OLYMPIAD CORNER

No. 435

The problems featured in this section have appeared in a regional or national mathematical Olympiad.

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To facilitate their consideration, solutions should be received by **November 15, 2025**.

OC741. A triple of positive real numbers (a, b, c) is called *mysterious* if

$$\sqrt{a^2+\frac{1}{a^2c^2}+2ab}+\sqrt{b^2+\frac{1}{b^2a^2}+2bc}+\sqrt{c^2+\frac{1}{c^2b^2}+2ca}=2(a+b+c).$$

Prove that if the triple (a, b, c) is mysterious, then the triple (c, b, a) is also mysterious.

OC742. Let $A \in \mathcal{M}_n(\mathbb{R})$ be an invertible matrix.

(a) Show that the matrix AA^T has real and positive eigenvalues.

(b) Suppose that there exist distinct positive integers p and q such that $(AA^T)^p = (A^TA)^q$. Prove that $A^T = A^{-1}$.

OC743. Let $(K, +, \cdot)$ be a division ring such that $x^2y = yx^2$ for all $x, y \in K$. Prove that $(K, +, \cdot)$ is a field.

OC744. Given a rectangle $ABCD$ and a point X lying inside it. The bisectors of angles DAX and CBX intersect at point P. Point Q satisfies the equalities $\angle QAP = \angle QBP = 90^\circ$. Prove that $PX = QX$.

OC745. Let n be a positive integer. Bolek draws $2n$ points on the plane, no two of which define a vertical or horizontal line. Then for each of these $2n$ points, Lolek draws two rays starting at that point, one of which is vertical and the other horizontal. Lolek wants to do this in such a way that the rays drawn divide the plane into as many areas as possible. Determine the largest integer k such that Lolek can obtain at least k areas regardless of the position of the points chosen by Bolek.