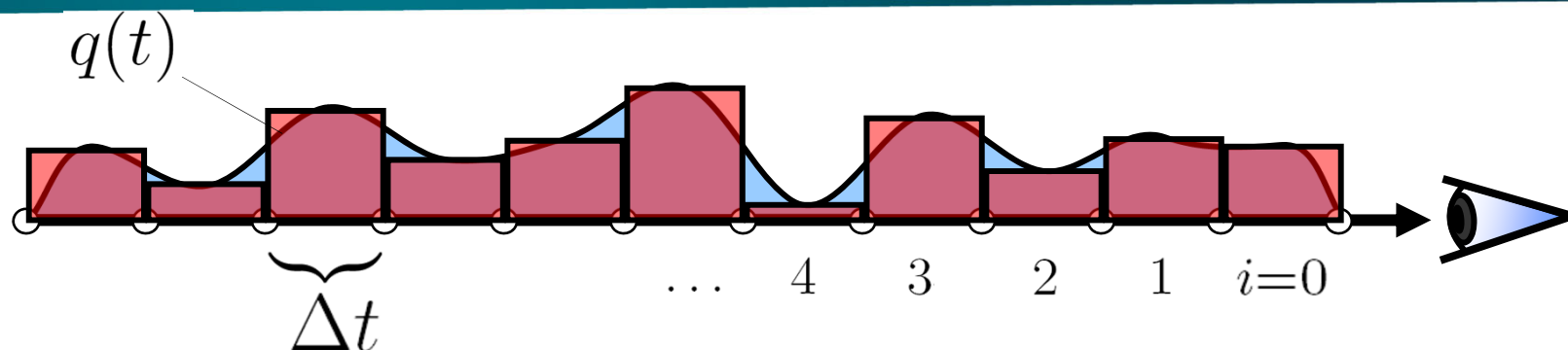


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Volume Rendering Integral: Numerical Solution

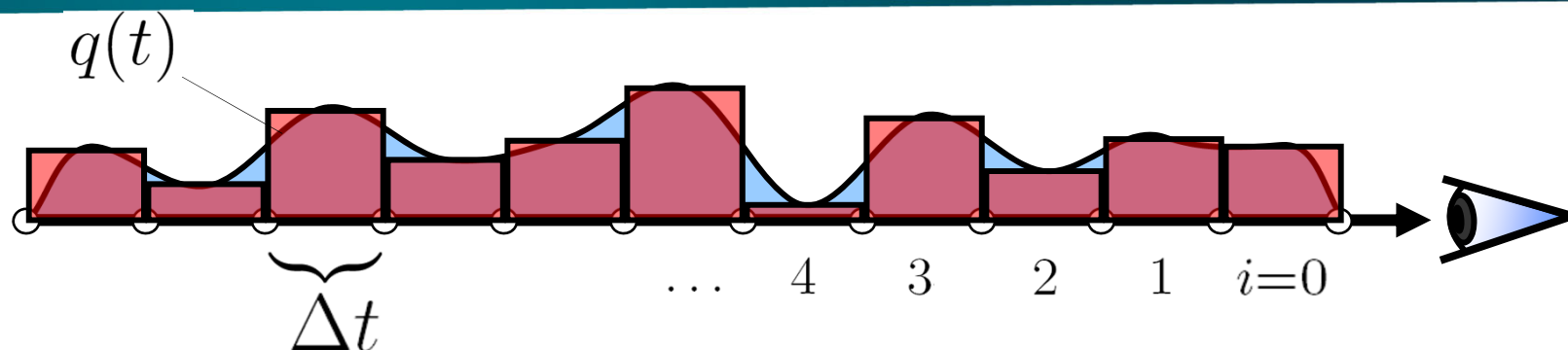


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Volume Rendering Integral: Numerical Solution



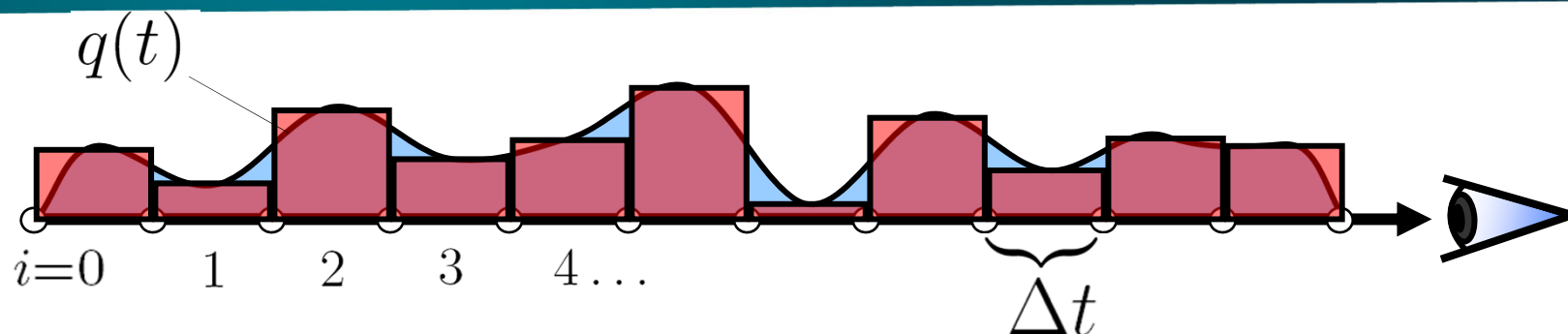
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can be computed iteratively/recursively!

