PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **January 15, 2026**.

5081. *Proposed by Alex Roberts.*

Oscar bought a set of blank playing cards. He puts stamps on each card such that each card has $k \geq 4$ different stamps and

1. every two cards have exactly one stamp in common;
2. every stamp is used at least twice.

Show that the number of different stamps v he can use is in the range

$$\binom{k+1}{2} \leq v \leq k^2 - k + 1,$$

the number of different cards b he can use is in the range

$$k + 1 \leq b \leq k^2 - k + 1,$$

and that $V = \max v$ is at least $k^2 - 2k + 5$ for any k and equals $k^2 - k + 1$ for infinitely many k.

5082. *Proposed by Michel Bataille.*

Let $H_m = \sum_{k=1}^{m} \frac{1}{k}$. Evaluate

$$\lim_{n\to\infty} \frac{1}{(\ln n)^2} \sum_{k=1}^{n} \binom{n}{k} \frac{(-1)^{k-1} H_k}{k^3}.$$

5083. *Proposed by Vasile Cîrtoaje.*

Let a, b, c, d be nonnegative real numbers such that $a \geq b \geq c \geq d$ and $ab + bc + cd + da = 4$. Prove that

$$abc + bcd + cda + dab \leq 4.$$

5084. *Proposed by Xicheng Peng.*

Let ABC be a triangle. Let its incircle O touch the side BA at point F. Prove that the area of ABC is $AF \cdot FB \cdot \cot \frac{C}{2}$.

5085. *Proposed by Tran Ngoc Khuong Trang.*

Prove that the following inequality

$$\frac{\sqrt{2-bc}}{a+1}+\frac{\sqrt{2-ca}}{b+1}+\frac{\sqrt{2-ab}}{c+1}\geq 1+\sqrt{2}$$

holds for all non-negative real numbers a, b, c satisfying $ab+bc+ca=1$. When does equality occur?

5086. *Proposed by Mihaela Berindeanu.*

Let ABC be an acute triangle. The bisector of angle B cuts the perpendicular bisector of segment AB at C_1. The projection of point C_1 onto AB, respectively AC is C_2, respectively C_3. The points $B_1,\ B_2,\ B_3$ and $A_1,\ A_2,\ A_3$ are defined similarly. Show that:

$$\frac{AC_3}{AC_2}+\frac{BA_3}{BA_2}+\frac{CB_3}{CB_2}\leq\frac{BC}{BA}+\frac{CA}{CB}+\frac{AB}{AC}.$$

5087. *Proposed by Tatsunori Irie.*

Let $n\geq 2$ be an integer and let $a_1, a_2, \ldots, a_n$ be integers with $1\leq a_i\leq n$ $(1\leq i\leq n)$ such that $a_1+a_2+\cdots+a_n\equiv 0 \pmod{n}$. Show that if the a_i are not all equal, then there exists a non-empty proper subset $\{i_1, i_2, \ldots, i_j\}\subset\{1, 2, \ldots, n\}$ with $1\leq j<n$ such that $a_{i_1}+a_{i_2}+\cdots+a_{i_j}\equiv 0 \pmod{n}$.

5088. *Proposed by To An Ky.*

Let $ABCD$ be a parallelogram satisfying $BC=BD$. Let ω be a circle centered at the midpoint of segment CD, touching side BD at E. From A, construct segment AK tangent to ω at K ($K\notin\overleftrightarrow{AD}$). Show that $\angle BKE=90^\circ$.

5089. *Proposed by Tatsunori Irie.*

Consider the infinite nested radical obtained by starting from the innermost term $\sqrt{2}$ and wrapping outward choosing either $+$ or $-$ sign independently:

$$\cdots\sqrt{2\pm\sqrt{2\pm\sqrt{2\pm\sqrt{2}}}}.$$

Can one, by a suitable choice of the signs, obtain values arbitrarily close to any prescribed number in the interval $(0,2)$?

5090. *Proposed by Ion Pătraşcu.*

Let ABC be a triangle where O and I are the centres of its circumscribed and inscribed circles, and R and r are the radii of these two circles respectively, such that $R = r(1+\sqrt{2})$. Let D, E and F be the points where the inscribed circle touches BC, CA and AB. Let Ω_1, Ω_2, Ω_3 be the circumscribed circles pertaining to triangles BOC, COA and AOB, and D_1, E_1, F_1 be the points where these circles meet rays OD, OE and OF respectively. Prove that D_1, E_1, F_1 are collinear.

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Cliquez ici afin de proposer de nouveaux problèmes, de même que pour offrir des solutions, commentaires ou généralisations aux problèmes proposés dans cette section.

Pour faciliter l'examen des solutions, nous demandons aux lecteurs de les faire parvenir au plus tard le **15 janvier 2026**.

5081. *Soumis par Alex Roberts.*

Oscar a acheté un jeu de cartes à jouer vierges. Il appose sur chaque carte des timbres de sorte que chaque carte porte $k \geq 4$ timbres différents et que :

1. deux cartes quelconques ont exactement un timbre en commun ;
2. chaque timbre est utilisé au moins deux fois.

Montrez que le nombre de timbres distincts v qu'il peut utiliser est compris dans l'intervalle

$$\binom{k+1}{2} \leq v \leq k^2 - k + 1$$

et que le nombre de cartes distinctes b qu'il peut utiliser est compris dans l'intervalle

$$k+1 \leq b \leq k^2 - k + 1$$

Enfin, montrez que $V = \max v$ est au moins égal à $k^2 - 2k + 5$ pour tout k, et qu'il est égal à $k^2 - k + 1$ pour une infinité de valeurs de k.

5082. *Soumis par Michel Bataille.*

Soit $H_m = \sum\limits_{k=1}^{m} \frac{1}{k}$. Évaluez

$$\lim_{n\to\infty} \frac{1}{(\ln n)^2} \sum_{k=1}^{n} \binom{n}{k} \frac{(-1)^{k-1} H_k}{k^3}.$$

5083. *Soumis par Vasile Cîrtoaje.*

Soient a, b, c et d des nombres réels non négatifs tels que $a \geq b \geq c \geq d$ et $ab + bc + cd + da = 4$. Montrez que

$$abc + bcd + cda + dab \leq 4.$$