PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **December 15, 2025**.

5071. *Proposed by Michel Bataille.*

Let triangle ABC be inscribed in a circle Γ with center O and let its incircle γ, with center I, touch BC at D. Let M be the midpoint of the arc BC of Γ containing A. If the line MD intersects γ at $U \neq D$ and Γ at $V \neq M$, prove that IU and OV are parallel.

5072. *Proposed by Seán M. Stewart.*

Consider the sequence of polynomials $\{P_n(x)\}_{n\geqslant 0}$ defined by the exponential generating function

$$\frac{1}{(1-x)e^t + x} = \sum_{n=0}^{\infty} P_n(x)\frac{t^n}{n!}.$$

Show that

$$\sum_{k=0}^{n} \binom{n}{k} \int_0^1 P_k(x)P_{n-k}(x)\,dx = (-1)^n.$$

5073. *Proposed by Yun Zhang.*

Let A_1, A_2, A_3, A_4 be the vertices of a tetrahedron with volume V, and let P be an arbitrary point in its interior. For each $k = 1, 2, 3, 4$, let M_k denote the midpoint of the segment A_kP. For each k, construct a plane that passes through M_k that is parallel to the face opposite A_k and intersects three edges of the tetrahedron. This divides the original tetrahedron into four internal tetrahedra of volumes V_k. Show that

$$V_1 + V_2 + V_3 + V_4 \geq \frac{27}{128}V,$$

with equality if and only if the point P is the centroid of the tetrahedron.

5074. *Proposed by Vasile Cîrtoaje.*

Let a, b, c, d be nonnegative real numbers such that at most one of them is larger than 1 and $ab + bc + cd + da = 4$. Prove that

$$\frac{1}{ab+2} + \frac{1}{ac+2} + \frac{1}{ad+2} + \frac{1}{bc+2} + \frac{1}{bd+2} + \frac{1}{cd+2} \geq 2.$$

5075. *Proposed by Matt Olechnowicz.*

Let m and k be positive integers. Show that

$$\binom{(m+1)k}{m} \leq (m+1)\binom{mk}{m}.$$

5076. *Proposed by Michael Friday.*

Let O and H be the circumcenter and orthocenter of triangle ABC satisfying $\angle B - \angle A = 90^\circ$. Prove that the circumcenters of triangles BCO, CAO, BCH, CAH, and vertex C, all lie on the same circle.

5077. *Proposed by Nguyen Viet Hung.*

Given a triangle ABC with the centroid G, let M be an arbitrary point inside the triangle. The lines that pass through M parallel to the median lines intersect the sides BC, CA, AB at X, Y, Z respectively. Prove that M, E, G are collinear, where E is the centroid of triangle XYZ.

5078. *Proposed by Tatsunori Irie.*

Let n be an integer such that $n \geq 2$ and let x be a positive integer. Show that the following holds:

$$\left(1+\frac{x}{n}\right)^n \geq x\left(\frac{1}{n-1}+1\right)^{n-1}.$$

5079. *Proposed by Torabi Dashti.*

For a square $ABCD$, let P be a point on AB and consider a triangle PQR, where Q is the point of intersection between DP and the diagonal AC, and R is on BC such that $\angle ADP = \angle BPR$. Prove that $PQ = PR$ if and only if $AP : PB = \frac{\sqrt{5}+1}{2}$.

5080. *Proposed by Nguyen Van Huyen.*

Let a, b, c, d be positive real numbers. Prove that

$$\frac{a^2b^2+a^2c^2+a^2d^2+b^2c^2+b^2d^2+c^2d^2}{abcd}+10 \geq (a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$

When does equality hold?

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