UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 01

Date: January 21, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 01 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

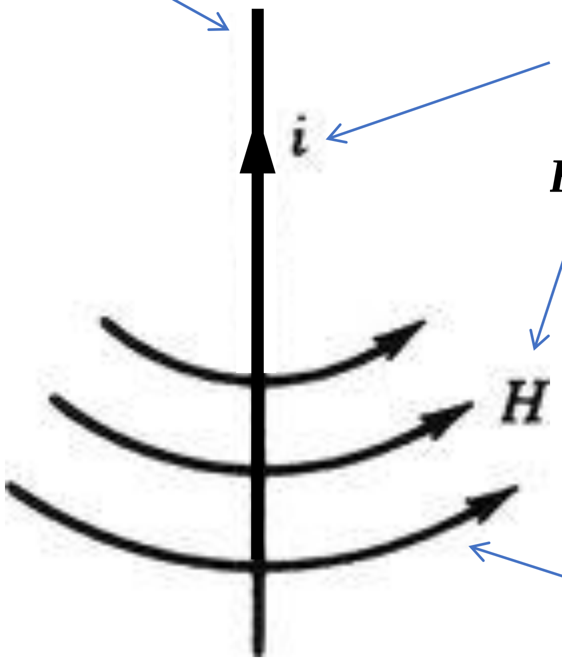
MAGNETIC CIRCUITS

i – *H* RELATION

Conductor (e.g., a copper wire)

i : Current through the conductor [ampere] or [A]

H : Magnetic field intensity [ampere/meter] or [A/m]



Alternative Names for Magnetic field intensity ***H***:

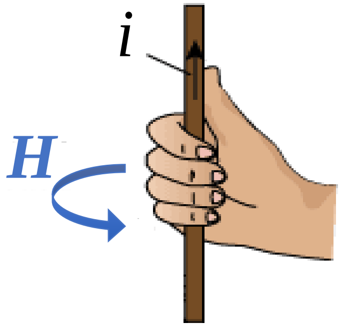
i) Magnetic field

ii) Flux

Flux line

Thumb Rule

If the conductor is held using the right hand with the thumb pointing in the direction of the current, then, the fingertips indicate the direction of the magnetic field.



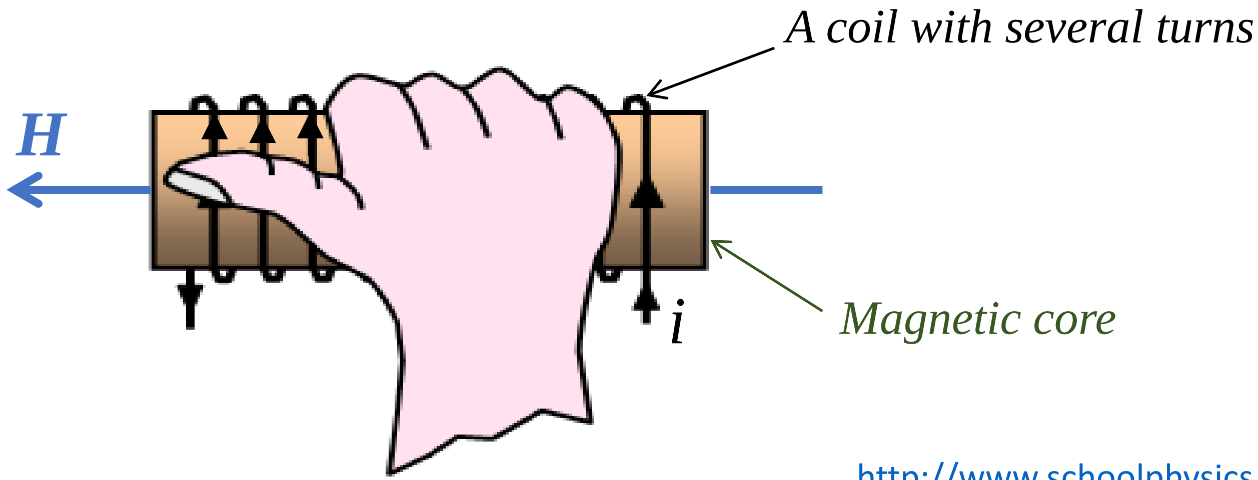
Right-Hand Rule for a Straight Wire

If you hold the wire in your right hand with the thumb pointing in the direction of the current *i*, then, the fingers point the direction of the magnetic field *H*.

https://www.brainscape.com

Right-Hand Rule for a Coil with Turns

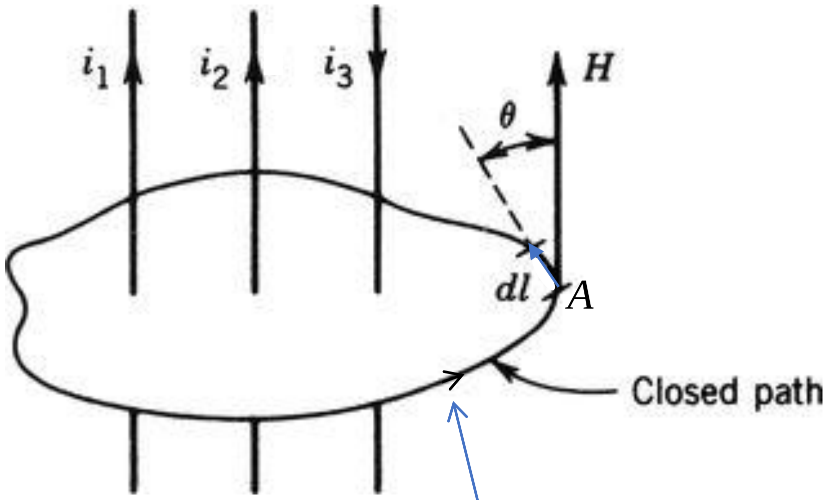
If you grip the coil in your right hand with the fingers pointing in the direction of the current *i*, the thumb points in the direction of the magnetic field *H*.



http://www.schoolphysics.co.uk

Ampere's Circuit Law

The line integral of the magnetic field intensity H around a closed loop (path) is equal to the algebraic some of the currents which pass through the loop.



$$\oint \underbrace{H \cdot \cos\theta \cdot dl}_{} = \sum i = i_1 + i_2 - i_3$$

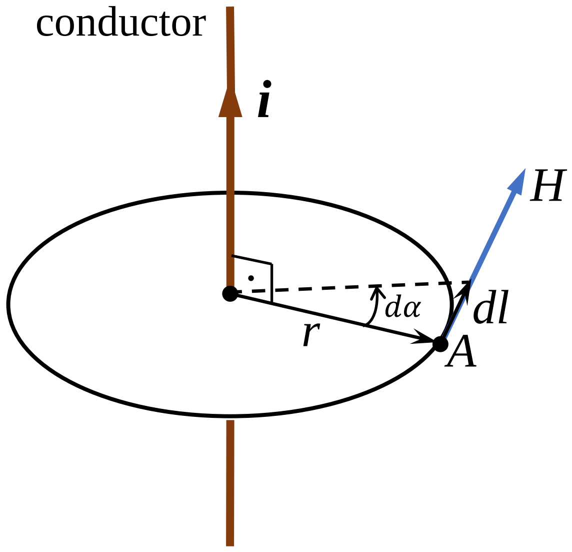
Projection of H onto dl.

In the algebraic sum, the positive direction for the currents is determined by the right-hand rule. With the fingers in the direction of dl (the incremental length at the point on the path), the thumb points in the positive direction.

The direction around the closed path, which is specified by the user.

dl : Differential tangent line at point A of the closed path.

Magnetic Field Intensity at a distance *r* from the straight conductor carrying current *i*



dl : Differential tangent line at point A of the circle.

Step 1: Draw a circle of radius r around the conductor.
Assumption: The conductor passes through the center of the circle at a right angle.

Step 2: Find the angle between H and dl.
Remark: Due to the above assumption, H and dl are in the same direction at each point on the circle. That is, $\theta = 0$.

Step 3: Use the symmetry.
H is the same at all points on the circle because of the symmetry.

$$\oint H \cdot \cos\theta \cdot dl = \oint H \cdot dl = i$$

$$\oint H \cdot dl = \int_{\alpha=0}^{2\pi} H \cdot r \cdot d\alpha = H \cdot 2\pi \cdot r$$

$$\Rightarrow \boxed{H = \frac{i}{2\pi \cdot r}}$$

$\boldsymbol{B-H}$ *RELATION*

$\boldsymbol{B}$: Magnetic flux density [tesla], [T], or, weber/m², [Wb/m²]

$\boldsymbol{H}$: Magnetic field intensity [ampere/meter], [A/m], or [ampere-turn/meter], [At/m]

$$B = \mu \cdot H$$

$\boxed{\text{Since } \mu \text{ is a positive scalar, } B \text{ and } H \text{ have the same direction.}}$

$$\mu = \mu_r \cdot \mu_0$$

μ : Permeability of the medium or material [henry/meter], [H/m]

Permeability is the degree of magnetization that a material obtains in response to an applied magnetic field.

μ_0 : Permeability of free space. It is $4 \cdot \pi \cdot 10^{-7}$ [H/m].

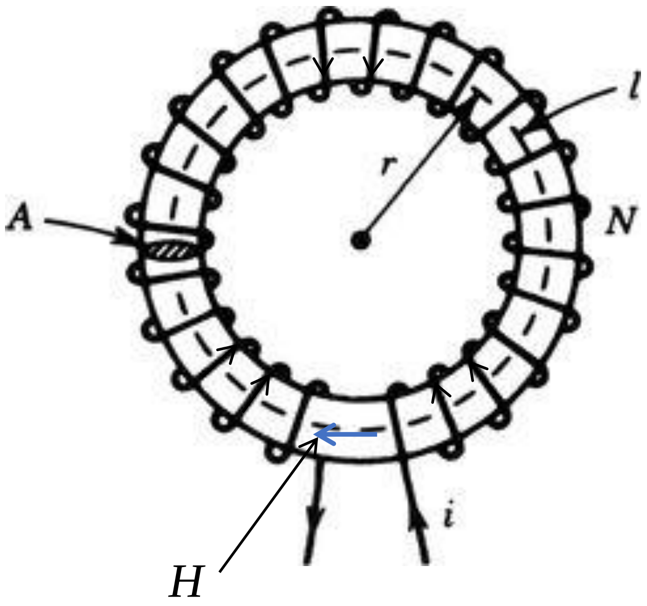
μ_r : Relative Permeability of the medium. No units.

μ_r = 1.0 for free space, electrical conductors such as copper, and insulators.

μ_r = 2000 – 6000 for ferromagnetic materials such as iron alloys (e.g., silicon steel used in electric machines).

A large value of μ_r implies that a small current can produce a large magnetic flux density in the electric machine.

MAGNETIC EQUIVALENT CIRCUIT OF TOROID



Toroid : A ring-shaped magnetic core

A coil of N turns is uniformly distributed around the toroid core.

r : The mean radius of the core.

l : Length of the closed path with radius r.

A : Cross sectional area of the core.

i : Current flowing through the coil.

H : Magnetic field intensity on the closed path.
Assumption: H is constant.

Ampere's Circuit Law $\Rightarrow$ $$H \cdot l = H \cdot 2\pi \cdot r = N \cdot i$$

Magnetomotive Force (MMF)

$F = N \cdot i$ [ampere-turn], [At]

MAGNETIC EQUIVALENT CIRCUIT OF TOROID

$$H = \frac{N \cdot i}{l} \qquad B = \frac{\mu \cdot N \cdot i}{l}$$

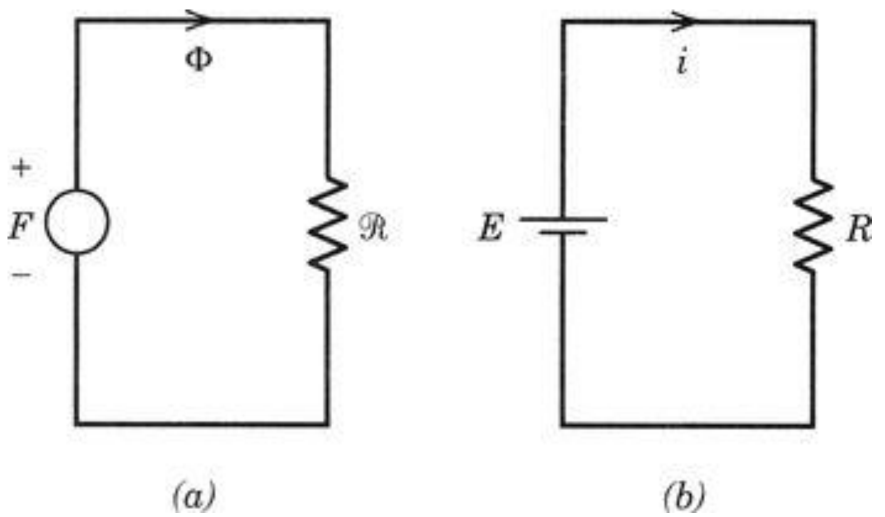
Φ : Magnetic flux crossing the cross section of the toroid [weber], [Wb]

$$\Phi = \int B \cdot dA = B \cdot A = \frac{\mu \cdot N \cdot i \cdot A}{l} = \frac{N \cdot i}{l/(\mu \cdot A)} = \frac{F}{\Re}$$

$\Re = \dfrac{l}{\mu \cdot A}$ Reluctance of the magnetic path [At/Wb]

$P = \dfrac{1}{\Re}$ Permeance of the magnetic path [Wb/At]

MAGNETIC EQUIVALENT CIRCUIT



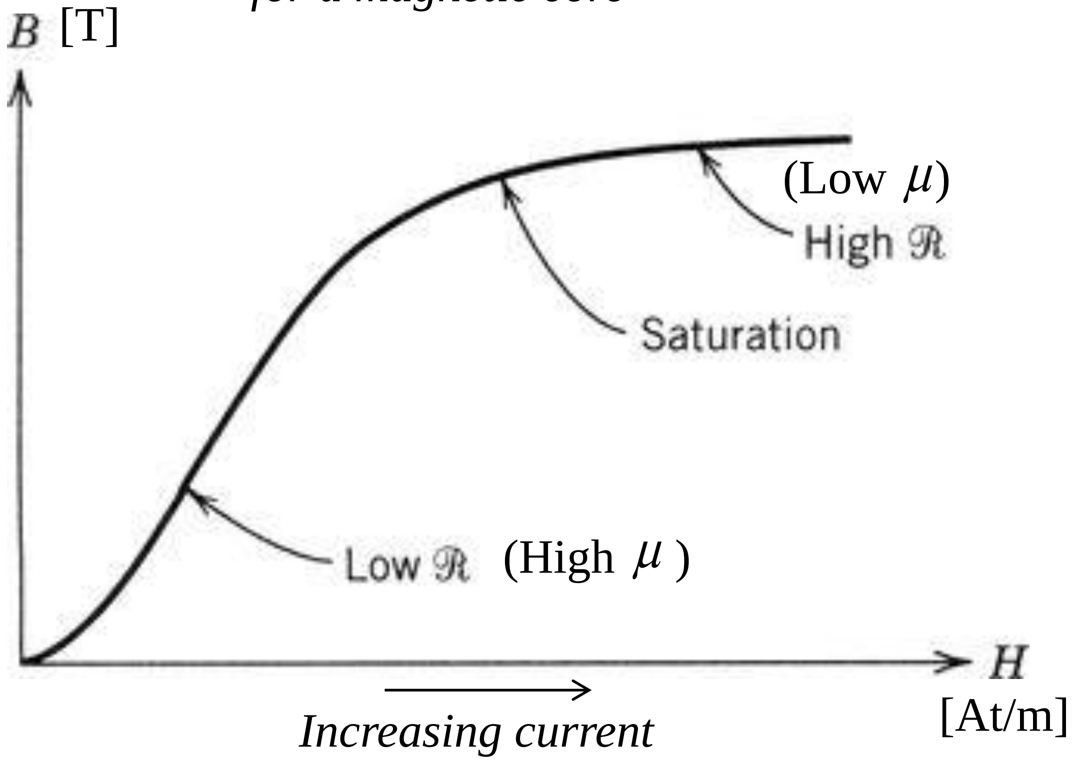
Analogy between *(a)* magnetic circuit and *(b)* electric circuit

	Electric Circuit	**Magnetic Circuit**
Driving force	Emf (E)	Mmf (F)
Produces	Current ($i = E/R$)	Flux ($\Phi = F/\mathscr{R}$)
Limited by	Resistance ($R = l/\sigma A$)[a]	Reluctance ($\mathscr{R} = l/\mu A$)[a]

[a] σ, Conductivity; μ, permeability.

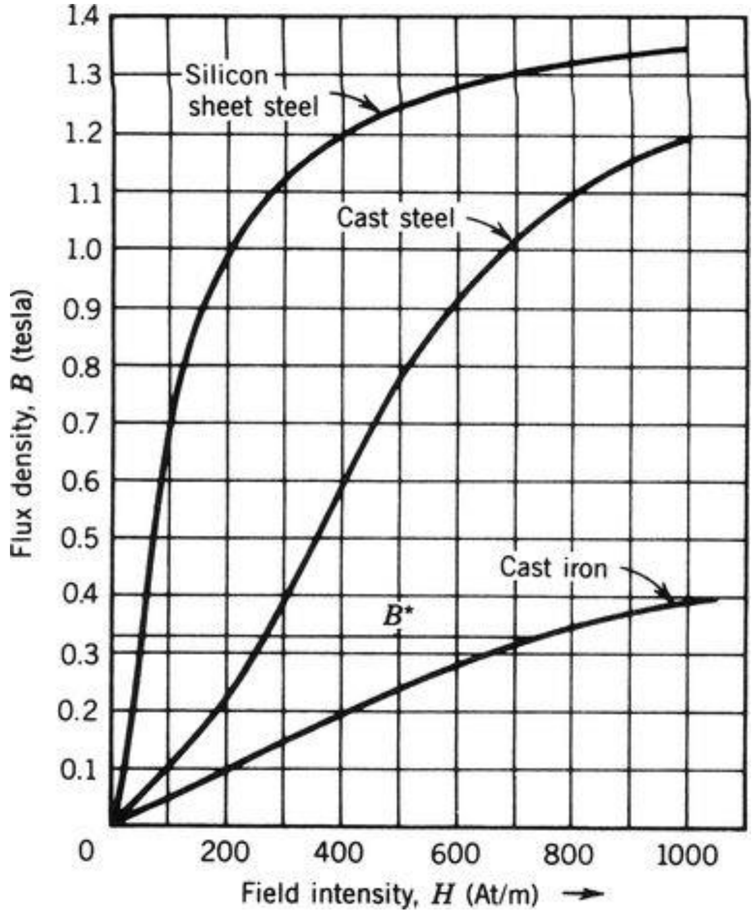
MAGNETIZATION CURVE

A typical magnetization curve for a magnetic core



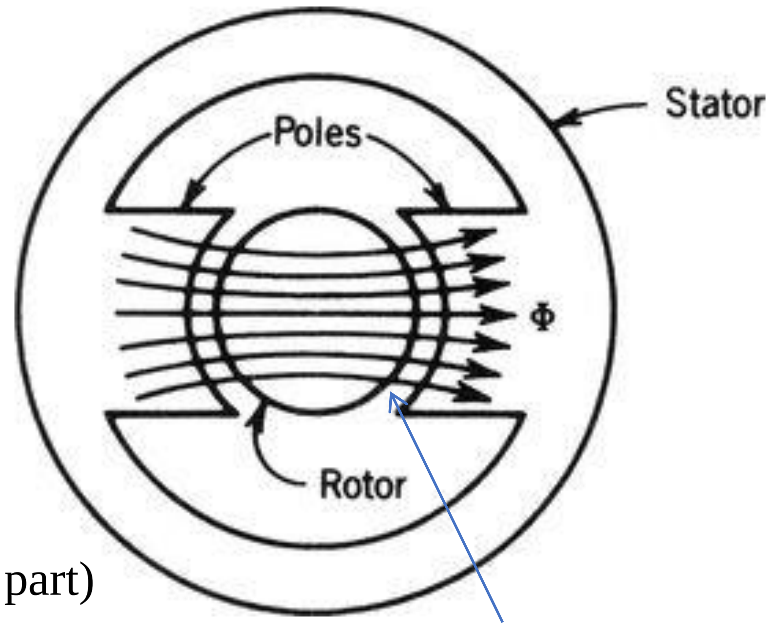
$$H \approx i$$

Magnetization curves for various magnetic cores



The magnetic materials are primarily iron alloys. The cast iron has more than 2 percent carbon (by weight) while the cast steel has 0.1-0.5 percent carbon. The silicon (electrical) steel is another iron alloy which has about 3 percent silicon and less than 0.005 percent carbon. The silicon increases the electrical resistivity of the steel and thereby reduces the eddy current component of the core losses. The silicon steel is used in electric machines due to its high permeability and low core loss.

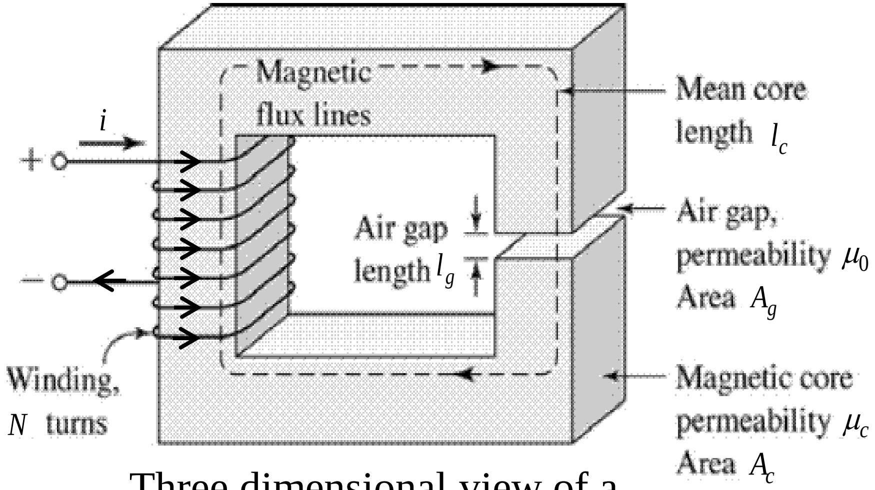
Cross Sectional View of a Rotating Machine



A rotating electric machine is a composite structure since its magnetic circuit has two or more media: i) Stator magnetic core, ii) Rotor magnetic core, and iii) Air gap.

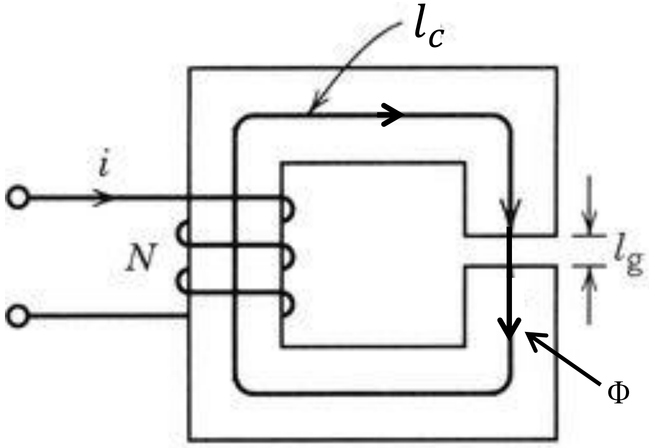
A composite structure

(A magnetic circuit having two or more media such as the magnetic core and air gap)



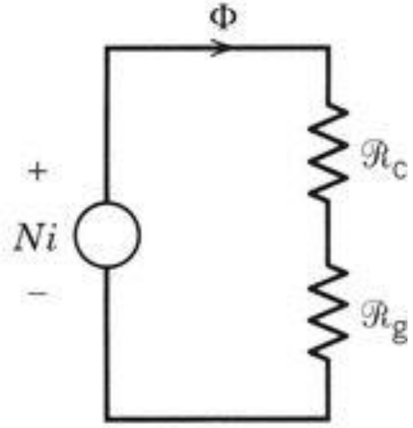
Three dimensional view of a magnetic circuit with air gap

https://physics.stackexchange.com



(a)

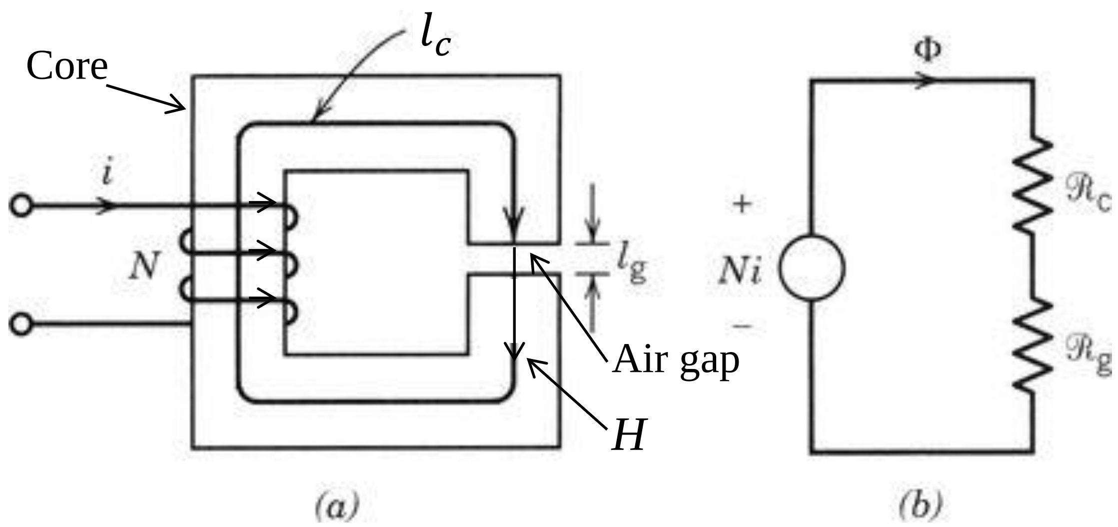
The simplified view



(b)

The magnetic equivalent circuit

Magnetic Equivalent Circuit of a Composite Structure



Magnetomotive force (MMF)

$$F = N \cdot i$$

Magnetic reluctance of the core

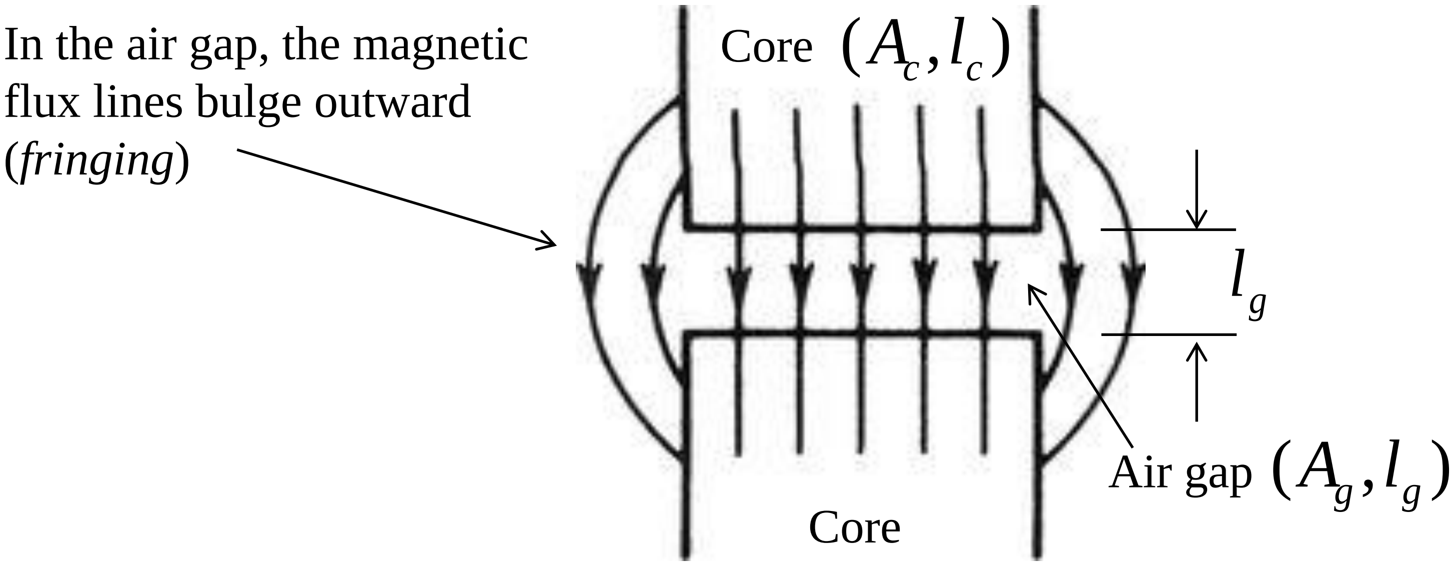
$$\Re_c = \frac{l_c}{\mu_c \cdot A_c}$$

Magnetic reluctance of the air gap

$$\Re_g = \frac{l_g}{\mu_0 \cdot A_g}$$

Magnetic flux $\Phi = \dfrac{N \cdot i}{\Re_c + \Re_g}$

Magnetic Flux Lines in the Air Gap



Magnetic Flux Densities in the Core and Air Gap

$$B_c = \frac{\Phi_c}{A_c} \qquad B_g = \frac{\Phi_g}{A_g} \qquad \Phi_g = \Phi_c = \Phi$$

For small air gaps, the fringing effect can be neglected. As a result, the cross-sectional areas of the core and the air gap are the same. That is, $A_c = A_g$. Then,

$$B_g = B_c = \frac{\Phi}{A_c}$$

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ECE 4363 – Electromechanical Energy Conversion

Lecture 02

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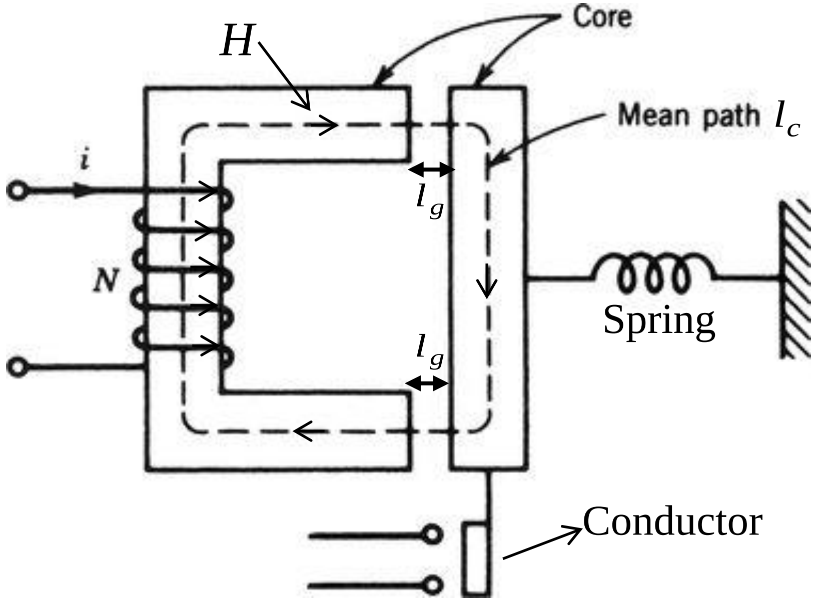
P.S. The pictures, notations, formulas, and statements in these lecture 02 notes have been adopted and adapted mainly from the course textbook "Principles of Electric Machines and Power Electronics" by P. C. Sen, Third Edition, 2013.

EXAMPLE 1.1 from the textbook

The figure on the right shows the magnetic circuit of a simple relay. In the magnetic circuit, the coil has 500 turns, the mean length of the core is 0.360m, and the air gap lengths are 0.0015m each. A flux density of 0.8 tesla is required to actuate the relay. The core is cast steel. a) Find the required current in the coil. b) Calculate the current to produce the same flux density (0.8 tesla) if the air gap is zero.



N = 500 , $l_c = 0.360\text{m}$, $l_g = 0.0015\text{m}$, $B = 0.8\text{T}$

$\mu_0 = 4\cdot\pi\cdot 10^{-7}$ [H/m] , $\mu_r = 1250$ for cast steel

Solution

a) $\Phi = B\cdot A_c = \dfrac{F}{\Re_c + \Re_g} = \dfrac{N\cdot i}{\Re_c + \Re_g}$, $\Re_c = \dfrac{l_c}{\mu_c \cdot A_c}$, $\Re_g = \dfrac{2\cdot l_g}{\mu_0 \cdot A_c}$, $\mu_c = \mu_r \cdot \mu_0$

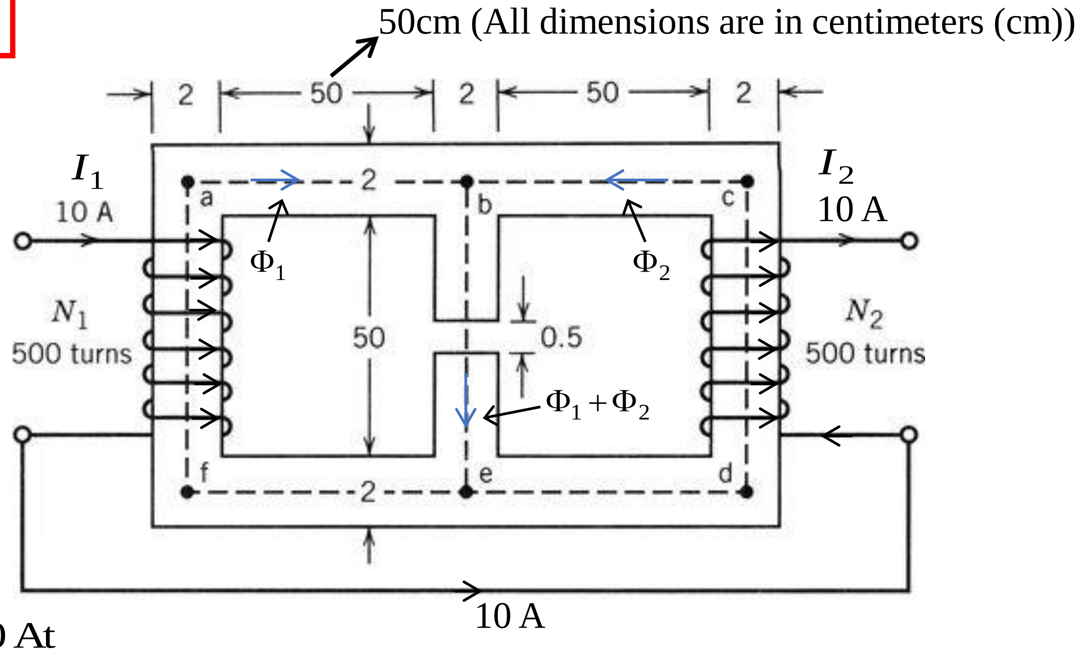
$$\Rightarrow \quad i = \frac{B\cdot\left(\dfrac{l_c}{\mu_r\cdot\mu_0} + \dfrac{2\cdot l_g}{\mu_0}\right)}{N} = 4.19A$$

b) $i = 0.367A$ for $l_g = 0.0$

EXAMPLE 1.3 from the textbook

In the figure on the right, the relative permeability of the core is 1200. Neglect magnetic leakage and fringing. All dimensions are in centimeters, and the core has a square cross-sectional area. Determine the magnetic flux, the magnetic flux density, and the magnetic field intensity in the air gap.

$\mu_r = 1200 \quad \mu_0 = 4\cdot\pi\cdot 10^{-7}$ [H/m]



Solution

$$F_1 = N_1 \cdot I_1 = 500 \times 10 = 5000 \text{ At}$$

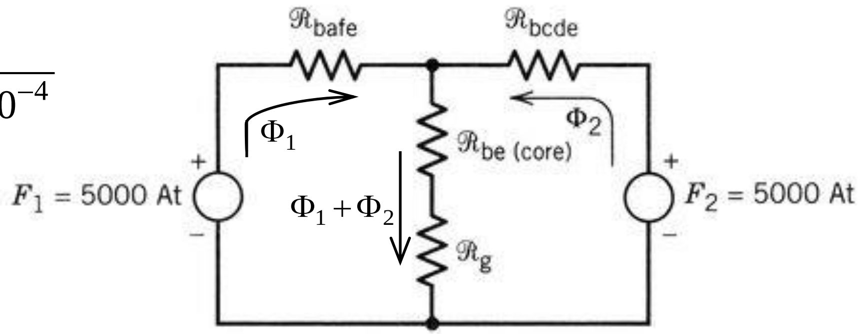
$$F_2 = N_2 \cdot I_2 = 500 \times 10 = 5000 \text{ At}$$

$$\mu_c = \mu_r \cdot \mu_0 = 1200 \times 4\cdot\pi\cdot 10^{-7}$$

$$\Re_{bcde} = \Re_{bafe} = \frac{l_{bafe}}{\mu_c \cdot A_c} = \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\cdot\pi\cdot 10^{-7} \times 4 \times 10^{-4}}$$

$$\Re_g = \frac{l_g}{\mu_0 \cdot A_g} = \frac{0.5 \times 10^{-2}}{4\cdot\pi\cdot 10^{-7} \times 4 \times 10^{-4}}$$

$$\Re_{be(core)} = \frac{l_{be(core)}}{\mu_c \cdot A_c} = \frac{51.5 \times 10^{-2}}{1200 \times 4\cdot\pi\cdot 10^{-7} \times 4 \times 10^{-4}}$$



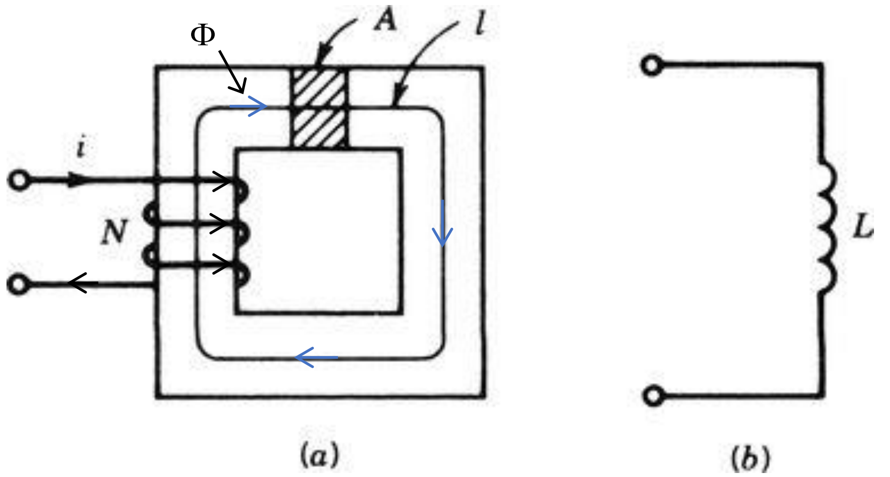
The equivalent magnetic circuit

$$\left.\begin{aligned} F_1 &= \Re_{bafe} \cdot \Phi_1 + \left(\Re_{be(core)} + \Re_g\right) \cdot \left(\Phi_1 + \Phi_2\right) \\ F_2 &= \Re_{bcde} \cdot \Phi_2 + \left(\Re_{be(core)} + \Re_g\right) \cdot \left(\Phi_1 + \Phi_2\right) \end{aligned}\right\} \Rightarrow$$

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{Wb}, \quad \Phi_g = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{Wb}$$

$$B_g = \frac{\Phi_g}{A_g} = 1.034\text{T}, \quad H_g = \frac{B_g}{\mu_0} = 0.822 \times 10^6 \text{ At/m}$$

INDUCTANCE



Coil-core assembly   Equivalent inductance

L: Inductance (The flux linkage of the coil per ampere of its current) [henrys] or [H].

$$\boxed{\text{Flux linkage} \quad \lambda = N \cdot \Phi \quad \text{[Wb·turns] or [Wb]}}$$

$$\boxed{\text{Inductance} \quad L = \frac{\lambda}{i} \quad \text{[H]}}$$

$$L = \frac{N \cdot \Phi}{i} = \frac{N \cdot B \cdot A}{i} = \frac{N \cdot \mu \cdot H \cdot A}{i}$$

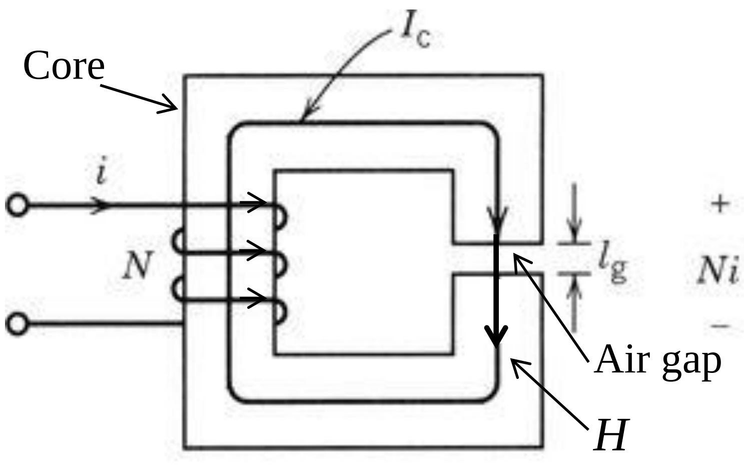
Ampere's Circuit Law $H \cdot l = N \cdot i$

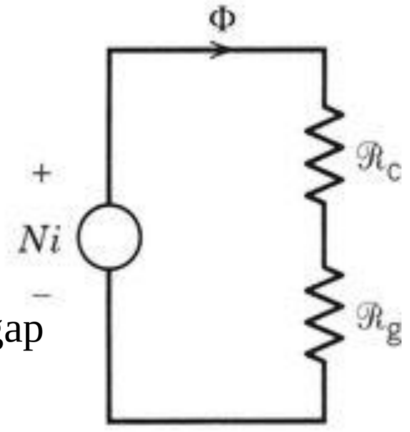
$$\boxed{L = \frac{N \cdot \mu \cdot H \cdot A}{H \cdot l / N} = \frac{N^2}{l / (\mu \cdot A)} = \frac{N^2}{\Re}}$$

EXAMPLE 1.4 from the textbook

For the magnetic circuit on the right, $N = 400$ turns, $l_c = 50\,\text{cm}$, $l_g = 1.0\,\text{mm}$, $A_c = A_g = 15\,\text{cm}^2$, $\mu_r = 3000$, $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$[H/m], $i = 1.0$ A.

Find (a) flux and flux density in the air gap and (b) inductance of the coil.





The equivalent magnetic circuit

Solution

(a) $$\Re_c = \frac{l_c}{\mu_r \cdot \mu_0 \cdot A_c} = \frac{50 \times 10^{-2}}{3000 \times 4 \cdot \pi \cdot 10^{-7} \times 15 \times 10^{-4}} = 88.42 \times 10^3 \text{ At/Wb}$$

$$\Re_g = \frac{l_g}{\mu_0 \cdot A_g} = \frac{1 \times 10^{-3}}{4 \cdot \pi \cdot 10^{-7} \times 15 \times 10^{-4}} = 530.515 \times 10^3 \text{ At/Wb}$$

$$\Phi = \frac{N \cdot i}{\Re_c + \Re_g} = \frac{400 \times 1.0}{(88.42 + 530.515) \times 10^3} = 0.6463 \times 10^{-3} \text{ Wb}$$

$$B_g = \frac{\Phi}{A_g} = \frac{0.6463 \times 10^{-3}}{15 \times 10^{-4}} = 0.4309\,\text{T}$$

(b) $$L = \frac{N^2}{\Re} = \frac{N^2}{\Re_c + \Re_g} = \frac{400^2}{(88.42 + 530.515) \times 10^3} = 258.52 \times 10^{-3} \text{ H}$$

CORE LOSSES

Power losses occur in a magnetic core (material) which is subject to a variable (alternating) magnetic field. These power losses are dissipated as heat by the core to the surrounding medium (e.g., air). There are two types of core losses as listed below:

1) Hysteresis Loss: It is due to the orientation of the magnetic dipole moments in the core in the direction of the applied (external) magnetic field. The motion of the magnetic dipoles causes a friction. As a result, a heat dissipation occurs in the core.

2) Eddy Current Loss: A voltage is induced in a closed path inside the core because of the time variation of flux enclosed by the path (*Faraday's law*). As a result, a current (known as *eddy current*) flows around the path, which causes a heat dissipation due to the electrical resistance of the core.

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ECE 4363 – Electromechanical Energy Conversion

Lecture 03

Date: January 28, 2021

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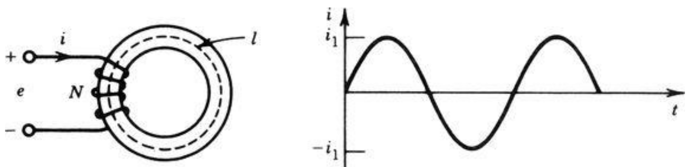
P.S. The pictures, notations, formulas, and statements in these lecture 03 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

HYSTERESIS

Consider the coil-core assembly on the right.
Assume that the core is initially unmagnetized.



(a)

As the field intensity H is increased by slowly increasing the current i, the flux density B will increase according to the curve 0a. At peak current $i = i_1$, $H = H_1$ and $B = B_1$.

If H is decreased by slowly decreasing i, the B-H curve follows a different path such as abc, and $B = B_r$ for $H = 0$ or $i = 0$. Note that the core retains flux density B_r (residual flux) even if $i = 0$.

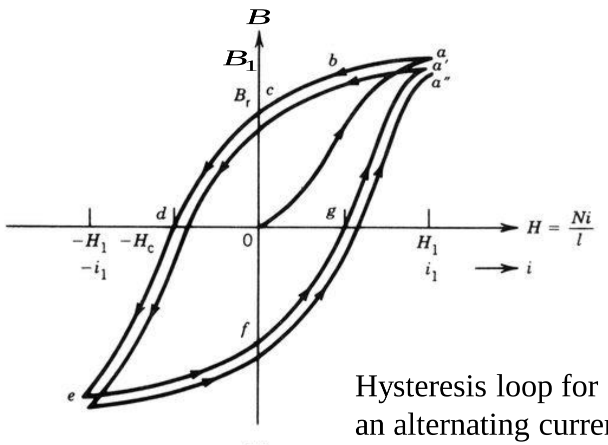
Let's reverse H by reversing i. Then, B will decrease by following the path cd. For $H = -H_c$, $B = 0$, and H_c is known as the *coercivity* or *coercive force*.

