PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **November 15, 2025**.

5061. *Proposed by Nguyen Van Huyen.*

Consider the polynomial $f(x) = x^4 - ax^3 + 6x^2 - bx + c$. Suppose that $f(x)$ has four distinct real roots. Prove that

$$a^3b + 4b^2 + 256 \leqslant 12a(a + 2b).$$

5062. *Proposed by Mihaela Berindeanu.*

Let ABC be an acute triangle with $AC > BC$. The midpoint of AB is M, the orthocenter of ΔABC is H, and the feet of the altitudes from A, B, C are D, E, F, respectively. Let X be the point of intersection of AB and ED. If O is the circumcenter of ΔCMX, then prove that

$$\overrightarrow{OH} = \frac{2(\overrightarrow{OC} + \overrightarrow{OX}) + \overrightarrow{OA} + \overrightarrow{OB}}{2}.$$

5063. *Proposed by Bing Jian.*

Given a circle ω with center O and a line ℓ not tangent to ω, let m be the line passing through O and perpendicular to ℓ, and denote by A one of the points where m intersects the circle. For points P and Q on ℓ, let P^* and Q^* be their respective reflections in the mirror m, and let P' and Q' be the second points of intersection of the lines P^*A and Q^*A with ω. Prove that the cross-joins PQ' and QP' intersect on ω.

5064. *Proposed by Michel Bataille.*

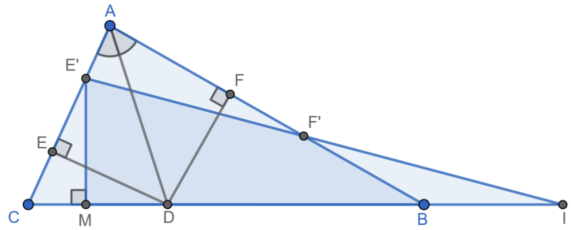
Let the sequence $(a_n)_{n\geq 1}$ be defined by $a_1 = 0$ and $a_{n+1} = a_n + \ln(2^n e^{a_n} - 1)$ for all $n \geq 1$. Evaluate

$$\ell = \lim_{n\to\infty} \frac{a_n}{2^n} \qquad \text{and} \qquad \lim_{n\to\infty} \frac{a_n - 2^n\ell}{n}.$$

5065. *Proposed by Yagub Aliyev.*

Let ABC be a triangle with acute angles at the vertices B and C, such that $\angle B < \angle C$. The angle bisector AD of the triangle ABC is drawn. Let DE and

DF be perpendiculars to the sides AC and AB, respectively. Let E' and F' be points on the sides AC and AB, respectively, such that $AE = CE'$ and $AF = BF'$. Let $E'M$ be perpendicular to the side BC. Prove that $E'M + E'I > AB + AC$.



5066. *Proposed by Tatsunori Irie.*

Let n be a positive integer. Initially, n stones – each coloured either white or black – are arranged in a single row. The game is played by repeatedly performing the following operation:

- Randomly select two white stones that are not adjacent (i.e. if two stones appear consecutively, they cannot be selected as a pair).
- Reverse the colour (i.e. switch from white to black or black to white) of every stone located between the two selected white stones.
- Finally, change the colours of the two chosen white stones to black.

The game terminates when no pair of white stones satisfying the above condition (that is, non-adjacent) can be selected.

Prove that, regardless of the initial configuration of the stones and irrespective of the order and combination in which the valid pairs of white stones are chosen, the game always terminates.

5067. *Proposed by Paul Bracken.*

Prove that

$$\sum_{n=1}^{\infty} \frac{4^n}{(2n-1)^2(4n+1)} \frac{\binom{2n}{n}}{\binom{4n}{2n}} = \frac{16}{9} \cdot \sqrt{2} - \frac{20}{9}.$$

5068. *Proposed by Nguyen Viet Hung.*

Prove that in any triangle ABC,

$$\frac{4R}{r} \geq \left(\frac{1}{r_a} + \frac{1}{r_b}\right) (\sqrt{r_a} + \sqrt{r_b})^2.$$

When does the equality happen?

5069. *Proposed by Michael Friday.*

Let ABC be a triangle in which $B - A = 90^\circ$. Let M and S be the feet of the median and symmedian, respectively, from vertex C. Prove that triangles ABC and CMS have the same orthocenter, and the circumcircle of CMS is internally tangent to the circumcircle of ABC at C.

5070. *Proposed by Vasile Cîrtoaje.*

Prove that 2 is the largest positive value of the constant k such that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}-4 \geq k(a+b+c+d-4)$$

for any positive real numbers a, b, c, d with at most one of them less than 1 and $ab+bc+cd+da=4$.

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Cliquez ici afin de proposer de nouveaux problèmes, de même que pour offrir des solutions, commentaires ou généralisations aux problèmes proposés dans cette section.

Pour faciliter l'examen des solutions, nous demandons aux lecteurs de les faire parvenir au plus tard le **15 novembre 2025**.

5061. *Soumis par Nguyen Van Huyen.*

Soit le polynôme $f(x) = x^4 - ax^3 + 6x^2 - bx + c$. Supposons que $f(x)$ ait quatre racines réelles distinctes. Montrez que

$$a^3b + 4b^2 + 256 \leqslant 12a(a+2b).$$

5062. *Soumis par Mihaela Berindeanu.*

Soit ABC un triangle aigu avec $AC > BC$. Le milieu de AB est M, l'orthocentre de ΔABC est H, et les pieds des hauteurs depuis A, B et C sont respectivement D, E et F. Soit X le point d'intersection de AB et ED. Si O est le centre du cercle circonscrit à ΔCMX, montrez que

$$\overrightarrow{OH} = \frac{2(\overrightarrow{OC}+\overrightarrow{OX})+\overrightarrow{OA}+\overrightarrow{OB}}{2}.$$