Multiplying and dividing by powers of 10

When multiplying by 10, we are increasing the *place value* of every digit, so ones become tens, tens become hundreds, and so forth.

Example 1. $43 \times 10 = 430$

(Notice that we needed to include the 0 at the end as a place-holder, so that we know what place value the 4 and the 3 refer to.)

Note: Don't think of multiplication by 10 as simply adding a 0. This is indeed the result when multiplying an integer by 10, but gives the wrong idea for more advanced problems such as decimal multiplication. Instead, focus on the idea of place value shift.

Another way to think about the process is that instead of moving the digits to the left along a place value grid, we can move the decimal point to the right (adding place-holder 0s as needed). This allows us to understand decimal multiplication by 10 much easier

Example 2. $1.78 \times 10 = 17.8$

Notice the decimal moved one place to the right. "Adding 0 to the end" is incorrect here.

When multiplying by larger powers of 10, we repeat the process for each factor of 10.

Example 3.

$$\begin{aligned} 2.81 \times 100 &= 281, \\ 0.0127 \times 1000 &= 12.7, \\ 182.1 \times 100 &= 18210 \end{aligned}$$

To divide, we move in the opposite direction.

Example 4.

$$\begin{aligned} 2.81 \div 100 &= 0.0281, \\ 0.0127 \div 1000 &= 0.0000127, \\ 182.1 \div 100 &= 1.821 \end{aligned}$$

Note: make sure you are shifting in the right direction! Use a sanity check: should your answer be bigger or smaller? On a related note, make sure you are dividing left-to-right: $2.81 \div 100$ is *not* the same as $100 \div 2.81$.

Powers of 10 are just copies of 10 multiplied together. In particular, the exponent tells us how many copies of 10 there are, and thus how much we need to shift the decimal point.

Example 5.

$$\begin{aligned} 0.091 \times 10^3 &= 91, \\ 8.1824 \div 10^4 &= 0.00081824, \\ 213.01 \times 10^3 &= 213010 \end{aligned}$$