If H is further increased in the reverse direction, B will increase in the reverse direction along the path de.

The lagging of B behind H in the magnetic core is called *hysteresis*.

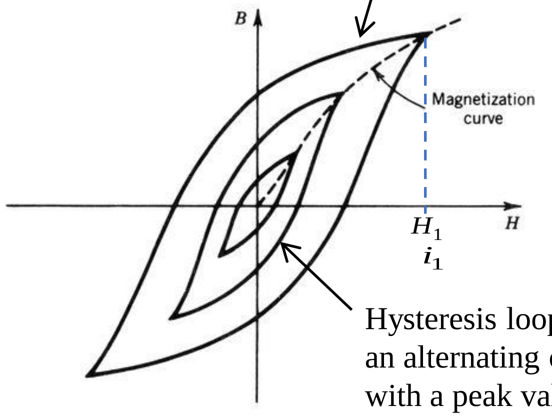
The closed loop showing the relationship between B and H is called *hysteresis loop*.

The locus of the tips of the hysteresis loops for different peak currents is the *magnetization curve*.



(b)

Hysteresis loop for an alternating current with a peak value i_1



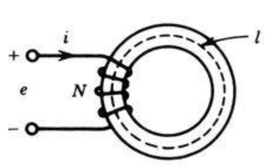
(c)

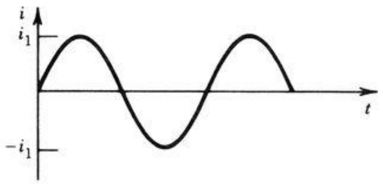
Hysteresis loop for an alternating current with a peak value smaller than i_1

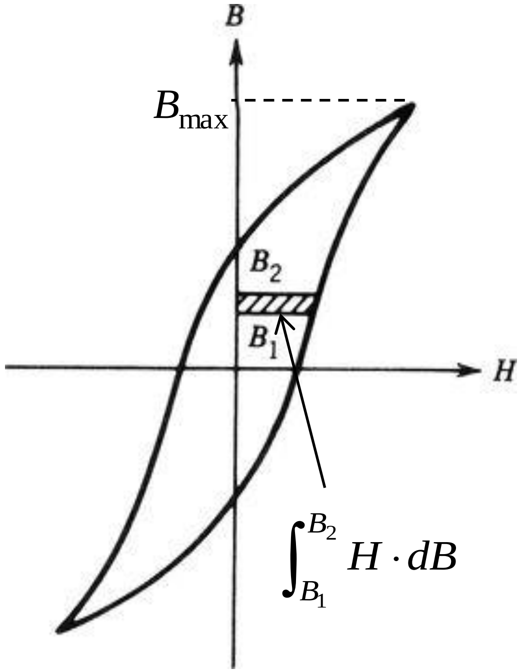
HYSTERESIS LOSS

According to Faraday's law, the voltage *e* across the coil is

$$e = N\frac{d\Phi}{dt}$$







$\int_{B_1}^{B_2} H \cdot dB$ is the hatched area

The energy transfer to the coil-core assembly during a time interval from t_1 to t_2 is given by

$$W = \int_{t_1}^{t_2} e \cdot i \cdot dt = \int_{\Phi_1}^{\Phi_2} N \cdot i \cdot d\Phi$$

Since

$$\Phi = B \cdot A \quad \text{and} \quad i = \frac{H \cdot l}{N}, \text{ then } W = V_{core} \cdot \int_{B_1}^{B_2} H \cdot dB$$

where $V_{core} = l \cdot A$ is the volume of the core.

The energy transfer to the coil-core assembly over one electrical cycle is

$$W = V_{core} \cdot \oint H \cdot dB = V_{core} \cdot W_h$$

where $W_h = \oint H \cdot dB$ is the energy density of the core. It is also equal to the area of the *B*-*H* loop.

With f being the frequency of the current *i*, the power loss in the core due to the hysteresis effect is

$$\boxed{P_h = V_{core} \cdot W_h \cdot f = K_h \cdot B_{\max}^n \cdot f}$$

where K_h is a constant whose value depends on the ferromagnetic material and volume of the core, $B_{\max}$ is the maximum flux density, and *n* is between 1.5 and 2.5. K_h and *n* are empirically determined.

Where does the hysteresis loss go ?

The hysteresis power loss:

$$\boxed{P_h = V_{core} \cdot W_h \cdot f = K_h \cdot B_{\max}^n \cdot f}$$

where K_h is a constant whose value depends on the ferromagnetic material and volume of the core, $B_{\max}$ is the maximum flux density, *n* is a constant between 1.5 and 2.5, and *f is the frequency of the current*. K_h and *n* are empirically determined.

The hysteresis power loss appears as heat in the core.

Part of the power supplied to the magnetic core goes to orientation of the magnetic dipole moments in the direction of the applied (external) magnetic field. This results in the residual flux density.

The magnetic dipoles are produced by the motion of the atomic particles in the material such as the orbiting electrons around the nucleus of the atom.

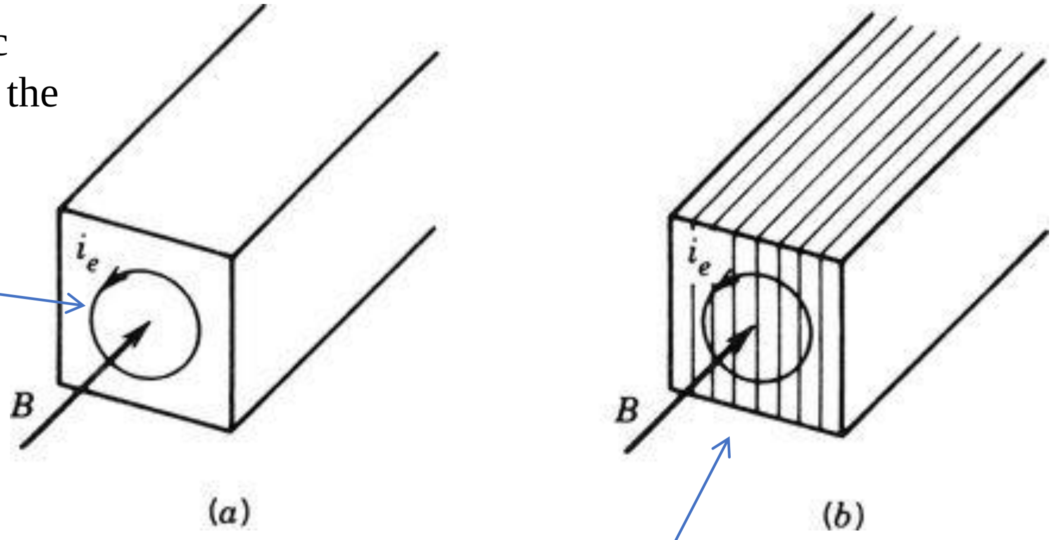
EDDY CURRENT LOSS

Another type of power loss occurs in a magnetic core when the flux density B changes rapidly in the core.

Consider a closed path in the cross section of the core.

A voltage will be induced in the path because of the time variation of flux enclosed by the path (Faraday's law). As a result, a current i_e (known as *eddy current*) will flow around the path.

Because core material has a resistance R, a power loss $i_e^2 \cdot R$ will be caused by the eddy current and will appear as heat in the core.



A laminated core structure to reduce eddy current loss. The thin layers of the metal are insulated from each other. The insulation is done by applying varnish coating over the surface of the laminations.

The eddy current power loss:

$$\boxed{P_e = K_e \cdot B_{\max}^2 \cdot f^2}$$

where K_e is a constant whose value depends on the ferromagnetic material and its lamination thickness, $B_{\max}$ is the maximum flux density, and f is the frequency of the current. The lamination thickness varies from 0.5 to 5 mm in electrical machines.

TOTAL CORE LOSSES

The core losses are the sum of the hysteresis loss and eddy current loss:

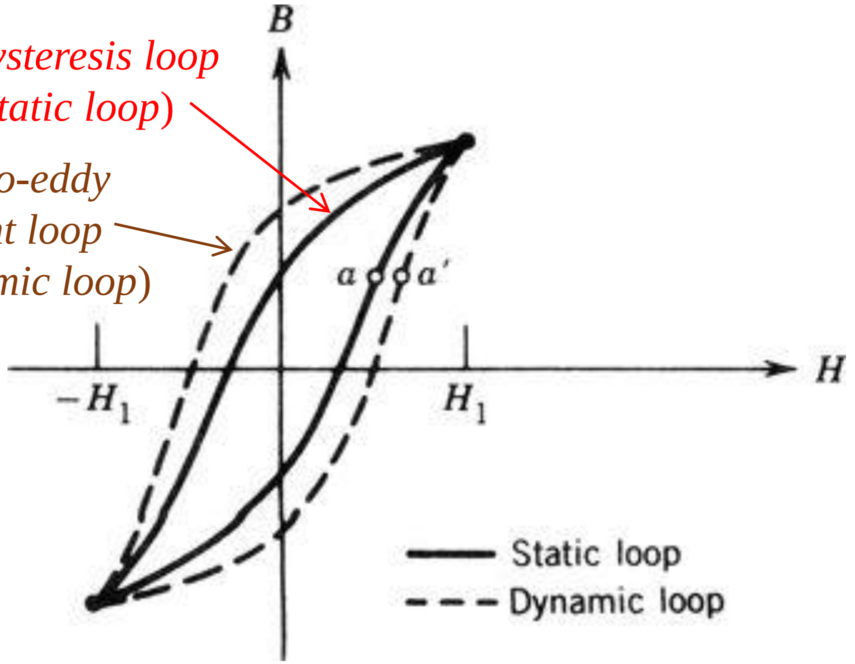
$$\boxed{P_c = P_h + P_e}$$

For the slowly-varying current (or, magnetic field intensity), the eddy currents induced in the core are negligible. The *B-H* loop for the slowly-varying current is called the *hysteresis loop* or *static loop*.

For the rapidly-varying current (or, magnetic field intensity), the eddy currents induced in the core are not negligible. The *B-H* loop becomes broader because of the pronounced effect of the eddy currents. This enlarged loop is called a *hystero-eddy current loop* or *dynamic loop*.

hysteresis loop (static loop)

hystero-eddy current loop (dynamic loop)



The eddy current in the core generates an mmf (magnetic field) that opposes the change in the magnetic flux produced by the coil current (*Lenz's law*). To maintain a given value of flux, the coil current must be increased by the amount necessary to overcome the effect of the eddy current mmf. Therefore, a point a on the static loop will be replaced by a point a' on the dynamic loop.

$$\boxed{P_c = V_{core} \cdot f \cdot \oint_{\substack{dynamic\\loop}} H \cdot dB}$$

PERMANENT MAGNET

A ***permanent magnet*** is a magnetic material which maintains a magnetic field without any excitation mmf provided to it.

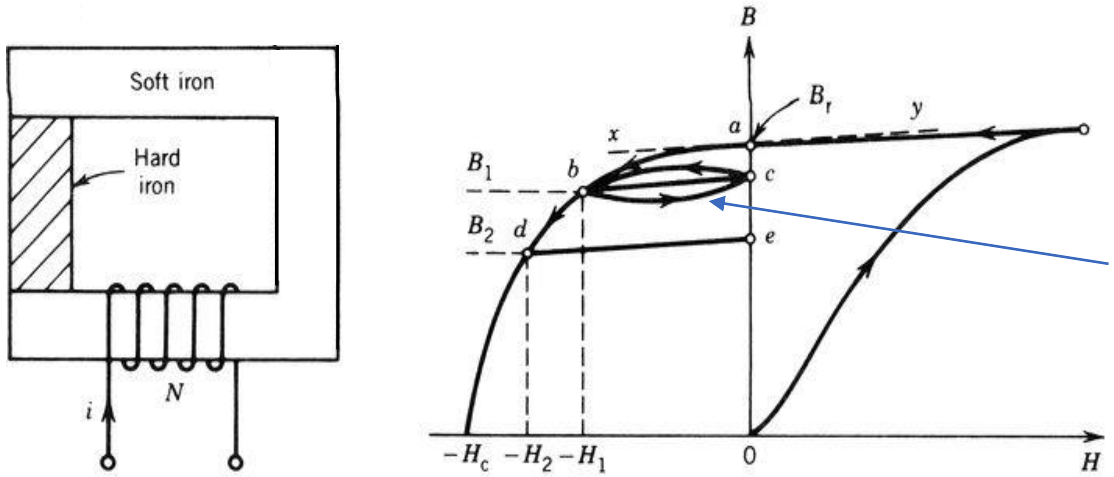
Examples of permanent magnets: i) *ferrite* (an alloy of iron oxide and barium or strontium), ii) *alnico* (an alloy of iron, aluminum, nickel and cobalt), and iii) *rare-earth* magnet material such as the neodymium-iron-boron (*NdFeB*) alloy.

Applications of permanent magnets: The *NdFeB* magnet is widely used for permanent magnet motors. The *ferrite* magnet is cheap and commonly used in household products such as refrigerator magnets.

Magnetization of Permanent Magnets

Soft iron: $B_r \approx 0, and\ H_c$ is small.

Hard iron (Permanent magnet): $Both\ B_r\ and\ H_c\ are\ large.$



Permanent magnet system and its *B*–*H* locus.

The magnetic material (hard iron) is initially unmagnetized. A large mmf is applied, and on its removal, the flux density will remain at the residual value B_r .

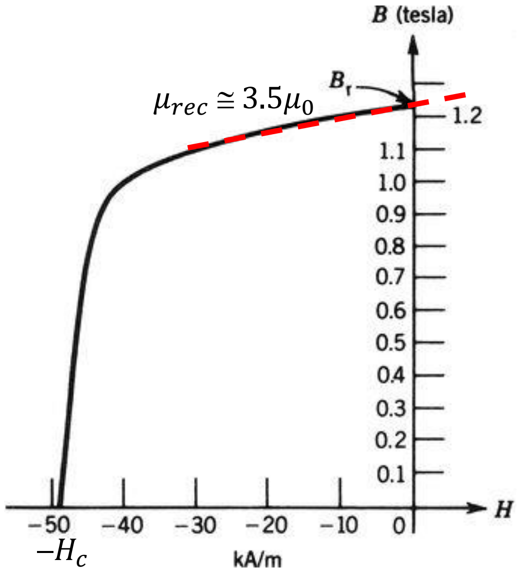
B_r: *Residual* (or, *remnant*, or, *remanent*) flux density.

H_c: *Coercive force*, the amount of opposing magnetic field that demagnetizes the permanent magnet.

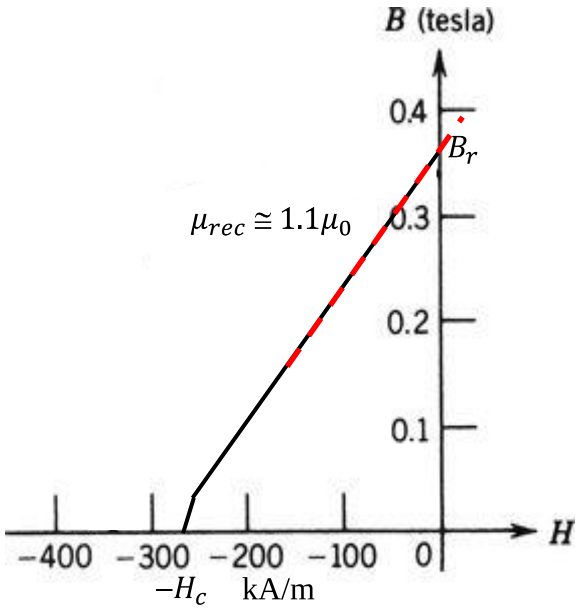
Operating region of the permanent magnet (hard iron). It is approximated by a straight line (*recoil line*) whose slope is called the *recoil permeability* μ_{rec} .

For hard iron such as ferrite magnets, $\mu_{rec} \cong 1.1\mu_0$.

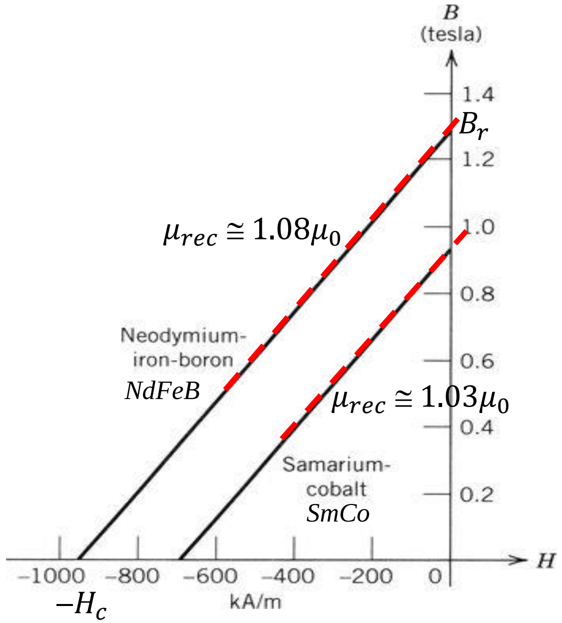
Permanent Magnet Materials



Demagnetization curve for *alnico 5*.



Demagnetization curve for *ferrite* magnet.



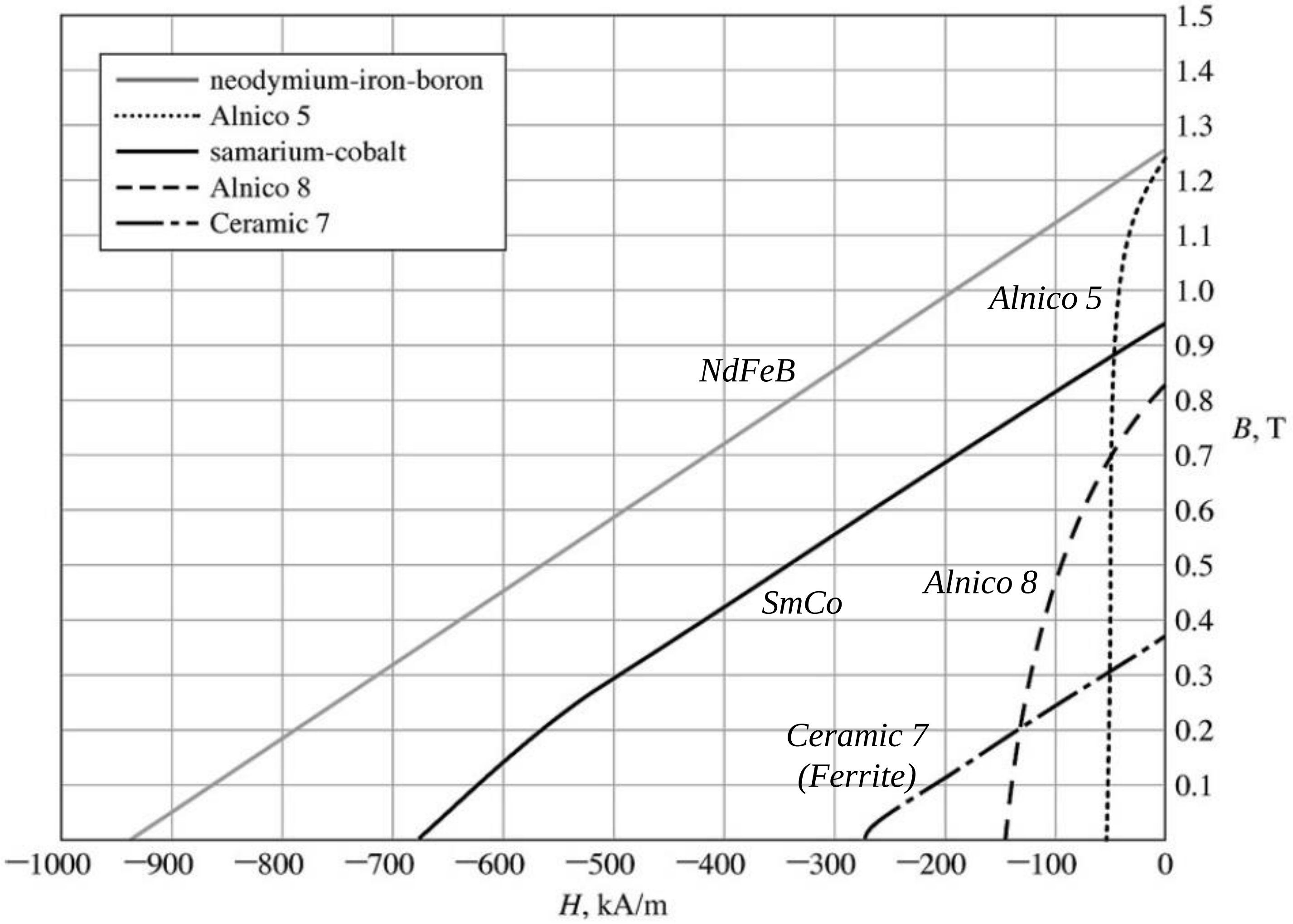
Demagnetization curve for *NdFeB and SmCo* magnets.

Properties of Permanent Magnet Materials

Material	B_r [T]	H_c [kA/m]	$\max(B \cdot H)$ [kJ/m^3]	*Relative* μ_{rec}
Alnico 5	1.23	49	40	3.50
Ferrite	0.37	270	25	1.10
SmCo	0.94	670	170	1.03
NdFeB	1.27	930	295	1.08

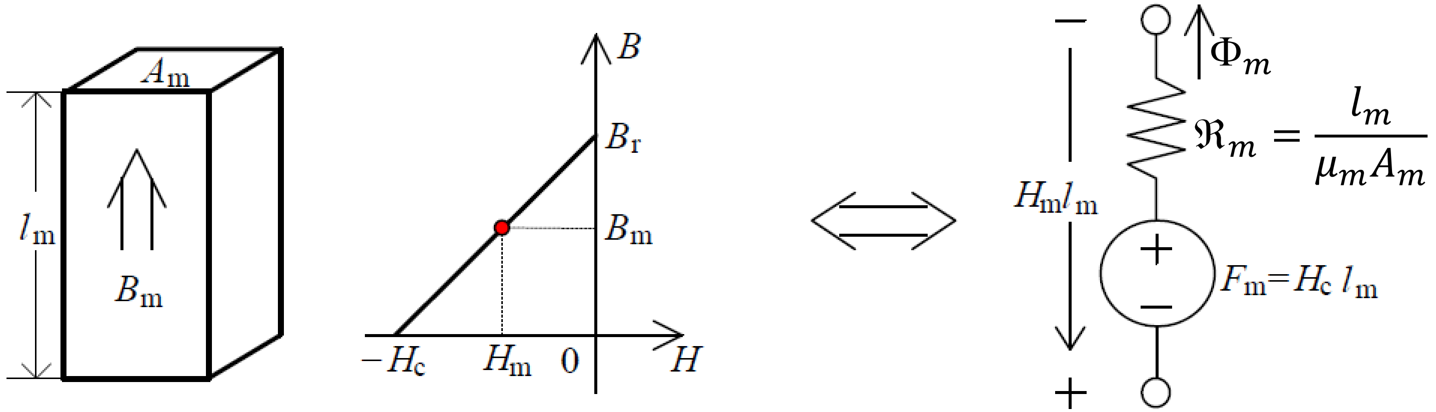
Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 5th Edition, 1990.

Demagnetization Curves of Common Permanent Magnet Materials



1- Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 5th Edition, 1990.
2- www.egr.msu.edu (fzpeng ECE320 Chapter 2).

Magnetic Circuit Model of Permanent Magnets



Magnetic circuit model of a magnet with linear demagnetization curve (www.ocw.nthu.edu.tw).

$$B_m = \frac{B_r}{H_c}(H_m + H_c) = \mu_m(H_m + H_c) \quad \text{for } H_c > 0,\ H_m < 0$$

$$\mu_m = \mu_{rec} = \frac{B_r}{H_c}, \qquad \mu_m = 1.08\mu_0 \ \text{ for } NdFeB$$

The magnetic voltage drop across the magnet (www.ocw.nthu.edu.tw):

$$H_m l_m = \left(\frac{B_m}{\mu_m} - H_c\right) l_m = \frac{l_m}{\mu_m A_m}\Phi_m - H_c l_m = \Re_m \Phi_m - F_m$$

$\Re_m = \dfrac{l_m}{\mu_m A_m}$ is the reluctance of the magnet. $F_m = H_c l_m$ is the mmf (voltage source) of the magnet.

www.ocw.nthu.edu.tw *(UTS Magnetic Circuit Analysis).*

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 04

Date: February 02, 2021

by

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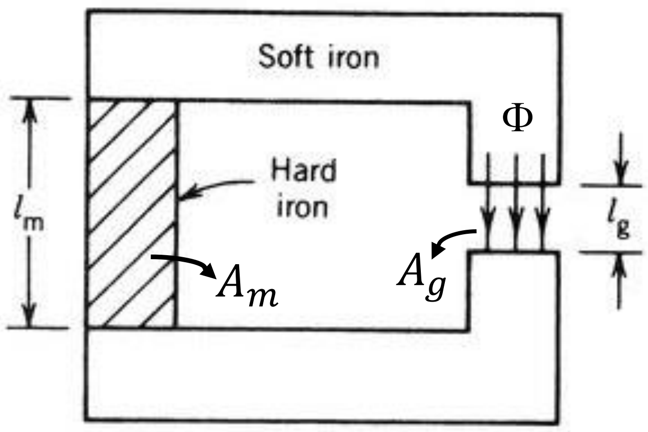
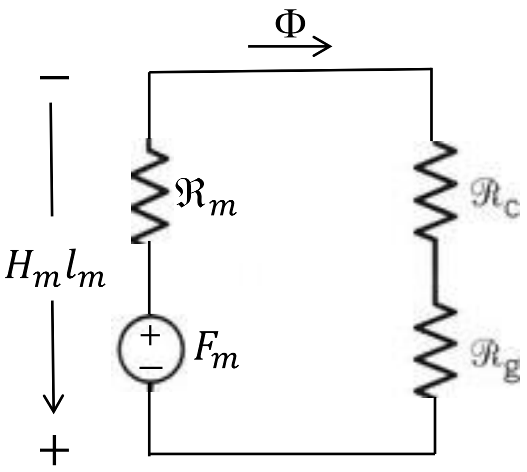
Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 04 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Magnetic Equivalent Circuit of a Composite Structure with Permanent Magnet





Mmf of the hard iron (permanent magnet such as *ferrite*): $F_m = H_c l_m$ where $H_c > 0$ is the coercivity.

Magnetic reluctance of the hard iron: $\Re_m = \dfrac{l_m}{\mu_m A_m}$, $\mu_m = \mu_{rec} \cong \mu_0$ $e.g.,\ \mu_m = 1.10\mu_0$

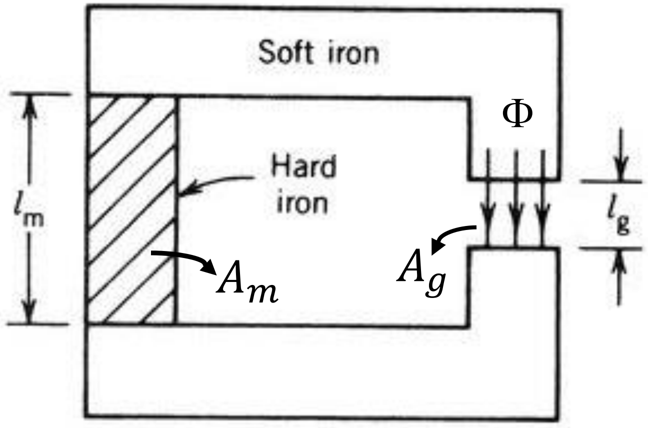
Magnetic reluctance of the soft iron such as silicon steel: $\Re_c = \dfrac{l_c}{\mu_c A_c}$, $\mu_c \gg \mu_0$ $e.g.,\ \mu_c = 2500\mu_0$

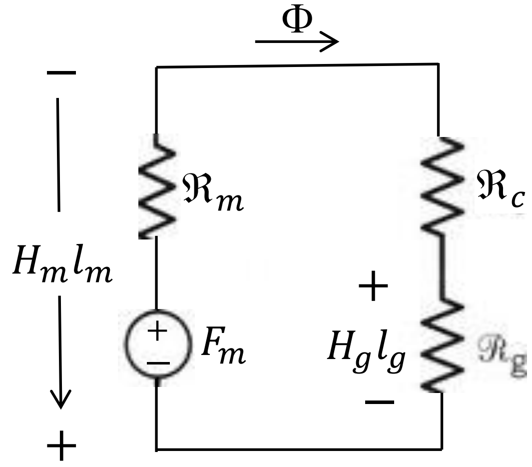
Magnetic reluctance of the air gap: $\Re_g = \dfrac{l_g}{\mu_0 A_g}$

Magnetic flux $\Phi = \dfrac{F_m}{\Re_m + \Re_c + \Re_g}$

$\Phi \cong \dfrac{F_m}{\Re_m + \Re_g}$ since $\Re_c \ll \Re_g$

Approximate Design of Permanent Magnets





Assumptions: 1) There is no leakage or fringing flux and 2) No mmf is required for the soft iron; that is, the magnetic reluctance of the soft iron is negligible ($\Re_c \cong 0$).

From the Ampere's circuit law, $(H_m l_m + H_g l_g) = 0 \Rightarrow l_m = -\dfrac{H_g l_g}{H_m}$ where $H_m < 0$.

For continuity of flux, $\Phi = B_m A_m = B_g A_g \Rightarrow A_m = \dfrac{B_g A_g}{B_m}$

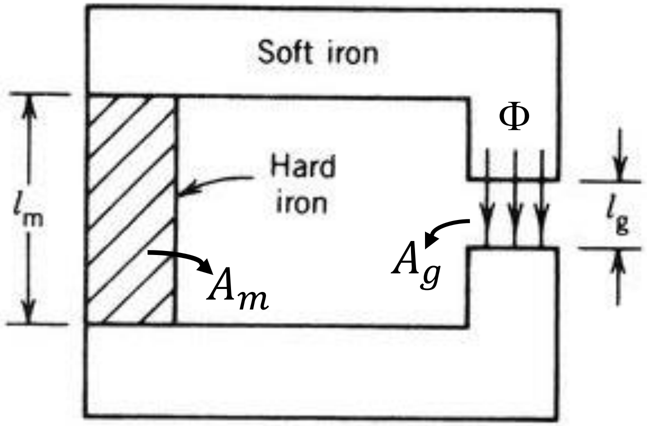
Also, $H_g = \dfrac{B_g}{\mu_0}$

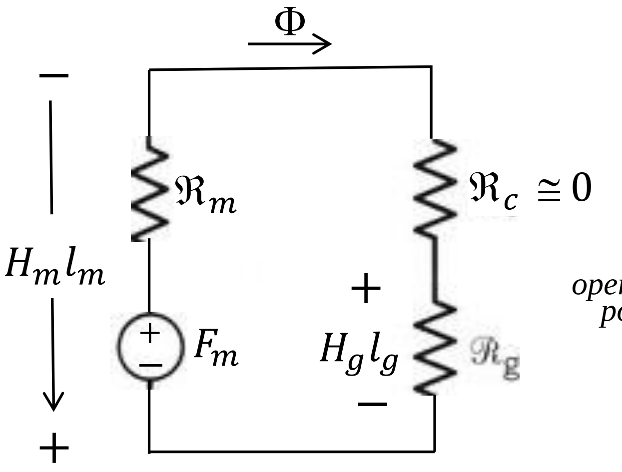
The volume of the permanent magnet: $V_m = A_m l_m = -\dfrac{B_g A_g}{B_m} \cdot \dfrac{H_g l_g}{H_m} = -\dfrac{B_g^2 V_g}{\mu_0 B_m H_m}$

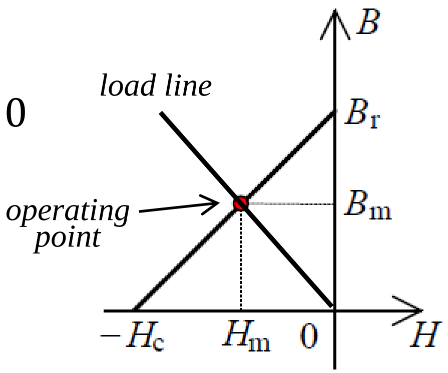
$V_g = A_g l_g$ is the volume of the air gap.

$-B_m H_m$ for $H_m < 0$ is the *energy product* of the permanent magnet.

Volume Minimization of Permanent Magnet in a Magnetic Circuit







Demagnetization curve

Assumptions: 1) There is no leakage or fringing flux and 2) No mmf is required for the soft iron; that is, the magnetic reluctance of the soft iron is negligible ($\Re_c \cong 0$).

$$l_m = -\frac{H_g l_g}{H_m} = -\frac{B_g l_g}{\mu_0 H_m}\ , \quad A_m = \frac{B_g A_g}{B_m} \quad \Rightarrow \quad \boxed{B_m = -\mu_0 \frac{A_g}{A_m}\frac{l_m}{l_g} H_m \quad \text{the } \textit{load (shear) line}}$$

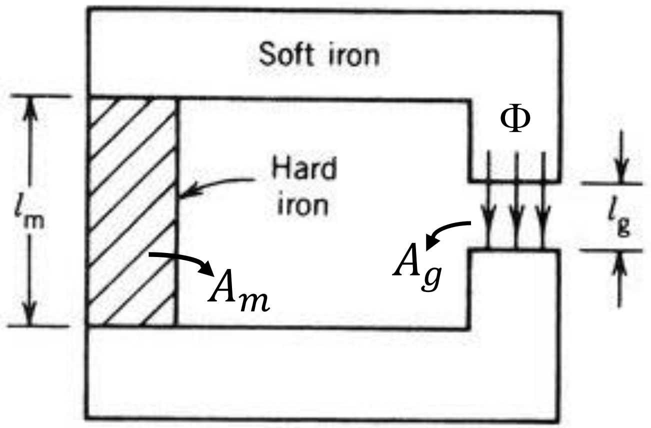
The volume of the permanent magnet: $\boxed{V_m = A_m l_m = -\frac{B_g^2 V_g}{\mu_0 B_m H_m}}$ where $V_g = A_g l_g$

$\boxed{-B_m H_m \quad \text{the } \textit{energy product} \text{ of the permanent magnet.}}$

$\boxed{V_m \text{ is minimized by maximizing the } \textit{energy product } -B_m H_m}$

EXAMPLE 1.8 from the textbook

The permanent magnet in the figure on the right is made of alnico 5, whose flux density and magnetic field intensity at its maximum energy product $(-B_m H_m)$ are $B_m = 0.95$ T and $H_m = -42$ kA/m. A flux density of $B_g = 0.8$ T is to be established in the air gap. The air gap has the dimensions $A_g = 2.5\ \text{cm}^2$ and $l_g = 0.4$ cm. Determine the dimensions (l_m and A_m) of the permanent magnet for which its volume is minimum. Assumption: The magnetic reluctance of soft iron is negligible, and the permeability of free space is $\mu_0 = 4\cdot\pi\cdot 10^{-7}$ H/m.



The permanent magnet system.

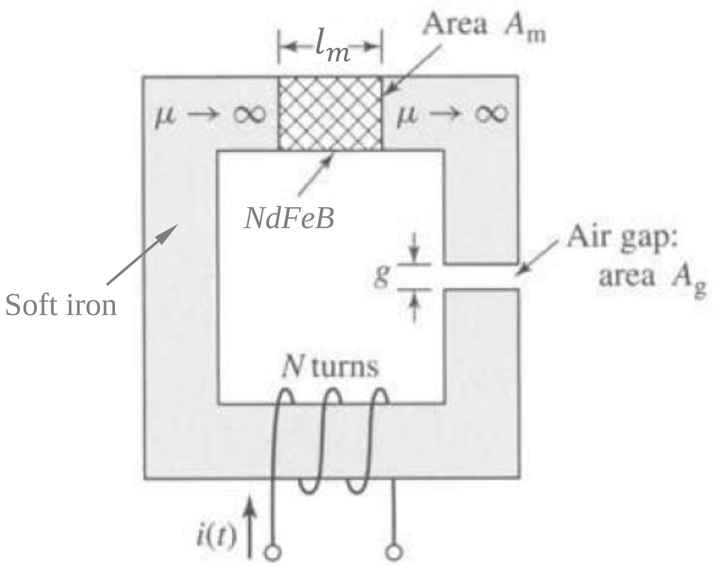
Solution

$$l_m = -\frac{B_g l_g}{\mu_0 H_m} = -\frac{0.8 \times 0.4 \times 10^{-2}}{4 \cdot \pi \cdot 10^{-7} \times (-42 \times 10^3)} = 0.0606\ \text{m} = 6.06\ \text{cm}$$

$$A_m = \frac{B_g A_g}{B_m} = \frac{0.8 \times 2.5 \times 10^{-4}}{0.95} = 0.0002105\ \text{m}^2 = 2.105\ \text{cm}^2$$

EXAMPLE *(Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 6th Edition, 2003.)*

In the permanent magnet-coil assembly below, the time-varying air-gap flux density is given by $B_g = B_o + B_1 sin(\omega t)$ where $B_o = 0.5$ T and $B_1 = 0.25$ T. The dc component B_o is created by *NdFeB* magnet and the time-varying component is created by the time-varying current $i(t)$. For $A_\mathrm{g} = 6\ \mathrm{cm}^2$, $\mathrm{g} = 0.4$ cm and $N = 200$ turns, find (a) the magnet length l_m and the magnet area A_m that will achieve the desired dc air-gap flux density B_o and minimize the magnet volume, and (b) the minimum and maximum values of the time-varying current $i(t)$ required to achieve the desired time-varying air-gap flux density B_g. The maximum energy product for the *NdFeB* magnet occurs at (approximately) $B_m = 0.63$ T and $H_m = -469$ kA/m. The residual flux density and the recoil permeability of the *NdFeB* magnet are $B_r = 1.27$ T and $\mu_{rec} = 1.08 \cdot \mu_0$ [H/m] , respectively, where $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$[H/m] is the permeability of the air. The magnetic reluctance of the soft iron is negligible.



Permanent magnet-coil assembly.

Solution

The magnetic circuit must satisfy

$$N \cdot i = H_m \cdot l_m + H_g \cdot g \quad \text{and} \quad B_m A_m = B_g A_g \,.$$

(a) For $i = 0$ and $B_g = 0.5$ T, the minimum magnet volume is achieved when the magnet is operating at its maximum energy point.

$$A_m = \frac{B_g A_g}{B_m} = \frac{0.5 \times 6.0 \times 10^{-4}}{0.63} = 0.0004762 \text{ m}^2 = 4.762 \text{ cm}^2$$

$$l_m = -\frac{B_g g}{\mu_0 H_m} = -\frac{0.5 \times 0.4 \times 10^{-2}}{4 \cdot \pi \cdot 10^{-7} \times (-469 \times 10^3)} = 0.00339 \text{ m} = 3.39 \text{ mm}$$

(b) The demagnetization curve of the *NdFeB* magnet is represented by $B_m = \mu_{rec} \cdot H_m + B_r$. Then,

$$i = \frac{H_m \cdot l_m + H_g \cdot g}{N} = \frac{\left(\frac{B_m}{\mu_{rec}} - \frac{B_r}{\mu_{rec}}\right) \cdot l_m + \frac{B_g}{\mu_o} \cdot g}{N} = \frac{\left(\frac{B_g \cdot A_g}{\mu_{rec} \cdot A_m} - \frac{B_r}{\mu_{rec}}\right) \cdot l_m + \frac{B_g}{\mu_o} \cdot g}{N} = \frac{B_g \cdot \left(\frac{A_g \cdot l_m}{\mu_{rec} \cdot A_m} + \frac{g}{\mu_o}\right) - \frac{B_r \cdot l_m}{\mu_{rec}}}{N}$$

$$i = 31.652 \cdot B_g - 15.861 \;\; [\text{A}]$$

For the minimum value of the air-gap flux density $B_g = 0.25$ T, the current is $i = -7.95$ A.

For the maximum value of the air-gap flux density $B_g = 0.75$ T, the current is $i = 7.88$ A.

UNIVERSITY of HOUSTON
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ECE 4363 – Electromechanical Energy Conversion

Lecture 05

Date: February 04, 2021

by

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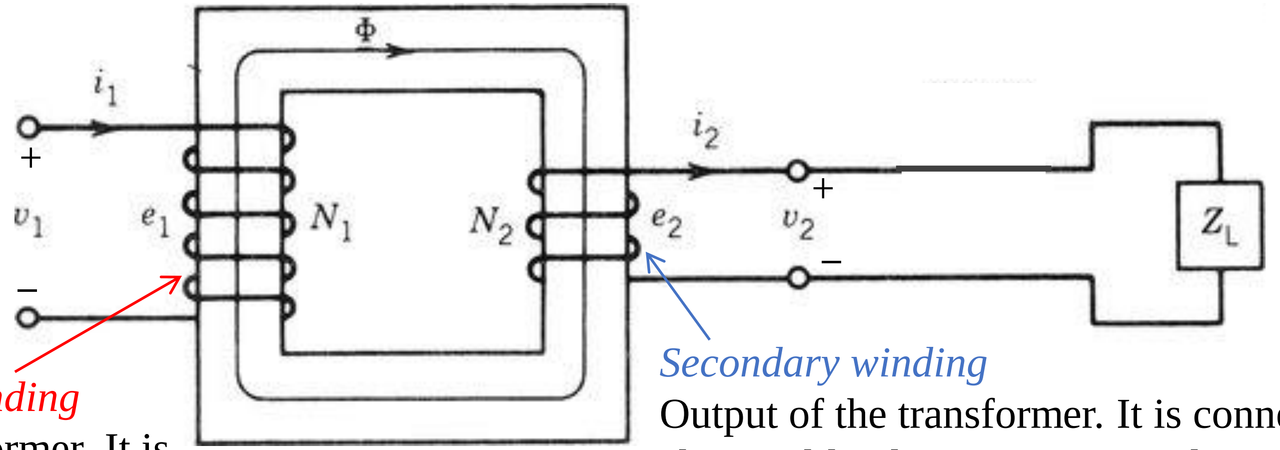
Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 05 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

TRANSFORMERS

A transformer is a static machine (no moving parts). It consists of two or more windings coupled by a mutual magnetic field. The main purpose of the transformer is to step up/down the voltage. It also enables the electrical isolation of low-power electronic or control circuits from the power supply. Analysis of transformers involves many principles that are basic to the understanding of the rotating electric machines.



Primary winding

Input of the transformer. It is connected to an ac supply v_1. For example, 120VAC, 60Hz electricity outlet.

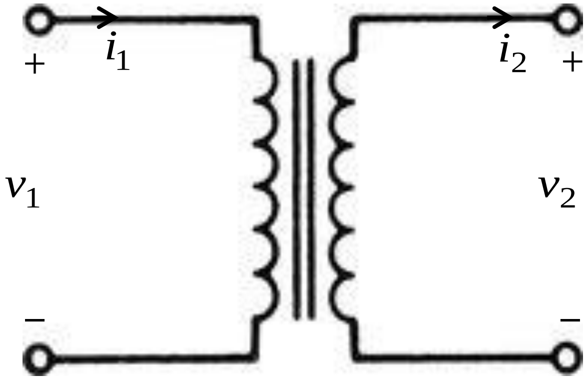
$v_2 < v_1$

for $N_2 < N_1$

Secondary winding

Output of the transformer. It is connected to an electrical load Z_L. For example, an ac/dc power supply to charge the battery of a laptop.

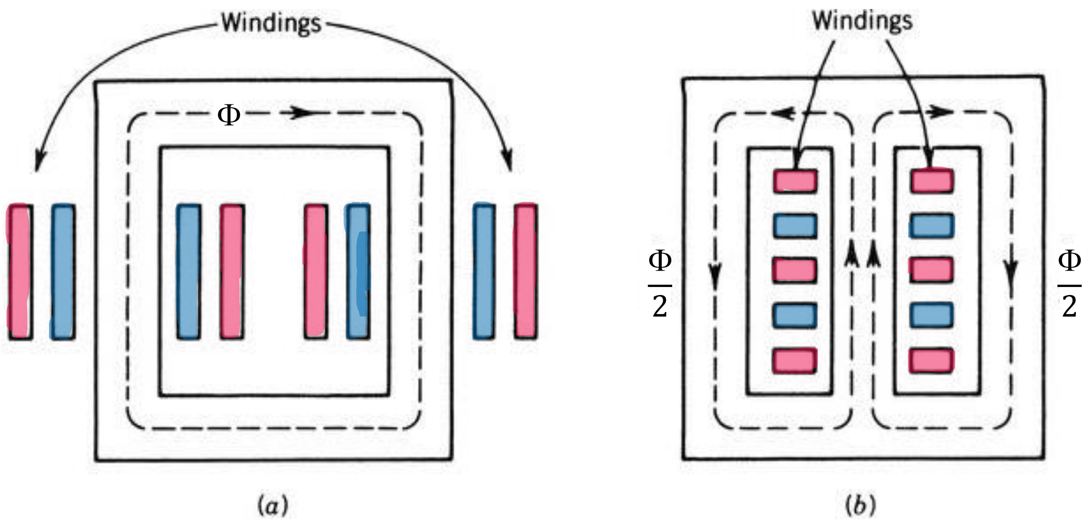
Schematic representation of a two-winding transformer



The two vertical bars indicate tight magnetic coupling between the windings via a magnetic core (such as iron or iron alloy)

CONSTRUCTION of TRANSFORMERS

Two types of core constructions are normally used in the transformers. These are, (a) *core-type* where the windings are wound around two legs of a magnetic core of a rectangular shape, and (b) *shell-type* where the windings are wound around the center leg of a three-legged magnetic core.



Transformer core construction: (a) Core-type, and (b) Shell-type.