Volume Rendering Integral: Numerical Solution



Note: we just changed the convention from $i=0$ is at the front of the volume (previous slides) to $i=0$ is at the back of the volume !

can be computed iteratively/recursively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

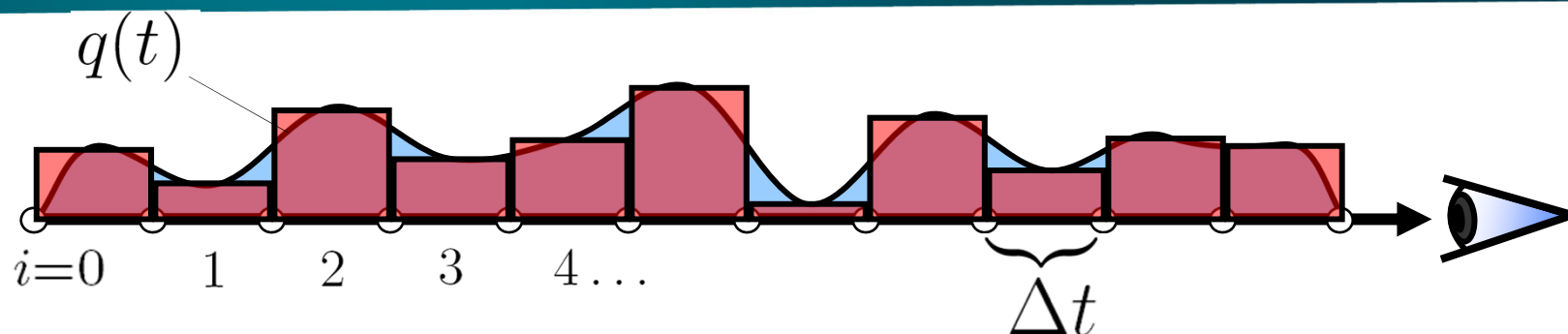
Radiant energy
observed at position i

Radiant energy
emitted at position i

Absorption at
position i

Radiant energy
observed at position $i-1$

Volume Rendering Integral: Numerical Solution



**Back-to-front
compositing**

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

iterate from $i=0$ (back) to $i=\max$ (front): i increases

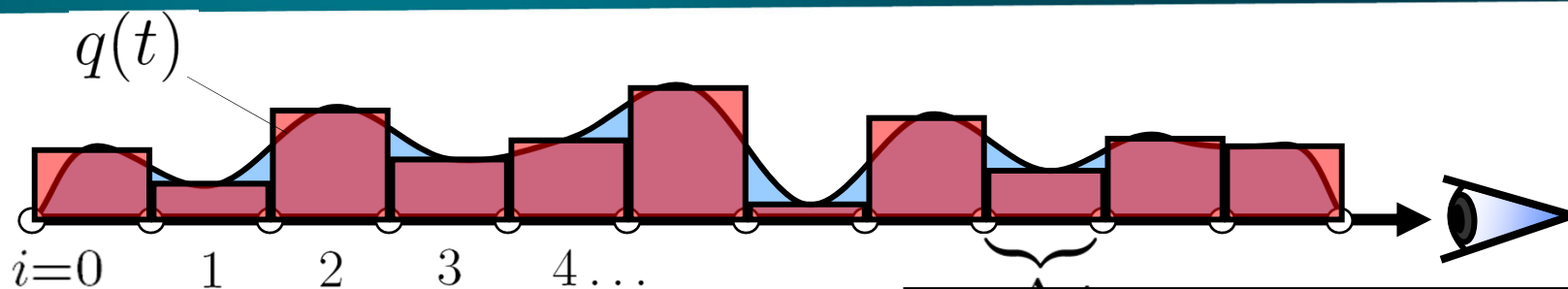
**Front-to-back
compositing**

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

iterate from $i=\max$ (front) to $i=0$ (back) : i decreases

Volume Rendering Integral: Numerical Solution



**Back-to-front
compositing**

$$C'_i = C_i + (1 - A'_i)C_i$$

iterate from $i=0$ (back)

Early Ray Termination:
Stop the calculation when

$$A'_i \approx 1$$

**Front-to-back
compositing**

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

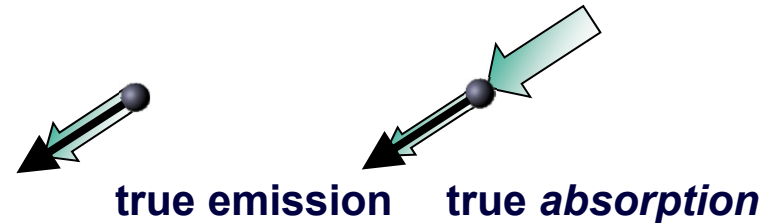
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iterate from $i=\text{max (front)}$ to $i=0$ (back) : i decreases

Volume Rendering Integral



Volume rendering integral
for *Emission Absorption* model



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Iterative/recursive numerical solutions:

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$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1})C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1})A_i \end{aligned}$$

here, all colors are *associated colors*!

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama