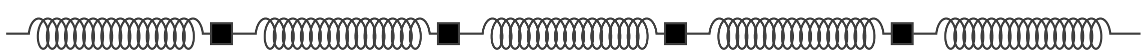
December 8, 2021

Interaction

- Interaction Picture
- **Time dependent perturbation theory
  - Perturbative solution: Dyson series
  - esp. for constant interaction term, specific time interval
  - Fermi's Golden Rule

Discrete to continuum



- Displacement of n^{th} mass : ϕ_n
- Corresponding discrete Lagrangian:
  L $= \sum_{n=1}^{N} \frac{1}{2} m \dot{\phi}_n^{\,2} - \frac{1}{2} k_s [\phi_{n+1}(t) - \phi_n(t)]$
- **Continuum Limit: $\sum_{n=1}^{N} \to \frac{1}{a} \int_0^{Na} dx$
  $\implies$ and $\phi_n(t) \to \sqrt{a}\phi(x,t)$ s.t. $\phi(x,t) \,\epsilon\, [0, L]$
- $L \to \int_0^L \mathcal{L} dx = \int_0^L dx \left[\frac{1}{2} m \dot{\phi}^2 - \frac{1}{2} k a^2 \left(\frac{\partial \phi}{\partial t} \right)^2 \right]$

Cl.FT

- **Lagrangian/Hamiltonian to Lagrangian/Hamltonian densities
  - Q: Require: Lagrangian (density) for variational principle and relativistic theories, Hamiltonian (density) for CM to QM transitions. But what if the Legendre Transformation connecting the Lagrangian to Hamiltonian doesn't exist?
- Noether theorem and conserved currents, corresponding stress energy tensor.
- Solved several problems from S.Govindrajan's (IITM) course and from Alan Guth's (MIT) course.

KG Equation

- $(\partial_\mu \partial^\mu + m^2)\, \phi(x) = 0$
  - negative currents and probabilities.
- **Minimal Coupling and Klein Paradox
  - Q: Do we always use minimal coupling in F.T.s to define a particle interacting with an em field?
  - Q: Where did this minimal coupling come from in the first place?

Dirac Equation

- $\not{p}\psi = m\psi \implies (i\not{\partial}_\mu - m)\psi = 0$, $H = \vec{\alpha} \cdot \vec{p} + \beta m$
- **$[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$

Representations

- SU(2): Isospin algebra
- SO(3): Angular Momentum (Lorentz) algebra

- SO(3) Lie Algebra $\equiv$ SO(2) Lie Algebra

- **weak

**Ordering

- Q: Why do we prefer Normal ordering over Weyl ordering?

KG Field

- Both Complex and Real Fields
- Wavefunction becomes a linear decomposition in Fourier space via annhilation and creation operators
  - While solving a problem, realised that $\exists$ as many annhilation/creation operators (in the decomposition) as there are available degrees of freedom.
- Spin 0 fields
- Massive and massless spin 1 fields don't follow from $\phi \to A_\mu$
  - For massive case:
    the real KG field decomposes into 4 KG fields if $\partial_\mu A_\mu \neq 0$ i.e. $4 = 1 \bigoplus 1 \bigoplus 1 \bigoplus 1$

    But if $\partial_\mu A_\mu = 0 \ \exists \ (4-1) DOFs \implies 4 = 3 \bigoplus 1$ (temporal gauge)

    $\mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu = -F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$
  - For massless case:
    putting m = 0 doesn't work as longitudinal DOF $\to \infty$,
    but from EMT DOF(photon) = 2 polarizations

    Additional constraint: Gauge fixing (1st Gauge theory that I encountered) i.e.
    $\mathcal{L} \to \mathcal{L}' = \mathcal{L}$ if $A_\mu \to A_\mu + \partial_\mu \Lambda$

EM Lagrangian: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \implies \partial_\mu \partial^\mu = 0$ i.e EM Wave equation

Dirac Field

- $\mathcal{L} = \bar{\psi}(i\gamma\partial_\mu - m)\psi$
- **Q: Why do we insert a factor of $i\epsilon$ in propagators? Somewhere: to maintain causality... mass shell and particle travelling outside the light cone. In 1 loop corrections if the particle lies on the mass shell $\implies D^r \to 0$.

Covariant Derivative

- Used while dealing with local symmetries
- Follows a general procedure:
  1. Start with a free field Lagrangian $\mathcal{L}(\psi, \partial_\mu\psi)$
  2. $\mathcal{L}$ must be symmetric under $\mathcal{U}(1)$ global symmetries
  3. $\mathcal{L}$ assymetric under local $\mathcal{U}(1)$ symmetries
  4. $\partial_\mu \to \mathcal{D}_\mu = \partial_\mu - i$ (coupling constant)(field variable)
  5. Other interaction terms could be added now.
- $\mathcal{D}_\mu{}'\psi'(x) = \mathcal{U}(x)\mathcal{D}_\mu\psi(x)$
- **Q: Does the procedure carries on in non-Abelian gauge theories since $\mathcal{U}(1)$ is Abelian? (Commutator relations to take care of)
- **Glanced through W fields: how amusing that W,Z vector bosons are described by scalar fields...

Scattering

- $2 \to 2$ scattering

- $\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{64\pi^2 E_{CM}^2}\frac{|\vec{p_f}|}{|\vec{p_i}|}|\mathcal{M}^2|\theta(E_{CM} - m_3 - m_4)$

- $2 \to N$ scattering: extend the result over the phase space of product particles

- **Not entirely comfortable with this procedure, esp. the derivation using S matrix and transfer matrix

Feynman Diagrams Wick's Theorem

- QED Lgrangian: $\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\mathcal{L})_{\text{fix}}$
  where $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$

- **Couldn't study beyond tree level diagrams (mathematically)... trouble with Wick's theorem and Dyson series' perturbative corrections in the diagrams

- But, now I understand what LO, NLO, NNLO, ... mean. Can(?) compute the matrix element squared for QED lagrangian using the direct Feynman rules.

Further Protocol for the theoretical background

- Representation Theory (remaining parts)

- Wick's theorem and applications of perturbative corrections in Feynman Diagrams and looped connections.

- QCD Lagrangian & Parton Distributions

- Deep Inelastic Scattering (prev s.in.)

- Standard Model