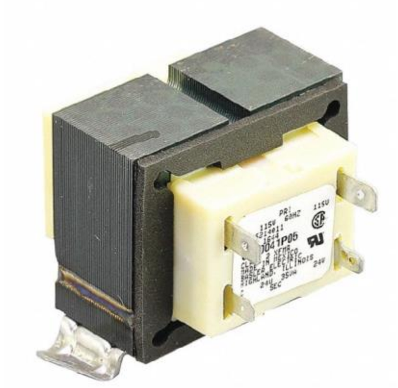
In the above figures (a) and (b), each transformer has two windings (primary winding in red and secondary winding in blue). The winding coils are wound around the transformer legs in certain configurations to achieve tighter magnetic coupling between the windings. For example, in the core-type constructure (figure (a) above), the primary and secondary coils are placed on top of one another. On the other hand, in the shell-type constructure (figure (b) above), the primary and secondary coils are placed side by side and they are interleaved. Furthermore, in the core-type structure, the low-voltage winding (the winding with the lower number of turns) is placed nearer the core, and the high voltage winding (the winding with the higher number of turns) on top.

To reduce the core losses, the magnetic core is formed of a stack of thin laminations. For example, silicon-steel laminations of 0.014-inch (0.36 mm) thickness are usually used for transformers operating at frequencies below a few hundred hertz.

VARIOUS TYPES of TRANSFORMERS

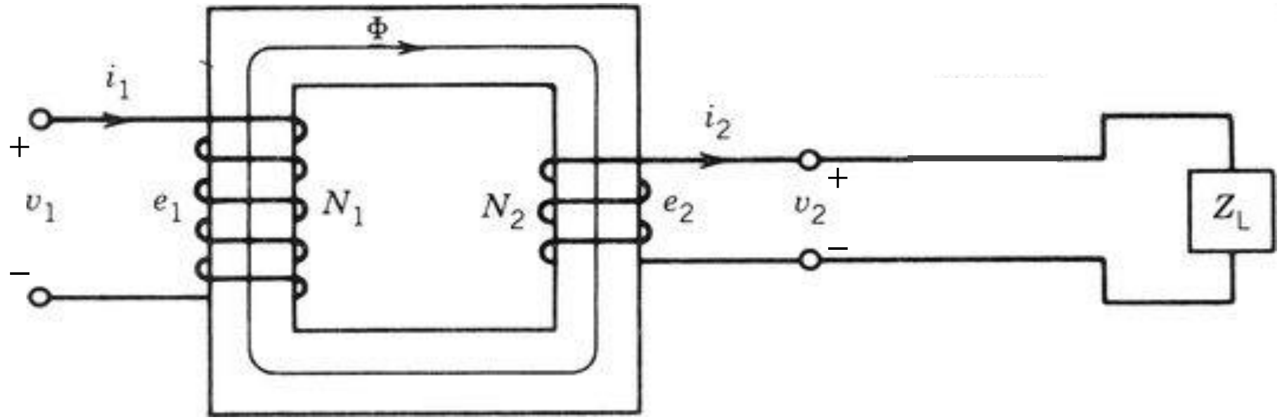
A single-phase pole-type distribution transformer used in a public utility system.
Rating: 10 kVA.
High voltage: 2.4 kV
Low voltage: 240/120 V and 480/240 V.
Frequency: 60Hz
Tank size (hxd): ≈ 22x15 inch
(www.prolecge.com)

A three-phase substation transformer.
Rating: Up to 12,000 kVA.
High voltage: 69.0 kV
Low voltage: 2.4 kV.
Frequency: 60Hz
Size (hxlxw): 164x263x177 inch
(www.prolecge.com)



A single-phase transformer for use in low-voltage control circuits.
Rating: 35 VA.
High voltage: 115 V
Low voltage: 24 V
Frequency: 60Hz
Stack size: 29x57 mm
(www.basler.com)

IDEAL TRANSFORMER

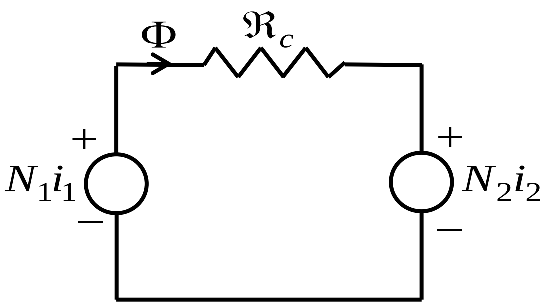


Assumptions: 1) The winding resistances are negligible. 2) All fluxes are confined to the core and link both windings. 3) Core losses are negligible. 4) Permeability of the core is infinite ($\mu_c \rightarrow \infty$).

$$N_1 \cdot i_1 - N_2 \cdot i_2 = \Phi \cdot \Re_c$$

$$\Re_c = \frac{l_c}{\mu_c \cdot A_c} \rightarrow 0 \text{ since } \mu_c \rightarrow \infty$$

$$N_1 \cdot i_1 - N_2 \cdot i_2 = 0 \quad \text{or} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$



Equivalent magnetic circuit of ideal transformer

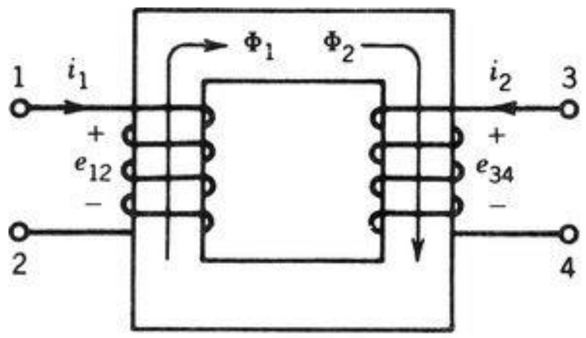
Power Conservation for Ideal Transformer (Instantaneous Input Power = Instantaneous Output Power):

$$v_1 \cdot i_1 = v_2 \cdot i_2 \quad \Rightarrow \quad \frac{v_1}{v_2} = \frac{i_2}{i_1} = \frac{N_1}{N_2} = a \quad \text{where } a \text{ is the turns ratio.}$$

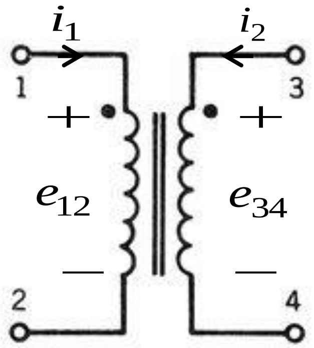
For a sinusoidal supply voltage, $\frac{V_1}{V_2} = \frac{I_2}{I_1} = a$ and $V_1 \cdot I_1 = V_2 \cdot I_2$ where V_1, I_1, V_2 and I_2 are the rms values.

Input volt-amperes Output volt-amperes

POLARITY of TRANSFORMER



(a)



(b)

$$\boxed{\begin{array}{l} e_{12} > 0 \Rightarrow e_{34} > 0 \\ e_{12} < 0 \Rightarrow e_{34} < 0 \end{array}}$$

The currents entering terminals 1 and 3 produce fluxes in the same direction in the core.

i_1 produces Φ_1 and i_2 produces Φ_2 where Φ_1 and Φ_2 support each other.

The time-varying common flux ($\Phi_1 + \Phi_2$) induces voltages e_{12} and e_{34} in the windings such that e_{12} and e_{34} are in phase. That is, if $e_{12} > 0$, then, $e_{34} > 0$.

In the schematic representation of the transformer, the terminals are marked by dots to indicate that the dot terminals have the same polarity.

PRACTICAL TRANSFORMER

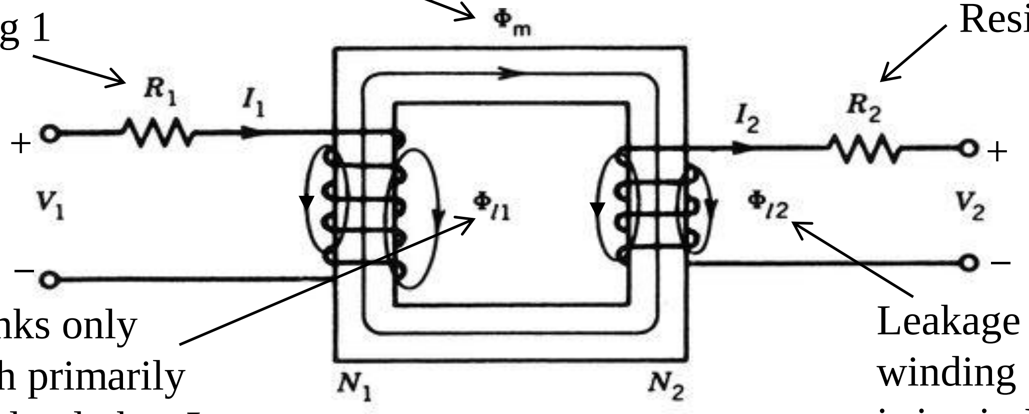
1) The windings have resistance. 2) Not all windings link the same flux. 3) Core losses occur.
4) Permeability (μ_c) of the core is not infinite.

Mutual (common) flux Φ_m is confined to the core and linked by both windings. It is produced by both winding currents I_1 and I_2.

Mutual flux

Resistance of winding 1

Resistance of winding 2



Leakage flux Φ_{l1} links only winding 1 and its path primarily is in air. It is produced only by I_1.

(a)

Leakage flux Φ_{l2} links only winding 2 and its path primarily is in air. It is produced only by I_2.

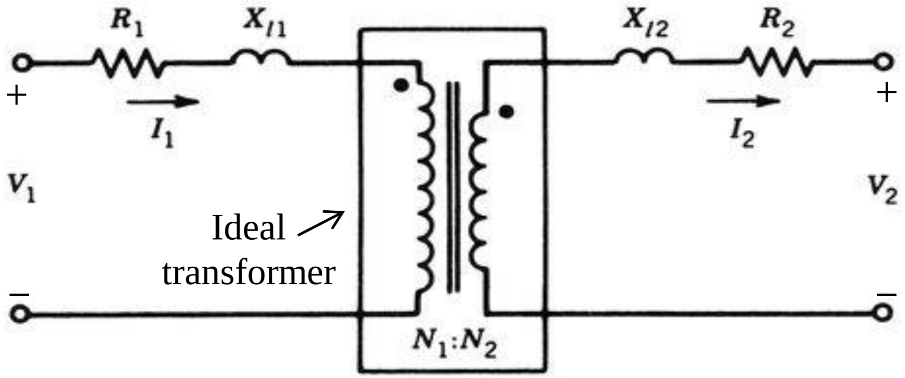
Transformer equivalent circuit that takes into account i) winding resistances and ii) winding leakage fluxes.

Leakage inductance of winding 1 [H]

$$L_{l1} = \frac{N_1 \cdot \Phi_{l1}}{i_1}$$

Leakage reactance of winding 1 [Ω]

$$X_{l1} = 2 \cdot \pi \cdot f \cdot L_{l1}$$



Tightly coupled transformer

(b)

Leakage inductance of winding 2 [H]

$$L_{l2} = \frac{N_2 \cdot \Phi_{l2}}{i_2}$$

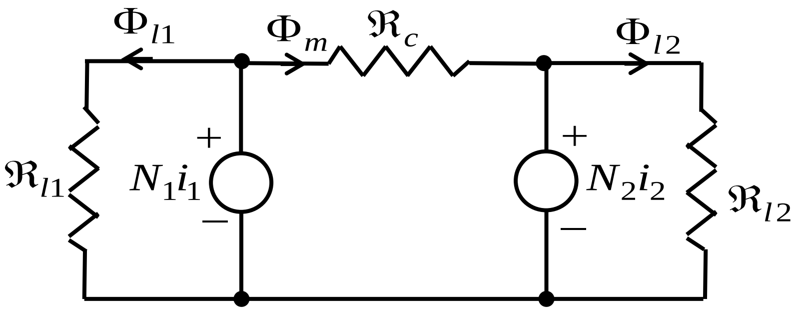
Leakage reactance of winding 2 [Ω]

$$X_{l2} = 2 \cdot \pi \cdot f \cdot L_{l2}$$

Equivalent magnetic circuit of a practical transformer

$\Re_{l1}$: Reluctance of air path that Φ_{l1} follows

$\Re_c$: Reluctance of core mean path that Φ_m follows



$\Re_{l2}$: Reluctance of air path that Φ_{l2} follows

$$N_1 \cdot i_1 = \Re_{l1} \cdot \Phi_{l1} \qquad N_2 \cdot i_2 = \Re_{l2} \cdot \Phi_{l2}$$

$$N_1 \cdot i_1 = N_2 \cdot i_2 + \Re_c \cdot \Phi_m$$

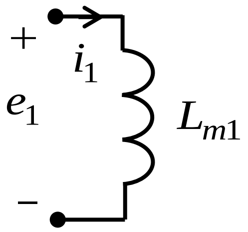
$$i_1 = \frac{N_2}{N_1} \cdot i_2 + \frac{\Re_c}{N_1} \cdot \Phi_m = \frac{N_2}{N_1} \cdot i_2 + i_{m1}$$

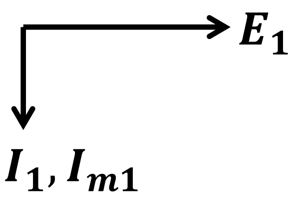
Magnetizing current $i_{m1} = \dfrac{\Re_c}{N_1} \cdot \Phi_m$

Magnetizing inductance $L_{m1} = \dfrac{N_1 \cdot \Phi_m}{i_{m1}} = \dfrac{N_1^2}{\Re_c}$

Voltage induced across L_{m1}: $e_1 = N_1 \cdot \dfrac{d\Phi_m}{dt} = L_{m1} \cdot \dfrac{di_{m1}}{dt}$

In theory, $i_1 = i_{m1}$ for $i_2 = 0$

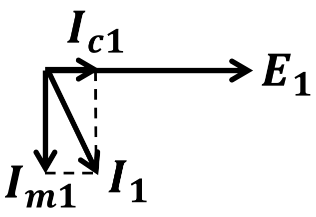


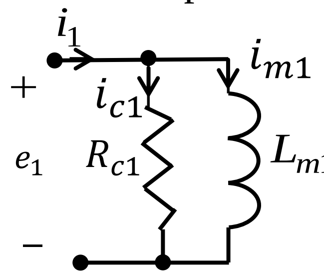


The phasors are denoted by bold letters.

In practice and by observations, $i_1 \neq i_{m1}$ for $i_2 = 0$

It is due to the core losses which is represented by R_{c1}





UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 06

Date: February 09, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 06 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Transformer equivalent circuit that takes into account core losses

$$\boxed{i_1 = \frac{N_2}{N_1} \cdot i_2 + i_{m1}}$$

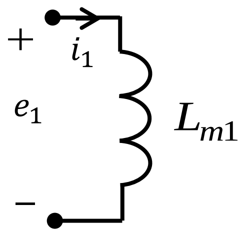
$\boldsymbol{i_1}$: Primary current, i_2: Secondary current, i_{m1} : Magnetizing current.

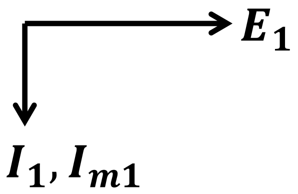
$$N_1 \cdot \Phi_m = L_{m1} \cdot i_{m1} \qquad L_{m1} = \frac{N_1^2}{\Re_c}$$

Φ_m : Mutual flux, L_{m1} : Magnetizing inductance, $\Re_c$: Reluctance of core, N_1 : Primary winding turns.

Voltage induced across the primary winding due to the mutual flux Φ_m : $e_1 = N_1 \cdot \frac{d\Phi_m}{dt} = L_{m1} \cdot \frac{di_{m1}}{dt}$

In theory, $i_1 = i_{m1}$ for $i_2 = 0$

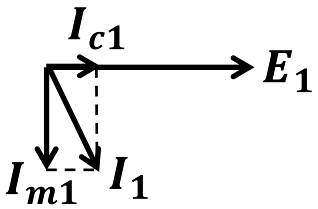


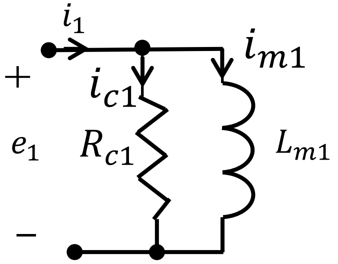


The phasors are denoted by bold letters.

In practice and by observations, $i_1 \neq i_{m1}$ for $i_2 = 0$

It is due to the core losses which is represented by R_{c1}





i_{c1} : Core-loss current

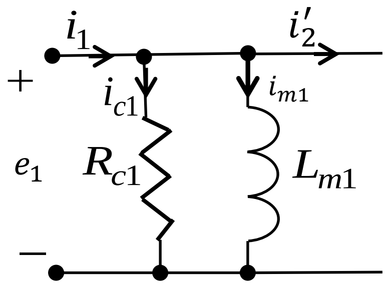
R_{c1} : Core-loss resistance

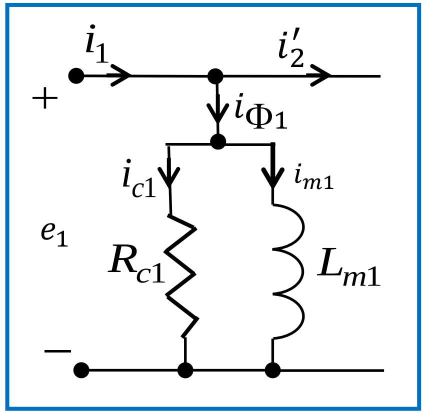
Then, for $i_2 \neq 0$,

$$i_1 = \frac{N_2}{N_1} \cdot i_2 + i_{m1} + i_{c1}$$

$$i_1 = i_2' + i_{m1} + i_{c1}$$

$$i_2' = \frac{1}{a} \cdot i_2 \ , \quad a = \frac{N_1}{N_2} \ \text{Turns ratio}$$

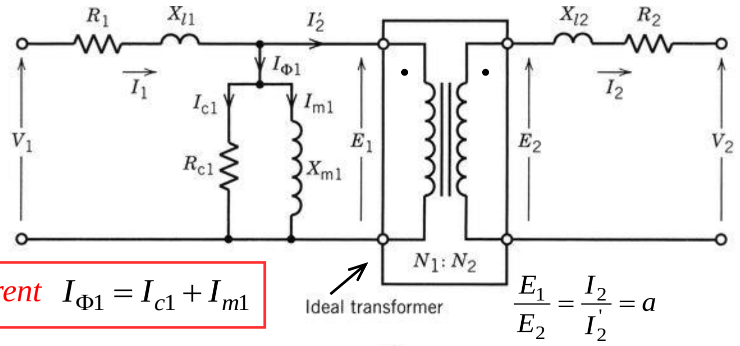




Exciting current

$$i_{\Phi1} = i_{c1} + i_{m1}$$

Transformer equivalent circuit that takes into account i) winding resistances, ii) winding leakage fluxes, and ii) core losses:



(c)

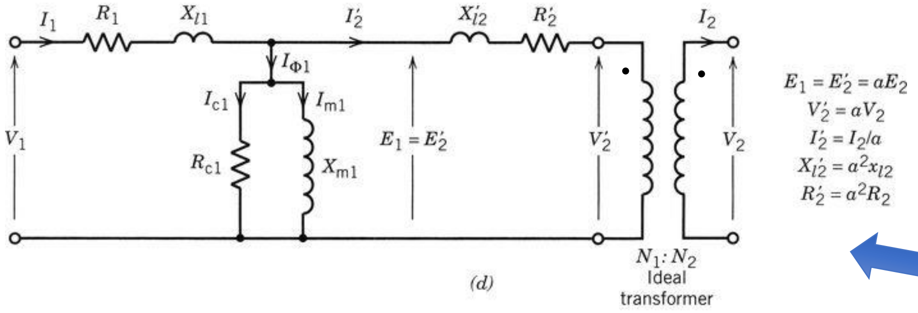
Exciting current $I_{\Phi 1} = I_{c1} + I_{m1}$

$$\begin{cases} \boldsymbol{E_2} = \boldsymbol{I_2} \cdot [j \cdot X_{l2} + R_2] + \boldsymbol{V_2} \\ \dfrac{\boldsymbol{E_1}}{a} = a \cdot \boldsymbol{I_2'} \cdot [j \cdot X_{l2} + R_2] + \boldsymbol{V_2} \\ \boldsymbol{E_1} = \boldsymbol{I_2'} \cdot [j \cdot a^2 \cdot X_{l2} + a^2 \cdot R_2] + a \cdot \boldsymbol{V_2} \end{cases}$$

Definitions: The referred quantities

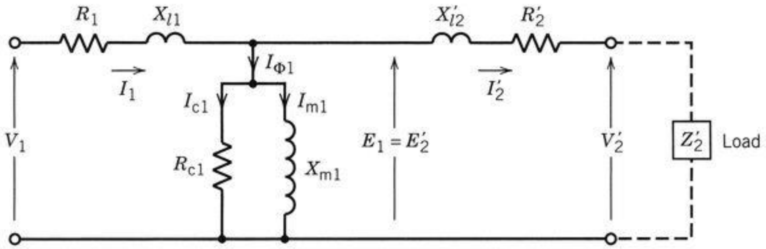
$\boxed{\boldsymbol{E_1} = \boldsymbol{E_2'} = a \cdot \boldsymbol{E_2}}$ $\boxed{\boldsymbol{V_2'} = a \cdot \boldsymbol{V_2}}$ $\boxed{\boldsymbol{I_2'} = \dfrac{\boldsymbol{I_2}}{a}}$

$\boxed{X'_{l2} = a^2 \cdot X_{l2}}$ $\boxed{R'_2 = a^2 \cdot R_2}$



(d)

$$\boldsymbol{E_2'} = \boldsymbol{I_2'} \cdot [j \cdot X_{l2}' + R_2'] + \boldsymbol{V_2'}$$



(e)

Load Impedance $\boldsymbol{Z_2} = \dfrac{\boldsymbol{V_2}}{\boldsymbol{I_2}} = \dfrac{\boldsymbol{V_2'}}{a^2 \cdot \boldsymbol{I_2'}}$

$$\boxed{\frac{\boldsymbol{V_2'}}{\boldsymbol{I_2'}} = a^2 \cdot \boldsymbol{Z_2} = \boldsymbol{Z_2'}}$$

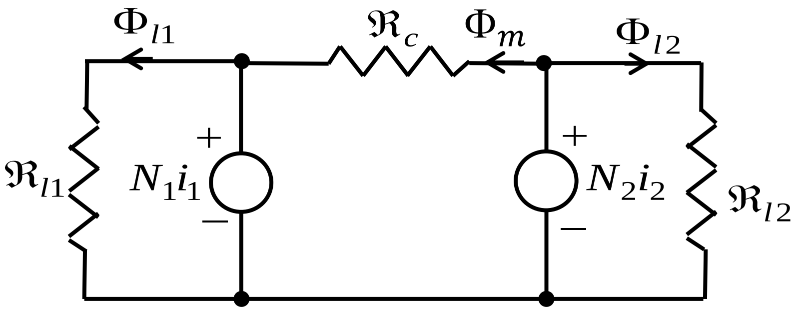
The transformer equivalent circuit with the primed (secondary side) quantities referred to the primary side.

For example: V_2' is the secondary voltage referred to the primary side.

In the equivalent magnetic circuit of the transformer, the direction of the mutual flux Φ_m can be chosen as depicted below. Then, the equations and definitions are rearranged to reflect the direction of the mutual flux.

$\Re_{l1}$: Reluctance of air path that Φ_{l1} follows

$\Re_c$: Reluctance of core mean path that Φ_m follows



$\Re_{l2}$: Reluctance of air path that Φ_{l2} follows

$$N_1 \cdot i_1 = \Re_{l1} \cdot \Phi_{l1}\,,\quad N_2 \cdot i_2 = N_1 \cdot i_1 + \Re_c \cdot \Phi_m\,,\quad N_2 \cdot i_2 = \Re_{l2} \cdot \Phi_{l2}$$

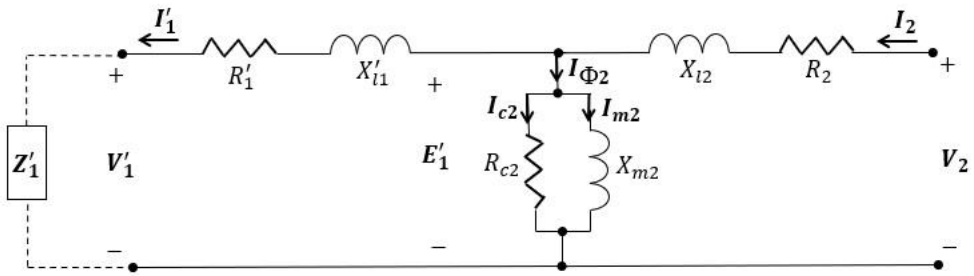
$$i_2 = \frac{N_1}{N_2} \cdot i_1 + \frac{\Re_c}{N_2} \cdot \Phi_m = \frac{N_1}{N_2} \cdot i_1 + i_{m2}$$

Magnetizing current $\quad i_{m2} = \frac{\Re_c}{N_2} \cdot \Phi_m$

Magnetizing inductance $\quad L_{m2} = \frac{N_2 \cdot \Phi_m}{i_{m2}} = \frac{N_2^2}{\Re_c}$

Voltage induced across L_{m2} : $\quad e_2 = N_2 \cdot \frac{d\Phi_m}{dt} = L_{m2} \cdot \frac{di_{m2}}{dt}$

The transformer equivalent circuit with the primed (primary side) quantities referred to the secondary side.



$$a = \frac{N_1}{N_2}\,,\quad V_1' = \frac{V_1}{a}\,,\quad E_1' = \frac{E_1}{a} = E_2\,,\quad I_1' = a \cdot I_1\,,\quad R_1' = \frac{R_1}{a^2}\,,\quad X_{l1}' = \frac{X_{l1}}{a^2}\,,\quad Z_1' = \frac{Z_1}{a^2}\,,\quad R_{c2} = \frac{R_{c1}}{a^2}\,,\quad X_{m2} = \frac{X_{m1}}{a^2}$$

Transformer Rating

The kilovolt-ampere (kVA) rating and voltage ratings of a transformer are marked on its nameplate.

For example: 10 kVA, 1100/110 volts. What are the meanings of these ratings ?

The voltage ratings indicate that the primary winding is rated for $V_1 = 1100$ V and the secondary winding is rated for $V_2 = 110$ V.

Furthermore, the transformer turns ratio is $a = V_1/V_2 = 1100/110 = 10$.

The 10 kVA rating means that each winding is designed for 10 kVA. Therefore, the current rating for the primary winding is

$I_1 = \text{kVA}/V_1 = 10000/1100 = 9.09$ A, and for the secondary winding is $I_2 = \text{kVA}/V_2 = 10000/110 = 90.9$ A.

Note also that the above given values of the voltages and currents are the rms (effective) values.

Determination of Equivalent Circuit Parameters

In order to use the equivalent circuit model for the actual transformer, the parameters R_1, X_{l1}, R_{c1}, X_{m1}, R_2, and X_{l2} must be known.

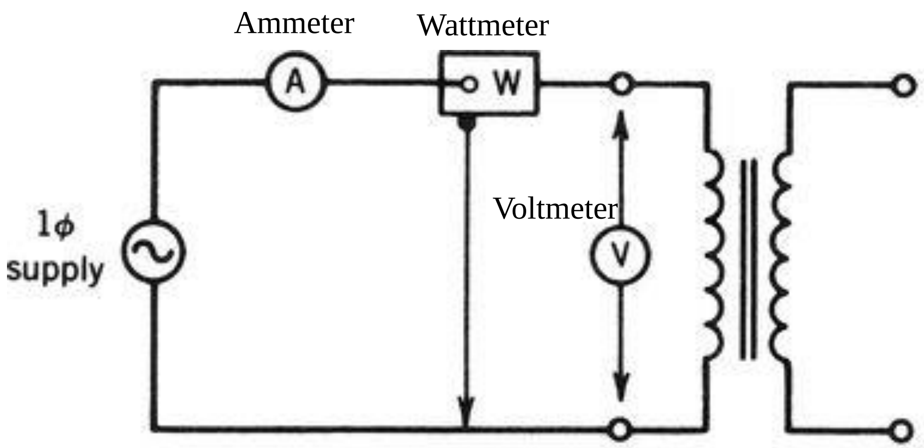
These parameters can be determined by performing two tests: 1) No-load (Open-circuit) test and 2) Short-circuit test.

No-Load Test (or Open-Circuit Test) of Transformer

Assumptions: 1) The secondary winding is open. That is, no load is connected. 2) The impedance of the series branch ($R_1 + j \cdot X_{l1}$) is much smaller than the impedance of the shunt branch ($R_{c1} // j \cdot X_{m1}$).

Since the secondary winding is open, $\boldsymbol{I_2} = 0 \;\Rightarrow\; \boldsymbol{I_2'} = 0 \;\Rightarrow\; \boldsymbol{I_{\Phi 1}} = \boldsymbol{I_1}$

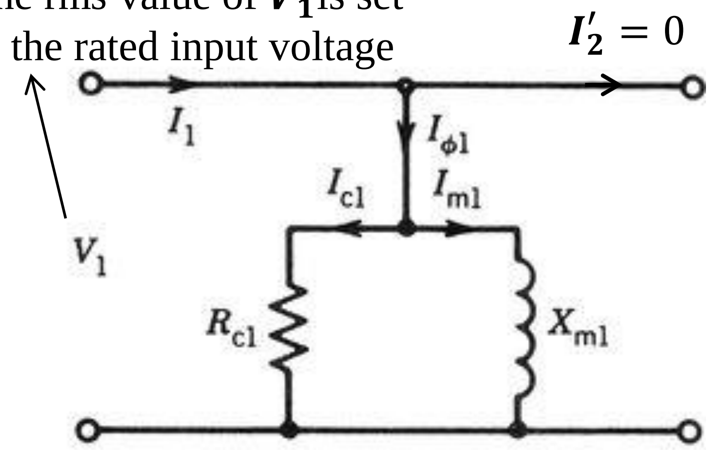
Since $\left|R_1 + j \cdot X_{l1}\right| << \left|R_{c1} // j \cdot X_{m1}\right| \Rightarrow (R_1 + j \cdot X_{l1}) + (R_{c1} // j \cdot X_{m1}) \cong R_{c1} // j \cdot X_{m1}$



(a)

Wiring diagram for no-load test

The rms value of $\boldsymbol{V_1}$ is set to the rated input voltage



(b)

Equivalent circuit under no-load

The parameters R_{c1} and X_{m1} can be determined by the voltmeter, ammeter, and wattmeter readings (V_1, I_1 and P, respectively) from the no-load test :

$$\boxed{R_{c1} = \frac{V_1^2}{P}} \Longrightarrow I_{c1} = \frac{V_1}{R_{c1}} \Longrightarrow I_{m1} = \left(I_1^2 - I_{c1}^2\right)^{1/2} \Longrightarrow \boxed{X_{m1} = \frac{V_1}{I_{m1}}}$$

Short-Circuit Test of Transformer

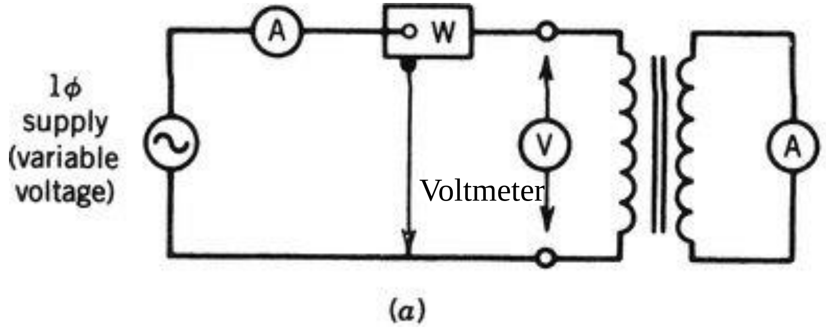
Assumptions: 1) The secondary terminals are shorted. 2) The impedance of the shunt branch (composed of R_{c1} and X_{m1}) is much larger than that of the series branch (composed of R'_2 and X'_{l2}).

Since the secondary winding is shorted, $\boldsymbol{V_2} = 0 \Rightarrow \boldsymbol{V'_2} = 0 \Rightarrow (R_{c1}//X_{m1})$ is in parallel with $\left(R'_2 + j \cdot X'_{l2}\right)$.

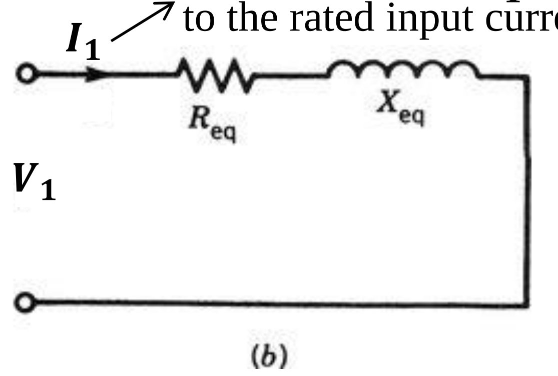
Since $\left|R_{c1} // j \cdot X_{m1}\right| >> \left|R'_2 + j \cdot X'_{l2}\right| \Rightarrow \left(R_{c1} // j \cdot X_{m1}\right) // \left(R'_2 + j \cdot X'_{l2}\right) \cong R'_2 + j \cdot X'_{l2}$

Then, $R_{eq} = R_1 + R'_2$ and $X_{eq} = X_{l1} + X'_{l2}$.

Furthermore, $R_1 = a^2 \cdot R_2 = R'_2$ and $X_{l1} = a^2 \cdot X_{l2} = X'_{l2}$ in a well-designed transformer.



Wiring diagram for short-circuit test



Equivalent circuit at short-circuit condition

The parameters R_{eq} and X_{eq} can be determined by the voltmeter, ammeter, and wattmeter readings (V_1, I_1 and P, respectively) from the short-circuit test :

$$\boxed{R_{eq} = \frac{P}{I_1^2}} \quad \text{and} \quad Z_{eq} = \frac{V_1}{I_1} \Rightarrow \boxed{X_{eq} = \left(Z_{eq}^2 - R_{eq}^2\right)^{1/2}}$$

EXAMPLE 2.2 from the textbook

Tests are performed on a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

	Open-Circuit Test (low-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	2200 V	150 V
Ammeter	0.25 A	4.55 A
Wattmeter	100 W	215 W

a) Derive the parameters for the approximate equivalent circuit referred to the high-voltage side.
b) Express the exciting current as a percentage of the rated current.
c) Determine the power factor for the no-load and short-circuit tests.

Solution

a) From the open-circuit test, $R_{c1} = \dfrac{V_1^2}{P} = \dfrac{2200^2}{100} = 48400\,\Omega$, $\quad I_{c1} = \dfrac{V_1}{R_{c1}} = \dfrac{2200}{48400} = 0.0455\,\text{A}$,

$$I_{m1} = \left(I_1^2 - I_{c1}^2\right)^{1/2} = \left(0.25^2 - 0.0455^2\right)^{1/2} = 0.2458\,\text{A}, \quad X_{m1} = \frac{V_1}{I_{m1}} = \frac{2200}{0.2458} = 8949\,\Omega, \quad I_{\Phi 1} = I_1 = 0.25\,\text{A}.$$

From the short-circuit test, $R_{eq} = \dfrac{P}{I_1^2} = \dfrac{215}{4.55^2} = 10.4\,\Omega$, $\quad Z_{eq} = \dfrac{V_1}{I_1} = \dfrac{150}{4.55} = 32.97\,\Omega$,

$$X_{eq} = \left(Z_{eq}^2 - R_{eq}^2\right)^{1/2} = \left(32.97^2 - 10.4^2\right)^{1/2} = 31.3\,\Omega.$$

b) $I_{1(rated)} = \dfrac{VA}{V_{1(rated)}} = \dfrac{10000\text{ VA}}{2200\text{ V}} = 4.55\text{ A}$, $\quad \dfrac{I_{\Phi 1}}{I_{1(rated)}} \times 100\% = \dfrac{0.25\text{ A}}{4.55\text{ A}} \times 100\% = 5.5\%$.

c) Power factor at no load

$$\frac{P}{VA} = \frac{100\text{ W}}{2200\text{ V} \times 0.25\text{ A}} = 0.182$$

Power factor at short-circuit

$$\frac{P}{VA} = \frac{215\text{ W}}{150\text{ V} \times 4.55\text{ A}} = 0.315$$

UNIVERSITY of HOUSTON
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ECE 4363 – Electromechanical Energy Conversion

Lecture 07

Date: February 11, 2021

by

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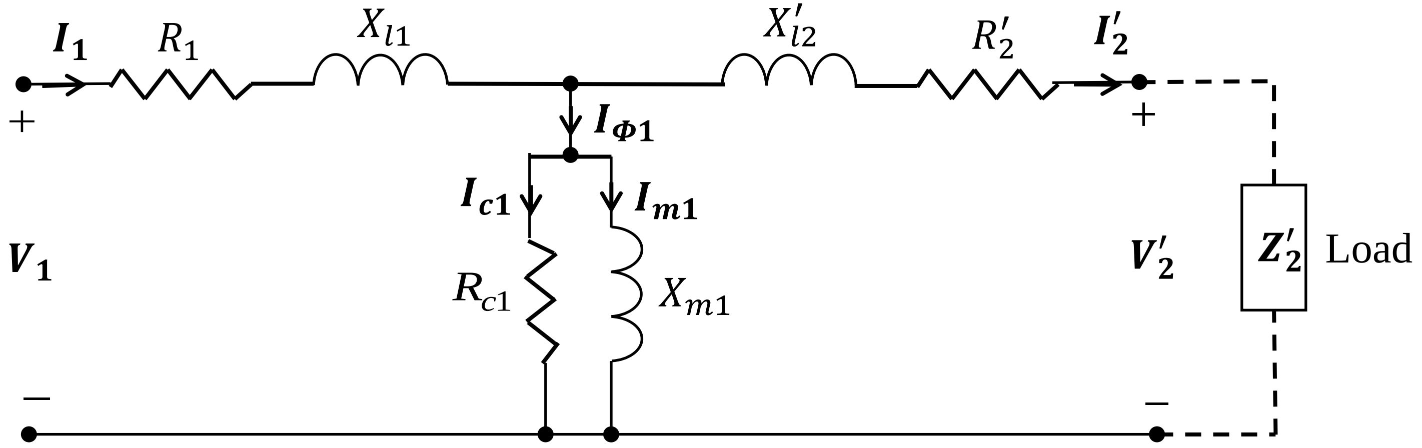
University of Houston, Houston, TX

Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 07 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

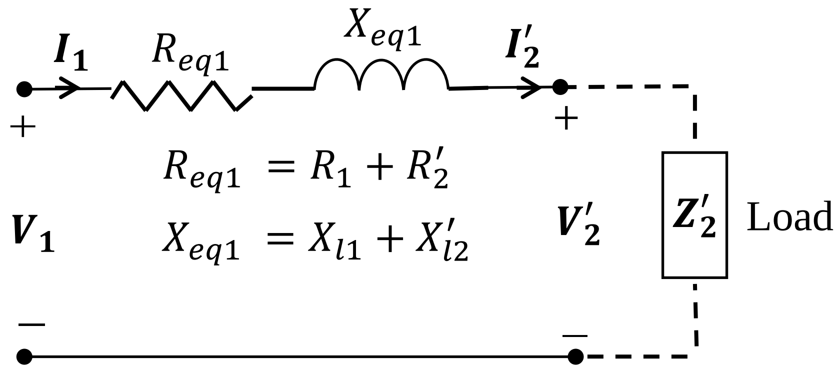
Approximate Equivalent Circuit of Transformer



Equivalent circuit of transformer with all quantities (voltages, currents and impedances) referred to the primary side.

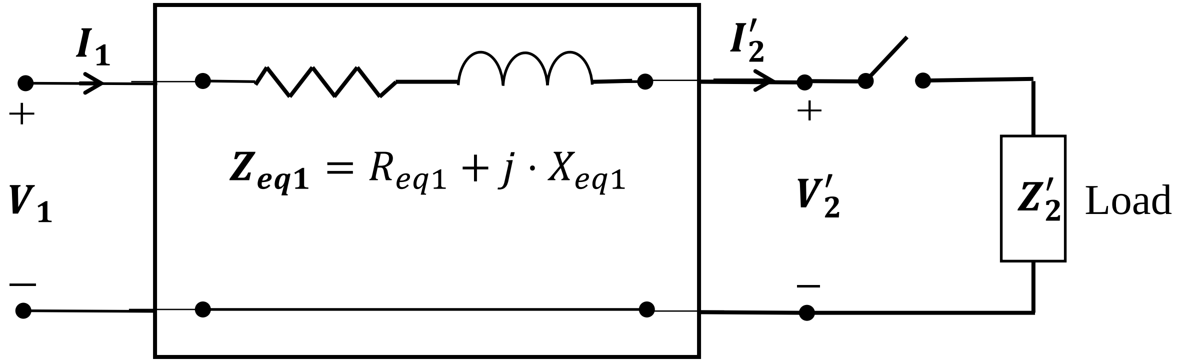
$$\boldsymbol{V}'_2 = a \cdot \boldsymbol{V}_2 \,, \quad \boldsymbol{I}'_2 = \frac{\boldsymbol{I}_2}{a} \,, \quad X'_{l2} = a^2 \cdot X_{l2} \,, \quad R'_2 = a^2 \cdot R_2 \,, \quad \boldsymbol{Z}'_2 = \frac{\boldsymbol{V}'_2}{\boldsymbol{I}'_2} = a^2 \cdot \boldsymbol{Z}_2 \,, \quad a = \frac{N_1}{N_2}$$

In a transformer, the exciting current $\boldsymbol{I}_{\Phi 1}$ *is a small percentage of the rated current of the transformer (e.g., about 5% or less). Then,* $\boldsymbol{I}_1 = \boldsymbol{I}'_2 + \boldsymbol{I}_{\Phi 1} \cong \boldsymbol{I}'_2$ *. That is, the approximate circuit is obtained by removing the excitation branch.*



Approximate equivalent circuit of transformer with all quantities referred to the primary side.

Voltage Regulation of Transformer



Transformer with equivalent internal impedance $\boldsymbol{Z_{eq1}} = R_{eq1} + j \cdot X_{eq1}$.

If a load is not applied to the transformer (open-circuit or no-load condition), there is no voltage drop across the internal impedance $\boldsymbol{Z_{eq1}}$ since $\boldsymbol{I'_2} = 0$. As a result, $\boldsymbol{V'_2} = \boldsymbol{V_1}$.

If the load is connected to the transformer secondary by closing the load switch, the load terminal voltage $\boldsymbol{V'_2}$ changes from the no-load voltage value. This is due to the voltage drop in the internal impedance of the transformer caused by the load current $\boldsymbol{I'_2}$. The voltage drop is given by $\boldsymbol{I'_2} \cdot \boldsymbol{Z_{eq1}}$.

A figure of merit called *voltage regulation* (VR) is used to identify the voltage change in a transformer with loading:

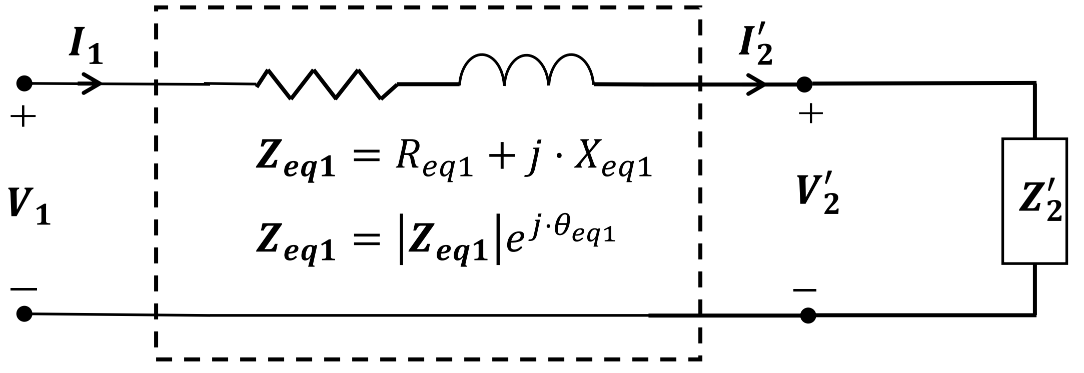
$$\boxed{\mathrm{VR} = \frac{|\boldsymbol{V_2}|_{\mathrm{NL}} - |\boldsymbol{V_2}|_{\mathrm{L}}}{|\boldsymbol{V_2}|_{\mathrm{L}}} = \frac{|\boldsymbol{V'_2}|_{\mathrm{NL}} - |\boldsymbol{V'_2}|_{\mathrm{L}}}{|\boldsymbol{V'_2}|_{\mathrm{L}}} = \frac{|\boldsymbol{V_1}| - |\boldsymbol{V'_2}|_{\mathrm{L}}}{|\boldsymbol{V'_2}|_{\mathrm{L}}} = \frac{|\boldsymbol{V_1}| - |\boldsymbol{V'_2}|_{\mathrm{rated}}}{|\boldsymbol{V'_2}|_{\mathrm{rated}}}}$$

Magnitude of the transformer secondary voltage with no load: $|\boldsymbol{V_2}|_{\mathrm{NL}}$

Magnitude of the transformer secondary voltage with load: $|\boldsymbol{V_2}|_{\mathrm{L}}$

The load voltage is taken as the rated voltage. That is, $|\boldsymbol{V_2}|_{\mathrm{L}} = |\boldsymbol{V_2}|_{\mathrm{rated}}$, or, $|\boldsymbol{V'_2}|_{\mathrm{L}} = |\boldsymbol{V'_2}|_{\mathrm{rated}}$.

