PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **February 15, 2026**.

5091. *Proposed by Tatsunori Irie.*

On a circle there are 2025 places. The numbers $1, 2, \ldots, 2025$ are written on them in some order (one number per place). A move consists of choosing three consecutive places with entries (x, y, z) in this cyclic order and replacing them by $(x+1, 2-y, z+1)$. In other words, we give 1 to each neighbour and then replace the middle entry by the negative of what remains. Show that, no matter how the numbers are arranged at the start, it is possible by finitely many moves to make all 2025 entries equal. Decide whether the common value is uniquely determined.

5092. *Proposed by Khuong Trang Tran Ngoc.*

Prove that the following inequality

$$\frac{1}{x^2+y^2+3z^2}+\frac{1}{y^2+z^2+3x^2}+\frac{1}{z^2+x^2+3y^2}\le\frac{3}{5}$$

holds for all positive real numbers $x \geq y \geq z$ such that $x^2+y^2+z^2+xyz=4$. When does equality occur?

5093. *Proposed by Michel Bataille.*

Let ABC be a triangle with $\angle A \neq 2\angle C$ and let D on side BC be such that AD bisects $\angle A$. Let Γ_1 be the circumcircle of ΔADB, Γ_2 be the circle through C and D orthogonal to Γ_1 and Γ_3 be the circle through A and C orthogonal to Γ_2. Let Γ_3 intersect the line BC at $M \neq C$ and its diameter through D intersect Γ_2 at $N \neq D$. Prove that the circumcenter of ΔDNM is on the line AC.

5094. *Proposed by Benjamin Braiman.*

Find a dense sequence of real numbers $\{x_n\}_{n\geq 1}$ such that

$$\lim_{n\to\infty}\frac{x_1+\cdots+x_n}{n}=0,$$

or show no such sequence exists. (Recall that a sequence is said to be dense if every open interval (a, b) contains an element of the sequence.)

5095. *Proposed by Nikolai Osipov.*

(a) Let x, y, z be positive integers such that $(x^2-1)(y^2-1) = z^2-1$. Show that x, y, z are pairwise coprime.

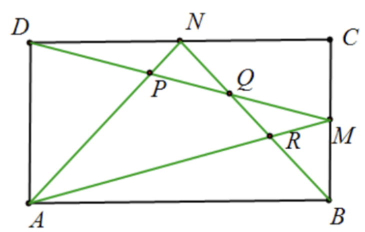
(b) Let $a \geqslant 2$ and $b \geqslant 1$ be integers such that b is a divisor of a^2-2. Prove that the equation
$$x^2-(a^2-1)y^2 = \frac{2-a^2}{b}$$
is solvable in integers x, y if and only if $b=1$.

5096. *Proposed by Vasile Cîrtoaje.*

Let a, b, c, d be nonnegative real numbers such that at most one of them is larger than 1 and $ab+ac+ad+bc+bd+cd=6$. Prove that
$$\frac{1}{(a+b+c)^2}+\frac{1}{(b+c+d)^2}+\frac{1}{(c+d+a)^2}+\frac{1}{(d+a+b)^2} \geq \frac{4}{9}.$$

5097. *Proposed by Xicheng Peng.*

Let $ABCD$ be a parallelogram. Let M and N be the midpoints of BC and CD, respectively. Let AM intersect BN at R, DM intersect BN at Q, and AN intersect DM at P. Prove that points A, P, Q, R are concyclic if and only if $BA \perp BC$.



5098. *Proposed by Mihaela Berindeanu, modified by the Editorial Board.*

Given a cyclic quadrilateral $BCAD$ (with A and B separating C from D) such that $CB = CD$, define A_1 and B_1 to be the feet of the altitudes from A and B in triangle ABC. Prove that the orthocenter H of ΔABC is the midpoint of AA_1 if and only if DB_1 is perpendicular to DB.

5099. *Proposed by Michel Bataille.*

Let s be a positive integer and let $a_0 = 1,\ a_k = \prod_{j=1}^{k}(s+j)$ for $k \geq 1$. Evaluate
$$\lim_{n\to\infty}\sum_{k=0}^{n}\binom{n}{k}\frac{a_k}{(2ns)^k}.$$

5100. *Proposed by Huseyin Yigit Emekci.*

Let $n \geq 2$ be an integer and let $a_i, \ldots, a_n$ be positive real numbers such that $a_1 + \cdots + a_n = 1$. Prove that

$$\sum_{k=2}^{n} \frac{a_k}{1-a_k}(a_1 + a_2 + \cdots + a_{k-1})^4 < \frac{1}{5}.$$

. . . .

Cliquez ici afin de proposer de nouveaux problèmes, de même que pour offrir des solutions, commentaires ou généralisations aux problèmes proposés dans cette section.

Pour faciliter l'examen des solutions, nous demandons aux lecteurs de les faire parvenir au plus tard le **15 fevrier 2026**.

5091. *Soumis par Tatsunori Irie.*

Sur un cercle, on dispose de 2025 emplacements. Les nombres $1, 2, \ldots, 2025$ y sont inscrits dans un certain ordre (un nombre par emplacement). Un coup consiste à choisir trois emplacements consécutifs portant les valeurs (x, y, z) dans cet ordre cyclique, et à les remplacer par $(x + 1, 2 - y, z + 1)$. Autrement dit, on ajoute 1 à chacun des voisins et on remplace la valeur du milieu par l'opposé de ce qui reste.

Montrez que, quelle que soit la disposition initiale des nombres, il est possible, en un nombre fini de coups, de rendre toutes les 2025 valeurs égales. Déterminez si la valeur commune obtenue est unique.

5092. *Soumis par Khuong Trang Tran Ngoc.*

Montrez que l'inégalité suivante

$$\frac{1}{x^2 + y^2 + 3z^2} + \frac{1}{y^2 + z^2 + 3x^2} + \frac{1}{z^2 + x^2 + 3y^2} \leq \frac{3}{5}$$

est vérifiée pour tous les réels positifs $x \geq y \geq z$ tels que $x^2 + y^2 + z^2 + xyz = 4$. Quand a-t-on égalité ?

5093. *Soumis par Michel Bataille.*

Soit ABC un triangle tel que $\angle A \neq 2\angle C$, et soit D un point du côté BC tel que AD soit la bissectrice de l'angle $\angle A$. Soit Γ_1 le cercle circonscrit au triangle ADB, Γ_2 le cercle passant par C et D et orthogonal à Γ_1, et Γ_3 le cercle passant par A et C et orthogonal à Γ_2. Soit $M \neq C$ le point d'intersection de Γ_3 avec la droite