Binomial Random Variable

Problem Statement:

A homeowner has just installed 20 light bulbs in a new home. Suppose that each has a probability of 0.2 of functioning for more than three months. What is the probability that at least five of these function more than three months?

Solution:

The probability of at least 5 out of 20 light bulbs functioning for more than three months can be calculated using the binomial probability formula. The formula for the binomial probability of having at least k successes in n trials is given by:

$$P(X \geq k) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

Here:
- n = 20 is the total number of trials (light bulbs).
- p = 0.2 is the probability of success (a bulb functioning for more than three months).
- k = 5 is the number of successes we are calculating the probability for (at least 5 bulbs functioning).

Thus, the probability is calculated as:

$$P(X \geq 5) = \sum_{i=5}^{20} \binom{20}{i} (0.2)^i (0.8)^{20-i}$$

In this expression:

- $\sum_{i=5}^{20}$ denotes the summation from $i = 5$ to 20.
- $\binom{20}{i}$ is the binomial coefficient, representing the number of ways to choose i successes out of 20 trials.
- $(0.2)^i$ is the probability of success raised to the power of i.
- $(0.8)^{20-i}$ is the probability of failure (1 minus the probability of success) raised to the power of $20 - i$.

After calculation, the probability is approximately 0.3704 or 37.04%.

Standard Normal Distribution

1. **Calculate Individual Probabilities:**
   - For $P(X = 0)$: Use $\binom{20}{0} \cdot (0.2)^0 \cdot (0.8)^{20}$
   - For $P(X = 1)$: Use $\binom{20}{1} \cdot (0.2)^1 \cdot (0.8)^{19}$
   - For $P(X = 2)$: Use $\binom{20}{2} \cdot (0.2)^2 \cdot (0.8)^{18}$
   - For $P(X = 3)$: Use $\binom{20}{3} \cdot (0.2)^3 \cdot (0.8)^{17}$
   - For $P(X = 4)$: Use $\binom{20}{4} \cdot (0.2)^4 \cdot (0.8)^{16}$
2. **Sum These Probabilities:**
   - Sum $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$
3. **Calculate the Final Probability:**
   - Subtract the sum from 1: $P(X \geq 5) = 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4))$

Problem Statement: The annual snowfall at a particular geographic location is modeled as a normal random variable with a mean of $= 60$ inches, and a standard

deviation of $= 20$. What is the probability that this year's snowfall will be at least 80 inches?

Solution:

Given that the annual snowfall is modeled as a normal random variable.

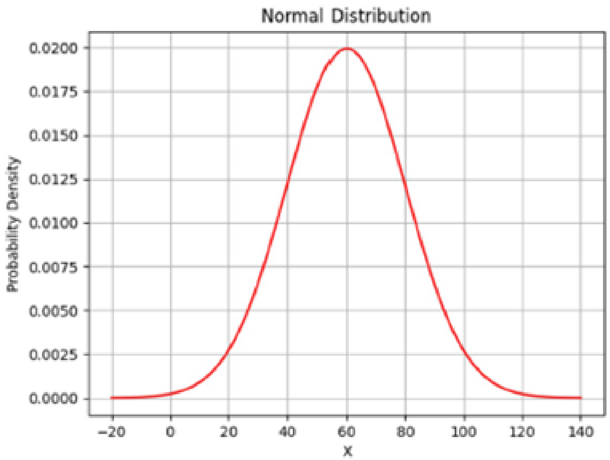
To find the probability that this year's snowfall will be at least 80 inches, we can use the standard normal distribution.

The standard normal distribution has a mean () of 0 and a standard deviation () of 1. We need to convert the value 80 inches to a z-score using the formula:

$$Z = \frac{X - \mu}{\sigma}$$

where:

- Z is the z-score,
- X is the value from the distribution (in this case, 80 inches),
- is the mean of the distribution,
- is the standard deviation of the distribution.



$\mu = 60$
$\sigma = 20$

For this problem:

$$\mathbf{Z} = \frac{\mathbf{80 - 60}}{\mathbf{20}}$$

$$\mathbf{Z = 1}$$

We need the area shaded in green in the following graph.

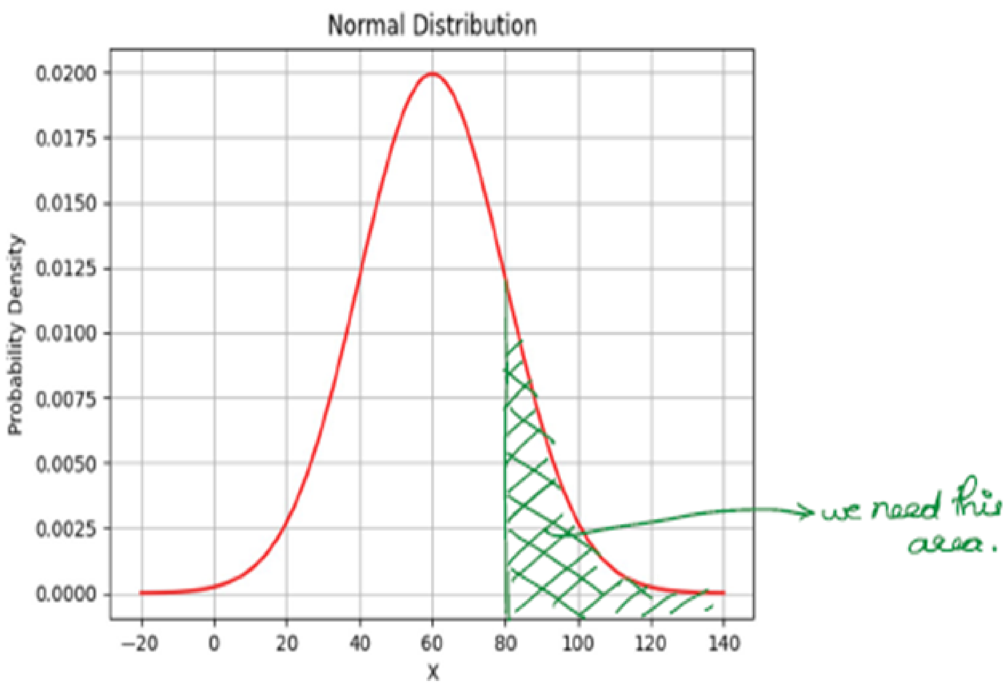
To find the area under the following curve:

$$\mathbf{F_X\left(20 \leq X \leq 100\right)} = \int_{\mathbf{20}}^{\mathbf{100}} \frac{\mathbf{1}}{\sigma\sqrt{\mathbf{2}\pi}} \exp\left(-\frac{\left(\mathbf{x}-\mu\right)^{\mathbf{2}}}{\mathbf{2}\sigma^{\mathbf{2}}}\right) \mathrm{dx}$$

$$\mathbf{P\left(X \geq 80\right) = P\left(\frac{X - 60}{20} \geq \frac{80 - 60}{20}\right) = P\left(Y \geq 1\right)}$$

From standard normal distribution table: $(1) = 0.84134$

$$\mathbf{P\left(X \geq 80\right) = 1 - 0.84134 = 0.1587}$$


Normal Distribution
Probability Density
0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
−20
0
20
40
60
80
100
120
140
X
we need this
area.