Phasor Diagram of Transformer

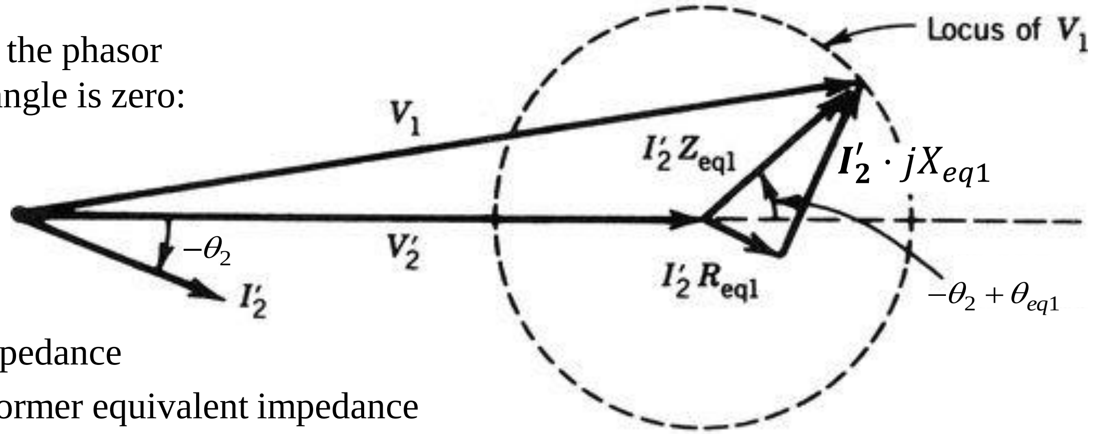


$$\boldsymbol{Z'_2} = |\boldsymbol{Z'_2}| e^{j \cdot \theta_2}$$

$$\boldsymbol{I_1} = \boldsymbol{I'_2} \qquad \boldsymbol{V'_2} = \boldsymbol{I'_2} \cdot \boldsymbol{Z'_2} \Rightarrow \boldsymbol{I'_2} = \frac{e^{-j \cdot \theta_2}}{|\boldsymbol{Z'_2}|} \cdot \boldsymbol{V'_2}$$

$$\boxed{\boldsymbol{V_1} = \boldsymbol{V'_2} + \boldsymbol{I'_2} \cdot \boldsymbol{Z_{eq1}} = \boldsymbol{V'_2} + \frac{e^{-j \cdot \theta_2}}{|\boldsymbol{Z'_2}|} \boldsymbol{V'_2} \cdot |\boldsymbol{Z_{eq1}}| e^{j \cdot \theta_{eq1}} = \boldsymbol{V'_2} + \boldsymbol{V'_2} \cdot \frac{|\boldsymbol{Z_{eq1}}|}{|\boldsymbol{Z'_2}|} e^{-j \cdot (\theta_2 - \theta_{eq1})}}$$

$\boldsymbol{V'_2}$ is the reference in the phasor diagram. That is, its angle is zero: $\boldsymbol{V'_2} = |\boldsymbol{V'_2}| e^{j \cdot 0}$



Phasor diagram

θ_2 : Angle of load impedance

θ_{eq1} : Angle of transformer equivalent impedance

EXAMPLE 2.3 from the textbook

Consider a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer with an equivalent internal impedance $\boldsymbol{Z_{eq1}} = 10.4 + j31.3\ \Omega$. a) Determine the voltage regulation (VR) in percent for the 75% full load with 0.6 power factor lagging. Assume that i) the rated value of the transformer output kVA is 10kVA, ii) the load voltage is at the rated value (220V), and iii) the load current is 75% of the rated current. b) Draw the phasor diagram for this load condition.

Solution

a) $I_{2(rated)} = \dfrac{VA|_{rated}}{V_{2(rated)}} = \dfrac{10000\text{ VA}}{220\text{ V}} = 45.5\text{ A}$ Turns ratio $a = \dfrac{2200\text{ V}}{220\text{ V}} = 10$

$$I_1 = I_2' = \frac{I_2}{a} = \frac{0.75 \cdot I_{2(rated)}}{a} = \frac{0.75 \cdot 45.5\text{ A}}{10} = 3.41\text{ A}$$

Power factor $\text{PF} = Cos(\theta_2) = 0.6 \Rightarrow \theta_2 = 53.13^\circ$, $\boldsymbol{I_1} = \boldsymbol{I_2'} = 3.41\angle -53.13^\circ\text{A}$

$$\boldsymbol{V_2'} = a \cdot V_2\angle 0^\circ\text{ V} = 10 \cdot 220\angle 0^\circ\text{ V} = 2200\angle 0^\circ\text{ V}$$

$$\boldsymbol{Z_{eq1}} = \sqrt{10.4^2 + 31.3^2}\angle(\arctan(31.3/10.4)) = 32.97\angle 71.62^\circ \Rightarrow \quad |\boldsymbol{Z_{eq1}}| = 32.97,\ \theta_{eq1} = 71.62^\circ$$

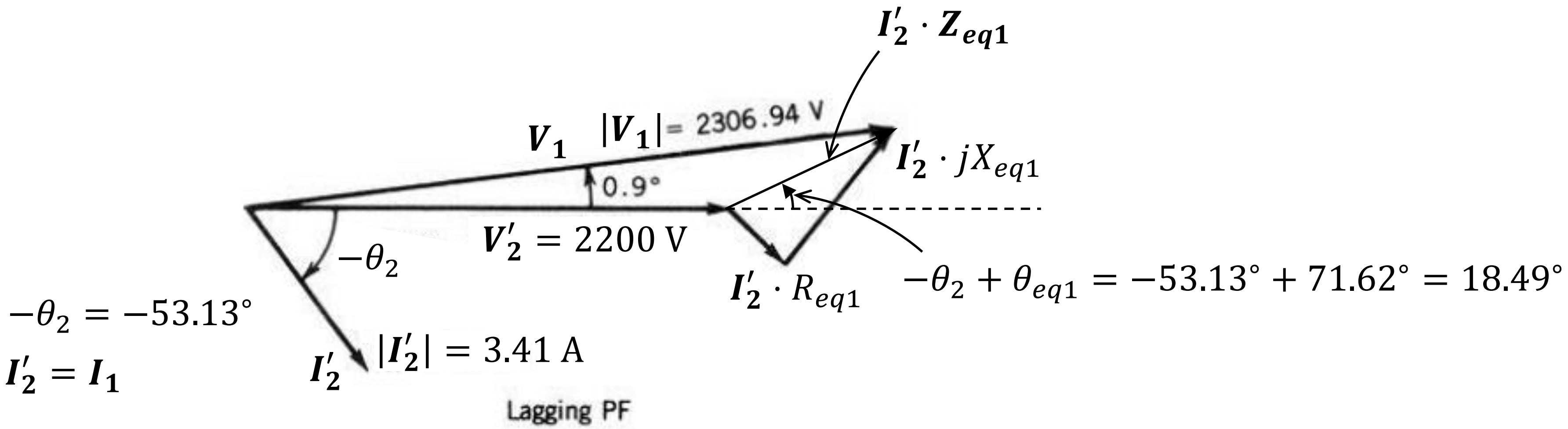
$$\boldsymbol{V_1} = \boldsymbol{V_2'} + \boldsymbol{I_2'} \cdot \boldsymbol{Z_{eq1}} = 2200\angle 0^\circ + (3.41\angle -53.13^\circ) \cdot (32.97\angle 71.62^\circ) = 2200\angle 0^\circ + 112.43\angle 18.49^\circ$$

$$\boldsymbol{V_1} = 2306.66 + j35.67 = 2306.94\angle 0.9^\circ$$

$$\text{VR in percent } = \frac{|\boldsymbol{V_1}| - |\boldsymbol{V_2'}|_{\text{rated}}}{|\boldsymbol{V_2'}|_{\text{rated}}} \times 100 = \frac{2306.94 - 2200}{2200} \times 100 = 4.86\%$$

That is, when the 75% full load at 0.6 lagging PF is connected to the load terminals of the transformer, the voltage drops from 230.69 to 220 volts.

b) The phasor diagram for this load condition



EFFICIENCY of TRANSFORMER

The efficiency is defined as following:

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}}}$$

P_{out} is the output power of the transformer in [W]. It is the power delivered to the load by the transformer.

P_{in} is the input power of the transformer in [W]. It is provided by a power supply to the primary of the transformer.

P_{loss} is the power losses in the transformer in [W], which include i) the core loss P_c and ii) copper loss P_{cu}.

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}}$$

$$P_{cu} = \underbrace{I_1^2 \cdot R_1 + I_2^2 \cdot R_2}_{\text{Power dissipated as heat by the winding resistances.}} = I_1^2 \cdot R_1 + I_2^2 \cdot \frac{R_2'}{a^2} = I_1^2 \cdot R_1 + I_2'^2 \cdot R_2' = I_1^2 \cdot R_{eq1} = I_2^2 \cdot \frac{R_{eq1}}{a^2}$$

$P_c \approx \frac{V_1^2}{R_{c1}}$ The core loss is almost constant since the primary voltage V_1 is essentially constant.

$P_{out} = V_2 \cdot I_2 \cdot \cos(\theta_2)$ Normally, the secondary voltage V_2 is almost constant. Then, the output power is determined by the load current I_2 and the load power factor $\cos(\theta_2)$.

EXAMPLE 2.4 from the textbook

A1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer has a core loss $P_c = 100\ \mathrm{W}$ at rated voltage and a copper loss $P_{cu} = 215\ \mathrm{W}$ at full load. Determine the efficiency at 75% rated output (0.75 × 10 kVA at 220 V) and 0.6 PF.

Solution

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}} \times 100\%$$

$$P_{out} = V_2 \cdot I_2 \cdot \cos\theta_2 = 0.75 \cdot 10000 \cdot 0.6 = 4500\ \mathrm{W}$$

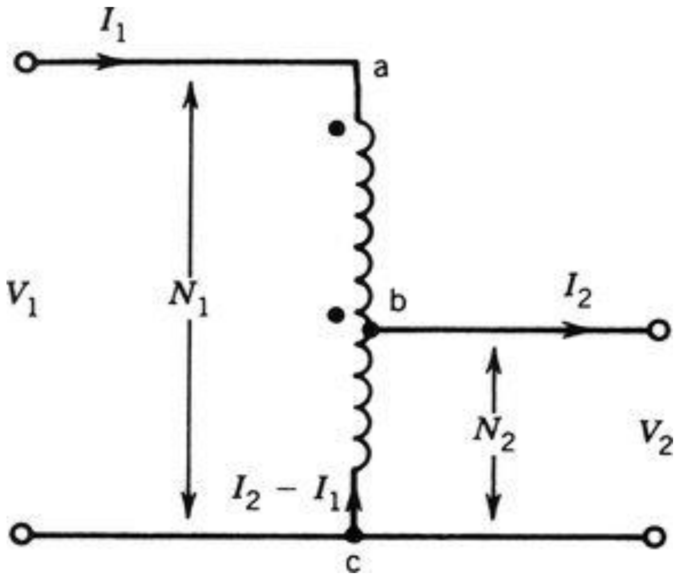
$$P_c = 100\ \mathrm{W}$$

$$P_{cu} = (0.75)^2 \cdot 215\ \mathrm{W} = 121\ \mathrm{W}$$

$$\eta = \frac{4500\ \mathrm{W}}{4500\ \mathrm{W} + 100\ \mathrm{W} + 121\ \mathrm{W}} \times 100\% = 95.32\%$$

AUTOTRANSFORMER

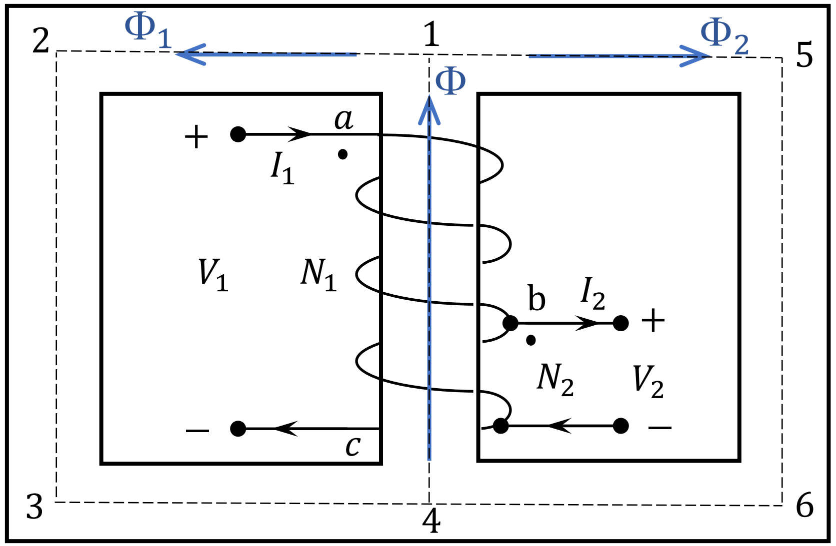
An autotransformer is a special transformer which has only single (common) winding mounted on a core. It has two end terminals (points a and c) on the primary (input) side and an intermediate tap (at point b) to form the secondary (output) as shown in the figure below. In brief, the portion of the winding (between points b and c) is shared by both the primary and secondary. Note also that the primary and secondary have a common terminal at point c. In addition, a continuous variable turns ratio, and hence a variable output, can be obtained by making the secondary connection at point b through a sliding brush (https://en.wikipedia.org/wiki/Autotransformer). An autotransformer with a variable output voltage is also called variac.



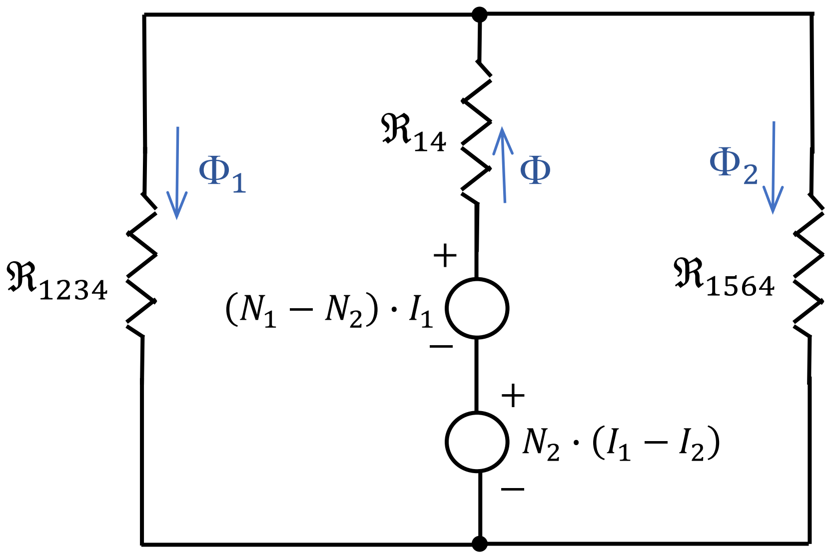
Autotransformer.

The advantages of an autotransformer are i) smaller, lighter, and cheaper than typical dual-winding transformers, ii) lower leakage reactances, lower losses (higher power efficiency), and increased VA rating for a given size and mass, and iii) a variable output voltage when the secondary connection is made through a sliding brush. The disadvantage with the autotransformer is that there is no electrical isolation between the primary and secondary sides (https://en.wikipedia.org/wiki/Autotransformer).

Equivalent Magnetic Circuit of Autotransformer



Equivalent Circuit



N_1: Number of turns between points a and c

N_2: Number of turns between points b and c

Primary and secondary have a common negative terminal at point c.

$\Re_{14}$: Reluctance of center leg between points 1 and 4.

$\Re_{1234}$: Reluctance of mean path through points 1, 2, 3 and 4.

$\Re_{1564}$: Reluctance of mean path through points 1, 5, 6 and 4.

$$V_1 = \frac{d(N_1 \cdot \Phi)}{dt} \quad \text{and} \quad V_2 = \frac{d(N_2 \cdot \Phi)}{dt}\,.$$

Then,

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

$\Phi = \Phi_1 + \Phi_2$

From the Ampere's circuit law,

$$(N_1 - N_2) \cdot I_1 + N_2 \cdot (I_1 - I_2) = \Re_{14} \cdot \Phi + \Re_{1234} \cdot \Phi_1$$

$$(N_1 - N_2) \cdot I_1 + N_2 \cdot (I_1 - I_2) = \Re_{14} \cdot \Phi + \Re_{1564} \cdot \Phi_2$$

For a magnetic core with very high permeability, $\Re_{14} \approx 0$, $\Re_{1234} \approx 0$ and $\Re_{1564} \approx 0$. Then,

$$(N_1 - N_2) \cdot I_1 + N_2 \cdot (I_1 - I_2) = 0$$

Or,

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}}$$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 08

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by

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P.S. The pictures, notations, formulas, and statements in these lecture 08 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

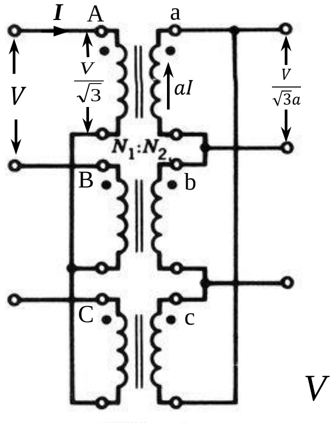
THREE-PHASE TRANSFORMERS

A set of three similar single-phase transformers may be connected to form a three-phase transformer. There are four possible connections for a three-phase transformer: 1) Y–Δ, 2) Δ–Y, 3) Δ–Δ, and 4) Y–Y.

1) Y–Δ: The primary windings are connected in wye (Y) configuration. The secondary windings are connected in delta (Δ) configuration.

On the primary side, three terminals of identical polarity are connected together to form the neutral of the Y connection. On the secondary side, the windings are connected in series.

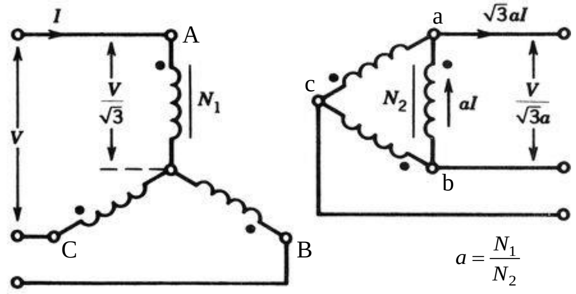
The Y–Δ connection is commonly used to step down a high voltage to a low voltage (e.g., for $a = \frac{N_1}{N_2} \geq 1$).



(a) Y - Δ

Y–Δ

≡

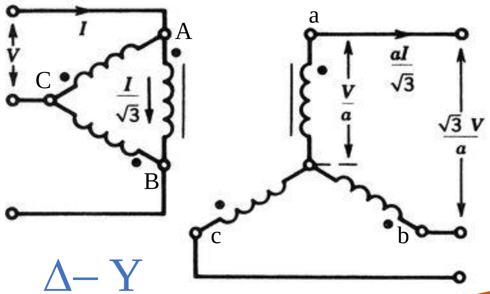


V is the primary line-to-line voltage

(b) Y – Δ

2) Δ–Y : The primary windings are connected in delta (Δ) configuration. The secondary windings are connected in wye (Y) configuration.

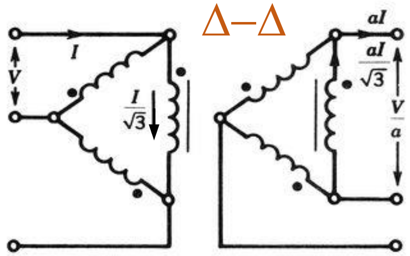
The Δ– Y connection is commonly used to step up voltage (e.g., for $a = \frac{N_1}{N_2} \leq 1$).



Δ– Y

(c) Δ - Y

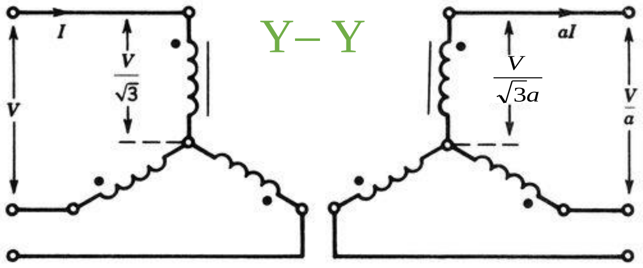
Δ–Δ



(d) Δ – Δ

3) Δ–Δ : The primary windings are connected in delta (Δ) configuration. The secondary windings are connected in Δ configuration as well.

Y– Y



(e) Y - Y

4) Y–Y : The primary windings are connected in wye (Y) configuration. The secondary windings are connected in Y configuration as well.

Phase Shift between the Primary and Secondary Line-to-Line Voltages of Three-Phase Transformer

The Y–Δ and Δ–Y connections of a three-phase transformer result in a 30° phase shift between the primary and secondary line-to-line voltages. On the other hand, the Δ–Δ and Y–Y connections have no phase shift in their line-to-line voltages.

Phase Shift in the Y–Δ connection

$$\boldsymbol{V}_{\mathbf{AN}} = V_p \cdot e^{j\cdot 0^\circ}$$

$$\boldsymbol{V}_{\mathbf{BN}} = V_p \cdot e^{-j\cdot 120^\circ}$$

$$\boldsymbol{V}_{\mathbf{CN}} = V_p \cdot e^{+j\cdot 120^\circ}$$

$$\boldsymbol{V}_{\mathbf{AB}} = \boldsymbol{V}_{\mathbf{AN}} - \boldsymbol{V}_{\mathbf{BN}} = \sqrt{3} \cdot V_p \cdot e^{+j\cdot 30^\circ}$$

$$\boldsymbol{V}_{\mathbf{AB}} = \sqrt{3} \cdot \boldsymbol{V}_{\mathbf{AN}} \cdot e^{+j\cdot 30^\circ}$$

$$\boldsymbol{V}_{\mathbf{BC}} = \boldsymbol{V}_{\mathbf{BN}} - \boldsymbol{V}_{\mathbf{CN}} = \sqrt{3} \cdot V_p \cdot e^{-j\cdot 90^\circ}$$

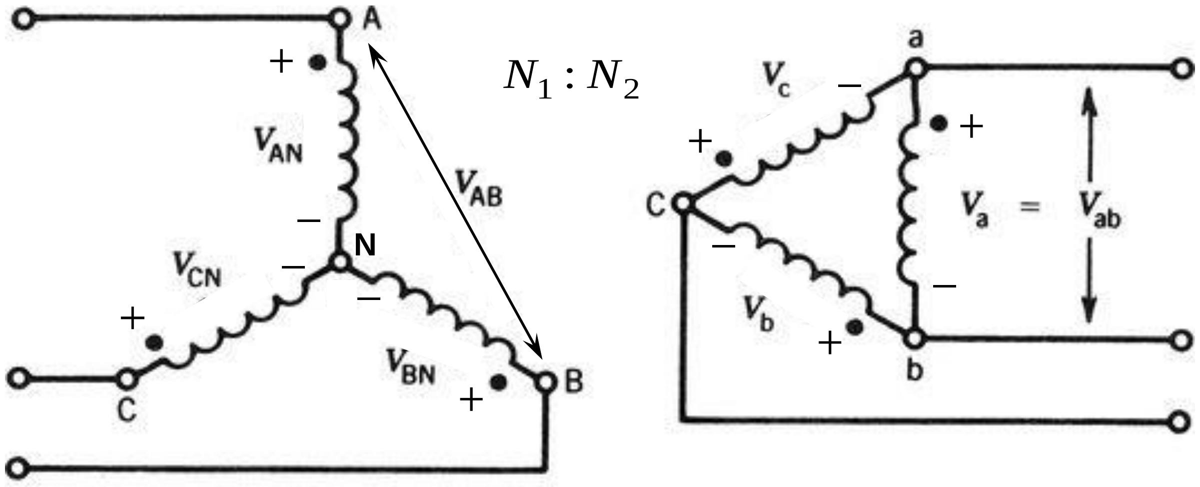
$$\boldsymbol{V}_{\mathbf{BC}} = \sqrt{3} \cdot \boldsymbol{V}_{\mathbf{BN}} \cdot e^{+j\cdot 30^\circ}$$

$$\boldsymbol{V}_{\mathbf{BC}} = \boldsymbol{V}_{\mathbf{AB}} \cdot e^{-j\cdot 120^\circ}$$

$$\boldsymbol{V}_{\mathbf{CA}} = \boldsymbol{V}_{\mathbf{CN}} - \boldsymbol{V}_{\mathbf{AN}} = \sqrt{3} \cdot V_p \cdot e^{+j\cdot 150^\circ}$$

$$\boldsymbol{V}_{\mathbf{CA}} = \sqrt{3} \cdot \boldsymbol{V}_{\mathbf{CN}} \cdot e^{+j\cdot 30^\circ}$$

$$\boldsymbol{V}_{\mathbf{CA}} = \boldsymbol{V}_{\mathbf{AB}} \cdot e^{+j\cdot 120^\circ}$$

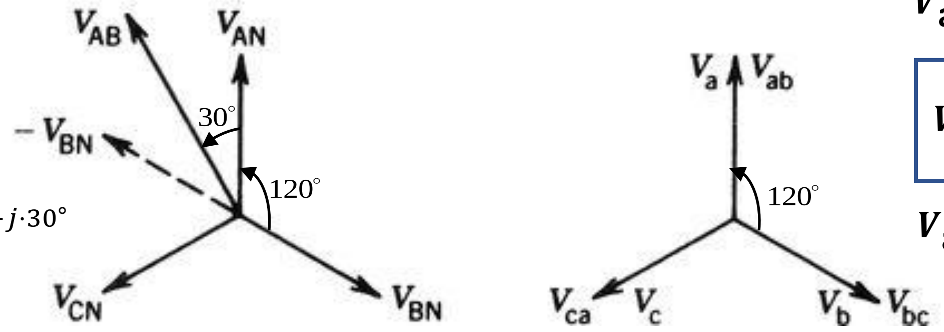


$$\boldsymbol{V}_{\mathbf{ab}} = \boldsymbol{V}_{\mathbf{a}} = \frac{\boldsymbol{V}_{\mathbf{AN}}}{a}, \quad a = \frac{N_1}{N_2}$$

$$\boxed{\boldsymbol{V}_{\mathbf{ab}} = \frac{1}{\sqrt{3} \cdot a} \cdot \boldsymbol{V}_{\mathbf{AB}} \cdot e^{-j\cdot 30^\circ}}$$

$\boldsymbol{V}_{\mathbf{ab}}$ is lagging $\boldsymbol{V}_{\mathbf{AB}}$ by 30°.

$$\boldsymbol{V}_{\mathbf{AB}} = \sqrt{3} \cdot \boldsymbol{V}_{\mathbf{AN}} \cdot e^{+j\cdot 30^\circ}$$

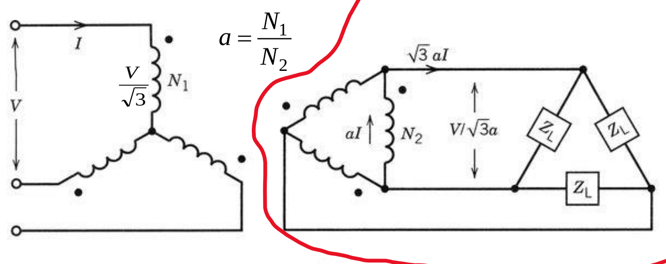


Single-Phase Equivalent Circuit of Three-Phase Transformer

Assumption: The three transformers forming the three-phase transformer are identical, and the source and load are balanced. Then, the voltages and currents on both primary and secondary sides are balanced. The voltages and currents in one phase are the same as those in the other phases except that there is a phase shift of 120°.

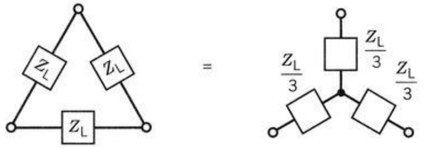
The actual circuit. The load is Δ–connected.

$$\frac{\text{Primary line} - \text{to} - \text{line voltage}}{\text{Secondary line} - \text{to} - \text{line voltage}} = \frac{V}{V/(\sqrt{3}\cdot a)} = \sqrt{3}\cdot a$$



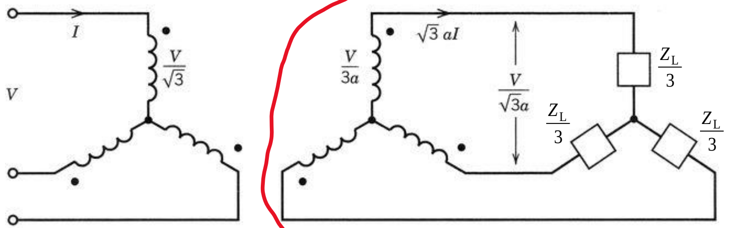
(a)

The equivalent Y load is obtained for the delta load by the Δ–Y transformation.



(b)

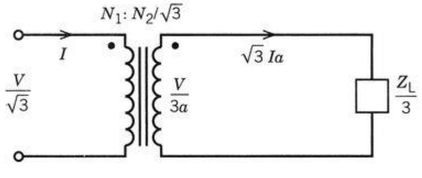
The equivalent Y representation of the Δ–connected secondary circuit. Note that the primary and secondary line currents and line-to-line voltages are identical to those of the actual circuit.



(c)

The turns ratio of the equivalent Y–Y transformer:

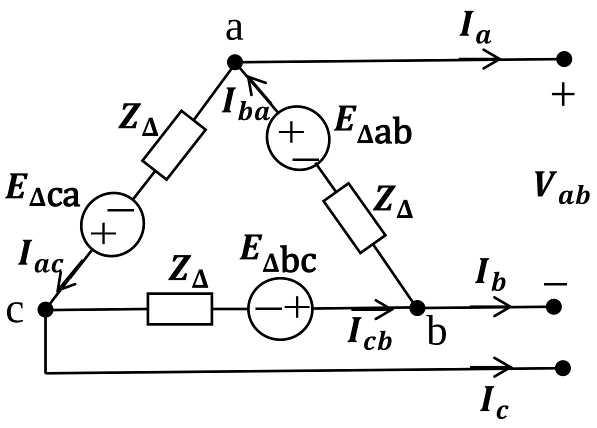
$$a' = \frac{V/\sqrt{3}}{V/(3\cdot a)} = \sqrt{3}\cdot a = \frac{N_1}{N_2/\sqrt{3}}$$



(d)

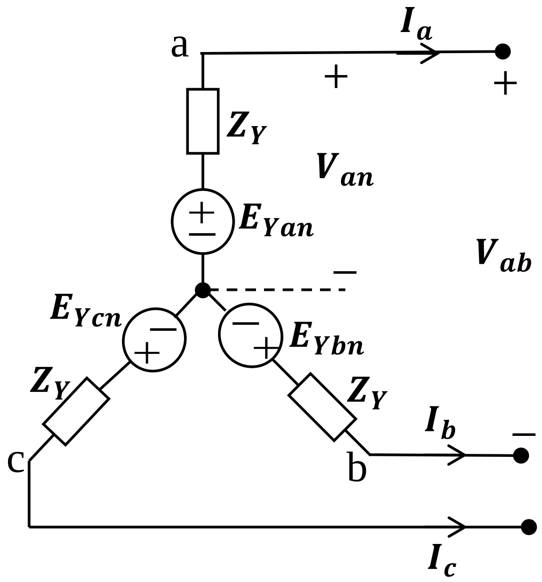
Δ-Y Transformation

Three-phase source with impedance in Δ Connection



Δ-Y Transformation

Three-phase source with impedance in equivalent Y Connection



Assumption: Three-phase source voltages and load currents are balanced.

$$\boldsymbol{E}_{\Delta\mathbf{ab}} = E_\Delta \cdot e^{j\cdot 0^\circ},\ \boldsymbol{E}_{\Delta\mathbf{bc}} = E_\Delta \cdot e^{-j\cdot 120^\circ},\ \boldsymbol{E}_{\Delta\mathbf{ca}} = E_\Delta \cdot e^{+j\cdot 120^\circ}$$

$$\boldsymbol{E}_{\Delta\mathbf{ab}} + \boldsymbol{E}_{\Delta\mathbf{bc}} + \boldsymbol{E}_{\Delta\mathbf{ca}} = 0$$

$$\boldsymbol{I}_{\mathbf{a}} = I_L \cdot e^{-j\cdot(\theta_2+30^\circ)},\ \boldsymbol{I}_{\mathbf{b}} = I_L \cdot e^{-j\cdot(\theta_2+150^\circ)},\ \boldsymbol{I}_{\mathbf{c}} = I_L \cdot e^{-j\cdot(\theta_2-90^\circ)}$$

$$\boldsymbol{I}_{\mathbf{a}} + \boldsymbol{I}_{\mathbf{b}} + \boldsymbol{I}_{\mathbf{c}} = 0$$

$$\boldsymbol{I}_{\mathbf{ba}} = \frac{I_L}{\sqrt{3}} \cdot e^{-j\cdot\theta_2},\ \boldsymbol{I}_{\mathbf{cb}} = \frac{I_L}{\sqrt{3}} \cdot e^{-j\cdot(\theta_2+120^\circ)},\ \boldsymbol{I}_{\mathbf{ac}} = \frac{I_L}{\sqrt{3}} \cdot e^{-j\cdot(\theta_2-120^\circ)}$$

$$\boldsymbol{I}_{\mathbf{ba}} + \boldsymbol{I}_{\mathbf{cb}} + \boldsymbol{I}_{\mathbf{ac}} = 0$$

$$\boldsymbol{E}_{Y\mathbf{an}} = E_Y \cdot e^{-j\cdot 30^\circ}$$

$$\boldsymbol{E}_{Y\mathbf{bn}} = E_Y \cdot e^{-j\cdot 150^\circ}$$

$$\boldsymbol{E}_{Y\mathbf{cn}} = E_Y \cdot e^{+j\cdot 90^\circ}$$

$$\boldsymbol{E}_{Y\mathbf{an}} + \boldsymbol{E}_{Y\mathbf{bn}} + \boldsymbol{E}_{Y\mathbf{cn}} = 0$$

$$E_Y = \frac{E_\Delta}{\sqrt{3}},\quad |\boldsymbol{V}_{an}| = \frac{|\boldsymbol{V}_{ab}|}{\sqrt{3}} \text{ and } \boldsymbol{Z}_Y = \frac{\boldsymbol{Z}_\Delta}{3}$$

Single-Phase Representation of Three-Phase Transformer with Y–Δ Connection by Considering Equivalent Y-Connection of Δ–Connected Secondary

$$\frac{E_{Y1}}{E_{\Delta 2}} = a\ ,\quad \boxed{a = \frac{N_1}{N_2}} \quad \text{and} \quad \frac{E_{Y1}}{E_{Y2}} = \frac{E_{Y1}}{\left(E_{\Delta 2}/\sqrt{3}\right)} = \frac{a \cdot E_{\Delta 2}}{\left(E_{\Delta 2}/\sqrt{3}\right)} = a' \quad \text{where} \quad a' = \sqrt{3} \cdot a$$

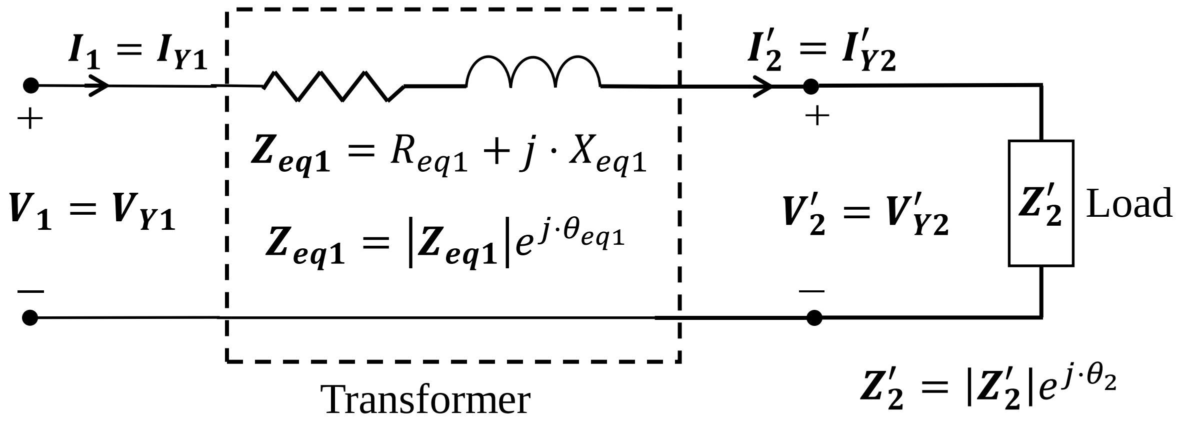
$$R_{Y2} = \frac{R_{\Delta 2}}{3} \quad \text{and} \quad R'_{Y2} = a'^2 \cdot R_{Y2} = a'^2 \cdot \frac{R_{\Delta 2}}{3} = a^2 \cdot R_{\Delta 2}$$

$$X_{Yl2} = \frac{X_{\Delta l2}}{3} \quad \text{and} \quad X'_{Yl2} = a'^2 \cdot X_{Yl2} = a'^2 \cdot \frac{X_{\Delta l2}}{3} = a^2 \cdot X_{\Delta l2}$$

$$I'_{Y2} = \frac{I_{Y2}}{a'} = \frac{\sqrt{3} \cdot I_{\Delta 2}}{a'} = \frac{I_{\Delta 2}}{a} = I_{Y1}$$

$$V'_{Y2} = a' \cdot V_{Y2} = a' \cdot \frac{V_{\Delta 2}}{\sqrt{3}} = a \cdot V_{\Delta 2}$$

Single phase representation with secondary quantities referred to the primary side



$$\boldsymbol{Z_{eq1}} = R_{eq1} + j \cdot X_{eq1}$$

$$\boldsymbol{Z_{eq1}} = R_{Y1} + R'_{Y2} + j \cdot (X_{Yl1} + X'_{Yl2})$$

$$\boxed{\begin{aligned} R_{eq1} &= R_{Y1} + a^2 \cdot R_{\Delta 2} \\ X_{eq1} &= X_{Yl1} + a^2 \cdot X_{\Delta l2} \end{aligned}}$$

$$\boldsymbol{Z'_2} = a'^2 \cdot \boldsymbol{Z_{Y2}} = a'^2 \cdot \frac{\boldsymbol{Z_L}}{3} = a^2 \cdot \boldsymbol{Z_L}$$

Assumptions: i) Transformer has Y–Δ connection and ii) load $\boldsymbol{Z_L}$ is Δ connected.

EXAMPLE 2.7 from the textbook

Three 1ϕ, 50 kVA, 2300/230 V, 60 Hz transformers are connected to form a 3ϕ, 4000/230 V transformer bank. The equivalent impedance of each transformer referred to the high voltage side is $\boldsymbol{Z_{eq1}} = 1.2 + j1.6\ \Omega$. The 3ϕ transformer supplies a 3ϕ, 120 kVA, 230 V, 0.85 PF (lagging) load.

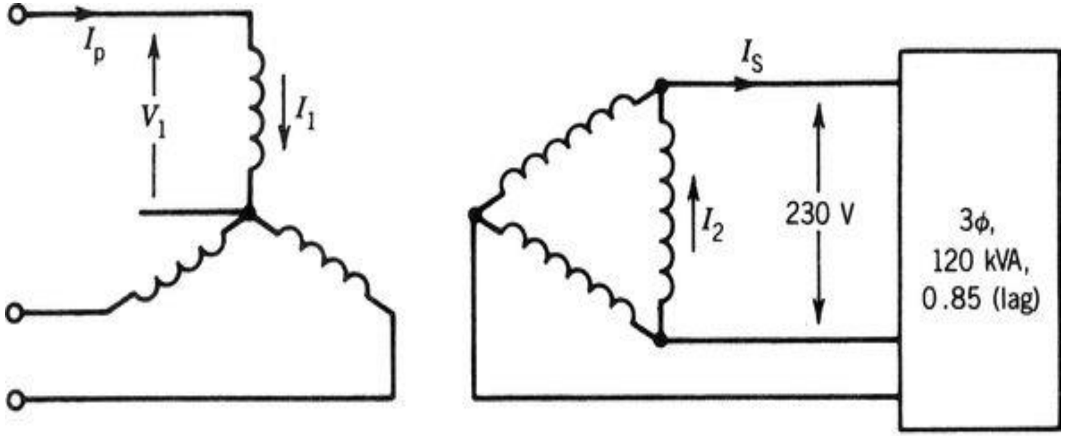
a) Draw a schematic diagram showing the transformer connection.
b) Determine the transformer winding currents.
c) Determine the primary line-to-line voltage required.
d) Determine the voltage regulation.

Solution

a) The connection diagram:

The high-voltage windings are connected in wye so that the primary can be connected to the 4000 V supply ($2300\text{V} \times \sqrt{3} \cong 4000\text{V}$).

The low-voltage winding is connected in delta to form a 230 V system for the load.



(a)

b) $$I_s = \frac{120000}{\sqrt{3}\cdot 230} = 301.24\text{ A}$$

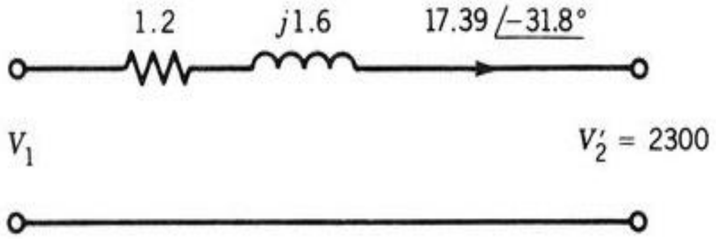
$$I_2 = \frac{301.24}{\sqrt{3}} = 173.92\text{ A}$$

$$a = \frac{2300\text{ V}}{230\text{ V}} = 10$$

$$I_1 = \frac{I_2}{a} = \frac{173.92\text{ A}}{10} = 17.39\text{ A}$$

c) The calculation is carried out on a per-phase basis.

$\boldsymbol{Z_{eq1}} = 1.2 + j1.6 = 2.0\angle 53.13°\ \Omega$



(b)

The load power factor:

$\mathrm{PF} = Cos(\theta_2) = 0.85 \Rightarrow \theta_2 = 31.8°$

$\boldsymbol{V_2'} = a \cdot \boldsymbol{V_2} = 10 \times 230\ \mathrm{V} = 2300\ \mathrm{V}$

$\boldsymbol{I_2'} = \boldsymbol{I_1} = 17.39\angle - 31.8°\mathrm{A}$

$\boldsymbol{V_1} = \boldsymbol{V_2'} + \boldsymbol{I_2'} \cdot \boldsymbol{Z_{eq1}} = 2300\angle 0° + (17.39\angle - 31.8°) \cdot (2.0\angle 53.13°) = 2300\angle 0° + 34.78\angle 21.33°$

$\boldsymbol{V_1} = 2332.40 + j12.65 = 2332.43\angle 0.31°\ \mathrm{V}$

$|\boldsymbol{V_1}| = 2332.43\ \mathrm{V}$

Primary line-to-line voltage= $\sqrt{3}|\boldsymbol{V_1}| = 4039.89\ \mathrm{V}$

d)

$$\mathrm{VR} = \frac{2332.43\mathrm{V} - 2300\mathrm{V}}{2300\mathrm{V}} \times 100 = 1.41\%$$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 09

Date: February 25, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 09 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

PER-UNIT (PU) SYSTEM

The per-unit (pu) quantities are used to simplify the calculations. The pu quantity is defined by

$$\text{Quantity in pu} = \frac{\text{actual quantity}}{\text{base (or reference) value of the quantity}}$$

Usually, the base values of power and voltage are selected, and then, the base values of current and impedance are obtained as follows:

$$P_{\text{base}}\ , V_{\text{base}}\ \ \text{selected}$$

$$I_{\text{base}} = \frac{P_{\text{base}}}{V_{\text{base}}}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}^2}{P_{\text{base}}}$$

Normally, the rated volt-amperes (VA) and rated voltage (V) are taken as the base values for power and voltage, respectively.

$$S_{\text{base}} = P_{\text{base}} = \text{rated volt–amperes (VA)}$$

$$V_{\text{base}} = \text{rated voltgae (V)}$$

In a transformer, the base power is the same for both primary and secondary. However, the values of base voltage are different on each side since the rated voltages are different for the two sides.

