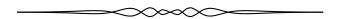
OLYMPIAD CORNER

No. 435

The problems featured in this section have appeared in a regional or national mathematical Olympiad.

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To facilitate their consideration, solutions should be received by November 15, 2025.



 $\mathbf{OC741}$. A triple of positive real numbers (a, b, c) is called *mysterious* if

$$\sqrt{a^2 + \frac{1}{a^2c^2} + 2ab} + \sqrt{b^2 + \frac{1}{b^2a^2} + 2bc} + \sqrt{c^2 + \frac{1}{c^2b^2} + 2ca} = 2(a+b+c).$$

Prove that if the triple (a,b,c) is mysterious, then the triple (c,b,a) is also mysterious.

OC742. Let $A \in \mathcal{M}_n(\mathbb{R})$ be an invertible matrix.

- (a) Show that the matrix AA^T has real and positive eigenvalues.
- (b) Suppose that there exist distinct positive integers p and q such that $(AA^T)^p = (A^TA)^q$. Prove that $A^T = A^{-1}$.

OC743. Let $(K, +, \cdot)$ be a division ring such that $x^2y = yx^2$ for all $x, y \in K$. Prove that $(K, +, \cdot)$ is a field.

OC744. Given a rectangle ABCD and a point X lying inside it. The bisectors of angles DAX and CBX intersect at point P. Point Q satisfies the equalities $\angle QAP = \angle QBP = 90^{\circ}$. Prove that PX = QX.

 $\mathbf{OC745}$. Let n be a positive integer. Bolek draws 2n points on the plane, no two of which define a vertical or horizontal line. Then for each of these 2n points, Lolek draws two rays starting at that point, one of which is vertical and the other horizontal. Lolek wants to do this in such a way that the rays drawn divide the plane into as many areas as possible. Determine the largest integer k such that Lolek can obtain at least k areas regardless of the position of the points chosen by Bolek.