

SOLUTIONS

1. (2 points) The base of a solid is the region bounded by $y = x^{1/3}$, $y = 3$, and $x = 1$. Cross-sections perpendicular to the x -axis are rectangles whose heights are twice their base. Set up an integral (or integrals) that represents the volume of the solid.

Solution:

$$\int_1^{27} 2(3 - x^{1/3})^2 dx$$

2. (2 points) Consider the region R bounded by $y = \arctan x$, $x = 1$, and $y = 1$. Find an integral (or integrals) that represents the volume of the solid of revolution if we revolve the region R about the line $x = -1$.

Solution:

$$\pi \int_{-\pi/4}^1 (\arctan y + 1)^2 dy$$

3. (2 points) Set up an integral (or integrals) that represents the volume of the solid generated by revolving the region above $y = x^3$, below the line $y = 8$, and between $x = 0$ and $x = 2$ around the x -axis.

Solution:

$$\pi \int_0^2 [(4 - x^3)^2 - (x^3)^2] dx$$

4. (2 points) Consider the region R that is bounded by $y^2 = 7x + 8$ and $y = x + 2$. If R is the base of a solid and cross-sections perpendicular to the y -axis are semi-circles, set up an integral (or integrals) that represents the volume of the solid.

Solution:

$$\int_1^6 \frac{\pi}{2} \left(\frac{(y-2) - \frac{1}{7}(y^2-8)/2}{2} \right)^2 dy$$

5. (2 points) Let R be the region enclosed by $y = x^2 - 4$ and $y = 2x + 4$. Find an integral (or integrals) that represents the perimeter of the region R .

Solution:

$$\int_{-2}^4 \sqrt{1 + (2)^2} dx + \int_{-2}^4 \sqrt{1 + (2x)^2} dx \quad \text{or} \quad 6\sqrt{5} + \int_{-2}^4 \sqrt{1 + 4x^2} dx$$

6. (2 points) Let R be the region in the first quadrant below $y = 4 - x^2$. Find an integral (or integrals) that represents the volume of the solid of revolution if we revolve the region R about the line $y = 4$.

Solution:

$$\pi \int_0^2 [4^2 - (4 - (4 - x^2))^2] dx = \pi \int_0^2 [4^2 - x^4] dx$$