Primary side:

V_{base}, $V_{\text{B1}} = V_{\text{R1}} =$ rated voltage of primary

I_{base}, $I_{\text{B1}} = I_{\text{R1}} =$ rated current of primary

Z_{base}, $Z_{\text{B1}} = \dfrac{V_{\text{R1}}}{I_{\text{R1}}}$

$\boldsymbol{Z_{eq1}}$, the equivalent impedance of the transformer referred to the primary side

Per unit value of $\boldsymbol{Z_{eq1}}$ is $\boldsymbol{Z_{\text{eq1,pu}}} = \dfrac{\boldsymbol{Z_{eq1}}}{Z_{\text{B1}}} = Z_{\text{eq1}} \cdot \dfrac{I_{\text{R1}}}{V_{\text{R1}}}$

Secondary side:

V_{base}, $V_{\text{B2}} = V_{\text{R2}} =$ rated voltage of secondary

I_{base}, $I_{\text{B2}} = I_{\text{R2}} =$ rated current of secondary

Z_{base}, $Z_{\text{B2}} = \dfrac{V_{\text{R2}}}{I_{\text{R2}}}$

$\boldsymbol{Z_{\text{eq2}}}$, the equivalent impedance of the transformer referred to the secondary side

Per unit value of $\boldsymbol{Z_{\text{eq2}}}$ is $\boldsymbol{Z_{\text{eq2,pu}}} = \dfrac{\boldsymbol{Z_{\text{eq2}}}}{Z_{\text{B2}}} = \boldsymbol{Z_{\text{eq2}}} \cdot \dfrac{I_{\text{R2}}}{V_{\text{R2}}} = \dfrac{\boldsymbol{Z_{eq1}}}{a^2} \cdot \dfrac{a^2 \cdot I_{\text{R1}}}{V_{\text{R1}}} = \dfrac{\boldsymbol{Z_{eq1}}}{Z_{\text{B1}}} = \boldsymbol{Z_{\text{eq1},pu}}$

In brief, the per-unit impedances referred to the primary and secondary sides are the same in a transformer. Furthermore,

$$\boldsymbol{I}_{\mathbf{1},\mathbf{pu}} = \frac{\boldsymbol{I}_{\mathbf{1}}}{I_{\mathrm{B1}}} = \frac{\boldsymbol{I}_{\mathbf{1}}}{I_{\mathrm{R1}}} = \frac{\boldsymbol{I}'_{\mathbf{2}}}{I_{\mathrm{R1}}} = \frac{\boldsymbol{I}_{\mathbf{2}}/a}{I_{\mathrm{R2}}/a} = \frac{\boldsymbol{I}_{\mathbf{2}}}{I_{\mathrm{R2}}} = \boldsymbol{I}_{\mathbf{2},\boldsymbol{pu}}$$

$$\boldsymbol{V}_{\mathbf{1},\mathbf{pu}} = \frac{\boldsymbol{V}_{\mathbf{1}}}{V_{\mathrm{B1}}} = \frac{\boldsymbol{V}_{\mathbf{1}}}{V_{\mathrm{R1}}} \quad \text{and} \quad \boldsymbol{V}_{\mathbf{2},\boldsymbol{pu}} = \frac{\boldsymbol{V}_{\mathbf{2}}}{V_{\mathrm{B2}}} = \frac{\boldsymbol{V}_{\mathbf{2}}}{V_{\mathrm{R2}}}$$

Transformer Equivalent Circuit in Per-Unit Form

From the equivalent circuit of a transformer referred to the primary shown in figure (a) on the right,

$$\boldsymbol{V}_{\mathbf{1}} = \boldsymbol{V}'_{\mathbf{2}} + \boldsymbol{I}'_{\mathbf{2}} \cdot \boldsymbol{Z}_{\boldsymbol{eq1}}$$

Let's divide the both sides of the above equation by V_{R1}.

$$\frac{\boldsymbol{V}_{\mathbf{1}}}{V_{\mathrm{R1}}} = \frac{\boldsymbol{V}'_{\mathbf{2}}}{V_{\mathrm{R1}}} + \frac{\boldsymbol{I}'_{\mathbf{2}} \cdot \boldsymbol{Z}_{\boldsymbol{eq1}}}{V_{\mathrm{R1}}} = \frac{a \cdot \boldsymbol{V}_{\mathbf{2}}}{a \cdot V_{\mathrm{R2}}} + \frac{\boldsymbol{I}_{\mathbf{1}} \cdot \boldsymbol{Z}_{\boldsymbol{eq1}}}{I_{\mathrm{R1}} \cdot Z_{\mathrm{B1}}}$$

Or,

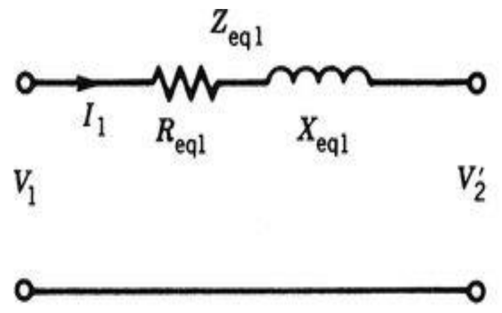
$$\boldsymbol{V}_{\mathbf{1},\mathbf{pu}} = \boldsymbol{V}_{\mathbf{2},\mathbf{pu}} + \boldsymbol{I}_{\mathbf{1},\mathbf{pu}} \cdot \boldsymbol{Z}_{\mathbf{eq1},\mathbf{pu}}$$

Then, the equivalent circuit in per-unit form is given in figure (b) on the right.

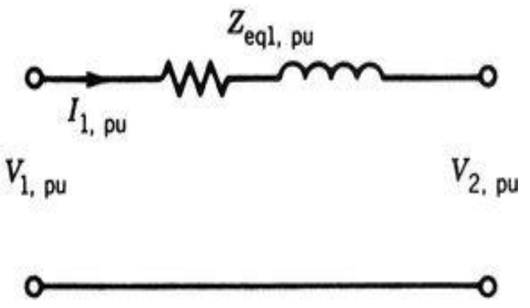
With

$$\boldsymbol{I}_{\mathbf{pu}} = \boldsymbol{I}_{\mathbf{1},\mathbf{pu}} = \boldsymbol{I}_{\mathbf{2},\boldsymbol{pu}} \quad \text{and} \quad \boldsymbol{Z}_{\mathbf{eq},\mathbf{pu}} = \boldsymbol{Z}_{\mathbf{eq1},\boldsymbol{pu}} = \boldsymbol{Z}_{\mathbf{eq2},\boldsymbol{pu}},$$

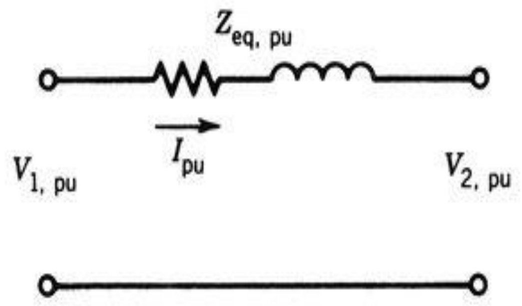
The transformer equivalent circuit in per-unit form can be depicted as in figure (c) on the right.



(a)



(b)



(c)

There are two advantages in using a per-unit system: 1) The parameters and variables fall in a narrow numerical range when expressed in a per-unit system; this simplifies computations and makes it possible to quickly check the correctness of the computed values. 2) The transformer equivalent circuit in per-unit form is the same whether the quantities are referred to the primary or secondary. This eliminates the need to refer impedances to one side or the other of transformer (*Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 6th Edition, 2003*).

EXAMPLE 2.9 from the textbook

A 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer has an equivalent impedance $\boldsymbol{Z_{eq1}} = 10.4 + j31.3\ \ \Omega$ which is referred to the primary side. a) Obtain the equivalent circuit in per-unit form. b) Find the full-load copper loss in per-unit form and in [W]. c) Determine the per-unit voltage regulation when the transformer delivers 75% full load at 0.6 lagging power factor.

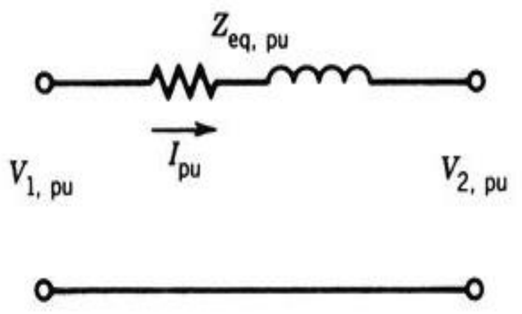
Solution

a) The base values are

$$S_{\text{base}} = P_{\text{base}} = 10000\text{ VA}$$

$$V_{\text{B1}} = V_{\text{R1}} = 2200\text{ V}, \quad I_{\text{B1}} = I_{\text{R1}} = \frac{P_{\text{base}}}{V_{\text{B1}}} = \frac{10000}{2200} = 4.55\text{ A} \quad \text{and} \quad Z_{\text{B1}} = \frac{V_{\text{R1}}}{I_{\text{R1}}} = \frac{2200}{4.55} = 483.52\ \Omega.$$

$$V_{\text{B2}} = V_{\text{R2}} = 220\text{ V}, \quad I_{\text{B2}} = \frac{P_{\text{base}}}{V_{\text{B2}}} = \frac{10000}{220} = 45.45\text{ A} \quad \text{and} \quad Z_{\text{B2}} = \frac{V_{\text{R2}}}{I_{\text{R2}}} = \frac{220}{45.45} = 4.84\ \Omega.$$



The equivalent circuit in per-unit form is shown on the right where

$$\boldsymbol{V}_{\mathbf{1,pu}} = \frac{\boldsymbol{V}_{\mathbf{1}}}{V_{\text{R1}}}, \quad \boldsymbol{V}_{\boldsymbol{2,pu}} = \frac{\boldsymbol{V}_{\mathbf{2}}}{V_{\text{R2}}}, \quad \boldsymbol{I}_{\boldsymbol{pu}} = \boldsymbol{I}_{\mathbf{1,pu}} = \boldsymbol{I}_{\boldsymbol{2,pu}} \text{ and } \boldsymbol{Z}_{\mathbf{eq,pu}} = \boldsymbol{Z}_{\mathbf{eq1,pu}} = \boldsymbol{Z}_{\mathbf{eq2,pu}}$$

In the equivalent circuit,

$$\boldsymbol{Z}_{\mathbf{eq},\boldsymbol{pu}} = R_{eq,pu} + j \cdot X_{eq,pu} = \frac{10.4 + j31.3}{483.52} = 0.0215 + j0.0647 \text{ pu}.$$

b) At full load,

$$I_{pu} = \frac{I_1}{I_{\text{R1}}} = \frac{I_{\text{R1}}}{I_{\text{R1}}} = 1.0 \text{ pu}$$

The copper loss in pu is

$$P_{cu,pu} = R_{eq,pu} \cdot I_{pu}^2 = 0.0215 \cdot (1.0)^2 = 0.0215 \text{ pu}.$$

The copper loss in W is

$$P_{cu} = R_{eq1} \cdot I_1^2 = 10.4 \cdot (4.55)^2 = 215 \text{ W}.$$

Or,

$$P_{cu} = P_{cu,pu} \cdot P_{\text{base}} = 0.0215 \cdot 10000 = 215 \text{ W}.$$

c) At 75% full load at 0.6 lagging power factor, the per-unit current is

$$\boldsymbol{I}_{\boldsymbol{pu}} = 0.75\angle(-\cos^{-1}(0.6)) = 0.75\angle(-53.13°) = 0.45 - j0.60 \text{ pu}.$$

The per-unit secondary voltage is

$$\boldsymbol{V}_{\mathbf{2},\boldsymbol{pu}} = 1.0\angle 0° \text{ pu}.$$

Then, from the equivalent circuit in per-unit form,

$$\boldsymbol{V}_{\mathbf{1,pu}} = \boldsymbol{V}_{\mathbf{2},\boldsymbol{pu}} + \boldsymbol{I}_{\boldsymbol{pu}} \cdot \boldsymbol{Z}_{\mathbf{eq},\boldsymbol{pu}} = 1.0\angle 0° + (0.45 - j0.60) \cdot (0.0215 + j0.0647) = 1.0485 + j0.0162 = 1.0486\angle 0.89° \text{ pu}\,.$$

The per-unit voltage regulation (VR) is

$$\text{VR} = \frac{1.0486 - 1.0}{1.0} = 0.0486 \text{ pu}\,.$$

Or,

$$\text{VR} = 4.86\%\,.$$

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ECE 4363 – Electromechanical Energy Conversion

Lecture 10

Date: March 02, 2021

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Spring 2021

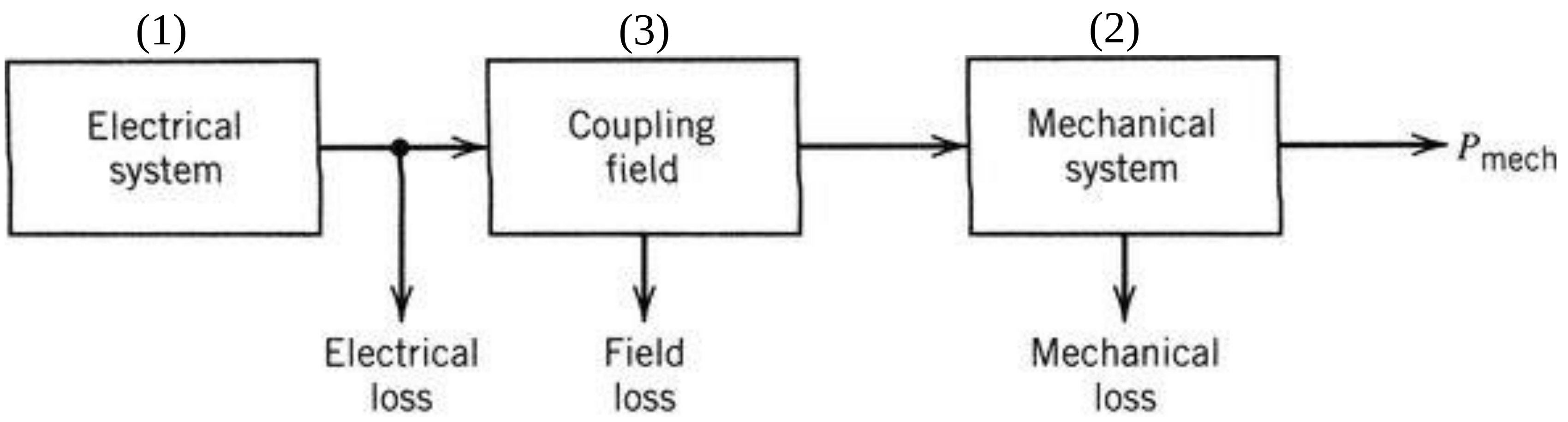
P.S. The pictures, notations, formulas, and statements in these lecture 10 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Electromechanical Energy Conversion

Conversion from Electrical Energy to Mechanical Energy and vice versa

Calculation of Force or Torque developed in an energy conversion device is based on the principle of conservation of energy: Energy can neither be created nor destroyed; it can only be changed from one form to another.

An electromechanical system has three essential parts:



Energy Balance Equation

Electrical energy input from source = Mechanical energy output + Increase in stored energy in coupling field + Energy losses

Energy Losses

i) Electrical energy loss: $i^2 \cdot R$ loss in the resistance R of winding. It is dissipated as heat to the surrounding area of the winding.

ii) Mechanical energy loss: Friction loss due to the motion of the moving components. For example, bearing loss given by $\omega^2 \cdot f$ where ω and f are the angular speed and friction coefficient, respectively. It is dissipated as heat.

iii) Field loss: Core loss due to changing magnetic field in the magnetic core. It is dissipated as heat.

Energy losses = Electrical energy loss + Mechanical energy loss + Field loss

Incremental Energy Balance Equation

$$dW_e = dW_m + dW_f$$

dW_e: Incremental electrical energy supplied to the system during differential time interval dt. It is after the $(i^2 \cdot R)$ loss is subtracted.

dW_m: Energy converted to the mechanical form during differential time interval dt. It is in useful form or as loss, or part useful and part as loss.

dW_f: Energy supplied to the magnetic field during differential time interval dt. It is either stored or lost, or part stored and part lost.

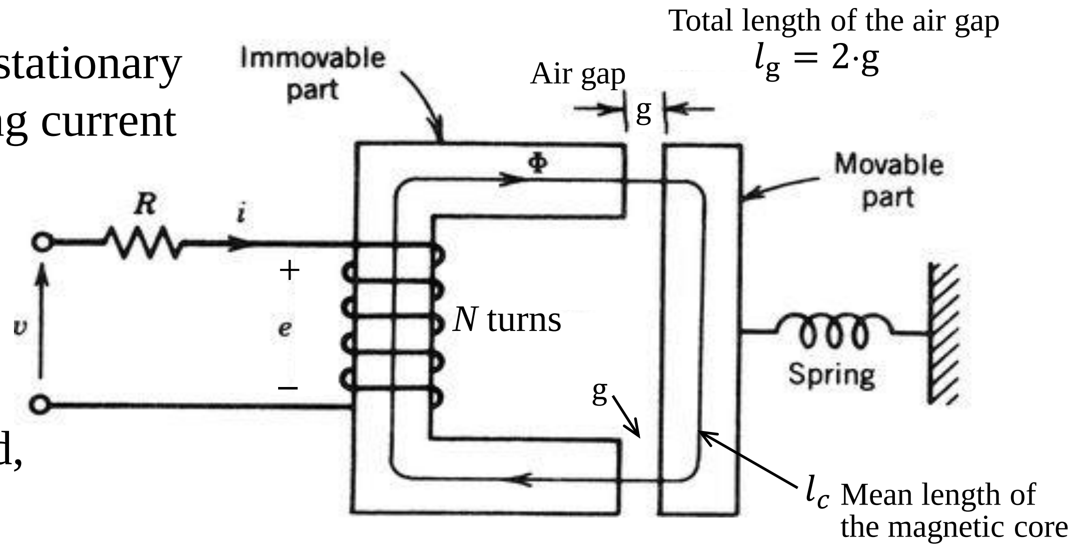
Field (Magnetic Field) Energy

Assumption: The movable part is held stationary at some air gap length g and the winding current is increased from 0 to a value i.

$dW_m = 0$ since there is no motion.

$\Rightarrow \ dW_e = dW_f$

If winding and core losses are neglected, then, the incremental electrical energy input is stored as incremental field energy.



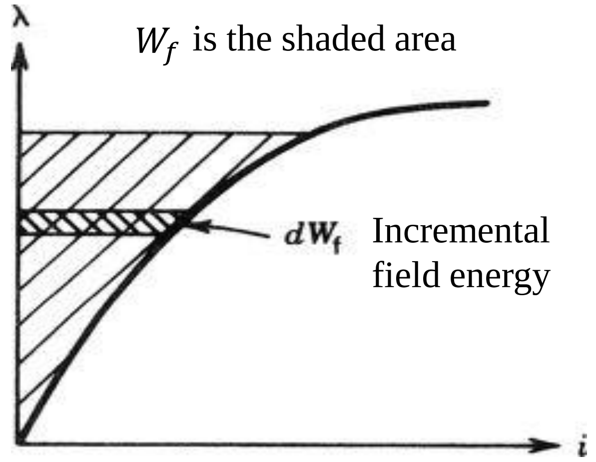
An electromechanical system

$e = \dfrac{d\lambda}{dt}$ where λ is total flux linkage by the winding.

$dW_e = e \cdot i \cdot dt = i \cdot d\lambda$

$\Rightarrow \ dW_f = i \cdot d\lambda \ \Rightarrow \ W_f = \int_0^{\lambda} i \cdot d\lambda$

W_f is the energy stored in the field.



$\lambda - i$ characteristic of the above electromechanical system for a specific air gap length g.

Ampere's circuit law: $N \cdot i = H_c \cdot l_c + H_g \cdot l_g$

H_c: Magnetic field intensity in the core.

H_g: Magnetic field intensity in the air gap.

l_c: Mean length of the magnetic core. l_g: Total length of the air gap.

Expression of Field Energy for an Electromechanical System

$$\lambda = N \cdot \Phi = N \cdot A \cdot B$$

Φ : Magnetic flux crossing the cross section of the core [Wb]

λ : Flux linkage of the coil (winding) [Wb·turns]

A : Cross-sectional area of the core [m²]

B : Flux density, assumed same throughout [Wb/m²]

$$\Rightarrow \quad W_f = \int_0^B \frac{H_c \cdot l_c + H_g \cdot l_g}{N} \cdot N \cdot A \cdot dB$$

For the air gap, $H_g = \dfrac{B}{\mu_0}$

Then, $$W_f = \int_0^B \left(H_c \cdot l_c \cdot A + \frac{B}{\mu_0} \cdot l_g \cdot A \right) \cdot dB = V_c \cdot \underbrace{\int_0^B H_c \cdot dB}_{w_{fc}} + V_g \cdot \underbrace{\frac{B^2}{2 \cdot \mu_0}}_{w_{fg}}$$

V_c: Volume of the magnetic core [m³]

V_g: Volume of the air gap [m³]

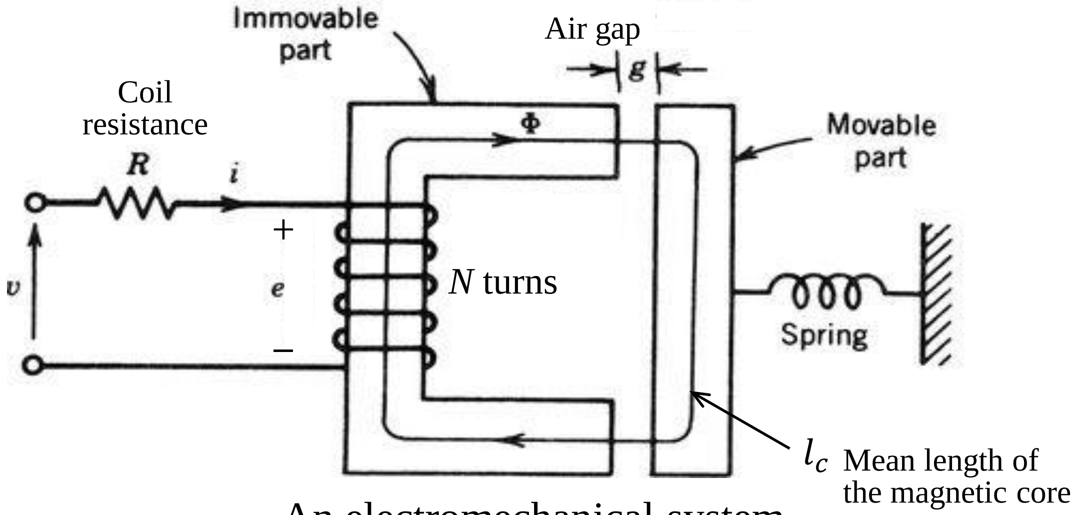
w_{fc}: Energy density in the magnetic core [J/m³]

w_{fg}: Energy density in the air gap [J/m³]

For a linear magnetic core (no magnetic saturation), μ_c is constant and much greater than μ_0. Then, $w_{fc} = \dfrac{B^2}{2 \cdot \mu_c} \ll w_{fg}$.

EXAMPLE 3.1 from the textbook

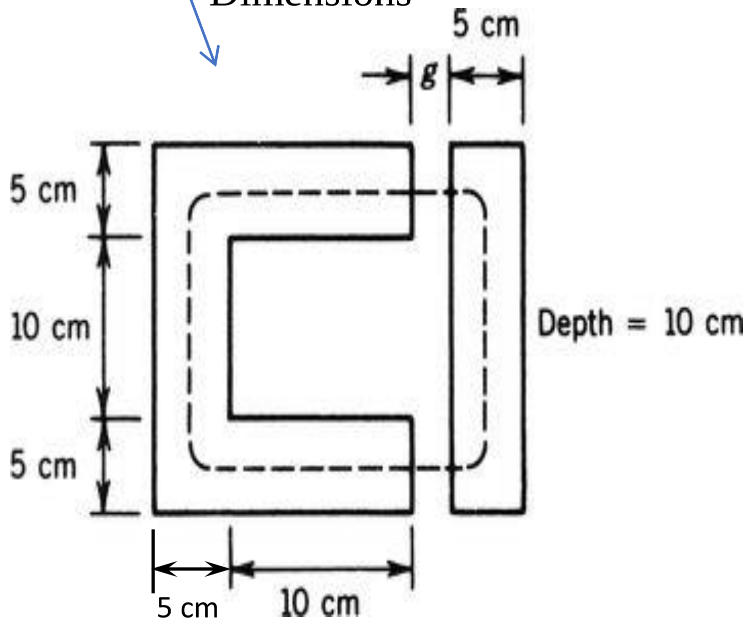
The dimensions of the actuator system on the right is shown in the figure below. The magnetic core is made of cast steel. The coil has 250 turns, and the coil resistance is $R = 5\Omega$. For a fixed air gap length $g = 5\text{mm}$, a dc source is connected to the coil to produce a flux density of 1.0 tesla in the air gap. a) Find the voltage of the dc source. b) Find the stored field energy.



An electromechanical system.

$$\mu_0 = 4\cdot\pi\cdot10^{-7}\ \text{[H/m]},\quad \mu_r = 1187 \text{ for cast steel}$$

Dimensions



Solution

$$\text{a) } H_c = \frac{B}{\mu_c} = \frac{B}{\mu_r\cdot\mu_0} = \frac{1.0}{1187\times4\cdot\pi\cdot10^{-7}} = 670\ \text{At/m}$$

$$l_c \cong 2\times(10+5)+2\times(10+5) = 60\ \text{cm} = 0.6\,\text{m}$$

$$H_g = \frac{B}{\mu_0} = \frac{1.0}{4\cdot\pi\cdot10^{-7}} = 795.8\times10^3\ \text{At/m}$$

The mmf required is

$$N\cdot i = H_c\cdot l_c + H_g\cdot l_g$$

$$\Rightarrow i = \frac{H_c\cdot l_c + H_g\cdot l_g}{N} = \frac{670\times0.6+795.8\times10^3\times2\times0.005}{250} = 33.44\ \text{A}$$

The voltage of the dc source is $V_{dc} = 33.44\,\text{A}\times5\,\Omega = 167.2\,\text{V}$

b) Energy density in the core is

$$w_{fc} = \frac{B^2}{2 \cdot \mu_c} = \frac{1^2}{2 \times 1187 \times 4 \cdot \pi \cdot 10^{-7}} = 335 \ \mathrm{J/m^3}$$

The volume of the core is

$$V_c = l_c \cdot A_c = 0.6 \text{ m} \times 0.05 \text{ m} \times 0.1 \text{ m} = 0.003 \text{ m}^3$$

The stored energy in the core is

$$W_{fc} = 335 \ \mathrm{J/m^3} \times 0.003 \text{ m}^3 = 1.005 \text{ J}$$

Energy density in the air gap is

$$w_{fg} = \frac{B^2}{2 \cdot \mu_0} = \frac{1^2}{2 \times 4 \cdot \pi \cdot 10^{-7}} = 397.9 \times 10^3 \ \mathrm{J/m^3}$$

Note that $w_{fc} \ll w_{fg}$.

The volume of the air gap is

$$V_g = l_g \cdot A_g = 2 \times 0.005 \text{ m} \times 0.05 \text{ m} \times 0.1 \text{ m} = 0.05 \times 10^{-3} \text{ m}^3$$

The stored energy in the air gap is

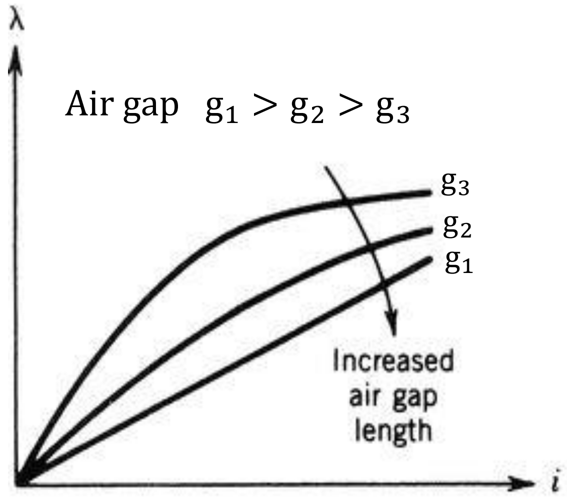
$$W_{fg} = 397.9 \times 10^3 \ \mathrm{J/m^3} \times 0.05 \times 10^{-3} \text{ m}^3 = 19.895 \text{ J}$$

Note again that $W_{fc} \ll W_{fg}$.

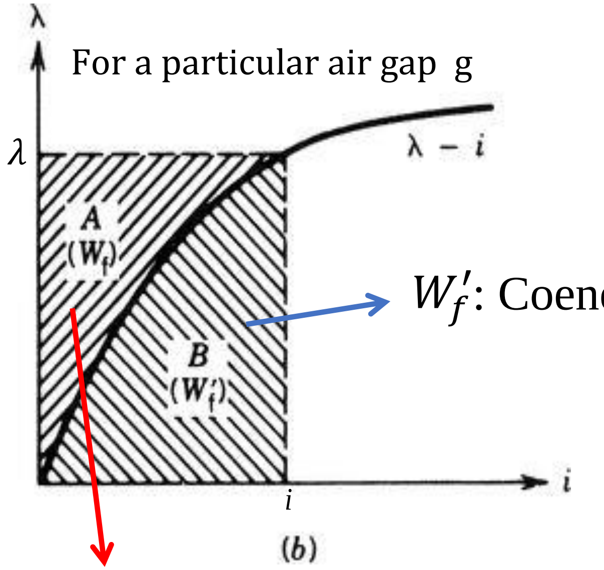
The total field energy is

$$W_f = W_{fc} + W_{fg} = 1.005 \text{ J} + 19.895 \text{ J} = 20.9 \text{ J}$$

Energy and Coenergy



(a)



(b)

W_f': Coenergy (Area B)

W_f : Field energy or Energy (Area A)

The $\lambda - i$ characteristic of an electromechanical system depends on i) the air-gap length and ii) the $B - H$ characteristic of the magnetic core material. For larger air-gap, the characteristic is essentially linear.

$$\boxed{W_f = \int_0^{\lambda} i \cdot d\lambda \quad \text{[joules] or [J]}}$$

$$\boxed{W_f' = \int_0^{i} \lambda \cdot di \quad \text{[joules] or [J]}}$$

$$W_f' + W_f = \lambda \cdot i$$

For a nonlinear $\lambda - i$ characteristic, $W_f' > W_f$

For a linear $\lambda - i$ characteristic, $W_f' = W_f$

The coenery W_f' has no physical meaning. But, it is useful to obtain expressions for force (or torque) in an electromechanical system.

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ECE 4363 – Electromechanical Energy Conversion

Lecture 11

Date: March 04, 2021

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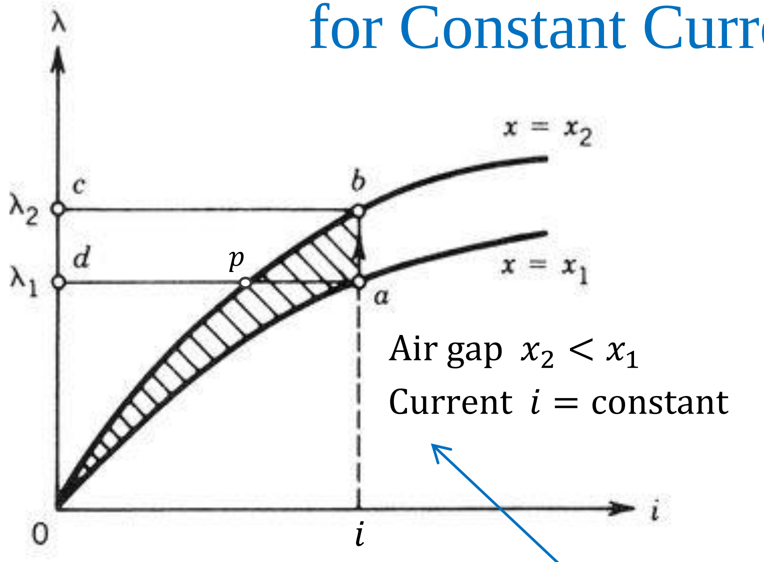
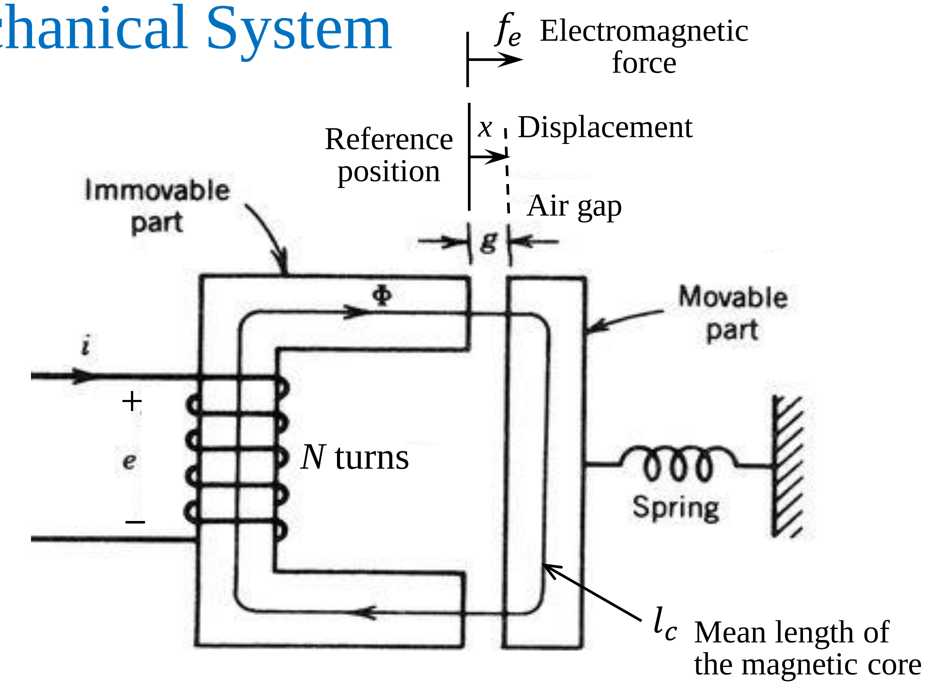
Spring 2021

P.S. The pictures, notations, formulas, and statements in these lecture 11 notes have been adopted and adapted mainly from the course textbook "Principles of Electric Machines and Power Electronics" by P. C. Sen, Third Edition, 2013.

Electromagnetic Force in an Electromechanical System for Constant Current





Let's move the operating point from *a* to *b* by keeping current *i* constant.
The flux linkage increases since the air gap length *x* has decreased.

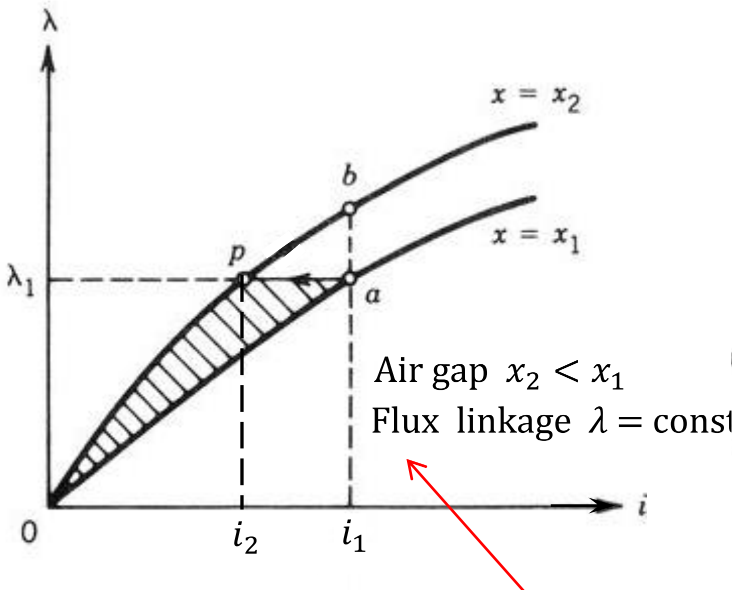
$$\Delta W_e = \int e \cdot i \cdot dt = \int_{\lambda_1}^{\lambda_2} i \cdot d\lambda = \text{ area } abcd \;, \quad \Delta W_f = (\text{area } 0bc) - (\text{area } 0ad)$$

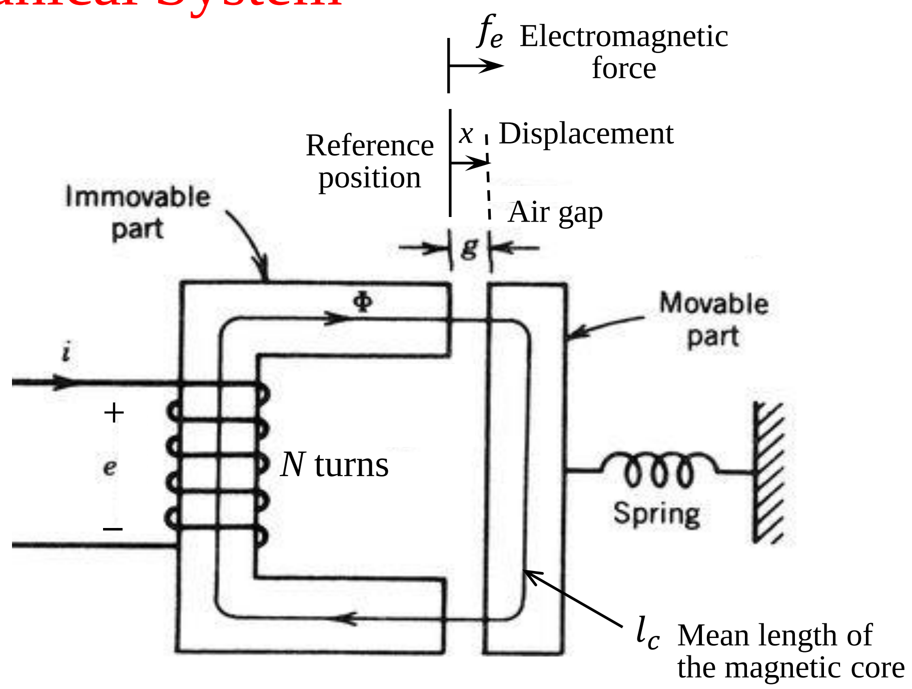
$$\Delta W_m = \Delta W_e - \Delta W_f = (\text{area } abcd) + (\text{area } 0ad) - (\text{area } 0bc) = \text{area } 0ab = \Delta W_f'$$

If the motion has occurred under constant current (*i* = constant), the mechanical work done is the increase in the coenergy. If f_e is the electromagnetic (mechanical) force causing the differential displacement dx, then,

$$f_e \cdot dx = dW_m = dW_f' \quad \Rightarrow \quad f_e = \left.\frac{\partial W_f'(i,x)}{\partial x}\right|_{i\,=\,\text{constant}}$$

Electromagnetic Force in an Electromechanical System for Constant Flux Linkage





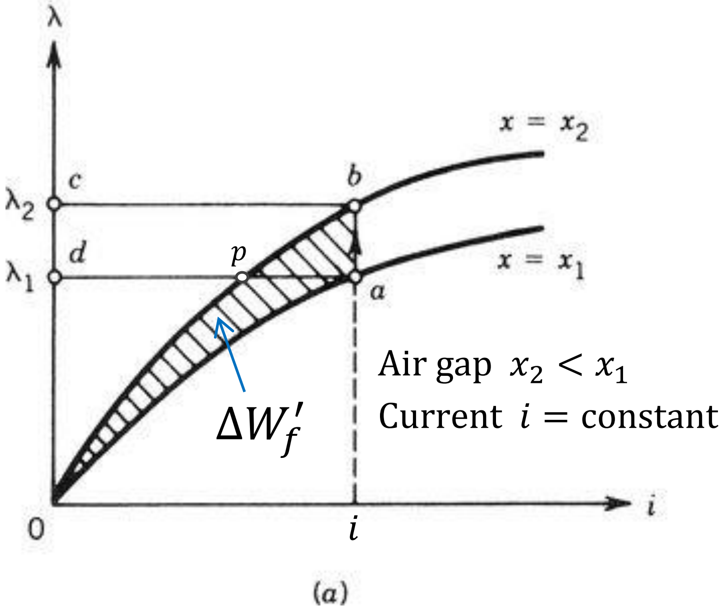
Let's move the operating point from *a* to *p* by keeping flux linkage λ constant. The current *i* decreases since the air gap length *x* has decreased. Note also that

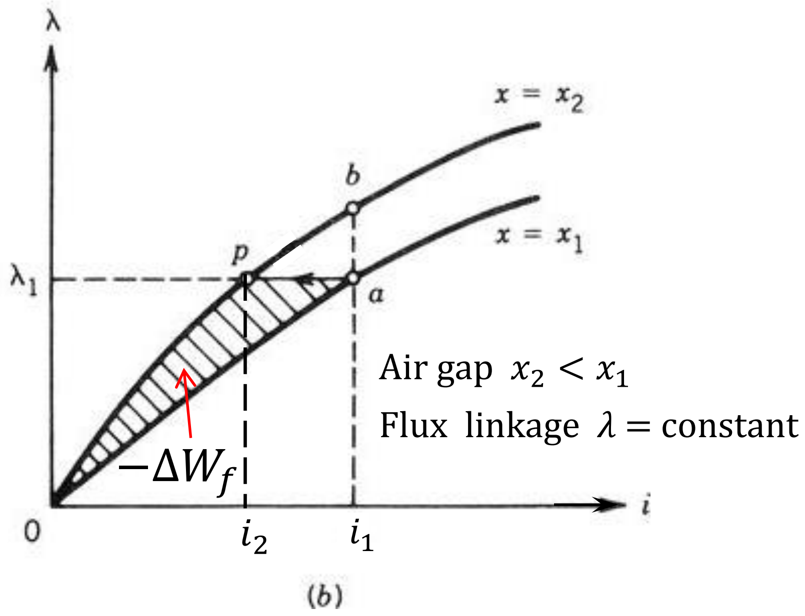
$$\Delta W_e = \int e \cdot i \cdot dt = 0 \quad \text{since } e = \frac{d\lambda}{dt} = 0. \quad \text{Then, } \Delta W_m = \Delta W_e - \Delta W_f = -\Delta W_f$$

During the motion under the constant flux linkage (λ = constant), the mechanical work done is represented by the shaded area 0*ap*, which is the decrease in the field energy. Therefore,

$$f_e \cdot dx = dW_m = -dW_f \quad \Rightarrow \quad f_e = -\left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda = \text{constant}}$$

Electromagnetic Force in an Electromechanical System: Summary





In the limit when the displacement $\Delta x = x_2 - x_1$ is small, the shaded areas $0ab$ in Figure (a) and $0ap$ in Figure (b) above will be essentially the same. That is,

$$\lim_{\Delta x \to 0} \Delta W_f' = \lim_{\Delta x \to 0} -\Delta W_f$$

Therefore, the electromagnetic (mechanical) force computed from the coenery and the field energy will be the same:

$$f_e = \left.\frac{\partial W_f'(i, x)}{\partial x}\right|_{\mathrm{i\,=\,constant}} = -\left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda\,=\,\mathrm{constant}}$$

EXAMPLE 3.2 from the textbook

The $\lambda - i$ relationship for an electromagnetic system is given by

$$i = \left(\frac{\lambda \cdot \mathrm{g}}{0.09}\right)^2$$

which is valid for the limits 0 < i < 4 A and 3 < g < 10 cm. For current i = 3 A and air gap length g = 5 cm, find the electromagnetic force on the moving part, using energy and coenergy of the field.

Solution

$$\lambda = \frac{0.09 \cdot i^{1/2}}{\mathrm{g}} \; , \qquad W_f' = \int_0^i \lambda \cdot di = \int_0^i \frac{0.09 \cdot i^{1/2}}{\mathrm{g}} \cdot di = \frac{0.09}{\mathrm{g}} \cdot \frac{2}{3} \cdot i^{3/2} \text{ joules}$$

$$f_e = \left.\frac{\partial W_f'(i,\mathrm{g})}{\partial \mathrm{g}}\right|_{i\,=\,\text{constant}} = -0.09 \times \frac{2}{3} \cdot i^{3/2} \cdot \frac{1}{\mathrm{g}^2} = -0.09 \times \frac{2}{3} \times 3^{3/2} \times \frac{1}{0.05^2} = -124.7 \text{ N}$$

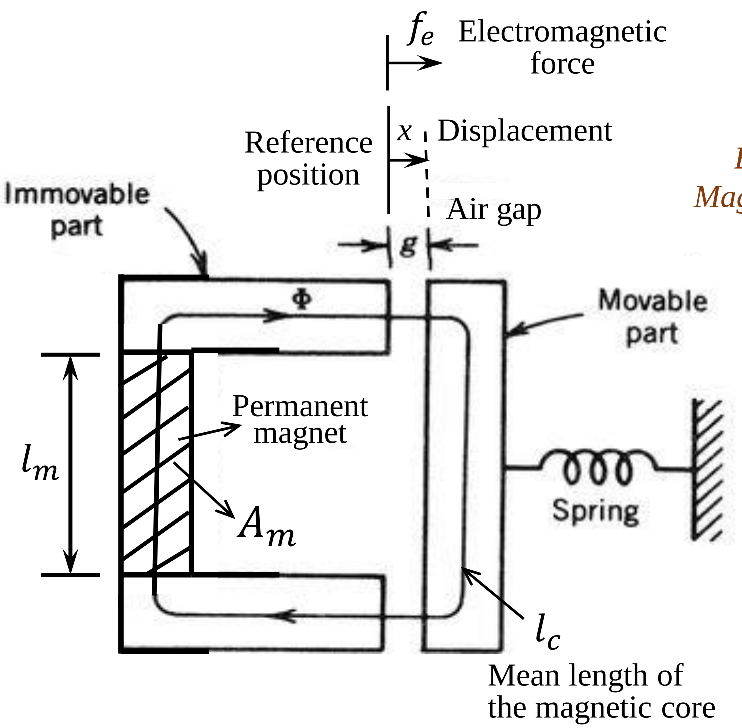
The negative sign for the force indicates that the force acts in such a direction as to decrease the air gap length.

The calculation of the force on the basis of energy function:

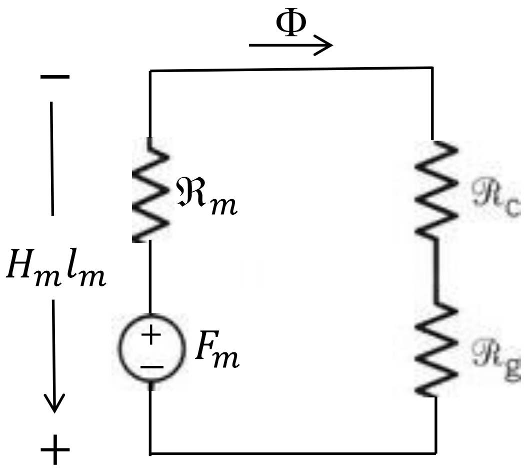
$$\lambda = \frac{0.09 \cdot i^{1/2}}{\mathrm{g}} = \frac{0.09 \times 3^{1/2}}{0.05} = 3.12 \text{ Wb−turns}\,, \qquad W_f = \int_0^\lambda i \cdot d\lambda = \int_0^\lambda \left(\frac{\lambda \cdot \mathrm{g}}{0.09}\right)^2 \cdot d\lambda = \frac{\mathrm{g}^2}{0.09^2} \cdot \frac{\lambda^3}{3} \text{ joules}$$

$$f_e = -\left.\frac{\partial W_f(\lambda,\mathrm{g})}{\partial \mathrm{g}}\right|_{\lambda\,=\,\text{constant}} = -\frac{1}{0.09^2} \times \frac{2}{3} \cdot \lambda^3 \cdot \mathrm{g} = -\frac{1}{0.09^2} \times \frac{2}{3} \cdot 3.12^3 \cdot 0.05 = -124.7 \text{ N}$$

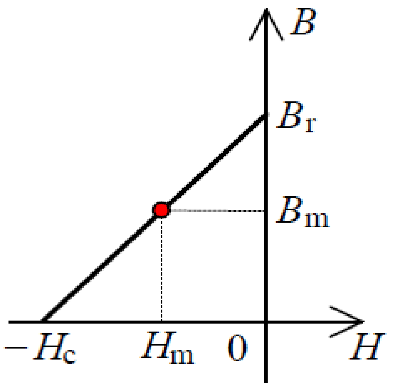
Electromagnetic Force in an Electromechanical System with Permanent Magnet



Equivalent Magnetic Circuit



$$F_m = \Phi \cdot (\Re_m + \Re_c + \Re_g)$$



Demagnetization curve of permanent magnet.

The magnetic voltage drop across the permanent magnet:

$$H_m l_m = \Re_m \Phi_m - F_m$$

The mmf of the permanent magnet:

$$F_m = H_c l_m$$

Permanent magnet flux:

$$\Phi_m = B_m A_m$$

The magnetic reluctance of the permanent magnet:

$$\Re_m = \frac{l_m}{\mu_m A_m}, \quad \mu_m = \frac{B_r}{H_c}$$

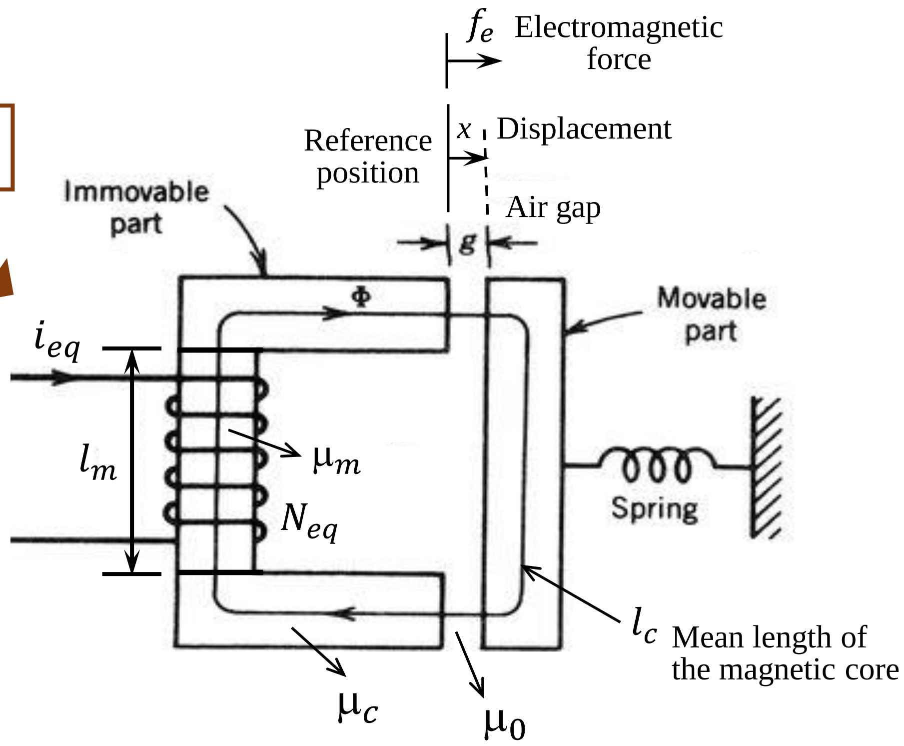
The magnetic reluctances of the magnetic core and air gap:

$$\Re_c = \frac{l_c}{\mu_c A_c}, \Re_g = \frac{l_g}{\mu_0 A_g}$$

$$F_m = \Phi \cdot (\Re_m + \Re_c + \Re_g), \quad F_m = H_c l_m$$

The permanent magnet is replaced by an equivalent (fictitious) coil that produces the same mmf as F_m.

$$\boxed{N_{eq} i_{eq} = \Phi \cdot (\Re_m + \Re_c + \Re_g), \quad N_{eq} i_{eq} = F_m = H_c l_m}$$



The coenergy of the system:

$$W_f' = \int_0^{i_{eq}} \lambda \cdot di_{eq}$$

where

$$\lambda = N_{eq} \cdot \Phi = \frac{N_{eq}^2 \cdot i_{eq}}{\Re_m + \Re_c + \Re_g}.$$

Then,

$$W_f' = \frac{1}{2} \frac{N_{eq}^2 \cdot i_{eq}^2}{\Re_m + \Re_c + \Re_g}$$

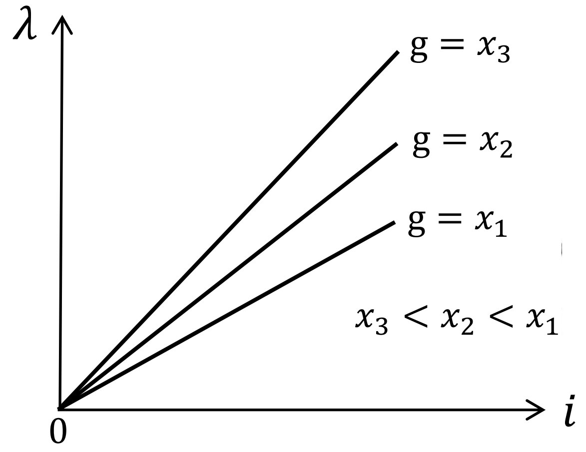
where

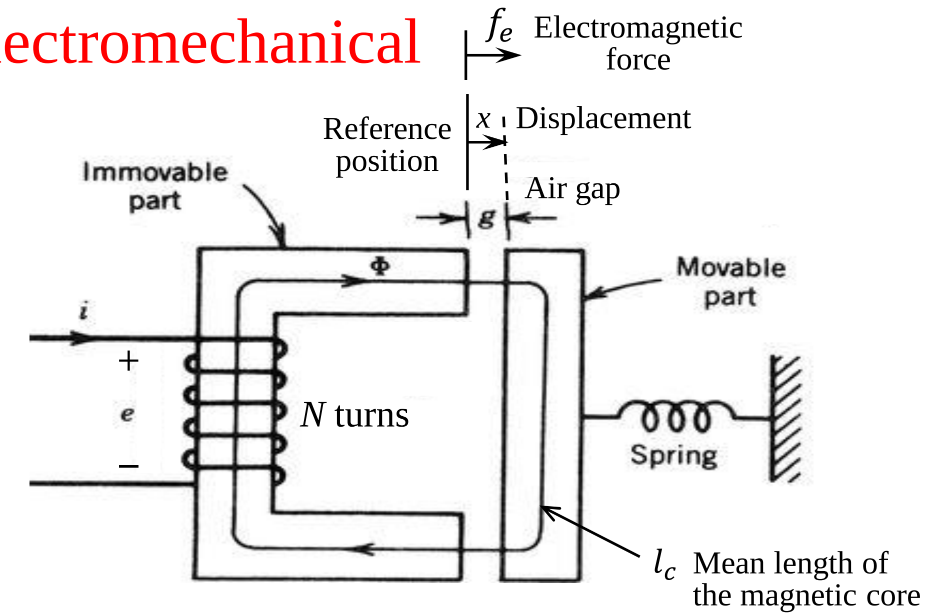
$$\Re_m = \frac{l_m}{\mu_m A_m} = \text{constant}, \quad \Re_c = \frac{l_c}{\mu_c A_c} = \text{constant}, \text{ and } \quad \Re_g = \frac{l_g}{\mu_0 A_g} = \frac{2 \cdot g}{\mu_0 A_g} = \frac{2 \cdot x}{\mu_0 A_g}.$$

The calculation of the force on the basis of coenergy function:

$$f_e = \left.\frac{\partial W_f'(i_{eq}, x)}{\partial x}\right|_{i_{eq} = \text{constant}} = -\left.\frac{N_{eq}^2 \cdot i_{eq}^2}{\mu_0 A_g (\Re_m + \Re_c + \Re_g)^2}\right|_{i_{eq} = \frac{H_c l_m}{N_{eq}}} = -\frac{(H_c l_m)^2}{\mu_0 A_g \left(\Re_m + \Re_c + \frac{2 \cdot x}{\mu_0 A_g}\right)^2}$$

Electromagnetic Force in a Linear Electromechanical System





Consider a linear electromechanical system whose $\lambda - i$ characteristic is a linear function and is given by:

$$\lambda = L(x) \cdot i$$

where $L(x)$ is the inductance of the coil whose value depends on the air gap length $g = x$.

$$W_f' = \int_0^i \lambda \cdot di = \int_0^i L(x) \cdot i \cdot di = \frac{1}{2} \cdot L(x) \cdot i^2 \quad \text{and} \quad f_e = \left. \frac{\partial W_f'(i,x)}{\partial x} \right|_{i\,=\,\text{constant}} = \frac{1}{2} \cdot i^2 \cdot \frac{dL(x)}{dx}$$

For the above electromechanical system with the reluctance of the magnetic core path neglected,

$$N \cdot i = H_g \cdot 2 \cdot g = \frac{B_g}{\mu_0} \cdot 2 \cdot g = \frac{\lambda}{\mu_0 \cdot N \cdot A_g} \cdot 2 \cdot g \;\Rightarrow\; L(x) = \frac{\lambda}{i} = \frac{\mu_0 \cdot N^2 \cdot A_g}{2 \cdot x} \text{ for } g = x$$

$$f_e = \frac{1}{2} \cdot i^2 \cdot \frac{dL(x)}{dx} = -\frac{\mu_0 \cdot A_g \cdot N^2}{4 \cdot x^2} \cdot i^2 \text{ for } x \neq 0$$

f_e is in N (newtons), and it is an attraction force due to the negative sign.

UNIVERSITY of HOUSTON
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ECE 4363 – Electromechanical Energy Conversion

Lecture 12

Date: March 09, 2021

by

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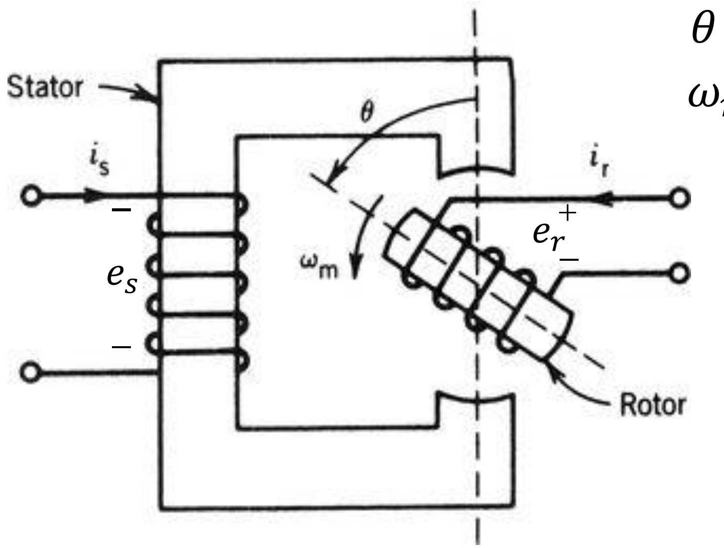
Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 12 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

ROTATING MACHINES

They produce rotational motion. The essential parts: 1) Stator (fixed part) and 2) Rotor (rotating part).



θ : Angular position of the rotor in radians (rad).

ω_m : Mechanical (angular) speed of the rotor in (rad/s).

$$\omega_m = \frac{d\theta}{dt}$$

Let's assume that there is no mechanical output power. That is, $\omega_m = 0$, or $\theta =$ constant.
Then, the differential energy stored in the field is calculated by the stator and rotor winding currents i_s and i_r as following:

$$dW_f = e_s \cdot i_s \cdot dt + e_r \cdot i_r \cdot dt = i_s \cdot d\lambda_s + i_r \cdot d\lambda_r$$

The flux linkage of the stator winding: $\lambda_s = L_{ss} \cdot i_s + L_{sr} \cdot i_r$

The flux linkage of the rotor winding: $\lambda_r = L_{rs} \cdot i_s + L_{rr} \cdot i_r$

The values of the inductances depend on the position θ of the rotor.

L_{ss} : The self inductance of the stator winding

L_{rr} : The self inductance of the rotor winding

L_{sr} , L_{rs} : The mutual inductances between stator and rotor windings

Torque Developed in a Rotational Electromechanical System

Let's assume that the magnetic system is linear and $L_{sr} = L_{rs}$. Then,

$$\begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{sr} & L_{rr} \end{bmatrix} \cdot \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

For $\omega_m = 0$, or $\theta =$ constant which implies that the inductances are constant as well,

$$dW_f = i_s \cdot d\lambda_s + i_r \cdot d\lambda_r = i_s \cdot d(L_{ss} \cdot i_s + L_{sr} \cdot i_r) + i_r \cdot d(L_{sr} \cdot i_s + L_{rr} \cdot i_r)$$

$$dW_f = L_{ss} \cdot i_s \cdot di_s + L_{rr} \cdot i_r \cdot di_r + L_{sr} \cdot d(i_s \cdot i_r)$$

The field energy is

$$W_f = L_{ss} \cdot \int_0^{i_s} i_s \cdot di_s + L_{rr} \cdot \int_0^{i_r} i_r \cdot di_r + L_{sr} \cdot \int_0^{i_s, i_r} d(i_s \cdot i_r)$$

$$W_f = \frac{1}{2} \cdot L_{ss} \cdot i_s^2 + \frac{1}{2} \cdot L_{rr} \cdot i_r^2 + L_{sr} \cdot i_s \cdot i_r$$

Following the procedure used to determine an expression of the electromagnetic force developed in a translational actuator, it may be shown that the torque (electromagnetic torque) developed in a rotational electromechanical system is

$$T_e = \left. \frac{\partial W_f'(i_s, i_r, \theta)}{\partial \theta} \right|_{i_s, i_r = \text{ constant}}$$

where W_f' is the coenery.

In a linear magnetic system, $W_f' = W_f$. Therefore,

Torque

$$\boxed{T_e = \frac{1}{2} \cdot i_s^2 \cdot \frac{dL_{ss}}{d\theta} + \frac{1}{2} \cdot i_r^2 \cdot \frac{dL_{rr}}{d\theta} + i_s \cdot i_r \cdot \frac{dL_{sr}}{d\theta}}$$

Components of Torque Developed in a Rotational Electromechanical System

Torque $$T_e = \underbrace{\frac{1}{2} \cdot i_s^2 \cdot \frac{dL_{ss}}{d\theta}}_{1} + \underbrace{\frac{1}{2} \cdot i_r^2 \cdot \frac{dL_{rr}}{d\theta}}_{2} + \underbrace{i_s \cdot i_r \cdot \frac{dL_{sr}}{d\theta}}_{3}$$

1- Torque produced solely by the stator magnetic field (or, stator current i_s), and it is due to the variation of the stator self-inductance L_{ss} with rotor position θ. It is called *reluctance torque*.

2- Torque produced solely by the rotor magnetic field (or, rotor current i_r), and it is due to the variation of the rotor self-inductance L_{rr} with rotor position θ. It is called *reluctance torque*.

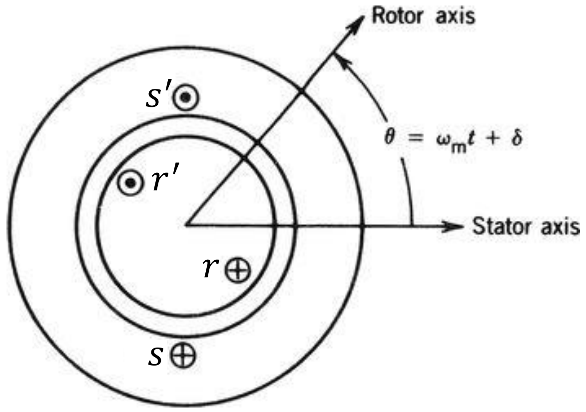
3- Torque produced both by the stator current i_s and rotor current i_r. It is due to i) the mutual interaction of the stator and rotor magnetic fields (or, currents) and ii) the variation of the mutual inductance L_{sr} between the stator and rotor windings with rotor position θ. It is called *mutual-interaction torque* (or, *torque*).

The unit of torque is N·m (newton meters).

Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 6th Edition, 2003.

Torque Produced in a Cylindrical Machine with a Uniform Air Gap

Let’s consider an elementary single-phase two-pole cylindrical machine with a uniform air gap, whose cross-sectional view is shown in the below figure.



For the above machine, it can be assumed that the self inductances L_{ss} and L_{rr} are constant, and therefore, no reluctance torques are produced. On the other hand, the mutual inductance L_{sr} varies with the rotor position θ and might be expressed by

$$L_{sr} = M \cdot \cos(\theta)$$

The rotor position is the angle between the magnetic axes of the rotor and stator windings, and it is given by

$$\theta = \omega_m \cdot t + \delta$$

where ω_m is the angular speed of the rotor and δ is the initial rotor position.

The torque produced by the cylindrical machine with a uniform air gap is calculated from

$$T_e = i_s \cdot i_r \cdot \frac{dL_{sr}}{d\theta}$$

Let the currents in the stator and rotor windings be

$$i_s = I_{sm} \cdot \cos(\omega_s \cdot t)$$

$$i_r = I_r = \text{constant}$$

where ω_s is the stator angular frequency. Then,

$$T_e = i_s \cdot i_r \cdot \frac{dL_{sr}}{d\theta} = -I_{sm} \cdot I_r \cdot M \cdot \cos(\omega_s \cdot t) \cdot \sin(\omega_m \cdot t + \delta)$$

$$T_e = -\frac{I_{sm} \cdot I_r \cdot M}{2} \cdot \left[\sin\big((\omega_m + \omega_s)t + \delta\big) + \sin\big((\omega_m - \omega_s)t + \delta\big)\right]$$

For $\omega_m = \mp\omega_s$,

$$T_e = -\frac{I_{sm} \cdot I_r \cdot M}{2} \cdot [\sin(\mp 2\omega_s \cdot t + \delta) + \sin(\delta)]$$

The instantaneous torque is pulsating. The average value of the torque at $\omega_m = \mp\omega_s$ is

$$T_{e,avg} = -\frac{I_{sm} \cdot I_r \cdot M}{2} \cdot \sin(\delta)$$

On the other hand, at $\omega_m \neq \mp\omega_s$ (e.g., at $\omega_m = 0$ and $\omega_s \neq 0$), the torque averaged over a sufficiently long time is zero.

The above analysis explains the principle of operation and torque generation in a synchronous machine which has dc excitation in the rotor and ac excitation in the stator. Specifically, the synchronous machine can develop an average torque and provide continuous energy conversion only at synchronous speed ($\omega_m = \omega_s$).

Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 5th Edition, 1990.

EXAMPLE

Consider a rotating electromechanical system in the figure below, where it has a linear magnetic characteristic. That is, the relative permeability of the magnetic core material is constant ($\mu_r =$ constant), and the self and mutual inductances ($L_{ss}, L_{rr}, L_{sr}, L_{rs}$) depend only on the angular position θ of the rotor. Show that the mutual inductances $L_{sr} = \left.\frac{\lambda_s}{i_r}\right|_{i_s=0}$ and $L_{rs} = \left.\frac{\lambda_r}{i_s}\right|_{i_r=0}$ are equal to each other. Assumption: The magnetomotive force of the rotor winding associated with the main (mutual) flux path can be expressed by $N_r \cdot \cos(\theta) \cdot i_r$.

i_s, λ_s: Current and flux linkage of stator winding

i_r, λ_r: Current and flux linkage of rotor winding

N_s : Number of stator winding turns

N_r: Number of rotor winding turns

$l_c(\theta)$: Length of magnetic core

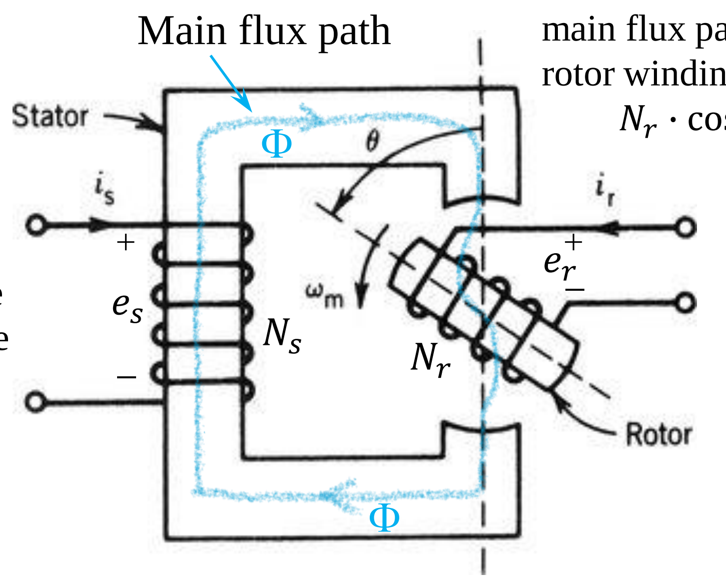
$l_g(\theta)$: Length of air gap

A_c : Cross sectional area of magnetic core

A_g : Cross sectional area of air gap

$A_c = A_g =$ constant

Algebraic sum of currents encircled by the main flux path due to the stator winding turns: $N_s \cdot i_s$



Algebraic sum of currents encircled by the main flux path due to the rotor winding turns: $N_r \cdot \cos(\theta) \cdot i_r$

A rotating electromechanical system.

$\boxed{\text{Solution}}$

Let's calculate the stator flux linkage λ_s produced by the rotor current i_r when the stator current $i_s = 0$.

$$N_r \cdot \cos(\theta) \cdot i_r = l_g(\theta) \cdot \frac{B_g}{\mu_0} + l_c(\theta) \cdot \frac{B_c}{\mu_0 \cdot \mu_r} = l_g(\theta) \cdot \frac{\Phi}{\mu_0 \cdot A_g} + l_c(\theta) \cdot \frac{\Phi}{\mu_0 \cdot \mu_r \cdot A_c} = \left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right) \cdot \frac{\Phi}{\mu_0}$$

$$N_r \cdot \cos(\theta) \cdot i_r = \left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right) \cdot \frac{\lambda_s}{\mu_0 \cdot N_s}$$

$$\Rightarrow L_{sr} = \left.\frac{\lambda_s}{i_r}\right|_{i_s = 0} = \frac{\mu_0 \cdot N_s \cdot N_r \cdot \cos(\theta)}{\left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right)}$$ where μ_r is constant (independent of i_r).

Let's calculate the rotor flux linkage λ_r produced by the stator current i_s when the rotor current $i_r = 0$.

$$N_s \cdot i_s = l_g(\theta) \cdot \frac{B_g}{\mu_0} + l_c(\theta) \cdot \frac{B_c}{\mu_0 \cdot \mu_r} = l_g(\theta) \cdot \frac{\Phi}{\mu_0 \cdot A_g} + l_c(\theta) \cdot \frac{\Phi}{\mu_0 \cdot \mu_r \cdot A_c} = \left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right) \cdot \frac{\Phi}{\mu_0}$$

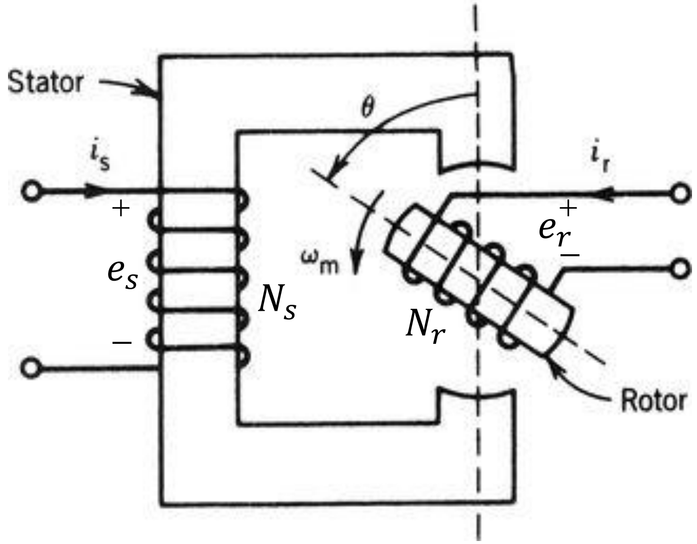
$$N_s \cdot i_s = \left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right) \cdot \frac{\lambda_r}{\mu_0 \cdot N_r \cdot \cos(\theta)}$$

$$\Rightarrow L_{rs} = \left.\frac{\lambda_r}{i_s}\right|_{i_r = 0} = \frac{\mu_0 \cdot N_s \cdot N_r \cdot \cos(\theta)}{\left(\frac{l_g(\theta)}{A_g} + \frac{l_c(\theta)}{\mu_r \cdot A_c}\right)}$$ where μ_r is constant (independent of i_s).

In brief, $L_{sr} = L_{rs}$.

EXAMPLE 3.5 from the textbook

In the rotating electromechanical system of the below figure, the rotor winding is open circuit (i.e., $i_r = 0$). The inductance of the stator is expressed by $L_{ss} = L_0 + L_2 \cdot \cos(2 \cdot \theta)$ where θ is the angular position of the rotor, and L_0 and L_2 are constants. The stator current is $i_s = I_{sm} \cdot \sin(\omega \cdot t)$ where ω is the angular frequency of the stator current in rad/s. (a) Obtain an expression of the torque acting on the rotor. (b) Let $\theta = \omega_m \cdot t + \theta_0$ where ω_m is the angular velocity of the rotor and θ_0 is the rotor angular position at $t = 0$. Obtain an expression of the average torque at $\omega_m = \omega$, where the average torque is calculated over the stator electrical period $(2 \cdot \pi)/\omega$.



A rotating electromechanical system.

Solution

(a) Since $i_r = 0$,

$$T_e = \frac{1}{2} \cdot i_s^2 \cdot \frac{dL_{ss}}{d\theta}$$

$$T_e = \frac{1}{2} \cdot I_{sm}^2 \cdot \sin^2(\omega \cdot t) \cdot \frac{d}{d\theta}(L_0 + L_2 \cdot \cos(2 \cdot \theta)) = -I_{sm}^2 \cdot L_2 \cdot \sin(2 \cdot \theta) \cdot \sin^2(\omega \cdot t)$$

(b) $$T_e = -I_{sm}^2 \cdot L_2 \cdot \sin(2 \cdot \theta) \cdot \frac{(1 - \cos(2 \cdot \omega \cdot t))}{2} = \frac{-I_{sm}^2 \cdot L_2}{2} \cdot (\sin(2 \cdot \theta) - \sin(2 \cdot \theta) \cdot \cos(2 \cdot \omega \cdot t))$$

$$T_e = \frac{-I_{sm}^2 \cdot L_2}{2} \cdot \left(\sin(2 \cdot \theta) - \frac{1}{2}(\sin(2 \cdot \theta + 2 \cdot \omega \cdot t) + \sin(2 \cdot \theta - 2 \cdot \omega \cdot t)) \right)$$

$$T_e = \frac{-I_{sm}^2 \cdot L_2}{2} \cdot \left(\sin(2(\omega_m \cdot t + \theta_0)) - \frac{1}{2}(\sin(2((\omega_m + \omega) \cdot t + \theta_0)) + \sin(2((\omega_m - \omega) \cdot t + \theta_0))) \right)$$

For $\omega_m = \omega$,

$$T_e = \frac{-I_{sm}^2 \cdot L_2}{2} \cdot \left(\sin(2 \cdot \omega \cdot t + 2 \cdot \theta_0) - \frac{1}{2}\sin(4 \cdot \omega \cdot t + 2 \cdot \theta_0) - \frac{1}{2}\sin(2 \cdot \theta_0) \right)$$

$$T_{e,avg} = \frac{1}{(2 \cdot \pi/\omega)} \int_0^{(2 \cdot \pi/\omega)} T_e \cdot dt = \frac{1}{4} \cdot I_{sm}^2 \cdot L_2 \cdot \sin(2 \cdot \theta_0)$$

UNIVERSITY of HOUSTON
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ECE 4363 – Electromechanical Energy Conversion

Lecture 13

Date: March 11, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 13 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Practice Questions

Question 1:

The toroidal (circular cross section) core shown in Figure 1 is made from cast steel whose relative permeability is $\mu_r = 955$. The permeability of the free space is $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ [H/m]. The coil has N = 200 turns. (a) Calculate the coil current i required to produce a core flux density of B =1.2 T at the mean radius of the toroid. (b) What is the core flux Φ in Wb? Assume that the flux density within the core is uniform and equal to that at the mean radius. (c) Find the inductance of the coil. (d) If a 2-mm-wide air gap is made in the toroid (across $A - A'$), determine the new coil current required to maintain a core flux density of 1.2 T.

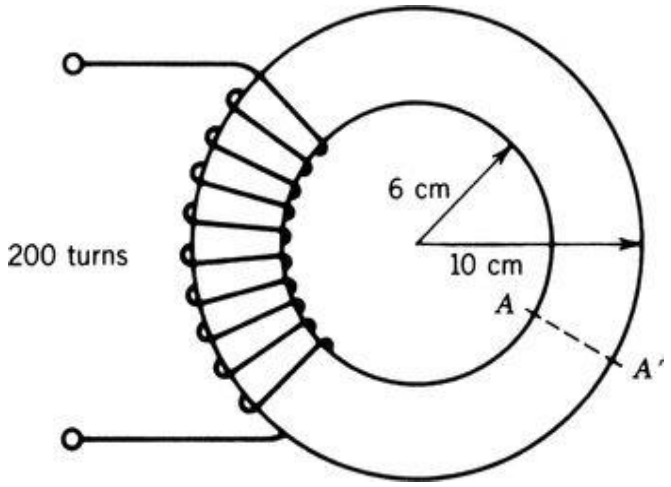


Figure 1. Toroidal core.

Solution to Question 1:

(a) Mean radius is $r = 0.5 \cdot (10 + 6) = 8$ cm, $\quad l_c = 2 \cdot \pi \cdot r = 2 \cdot \pi \cdot 0.08 = 0.503$ m

$$H_c = \frac{B_c}{\mu_r \cdot \mu_0} = \frac{1.2}{955 \cdot 4 \cdot \pi \cdot 10^{-7}} = 999.9 \text{ At/m}, \quad i = \frac{H_c \cdot l_c}{N} = \frac{999.9 \times 0.503}{200} = 2.51 \text{ A}$$

(b)

$$A_c = \pi \cdot \left(0.5 \cdot (0.1 - 0.06)\right)^2 = 0.00126 \text{ m}^2, \quad \Phi_c = B_c \cdot A_c = 1.2 \cdot 0.00126 = 0.00151 \text{ Wb.}$$

(c)

$$L = \frac{\lambda}{i} = \frac{N \cdot \Phi_c}{i} = \frac{200 \cdot 0.00151}{2.51} = 0.1203 \text{ H},$$

Or,

$$\Re_{core} = \frac{l_c}{\mu_r \cdot \mu_0 \cdot A_c} = \frac{0.503}{955 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 0.00126} = 332647 \text{ At/Wb}$$

$$L = \frac{N^2}{\Re_{core}} = \frac{200^2}{332647} = 0.1203 \text{ H}$$

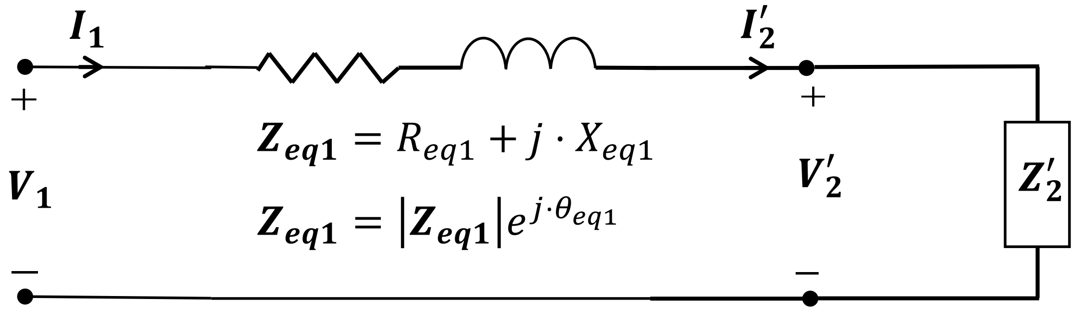
(d)

$$i = \frac{H_c \cdot l_c + H_g \cdot l_g}{N} = \frac{\frac{B_c}{\mu_r \cdot \mu_0} \cdot l_c + \frac{B_g}{\mu_0} \cdot l_g}{N} = \frac{\frac{1.2}{955 \cdot 4 \cdot \pi \cdot 10^{-7}} \cdot (0.503 - 0.002) + \frac{1.2}{4 \cdot \pi \cdot 10^{-7}} \cdot 0.002}{200}$$

$i = 12.05$ A.

Question 2:

A single-phase, 300 kVA, 11 kV/ 2.2 kV, 60 Hz transformer has the following equivalent circuit parameters referred to the high-voltage (primary) side: $R_{eq1} = 2.53\ \Omega$, $X_{eq1} = 8.45\ \Omega$. (a) Determine (i) the rated current, and (ii) full-load copper loss. (b) Assume that the load impedance on the low-voltage (secondary) side is $\boldsymbol{Z_{load}} = 15.0\angle 65.0°\ \Omega$. Then, (i) determine the voltage regulation using the equivalent circuit of the transformer shown in Figure 2, and (ii) draw the phasor diagram for this load condition.



$$\boldsymbol{Z_2'} = a^2 \cdot \boldsymbol{Z_{load}}$$

$$\boldsymbol{Z_2'} = a^2 \cdot |\boldsymbol{Z_{load}}| e^{j \cdot \theta_2}$$

Figure 2. Equivalent circuit of transformer.

Solution to Question 2:

(a) (i) The rated current is

$$I_{1(rated)} = \frac{300 \times 10^3}{11 \times 10^3} = 27.27 \text{ A}$$

(ii) The full-load copper loss is

$$P_{CU(FL)} = 27.27^2 \times 2.53 = 1882 \text{ W}$$

(b) (i) For $\boldsymbol{Z_{load}} = 15.0\angle 65.0° \ \Omega$,

$$\boldsymbol{I_2} = \frac{\boldsymbol{V_2}}{\boldsymbol{Z_{load}}} = \frac{2200\angle 0°}{15.0\angle 65.0°} = 146.7\angle -65° \text{ A}, \quad \text{Turns ratio } a = \frac{11000}{2200} = 5$$

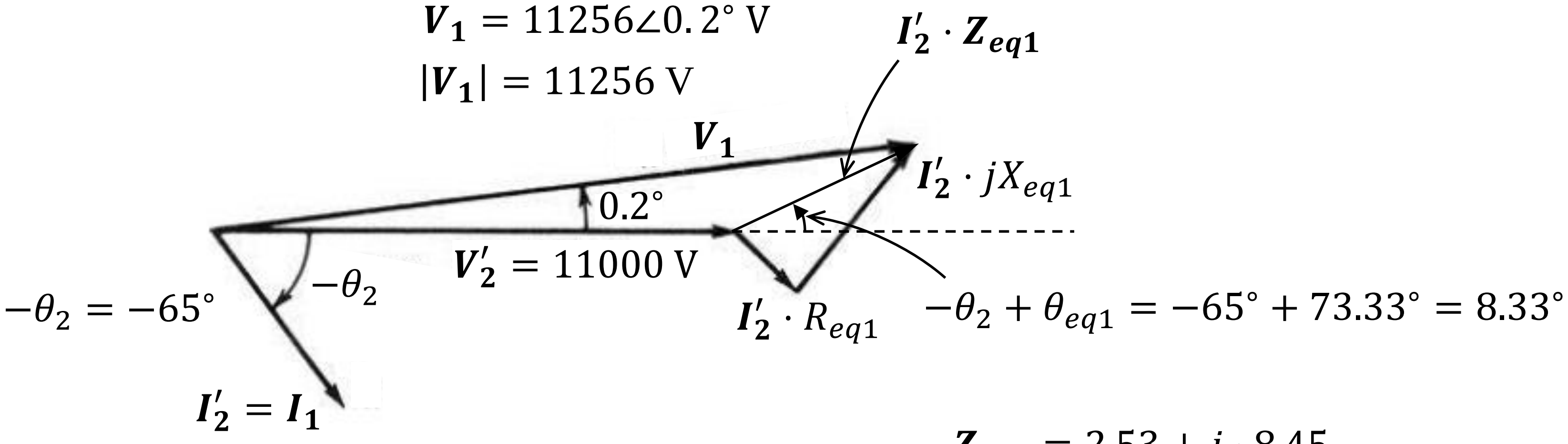
$$\boldsymbol{I_2'} = \frac{\boldsymbol{I_2}}{a} = \frac{146.7}{5}\angle -65.0° \text{ A} = 29.3\angle -65.0° \text{ A}$$

$$\boldsymbol{V_2'} = a \cdot \boldsymbol{V_2} = 5 \cdot 2200\angle 0° \text{ V} = 11000\angle 0° \text{ V}$$

$$\boldsymbol{V_1} = \boldsymbol{V_2'} + \boldsymbol{I_2'} \cdot \boldsymbol{Z_{eq1}} = 11000\angle 0° + (29.3\angle -65.0°) \cdot (2.53 + j \cdot 8.45) = 11256\angle 0.2° \text{ V}$$

$$VR = \frac{11256 - 11000}{11000} \times 100\% = 2.33\%$$

(ii) The phasor diagram for this load condition:



$$\boldsymbol{I'_2} = 29.3\angle -65.0°\ \text{A}$$

$$|\boldsymbol{I'_2}| = 29.3\ \text{A}$$

$$\boldsymbol{Z_{eq1}} = 2.53 + j \cdot 8.45$$

$$\boldsymbol{Z_{eq1}} = \left|\boldsymbol{Z_{eq1}}\right| e^{j \cdot \theta_{eq1}}$$

$$\left|\boldsymbol{Z_{eq1}}\right| = \sqrt{(2.53)^2 + (8.45)^2} = 8.82\ \Omega$$

$$\theta_{eq1} = \tan^{-1}\left(\frac{8.45}{2.53}\right) = 73.33°$$

Question 3:

The electromechanical actuator in Figure 3 includes a neodymium-iron-boron ($NdFeB$) permanent magnet and an excitation winding of $N = 1500$ turns. The winding direction is such that the positive winding current reduces the air gap flux produced by the permanent magnet. The reluctance of the magnetic core is negligible ($\mu_c \rightarrow \infty$). The dimensions of the permanent magnet are $A_m = 14.0\ \text{cm}^2$ and $l_m = 8.0$ mm. The coercivity and (recoil) permeability of the permanent magnet are $H_c = 930$ kA/m and $\mu_m = 1.08 \cdot \mu_0$, respectively, where the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ [H/m]. The cross sectional area of the magnetic core is $A_c = A_m$. (a) Find the force f_e acting on the movable part in the x direction when the winding current $i = 0$ A and $x = 3$ mm. (b) Calculate the winding current i required to reduce the force f_e to zero.

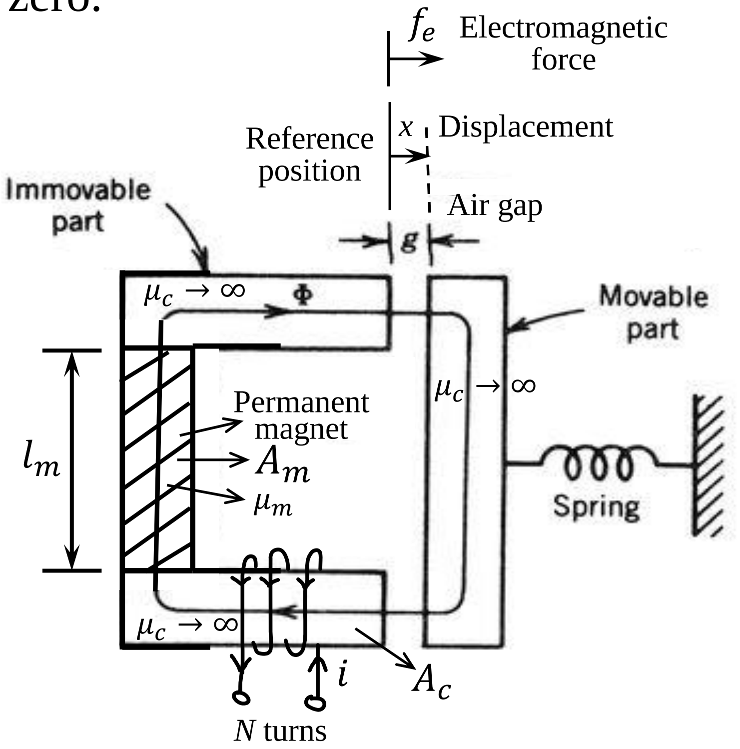
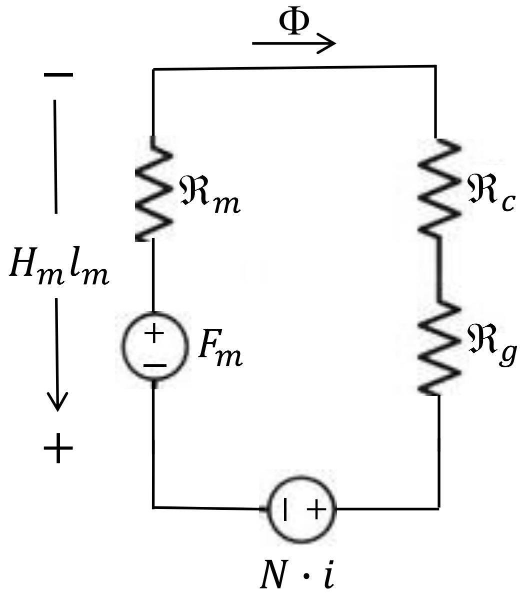


Figure 3. Electromechanical actuator with permanent magnet and excitation winding.

Solution to Question 3:

(a) The equivalent magnetic circuit:



The mmf of the permanent magnet: $F_m = H_c l_m = N_{eq} i_{eq}$

The mmf of the excitation winding: $N \cdot i$

The magnetic reluctance of the permanent magnet: $\Re_m = \dfrac{l_m}{\mu_m A_m}$

$$\Re_m = \frac{0.008}{1.08 \cdot 4\pi \times 10^{-7} \cdot 0.0014} = 4210448 \text{ At/Wb}$$

The magnetic reluctances of the magnetic core: $\Re_c = \dfrac{l_c}{\mu_c A_c} \to 0$ as $\mu_c \to \infty$

The magnetic reluctances of the air gap: $\Re_g = \dfrac{l_g}{\mu_0 A_g} = \dfrac{2 \cdot g}{\mu_0 A_c} = \dfrac{2 \cdot x}{\mu_0 A_m}$

From the equivalent circuit,

$$F_m - N \cdot i = (\Re_m + \Re_c + \Re_g) \cdot \Phi$$

$$N_{eq} i_{eq} - N \cdot i = (\Re_m + \Re_c + \Re_g) \cdot \Phi \implies \Phi = \frac{N_{eq} i_{eq} - N \cdot i}{(\Re_m + \Re_c + \Re_g)}$$

The differential field energy of the system is

$$dW_f = e_{eq} \cdot i_{eq} \cdot dt + e \cdot i \cdot dt = i_{eq} \cdot d\lambda_{eq} + i \cdot d\lambda = i_{eq} \cdot d(N_{eq} \cdot \Phi) + i \cdot d(-N \cdot \Phi) = N_{eq} \cdot i_{eq} \cdot d\Phi - N \cdot i \cdot d\Phi$$

$$dW_f = \frac{1}{(\Re_m + \Re_c + \Re_g)} \left[N_{eq}^2 \cdot i_{eq} \cdot di_{eq} - N_{eq} \cdot N \cdot i_{eq} \cdot di - N_{eq} \cdot N \cdot i \cdot di_{eq} + N^2 \cdot i \cdot di \right]$$

$$dW_f = \frac{1}{\left(\Re_m + \Re_c + \Re_g\right)}\left[N_{eq}^2 \cdot i_{eq} \cdot di_{eq} - N_{eq} \cdot N \cdot i_{eq} \cdot di - N_{eq} \cdot N \cdot i \cdot di_{eq} + N^2 \cdot i \cdot di\right]$$

$$W_f = \frac{1}{\left(\Re_m + \Re_c + \Re_g\right)}\left[\int_0^{i_{eq}} N_{eq}^2 \cdot i_{eq} \cdot di_{eq} - \int_{0,0}^{i_{eq},i} N_{eq} \cdot N \cdot d(i_{eq} \cdot i) + \int_0^{i} N^2 \cdot i \cdot di\right]$$

$$W_f = \frac{1}{2\left(\Re_m + \Re_c + \Re_g\right)}\left(N_{eq}^2 \cdot i_{eq}^2 - 2 \cdot N_{eq} \cdot N \cdot i_{eq} \cdot i + N^2 \cdot i^2\right)$$

$$W_f = \frac{1}{2\left(\Re_m + \Re_c + \Re_g\right)}\left(H_c^2 \cdot l_m^2 - 2 \cdot H_c \cdot l_m \cdot N \cdot i + N^2 \cdot i^2\right) = \frac{(H_c \cdot l_m - N \cdot i)^2}{2\left(\Re_m + \Re_c + \Re_g\right)}$$

Since the coenery W_f' is equal to the energy W_f in a linear magnetic system,

$$W_f' = \frac{(H_c \cdot l_m - N \cdot i)^2}{2\left(\Re_m + \Re_c + \Re_g\right)} = \frac{(H_c \cdot l_m - N \cdot i)^2}{2\left(\Re_m + \Re_c + \frac{2 \cdot x}{\mu_0 A_m}\right)}$$

The calculation of the force on the basis of coenergy function:

$$f_e = \left.\frac{\partial W_f'(i,x)}{\partial x}\right|_{i\,=\,\text{constant}} = -\frac{(H_c \cdot l_m - N \cdot i)^2}{\mu_0 A_m \left(\Re_m + \Re_c + \dfrac{2 \cdot x}{\mu_0 A_m}\right)^2}$$

(a) The force at $i = 0$ A and $x = 3$ mm:

$$f_e = -\frac{(H_c \cdot l_m - N \cdot i)^2}{\mu_0 A_m \left(\Re_m + \Re_c + \dfrac{2 \cdot x}{\mu_0 A_m}\right)^2} = -\frac{(930000 \cdot 0.008)^2}{4\pi \times 10^{-7} \cdot 0.0014 \left(4210448 + 0 + \dfrac{2 \cdot 0.003}{4\pi \times 10^{-7} \cdot 0.0014}\right)^2} = -541.7 \text{ N.}$$

The negative sign indicates that it is an attraction force. That is, the force f_e acts in the direction to reduce air gap x.

(b) The winding current i required to reduce the force f_e to zero:

$$f_e = -\frac{(H_c \cdot l_m - N \cdot i)^2}{\mu_0 A_m \left(\Re_m + \Re_c + \dfrac{2 \cdot x}{\mu_0 A_m}\right)^2} = 0 \quad \Rightarrow H_c \cdot l_m = N \cdot i \Rightarrow i = \frac{H_c \cdot l_m}{N} = \frac{930000 \cdot 0.008}{1500} = 4.96 \text{ A.}$$

UNIVERSITY of HOUSTON
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ECE 4363 – Electromechanical Energy Conversion

Lecture 14

Date: March 25, 2021

by

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University of Houston, Houston, TX

Spring 2021

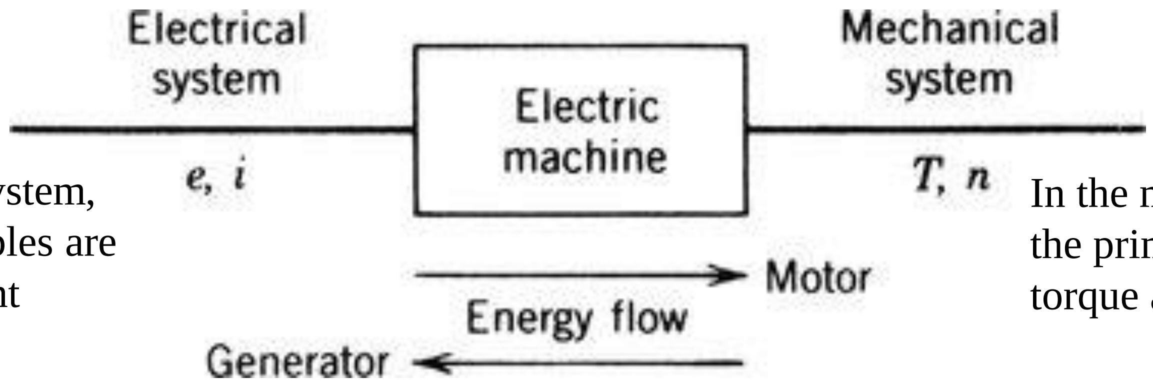
P.S. The pictures, notations, formulas, examples, and statements in these lecture 14 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

DC (Direct Current) MACHINES

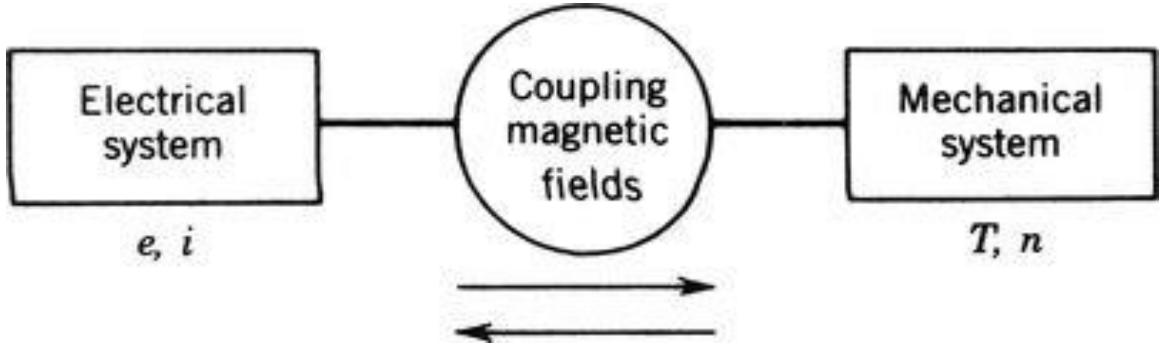
1- DC motor: A dc motor is a rotary electrical machine that converts dc electrical energy into mechanical energy.

2- DC generator A dc generator is a rotary electrical machine that converts mechanical energy into dc electrical energy.

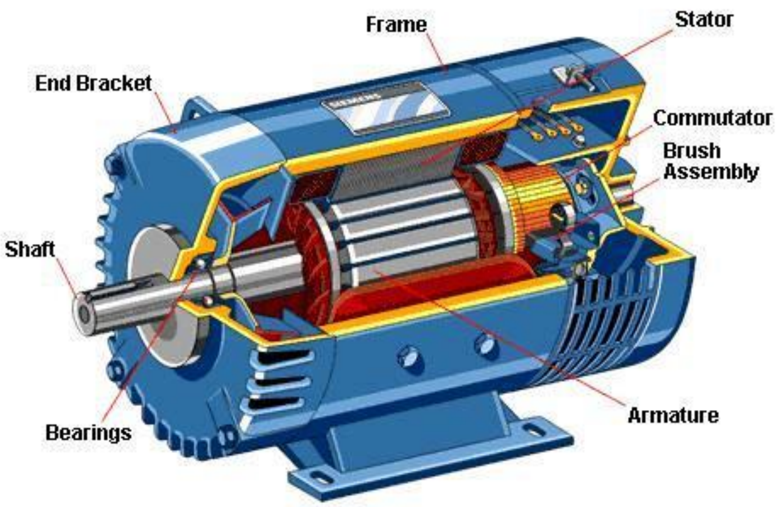
In the electrical system, the primary variables are voltage and current



In the mechanical system, the primary variables are torque and speed



Structure and Operating Principle of a DC Motor



Cutaway view of a dc motor

http://www.electrical-knowhow.com

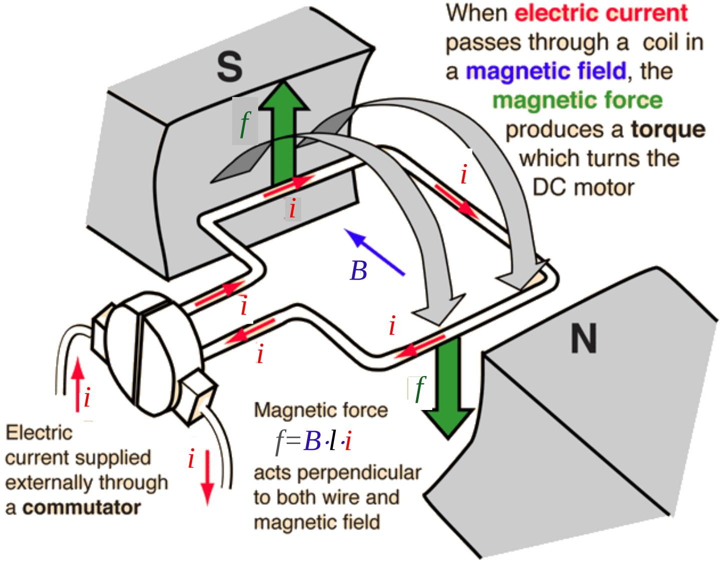
Stator: Static or fixed part. The field winding is placed on the stator and supplied by dc current to form stator north/south poles .

Armature: Rotor or rotating part is the armature in a dc machine. An external dc power is supplied to armature winding through the commutator/brush assembly.

Commutator reverses (commutates) the direction of the current in the armature winding with each half turn. It is made of a conductive material like copper. The commutator is mounted to the rotor and it rotates with the rotor.

A sliding contact is established by a brush that presses on the rotor mounted commutator or slip ring.

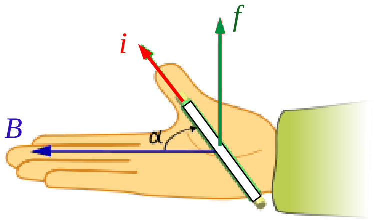
Brush is stationary and fixed to the stator. It is made of a soft conductive material like carbon/graphite (a form of coal).



Operating principle of a dc motor

http://hyperphysics.phy-astr.gsu.edu

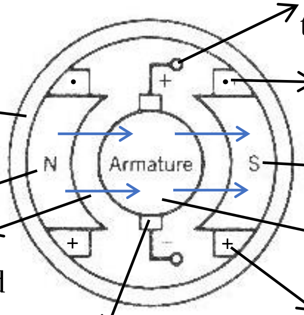
Lorentz force: An electromagnetic force is produced on a current carrying conductor in a magnetic field. The force in newton (or, kg·m/s²) is calculated by $f = B \cdot l \cdot i$ where B flux density in tesla, l is the length of the conductor in meter, and i is the current flowing through the conductor in ampere. In the formula, B and i are assumed to be mutually perpendicular. The direction of the force can be determined using the right hand rule.



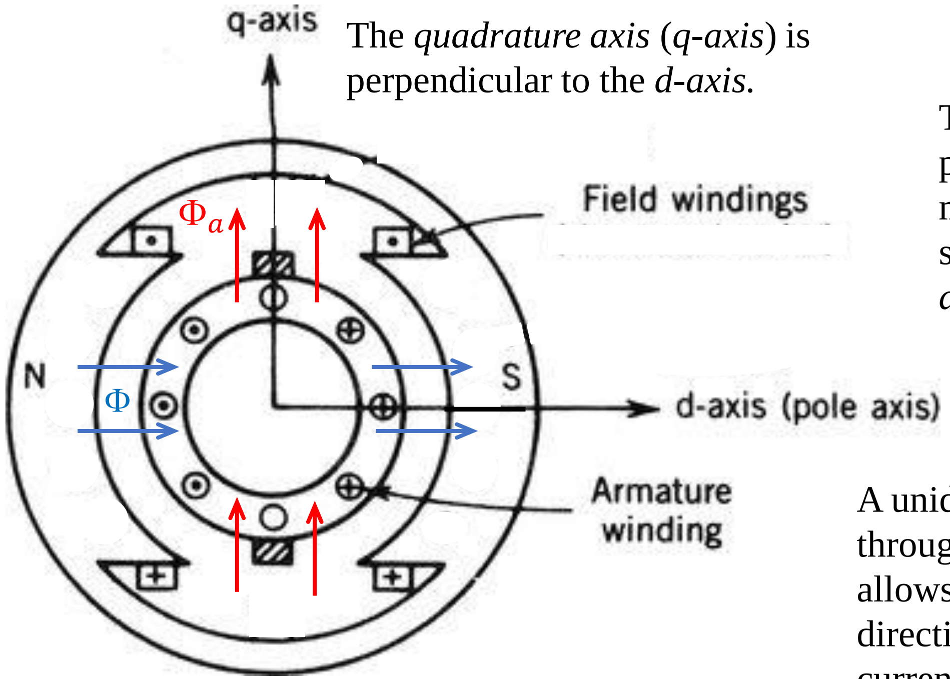
Right hand rule: Point the thumb in the direction of i and the fingers in the direction of B, then, the palm points in the direction of the electromagnetic force.

https://en.wikipedia.org/wiki/Lorentz_force

Schematic View of DC Machine



Schematic cross-sectional view of a 2-pole dc machine



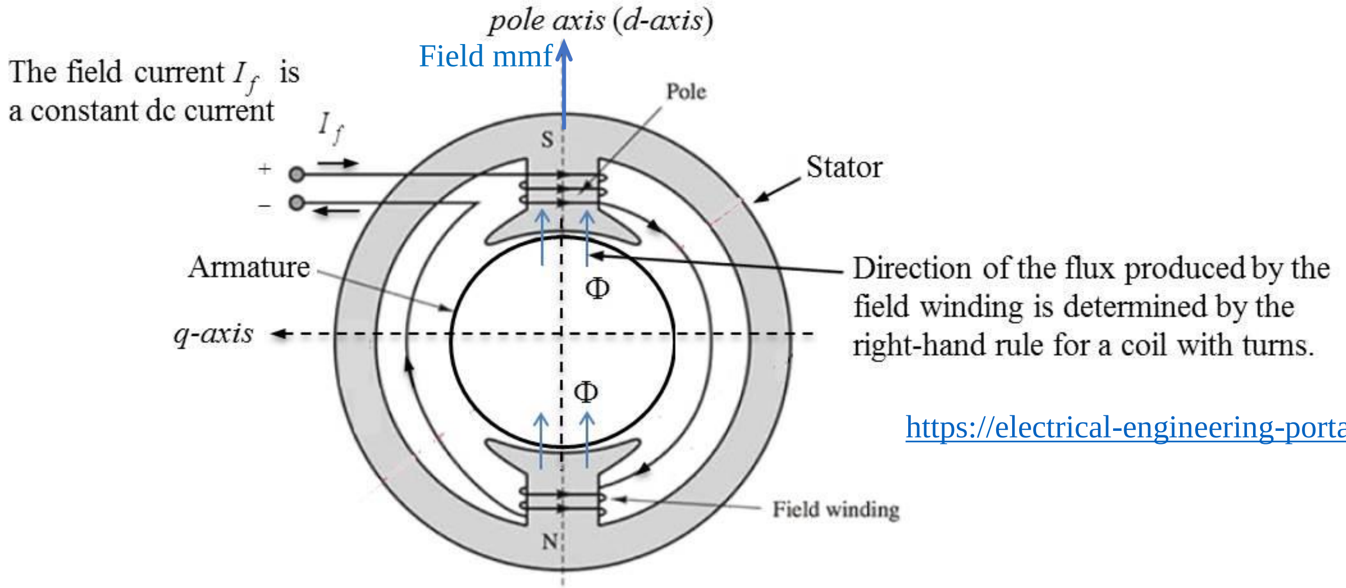
The *quadrature axis* (*q-axis*) is perpendicular to the *d-axis*.

Schematic diagram of a dc machine

The field winding is placed on the stator. A dc current is passed through the field winding to produce flux Φ in the machine. The resultant air gap flux distribution is symmetrical about the *pole axis* (also called the *field axis*, *direct axis*, or *d-axis*).

A unidirectional (DC) terminal voltage is applied to the armature through a mechanical commutator and a brush assembly which allows all the conductors under one pole carry current in one direction. As a result, the mmf (or, flux Φ_a) due to the armature current is along the *q-axis* which is in quadrature with the *d-axis*.

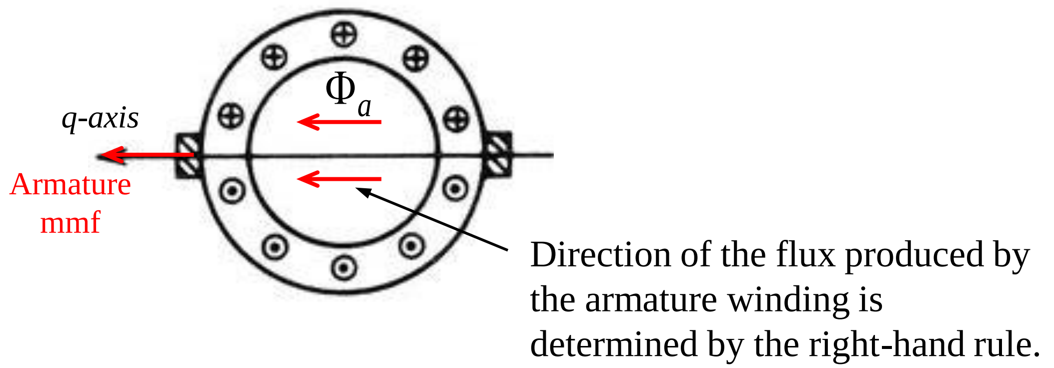
Field MMF of 2-Pole DC Machine



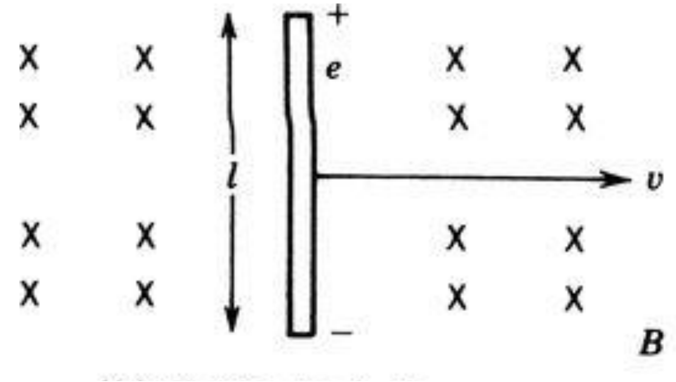
https://electrical-engineering-portal.com

Forming the stator north/south poles of DC machine by the field current I_f.

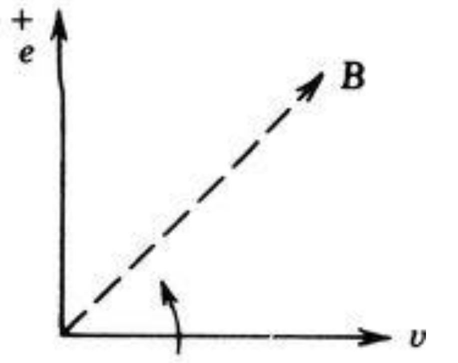
Armature MMF of 2-Pole DC Machine



Motional Voltage (or, Motional EMF)



A voltage *e* is induced between the ends of conductor (bar) when the conductor moves in the magnetic field *B*. The sign x indicates *B* is into the slide.



Right-hand screw rule to determine the polarity of induced voltage *e*.

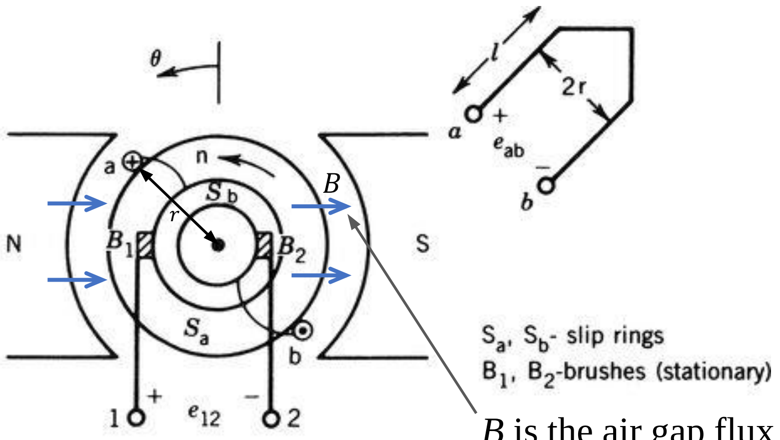
If a conductor of length *l* moves at a linear speed *v* in a magnetic field *B*, a voltage is induced in the conductor . The induced voltage is given by

$$e = B \cdot l \cdot v$$

where *B*, *l* and *v* are assumed to be mutually perpendicular. In the above formula, *e* is in volts, *B* is in teslas, *l* is in meters and *v* is in meters/second.

The polarity of the induced voltage can be determined by the right-hand screw rule: Turn the vector *v* toward the vector *B*. If a right-hand screw is turned in the same way, the motion of the screw will indicate the direction of positive polarity of the induced voltage.

Induced Voltage in the Armature Winding of DC Machine



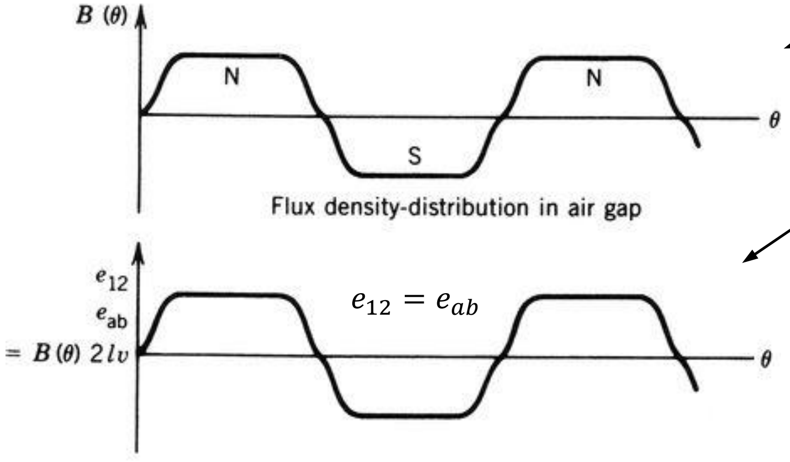
(a)

The turn a–b is placed in diametrically opposite slots of the rotor. The distance between the two sides of the turn is 2·r (twice the rotor radius). The length of each turn side is l.

The two terminals a and b of the turn are connected to two slip rings S_a (outer ring) and S_b (inner ring), respectively.

The two stationary brushes B_1 and B_2 pressing onto the two slip rings S_a and S_b, respectively, provide access to the rotating turn a–b.

B is the air gap flux density produced by the stator field poles.



(b)

Air gap flux density B produced by the stator field poles varies with position θ.

e_{ab} is the voltage induced in the turn a–b. It is due to the voltages induced in the two sides of the turn under the poles. These two voltages are induced based on the *motional voltage* principle. They are in series and aid each other.

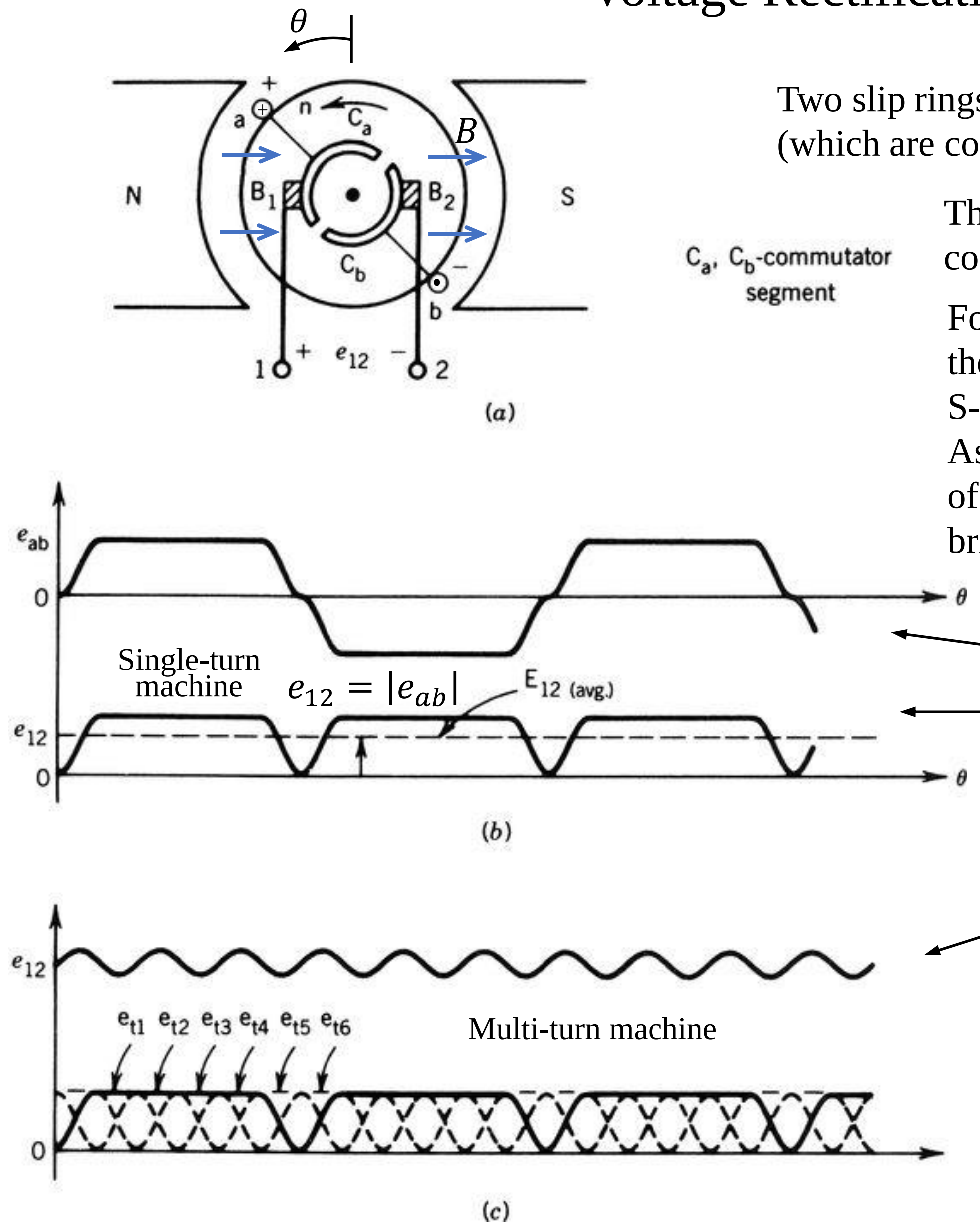
e_{12} is the voltage across the brushes. It is equal to e_{ab}.

$$e_{12} = e_{ab} = B(\theta) \cdot 2 \cdot l \cdot \omega_m \cdot r$$

e_{12} (or, e_{ab}) is alternating and has the same waveform as that of the flux density distribution $B(\theta)$ in air gap.

How can we make the voltage e_{12} across the brushes unidirectional ?

Voltage Rectification in DC Machine



Voltage rectification by a commutator and a brush assembly.

Two slip rings are replaced with two commutator segments C_a and C_b (which are copper segments separated by insulating materials).

The two terminals a and b of the turn are connected to two commutator segments C_a and C_b, respectively.

For counterclockwise rotation, the terminal of the turn under the N-pole is positive with respect to the terminal under the S-pole (from the principle of the induced motional voltage). As a result, brush B_1 is always connected to the positive end of the turn and brush B_2 to the negative end of the turn. In brief, the voltage e_{12} across the brushes is unidirectional.

The voltage e_{ab} induced in the turn a–b is alternating.

The voltage e_{12} across the brushes is unidirectional due to the rectification by the commutator and brush assembly. But, it contains a significant amount of ripple.

In an actual DC machine, a large number of turns connected in series are placed in several slots around the periphery of the rotor to obtain a dc-voltage e_{12} with a much smaller amount of ripple.

Current Reversal in a Turn by Commutators and Brushes in DC Machine

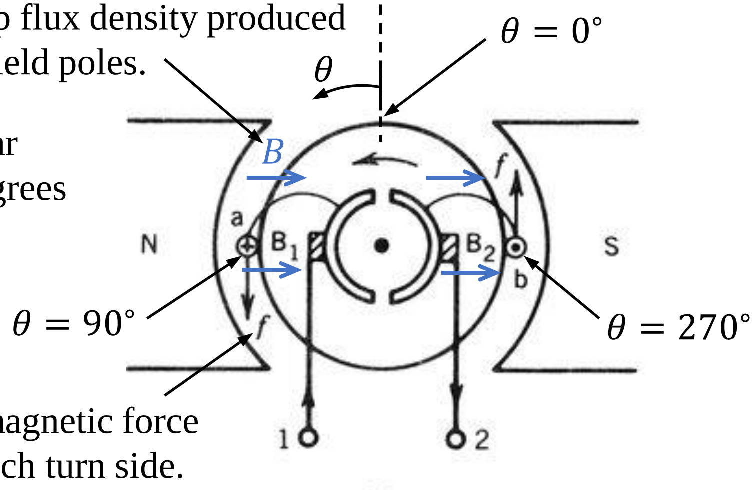
Terminal a of the turn a–b touches brush B_1 and terminal b touches brush B_2. The current flows into terminal a and flows out of terminal b.

The turn a–b is short-circuited by the brushes B_1 and B_2 when its sides pass midway between the field poles (i.e., the *q-axis*). There is no current in the turn a-b. There is no force acting on the sides of turn a-b.

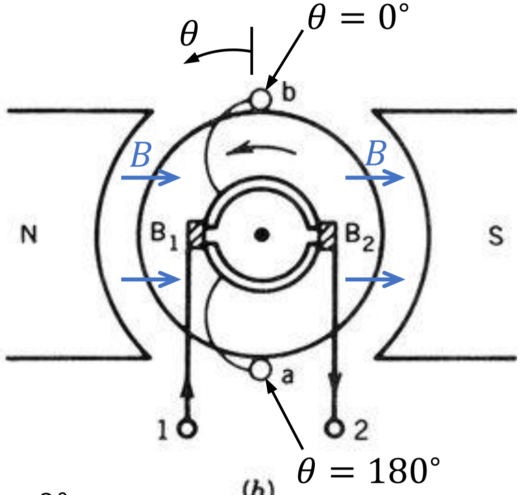
B is the air gap flux density produced by the stator field poles.

θ is the angular position in degrees or in radians.

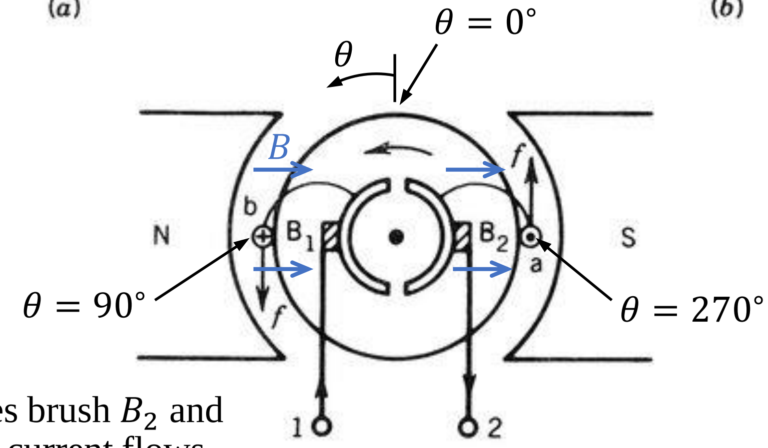
f is electromagnetic force acting on each turn side.



(a)



(b)



(c)

Terminal a of the turn a–b touches brush B_2 and terminal b touches brush B_1. The current flows into terminal b and flows out of terminal a.

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ECE 4363 – Electromechanical Energy Conversion

Lecture 15

Date: March 30, 2021

by

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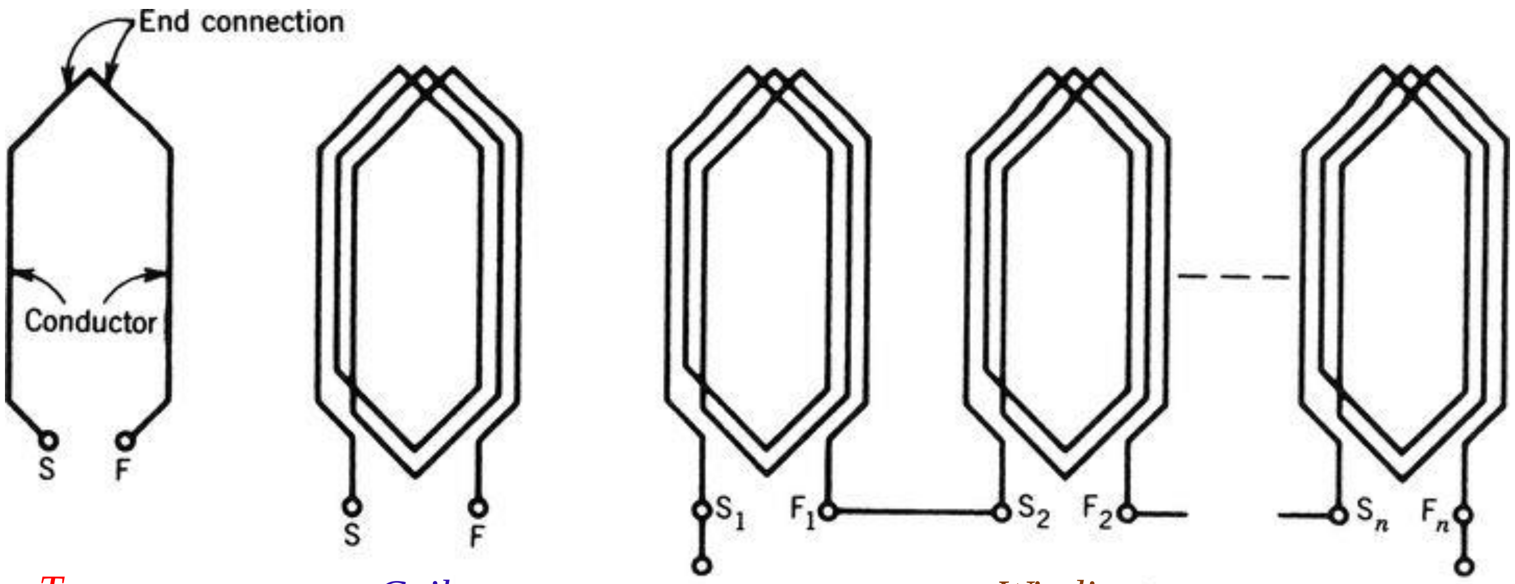
Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 15 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Armature Windings of DC Machine



Turn *Coil* *Winding*

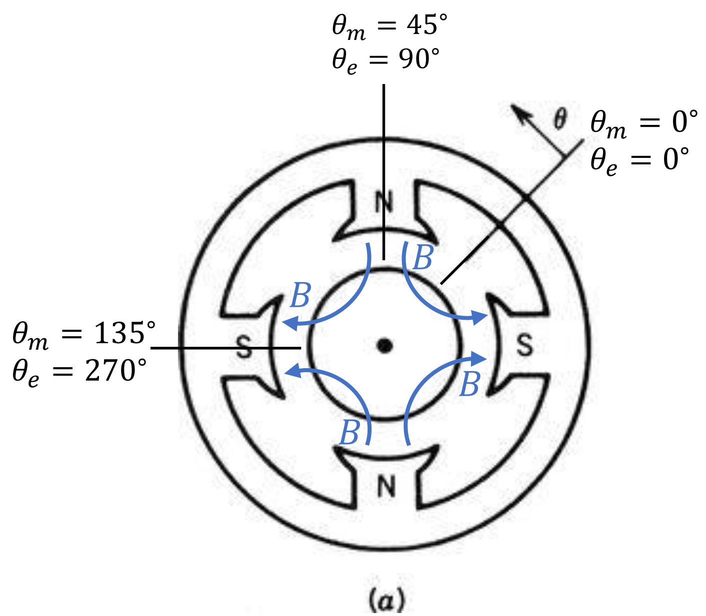
A *turn* consists of two conductors connected to one end by an end connector.

A *coil* is formed by connecting several turns in series.

A *winding* is formed by connecting several coils in series.

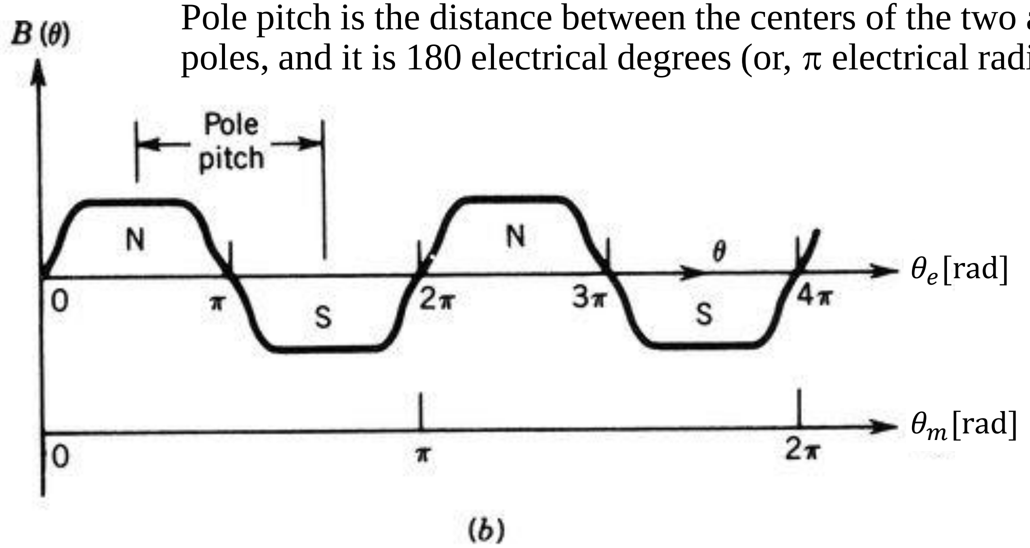
The beginning of the turn, or coil, is identified by the symbol S, and the end of the turn or coil by the symbol F.

Mechanical and Electrical Angles



Four-pole dc machine, and mechanical and electrical angles in degrees.

Pole pitch is the distance between the centers of the two adjacent poles, and it is 180 electrical degrees (or, π electrical radians).



Air gap flux density distribution due to the stator poles in a four-pole dc machine. The angle is in radians (2π radians = 360°).

θ_m = Mechanical angle in mechanical degrees (or, mechanical radians).

θ_e = Electrical angle in electrical degrees (or, electrical radians).

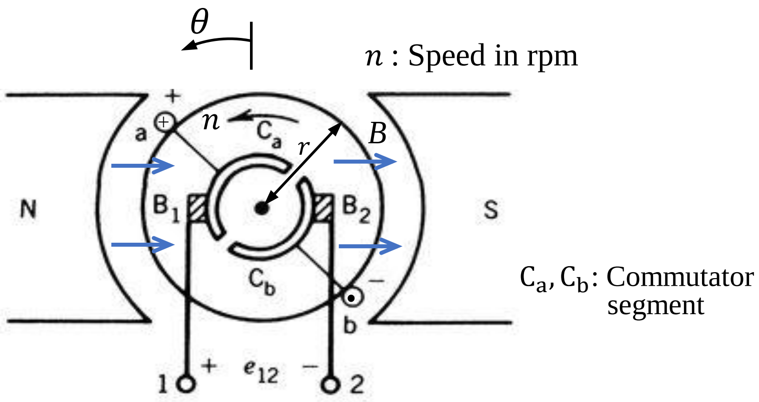
For a four-pole machine, going around the air gap once (i.e., one mechanical cycle), two cycles of variation of the flux density distribution B are encountered.

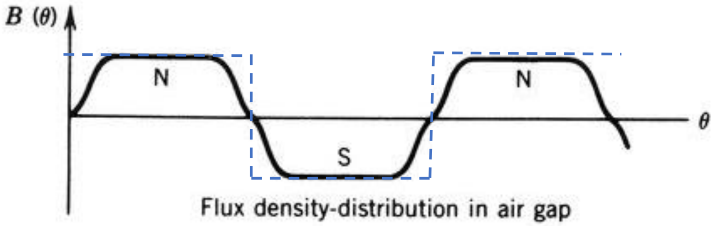
In general, for a *p*-pole machine,

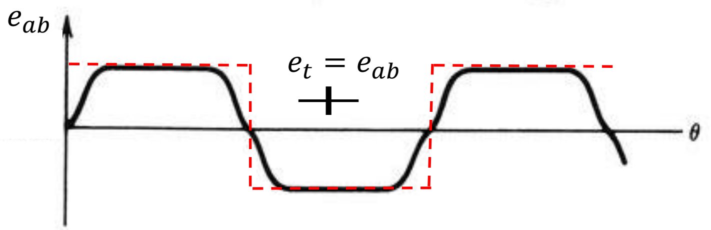
$$\boxed{\theta_e = \frac{p}{2} \cdot \theta_m}$$

The two sides of a coil are placed in two slots on the rotor surface. The distance between the two sides of a coil is called the *coil pitch*. If the coil pitch is one *pole pitch* (the distance between the centers of the two adjacent poles), it is called a *full-pitch coil*. If the coil pitch is less than one pole pitch, the coil is known as a *short-pitch* (or, *fractional-pitch*) coil.

Armature Voltage of DC Machine







Induced voltage in a single turn a–b.

B is in dashed blue and e_{ab} is in dashed red when the poles cover 100% of the armature periphery.

As the armature rotates in the magnetic field produced by the stator poles, voltage is induced in the armature winding.

The induced voltage (motional voltage) in a single turn a–b is given by

$$e_t = 2 \cdot B(\theta) \cdot l \cdot v_m = 2 \cdot B(\theta) \cdot l \cdot \omega_m \cdot r$$

l : Length of the conductor in the slot of the armature [m].

v_m : Tangential speed [m/s].

ω_m : Mechanical angular speed [rad/s]. $n = \frac{60}{2 \cdot \pi} \cdot \omega_m$ [rpm].

r : Radius of the armature [m].

Let Φ denote flux per pole in [Wb].

Assume that the poles cover 100% of the armature periphery.

$A = \frac{2 \cdot \pi \cdot r \cdot l}{p}$: Area under per pole [m²] where p is the number of poles.

$B(\theta) \cong \frac{\Phi}{A} = \frac{p \cdot \Phi}{2 \cdot \pi \cdot r \cdot l}$: Average flux density under a pole.

(Blue dashed line in the figure on the left)

$e_t \cong \frac{p \cdot \Phi}{\pi} \cdot \omega_m$ (Red dashed line in the figure on the left)

Let N denote total number of turns connected in series.

Then, the induced armature voltage in volts is obtained as

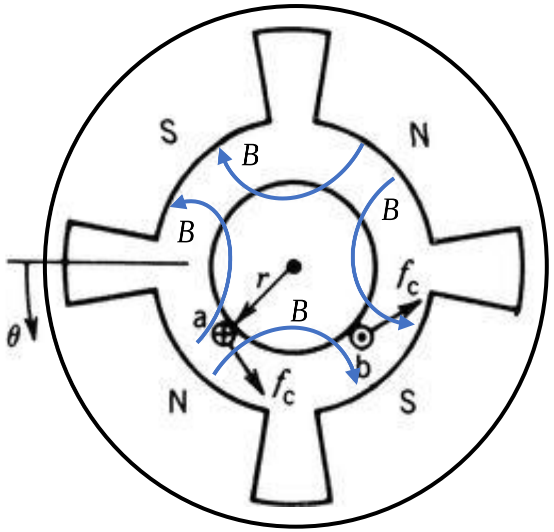
$$E_a = \frac{N \cdot p}{\pi} \cdot \Phi \cdot \omega_m$$

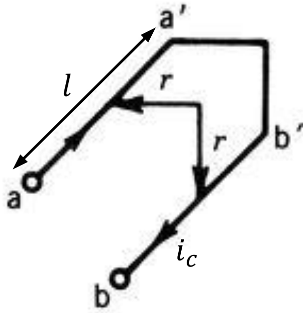
$$E_a = K_a \cdot \Phi \cdot \omega_m \quad \text{where} \quad K_a = \frac{N \cdot p}{\pi} \quad \text{is the machine (or, armature) constant.}$$

E_a is known as *back emf* (*electromotive force*) in motor operation, and *generated voltage* in generator operation.

Electromagnetic Torque of DC Machine

The expression of the torque developed by the dc machine is derived based on the concept of Lorentz force.





The force on conductor placed on the periphery of the armature is

$$f_c = B(\theta) \cdot l \cdot i_c = B(\theta) \cdot l \cdot I_a \quad \text{for} \quad i_c = I_a$$

i_c : Current in the conductor of the armature winding.

I_a : Armature terminal current.

The torque developed by the conductor is

$$T_c = f_c \cdot r = B(\theta) \cdot l \cdot I_a \cdot r$$

Assume that the poles cover 100% of the armature periphery. That is, $B(\theta) \cong \frac{\Phi}{A} = \frac{p \cdot \Phi}{2 \cdot \pi \cdot r \cdot l}$. Then,

$$T_c = \frac{p}{2 \cdot \pi} \cdot \Phi \cdot I_a$$

Let *N* be the number of turns connected in series. Then, the total torque developed by all the conductors in the armature winding is obtained by

$$T_e = 2 \cdot N \cdot T_c = \frac{N \cdot p}{\pi} \cdot \Phi \cdot I_a$$

$$T_e = K_a \cdot \Phi \cdot I_a \quad \text{where} \quad K_a = \frac{N \cdot p}{\pi} \quad \text{is the machine (or, armature) constant.}$$

T_e is the *torque* (or, *electromagnetic torque*, or, *electrical torque*) developed by the DC machine. It is in Nm.

EXAMPLE 4.1 from the textbook

A four-pole dc machine has an armature of radius 12.5 cm and an effective length of 25 cm. The poles cover 75% of the armature periphery. The average flux density under each pole is 0.75 T. The armature constant is $K_a = 73.53$ [V·s/ (Wb·rad)] (a) Determine the induced armature voltage when the armature rotates at 1000 rpm (rotations per minute). (b) Determine the electromagnetic torque T_e developed when the armature current is 400 A. (c) Determine the power developed by the armature.

Solution

(a) The pole area is

$$A_p = \frac{2 \cdot \pi \cdot r \cdot l}{p} \times 0.75 = \frac{2 \cdot \pi \cdot 0.125 \cdot 0.25}{4} \times 0.75 = 36.8 \times 10^{-3} \mathrm{m}^2$$

$$\Phi = B \cdot A_p = 0.75 \times 36.8 \times 10^{-3} = 0.0276 \text{ Wb}$$

$$E_a = K_a \cdot \Phi \cdot \omega_m = 73.53 \times 0.0276 \times \frac{1000}{60} \times 2 \cdot \pi = 212.5 \text{ V}$$

(b) The electromagnetic torque developed by the dc machine is

$$T_e = K_a \cdot \Phi \cdot I_a = 73.53 \times 0.0276 \times 400 = 811.8 \text{ N} \cdot \text{m}$$

(c) The power developed by the armature is

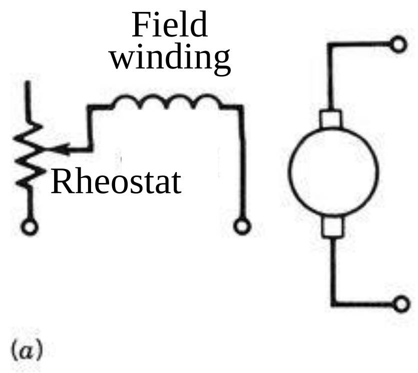
$$P_a = E_a \cdot I_a = 212.5 \times 400 = 85.0 \text{ kW}$$

or,

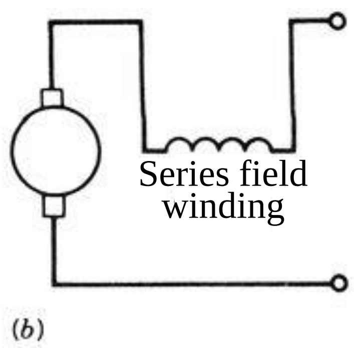
$$P_a = T_e \cdot \omega_m = 811.8 \times \frac{1000}{60} \times 2 \cdot \pi = 85.0 \text{ kW}$$

Classification of DC Machines

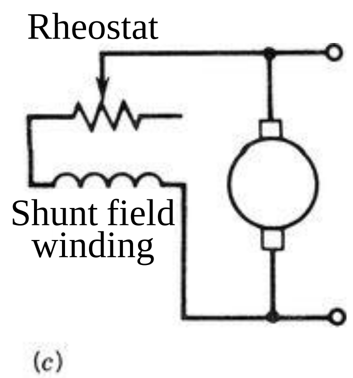
The DC machines are classified according to the way the field and armature windings are connected.



Separately excited dc machine



Series dc machine



Shunt dc machine

1- Separately excited dc machine: The field winding is excited from a separate dc source. The air gap flux can easily be adjusted by the field current.

2- Series dc machine: The field winding is connected in series with the armature winding.

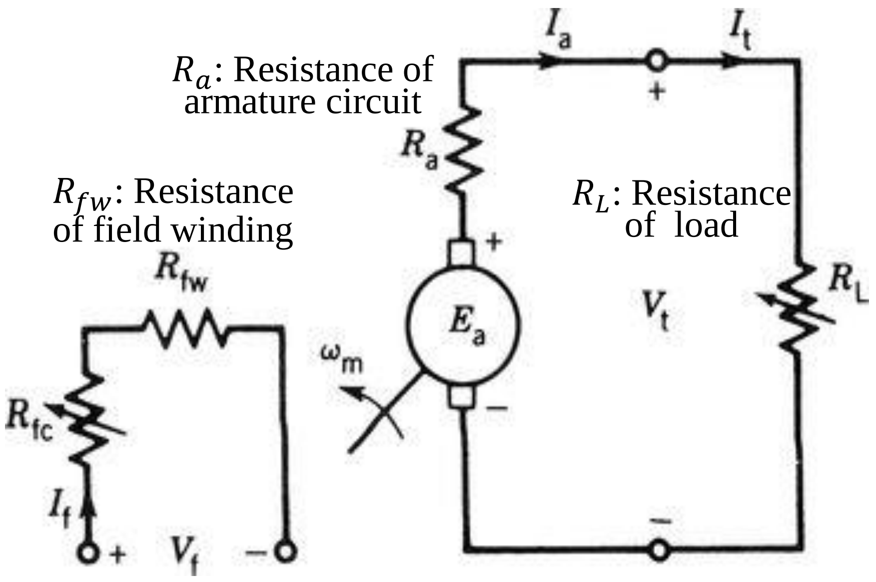
3- Shunt dc machine: The field winding is connected in parallel with the armature winding.

Note that the field current, in turn the air gap flux by the stator field poles, can be adjusted by a rheostat (variable resistor) in the separately excited and shunt dc machines. In the series dc machine, a rheostat is not used due to the high armature current that causes large power consumption.

DC GENERATORS

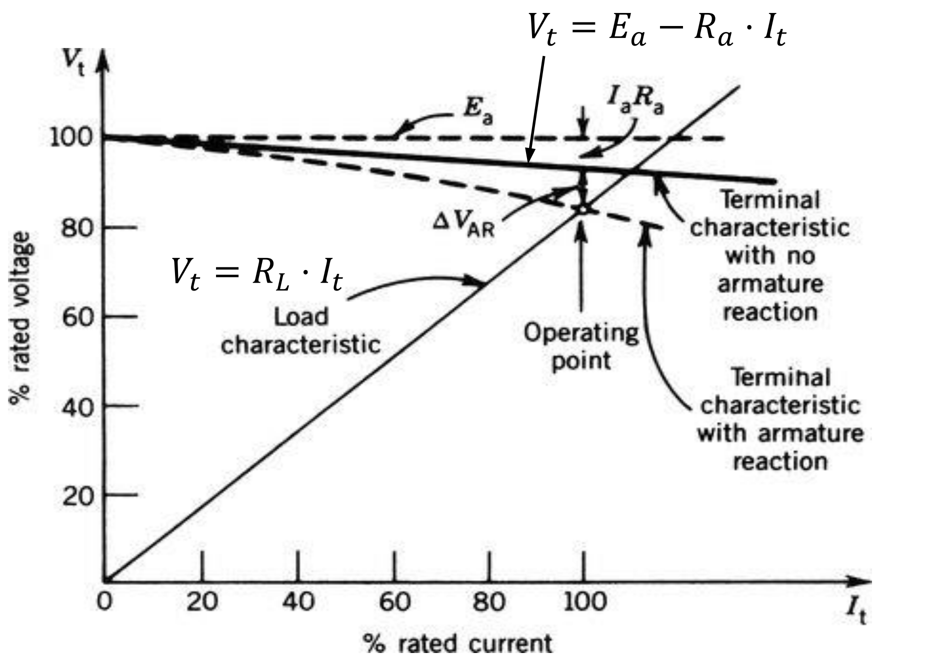
The DC machine operating as a generator is driven by a prime mover at a constant speed and the armature terminals are connected to a load. The prime mover can be a gas turbine or a diesel engine.

Separately Excited DC Generator



R_{fc}: Resistance of control rheostat

Steady-state model of a separately excited dc generator.



Terminal characteristic of a separately excited dc generator.

In the steady-state operation, the inductances of the field and armature windings are not considered as the currents are assumed to be constant.

$$V_f = (R_{fc} + R_{fw}) \cdot I_f = R_f \cdot I_f \quad \text{where} \quad R_f = R_{fc} + R_{fw}$$

$$E_a = V_t + R_a \cdot I_a \quad \Rightarrow \quad V_t = E_a - R_a \cdot I_a$$

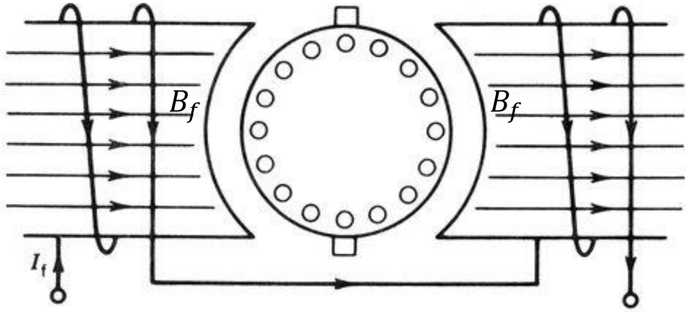
$$E_a = K_a \cdot \Phi \cdot \omega_m$$

$$V_t = R_L \cdot I_t \quad \text{where} \quad I_t = I_a$$

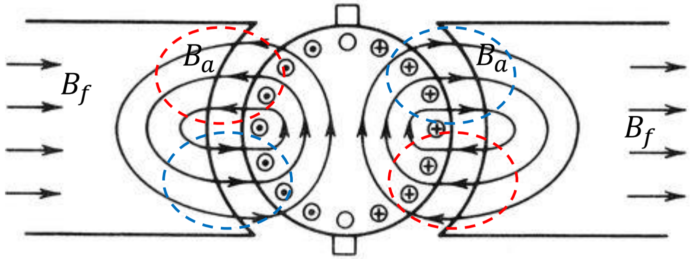
Armature Reaction in DC Machine

The generated voltage E_a in the armature decreases as the current in the armature increases. This is known as *armature reaction*. It is also known as *demagnetization effect* since the armature current causes a reduction of flux per pole.

With no current in the armature, the flux in the machine is established by field current I_f.

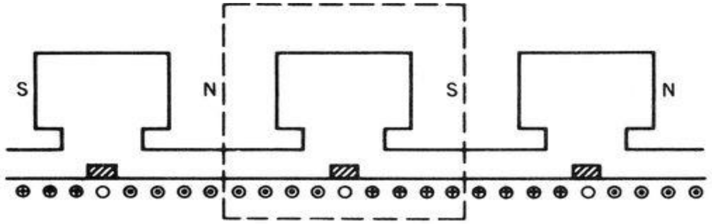


If the current flows in the armature, it produces its own mmf (hence flux) acting along the *q-axis*.



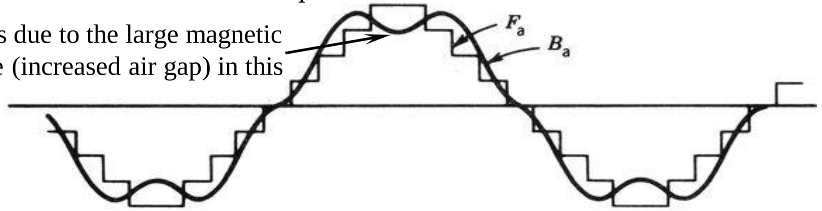
The flux produced by the armature mmf opposes flux in the pole under one half of the pole (the areas inside the red dashed circles). On the other hand, it aids under the other half of the pole (the areas inside the blue dashed circles). Consequently, flux density increases under one half of the pole and decreases under the other half of the pole.

Due to the magnetic saturation, the flux density is reduced in a greater amount under one half of the pole than it is increased under the other. As a result, the net effect is a reduction of flux per pole.

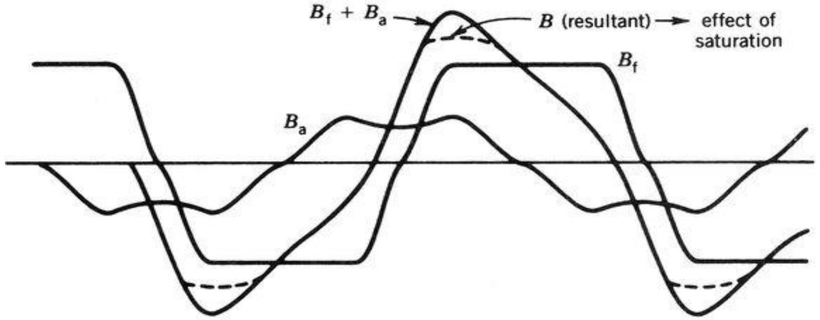


For the path shown by the dashed line, the net armature mmf is zero since it encloses equal numbers of dot and cross currents.

This dip is due to the large magnetic reluctance (increased air gap) in this region.



The flux density distribution B_a produced by the armature mmf F_a.



The flux density distributions caused by the field mmf, the armature mmf, and their resultant mmf.

The armature mmf is neutralized by using a compensating winding, which is fitted in slots cut on the pole faces.

PROBLEM 4.3 from the textbook

A dc machine rated at 6 kW, 120V, and 1200 rpm has the following magnetization characteristic at 1200 rpm.

I_f	(A)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2
E_a	(V)	5	20	40	60	79	93	102	114	120	125

The machine parameters are $R_a = 0.2\ \Omega$, $R_{fw} = 100\ \Omega$. The machine is driven at 1200 rpm and is separately excited. The field current is adjusted at $I_f = 0.8$ A. A load resistance $R_L = 2\ \Omega$ is connected to the armature terminals. Neglect armature reaction effect. (a) Determine the quantity $K_a \cdot \Phi$ for the machine. (b) Determine I_a. (c) Determine torque T_e and load power P_L.

Solution

(a) $$\omega_m = \frac{1200 \text{ rpm}}{60} \times 2 \cdot \pi = 125.66 \ \text{rad/s}.$$

$$K_a \cdot \Phi = \frac{E_a|_{I_f=0.8\text{A},\, n=1200\text{rpm}}}{\omega_m} = \frac{114 \text{ V}}{125.66 \text{ rad/s}} = 0.907 \ \text{V} \cdot \text{s/rad}$$

(b) $$I_a = \frac{E_a}{R_a + R_L} = \frac{114}{0.2 + 2} = 51.82 \text{ A}$$

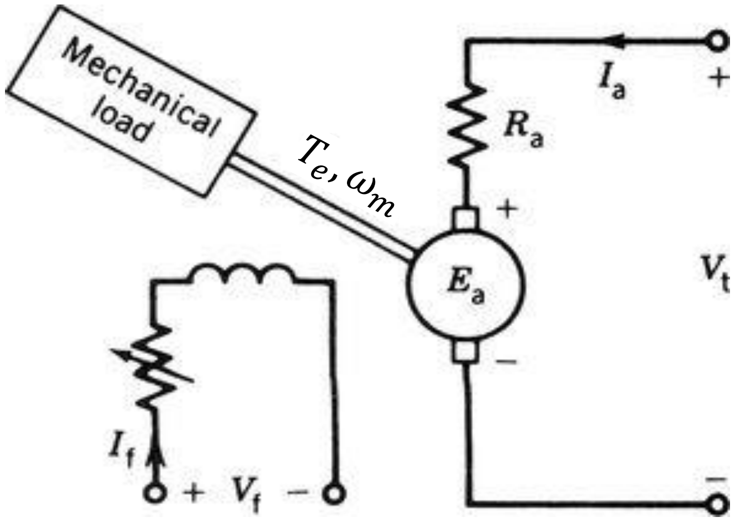
Note that it is a generator (not a motor) since a load resistance is connected to the armature !

(c) $T_e = K_a \cdot \Phi \cdot I_a = 0.907 \times 51.82 = 47$ Nm. $P_L = I_a^2 \cdot R_L = (51.82)^2 \times 2 = 5370.6$ W

DC MOTORS

When the dc machine operates as a motor, the input to the machine is electrical power, and the output is mechanical power. If the armature is supplied from a dc source, the motor will develop mechanical torque and power on its shaft. DC motors can provide a wide range of accurate speed and torque control.

Separately Excited DC Motor



$$E_a = K_a \cdot \Phi \cdot \omega_m = V_t - I_a \cdot R_a$$

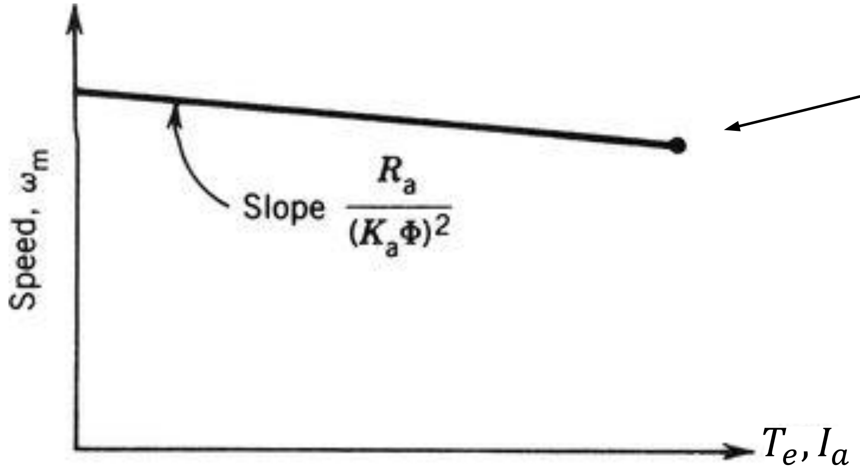
$$\Longrightarrow \omega_m = \frac{V_t}{K_a \cdot \Phi} - \frac{I_a \cdot R_a}{K_a \cdot \Phi}$$

$$T_e = K_a \cdot \Phi \cdot I_a$$

$$\Longrightarrow I_a = \frac{T_e}{K_a \cdot \Phi}$$

Then,

$$\boxed{\omega_m = \frac{V_t}{K_a \cdot \Phi} - \frac{R_a}{(K_a \cdot \Phi)^2} \cdot T_e}$$



If the terminal voltage V_t and machine flux Φ are kept constant, the torque-speed characteristic is linear as shown on the left.

CONTROL OF DC MOTORS

The relationship between the speed and torque of the dc motor is given by

$$\boxed{\omega_m = \frac{V_t}{K_a \cdot \Phi} - \frac{R_a}{(K_a \cdot \Phi)^2} \cdot T_e}$$

The speed control in a dc machine can be achieved by keeping the field current I_f (and, hence the flux Φ) constant and varying the armature (terminal) voltage V_t.

Armature Voltage Control

In this technique, the speed control is achieved by varying V_t while keeping Φ constant.

$$\omega_m = K_1 \cdot V_t - K_2 \cdot T_e$$

where

$$K_1 = \frac{1}{K_a \cdot \Phi} \quad \text{and} \quad K_2 = \frac{R_a}{(K_a \cdot \Phi)^2}$$

$E_a = K_a \cdot \Phi \cdot \omega_m$ increases linearly with ω_m.

$$T_e = K_a \cdot \Phi \cdot I_a$$

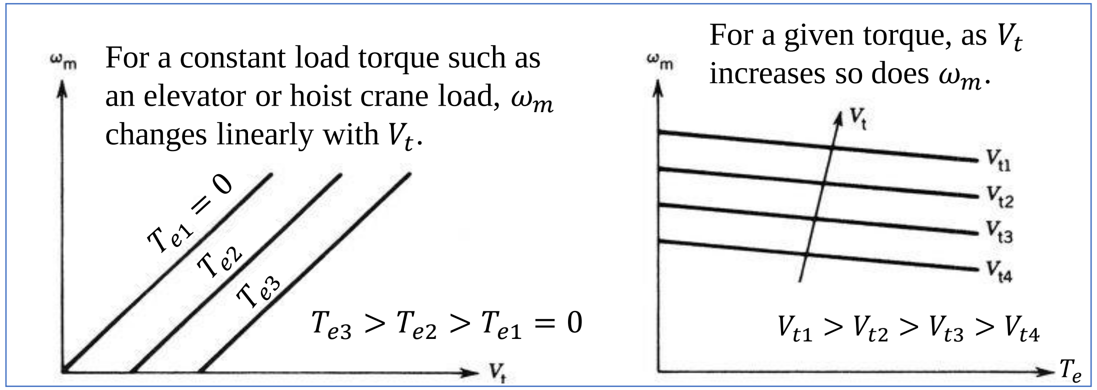
$$V_t = K_a \cdot \Phi \cdot \omega_m + I_a \cdot R_a$$

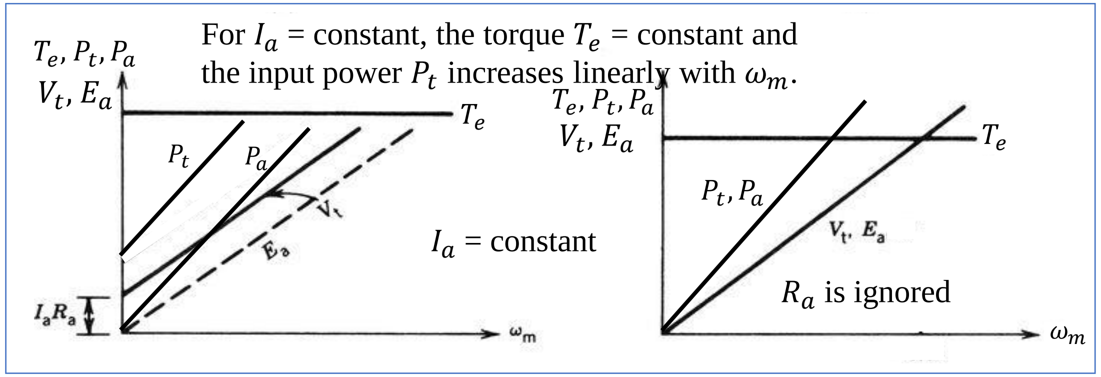
The terminal (electrical) power is

$$P_t = V_t \cdot I_a = K_a \cdot \Phi \cdot I_a \cdot \omega_m + I_a^2 \cdot R_a$$

The armature (mechanical) power is

$$P_a = E_a \cdot I_a = K_a \cdot \Phi \cdot \omega_m \cdot I_a = T_e \cdot \omega_m$$





UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 16

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by

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P.S. The pictures, notations, formulas, examples, and statements in these lecture 16 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Broad Range Speed Control of DC Motors

A wide speed range for a dc motor can be obtained under the rated (nominal) terminal voltage and rated (nominal) terminal current by implementing both the Armature Voltage Control and the Field Control techniques.

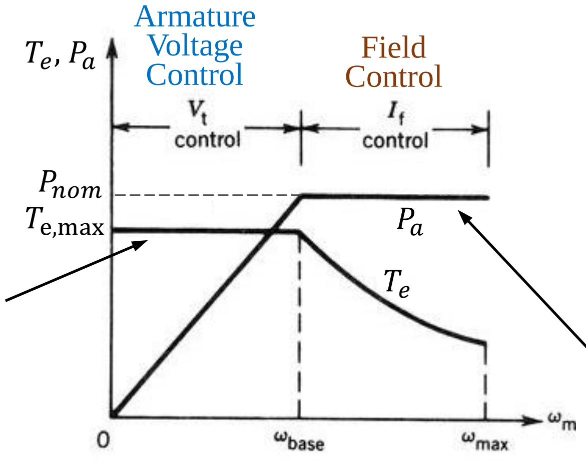
Armature Voltage Control

Maximum torque $T_{e,\max}$ is achieved for $0 < \omega_m \leq \omega_{base}$.

$$T_{e,\max} = K_a \cdot \Phi_{\max} \cdot I_{a,nom}$$

Constant Torque Operation

I_a is limited to $I_{a,nom}$ to prevent overheating of the machine !



ω_{base}: Base speed (rad/s).

The base speed is defined as the speed obtained at the nominal terminal voltage $V_{t,nom}$ and nominal armature current $I_{a,nom}$.

Field Control

Constant Power Operation for $\omega_m > \omega_{base}$

With the armature voltage control technique, a maximum torque $T_{e,\max}$ can be achieved from zero to the base speed ω_{base} by maintaining armature current at its nominal value $I_{a,nom}$ while holding the flux (or, the field current) at its maximum value $\Phi_{\max}$ (or, $I_{f,\max}$).

$$V_t = K_a \cdot \Phi_{\max} \cdot \omega_m + I_{a,nom} \cdot R_a \text{ at } \omega_m < \omega_{base} \text{ and } T_e = T_{e,\max}$$

$$V_{t,nom} = K_a \cdot \Phi_{\max} \cdot \omega_{base} + I_{a,nom} \cdot R_a \text{ at } \omega_m = \omega_{base} \text{ and } T_e = T_{e,\max}$$

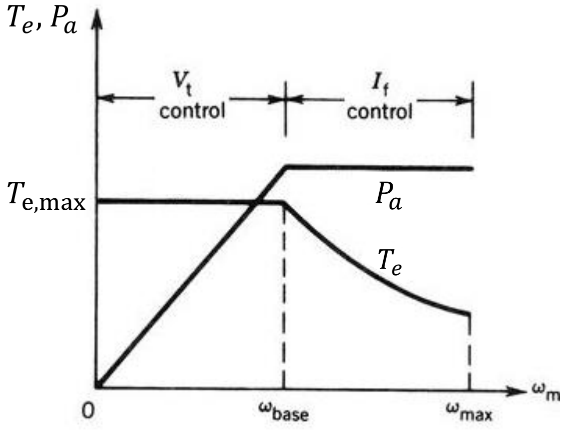
Beyond the base speed, the field control technique is implemented where the flux (or, field current) is decreased inversely with the speed. This is done so to prevent the terminal voltage from saturating or going above its rated value. As a result, the torque decreases inversely with the speed. The terminal (electrical) power and the armature (mechanical) power will remain constant at $P_t = V_{t,nom} \cdot I_{t,nom}$ and $P_a = P_{nom} = T_{e,\max} \cdot \omega_{base}$, respectively, beyond the base speed.

$$V_{t,nom} = K_a \cdot \Phi \cdot \omega_m + I_{a,nom} \cdot R_a \text{ at } \omega_m > \omega_{base} \text{ , } \Phi = \Phi_{\max} \cdot \frac{\omega_{base}}{\omega_m} \text{ and } T_e = K_a \cdot \Phi \cdot I_{a,nom}$$

EXAMPLE 4.8 from the textbook

A variable-speed drive uses a dc motor that is supplied from a variable-voltage source. The torque and power profiles are shown in the figure below. The drive speed is varied from 0 to 1500 rpm (base speed ω_{base}) by varying the terminal voltage from 0 to 500 V with the field current maintained constant. (a) Neglect the voltage drop across the armature resistance. Then, determine the motor armature current if the torque is held constant at $T_\mathrm{e} = T_\mathrm{e,max} = 300$ N·m up to the base speed. (b) The speed beyond the base speed is obtained by field weakening while the armature voltage is held constant at 500 V. Determine the torque available at a speed of 3000 rpm if the armature current is held constant at the value obtained in part (a). Neglect all losses.

Solution



The torque and power profiles of the dc motor in Example 4.8

(a)

$$\omega_{base} = \frac{1500\ \text{rpm}}{60} \times 2 \cdot \pi = 157.08\ \ \text{rad/s.}$$

$$E_a = V_t = 500\ \text{V}\ \ \text{since}\ \ I_a \cdot R_a\ \ \text{is}\ \ \text{omitted.}$$

$$K_a \cdot \Phi = \frac{E_a}{\omega_m} = \frac{500\ \text{V}}{157.08\ \text{rad/s}} = 3.183\ \ \text{V} \cdot \text{s/rad}$$

$$I_a = \frac{T_\mathrm{e}}{K_a \cdot \Phi} = \frac{300\ \text{N} \cdot \text{m}}{3.183\ \text{rad/s}} = 94.3\ \text{A}$$

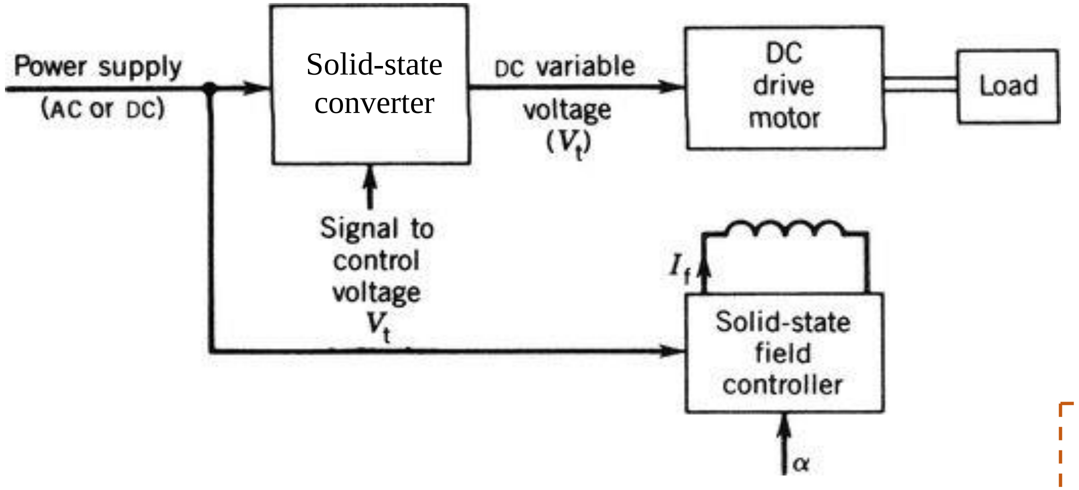
(b)

$$\omega_m = \frac{3000\ \text{rpm}}{60} \times 2 \cdot \pi = 314.16\ \ \text{rad/s.} \qquad K_a \cdot \Phi = \frac{E_a}{\omega_m} = \frac{500\ \text{V}}{314.16\ \text{rad/s}} = 1.592\ \ \text{V} \cdot \text{s/rad.}$$

$$T_\mathrm{e} = K_a \cdot \Phi \cdot I_a = 1.592\ \text{V} \cdot \text{s/rad}\ \times\ 94.3\ \text{A} = 150\ \text{N} \cdot \text{m.}\ \ \text{Or,}\ \ T_\mathrm{e} = \frac{P_a}{\omega_m} = \frac{E_a \cdot I_a}{\omega_m} = \frac{500 \times 94.13}{314.16} = 150\ \text{N} \cdot \text{m}$$

Solid-State Control of DC Motors

The solid-state converters are used to control the dc motors. It is called solid-state converter since the converter is made of solid-state switching devices such as the silicon controlled rectifier (SCR).



Block diagram of solid-state control of dc motor.

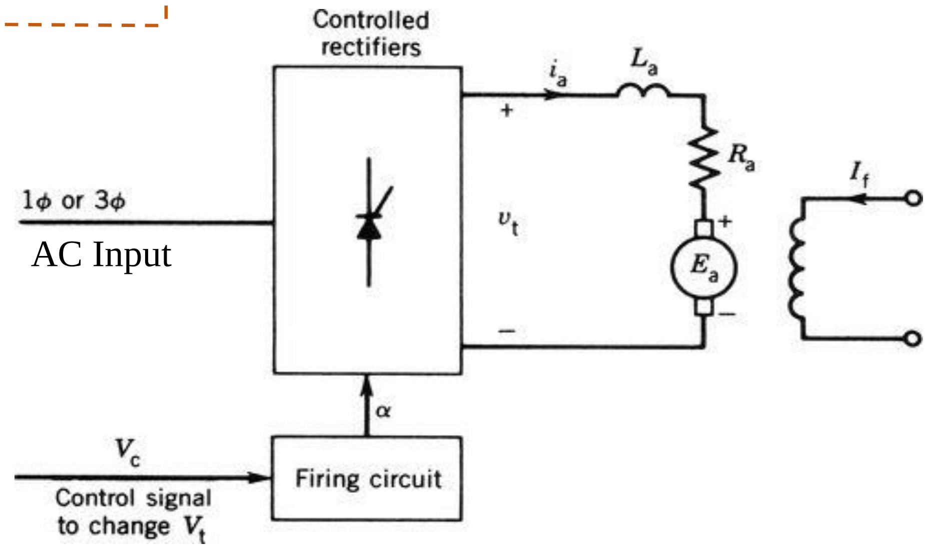
A converter converts either i) AC voltage into DC voltage (called Rectifier), or, ii) DC voltage into a lower DC voltage (called Chopper or Buck Converter) or higher DC voltage (called Boost Converter), or iii) DC voltage into AC voltage (called Inverter), or iv) AC voltage into variable-frequency AC voltage (Cycloconverter).

Controlled Rectifiers

If the supply is AC, a controlled rectifier can be used to convert a fixed AC supply into a variable-voltage DC supply. The firing angle α of the SCRs determines the average value of the rectifier output voltage V_t that is applied to the armature of the DC motor. Let denote the rms value of the ac supply phase voltage by V_{ph}. Then,

Single-phase AC Input: $$V_t = \frac{2 \cdot \sqrt{2} \cdot V_{ph}}{\pi} \cdot \cos(\alpha)$$

Three-phase AC Input: $$V_t = \frac{3 \cdot \sqrt{6} \cdot V_{ph}}{\pi} \cdot \cos(\alpha)$$



Speed control of dc motor by a controlled rectifier.

EXAMPLE 4.11 from the textbook

The speed of a 10 hp, 220 V (dc), 1200 rpm separately excited dc motor is controlled by a single-phase controlled rectifier. The rated armature current is 40 A. The armature resistance is $R_a = 0.25\ \Omega$. The ac supply voltage is 265 V (rms). The motor voltage constant is $K_a \cdot \Phi = 0.18\ \mathrm{V/rpm}$. For a firing angle $\alpha = 30^\circ$ and the rated motor current, determine (a) the speed of the motor, (b) the motor torque, and (c) the power to the motor.

Solution

(a) The output voltage of the single-phase controlled rectifier is

$$V_t = \frac{2 \cdot \sqrt{2} \cdot V_{ph}}{\pi} \cdot \cos(\alpha) = \frac{2 \cdot \sqrt{2} \cdot 265}{\pi} \cdot \cos(30°) = 206.6\ \mathrm{V}$$

The back emf is $E_a = V_t - I_a \cdot R_a = 206.6 - 40 \times 0.25 = 196.6\ \mathrm{V}$

The speed in rpm is $\omega_m = \dfrac{E_a}{K_a \cdot \Phi} = \dfrac{196.6\ \mathrm{V}}{0.18\ \mathrm{V/rpm}} = 1092.2\ \mathrm{rpm}.$

(b)

$$K_a \cdot \Phi = 0.18\ \mathrm{V/rpm} = \frac{0.18}{\left(\frac{2 \cdot \pi}{60}\right)}\ \mathrm{V \cdot s/rad} = 1.72\ \mathrm{V \cdot s/rad}$$

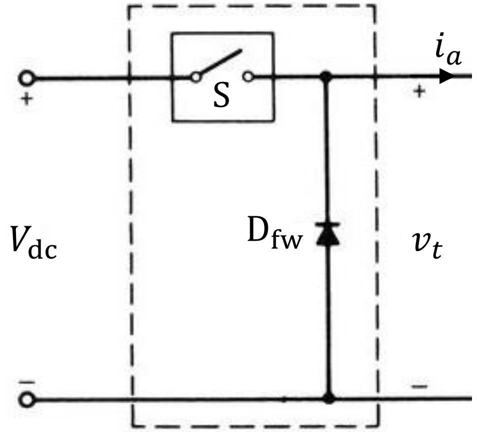
$$T_\mathrm{e} = K_a \cdot \Phi \cdot I_a = 1.72\ \mathrm{V \cdot s/rad} \times 40.0\ \mathrm{A} = 68.75\ \mathrm{N \cdot m}$$

(c) The power to the motor is

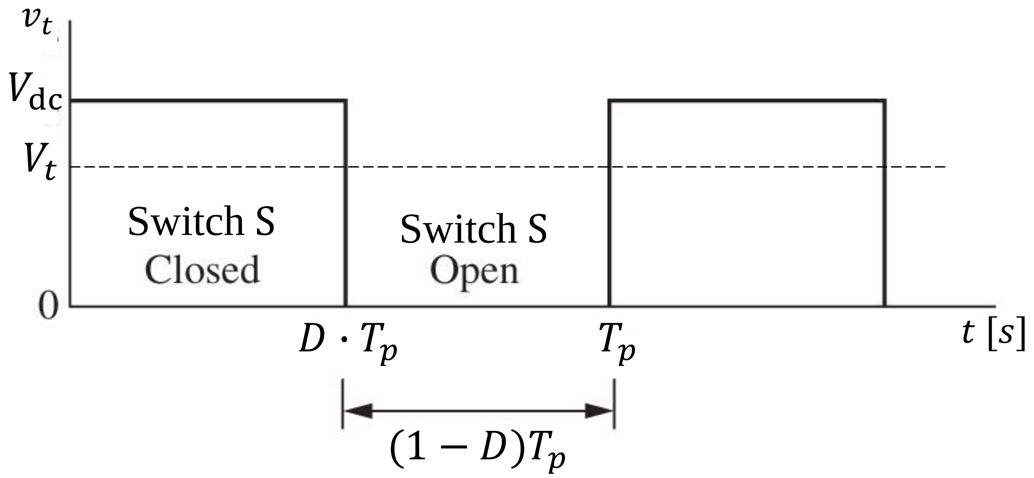
$$P_t = V_t \cdot I_a = 206.6\ \mathrm{V} \times 40.0\ \mathrm{A} = 8264\ \mathrm{W}$$

Chopper

Chopper is a dc-dc buck (step-down) converter. The average value of the dc output voltage v_t is lower than or equal to the fixed dc input voltage V_dc.



A chopper (dc-dc buck converter).
S: Switch (e.g., power transistor).
$\mathrm{D_{fw}}$: Freewheeling diode.

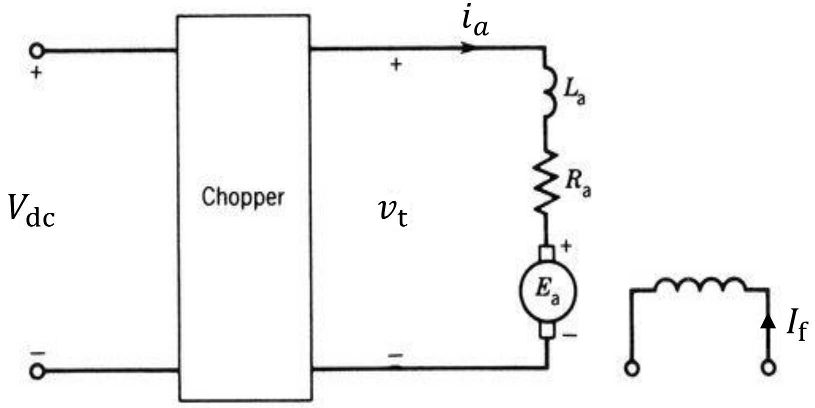


Output voltage v_t of a chopper with switching period T_p and duty ratio D for continuous output current i_a.

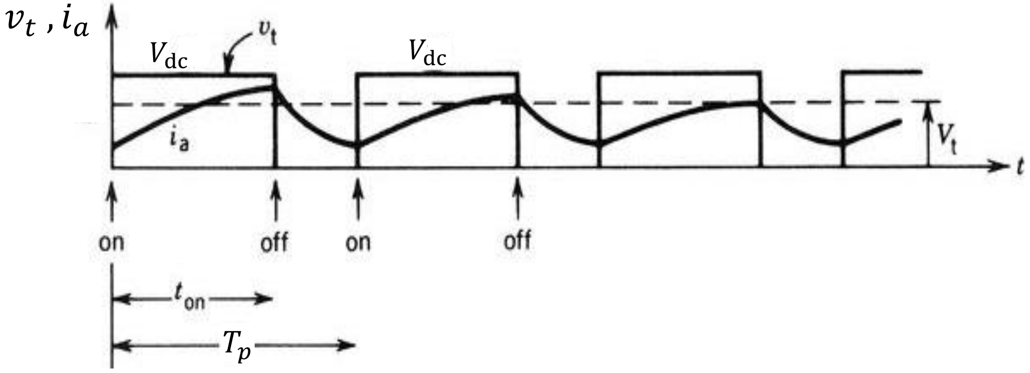
For continuous output current, the average output voltage of the chopper is

$$V_t = \frac{1}{T_p}\int_0^{T_p} v_t dt = D \cdot V_\mathrm{dc} \leq V_\mathrm{dc}$$

where D is the duty ratio of the chopper and T_p is the switching period in [s].
The range of the duty ratio is $0 \leq D \leq 1$.



Speed control of a dc motor by a chopper.



The average values: $V_t = \dfrac{1}{T_p}\displaystyle\int_0^{T_p} v_t dt\,,\;\; I_a = \dfrac{1}{T_p}\displaystyle\int_0^{T_p} i_a dt$

The average value of the motor armature terminal voltage is equal to the average value of the chopper output voltage. The average value of the motor armature current is equal to the average value of the chopper output current.

The motor speed and motor torque can be expressed, respectively, by

$$\omega_m = \frac{V_t}{K_a \cdot \Phi} - \frac{R_a}{(K_a \cdot \Phi)^2} \cdot T_e$$

and

$$T_e = K_a \cdot \Phi \cdot I_a$$

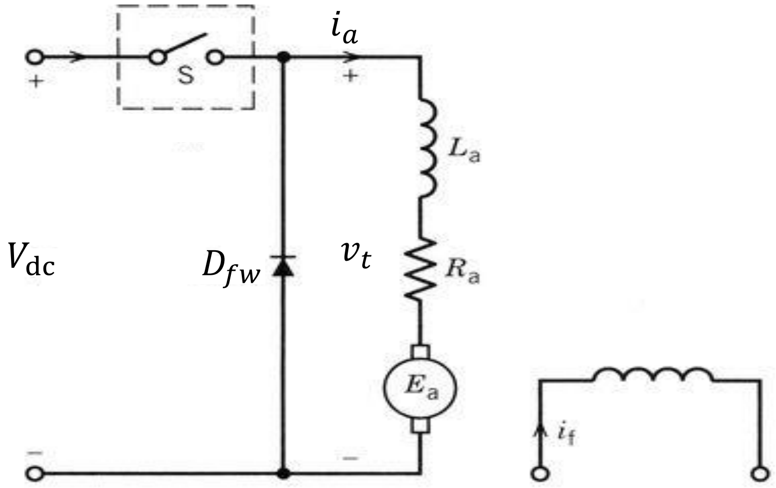
where

$$V_t = D \cdot V_{dc}$$

is the average output voltage of the chopper and D is the duty ratio of the chopper. The range of the duty ratio is $0 \le D \le 1$.

EXAMPLE

A separately excited dc motor is powered by a chopper (dc-dc buck converter) from a $V_{\mathrm{dc}} = 600$ V dc source as shown in the below figure. The motor armature resistance is $R_a = 0.05\ \Omega$. The motor voltage constant is $(K_a \cdot \Phi) = 3.818$ [Vs/rad]. Assume that the motor armature inductance is large enough to make the motor armature current continuous and ripple free. If the duty ratio of the chopper is $D = 0.60$ and the motor armature current is $I_a = 250$ A, determine (a) the power supplied to the the dc motor, (b) the motor speed, and (c) the torque developed by the motor.



A chopper with a dc motor load.

Solution to Question 3:

(a) The power supplied to the dc motor is

$$P_t = V_t \cdot I_a$$

where

$$V_t = D \cdot V_{\mathrm{dc}} = 0.6 \cdot 600 = 360 \text{ V}$$

and

$$I_a = 250 \text{ A}.$$

Then,

$$P_t = 360 \cdot 250 = 90 \text{ kW}$$

(b) The back emf of the motor is calculated by

$$E_a = V_t - R_a I_a = D \cdot V_{\mathrm{dc}} - R_a I_a = 360 - 0.05 \times 250 = 347.5 \text{ V}$$

Then, the motor speed is

$$\omega_m = \frac{E_{\mathrm{a}}}{K_a \cdot \Phi} = \frac{347.5}{3.818} = 91.02 \frac{\mathrm{rad}}{\mathrm{s}} = 91.02 \times \frac{60}{2\pi} \text{ rpm} = 869.2 \text{ rpm}$$

(c) The torque developed by the motor is

$$T_e = K_a \cdot \Phi \cdot I_a = 3.818 \times 250 = 954.5 \text{ Nm}$$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 17

Date: April 06, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 17 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

INDUCTION (ASYNCHRONOUS) MACHINES

Induction machine is an ac (alternating current) machine. It is the most common type of electrical machine used in industry. This is due to the following advantages of the induction machine:

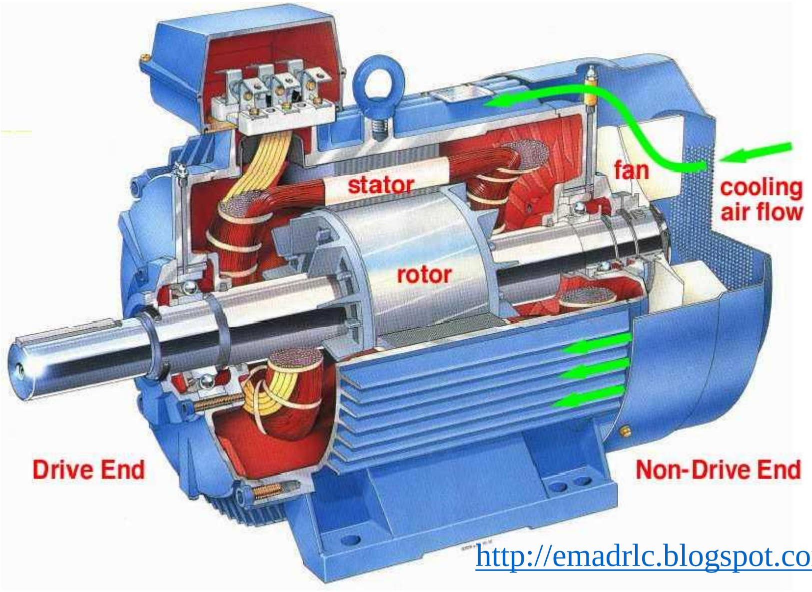
i) It has a rugged structure (unlike dc machines, the squirrel-cage induction machines don’t have brushes and commutators, and they do not require periodic maintenance.

ii) It can be operated directly from the ac mains without requiring intermediate converter.

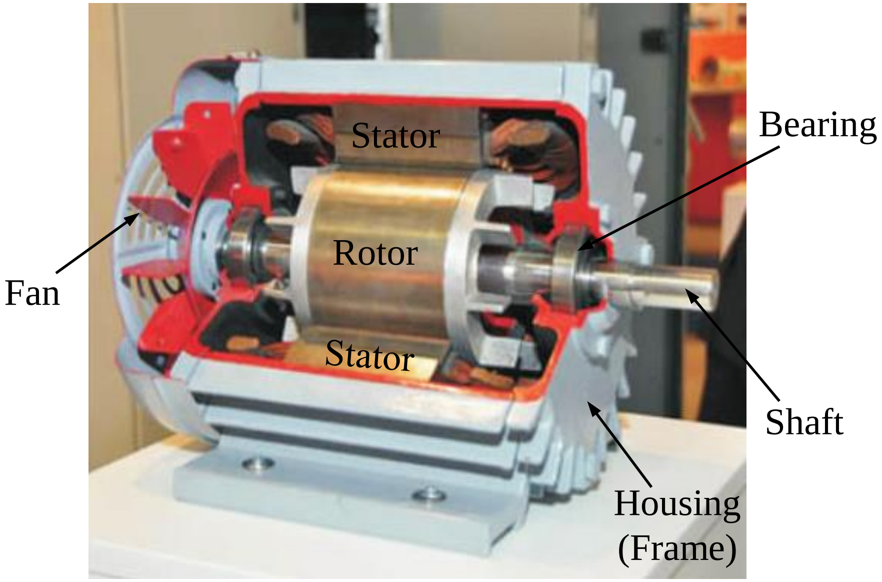
iii) It is cheap. The induction machines have been manufactured over a century and the manufacturers know how to optimize (minimize) the cost. Furthermore, the induction machines do not require any PMs (permanent magnets).

- Like the dc machine, the induction machine has a stator (fixed part) and a rotor (rotating part) mounted on bearings and separated from the stator by an air gap.
- However, in induction machine, both stator winding and rotor winding carry ac currents. The ac current is supplied to the stator winding directly, and to the rotor winding by induction – hence the name induction machine.
- An induction machine can be a three-phase machine, or a two-phase machine, or a single-phase machine. The three-phase induction motors are the most common.

Construction of Induction Machines



http://emadrlc.blogspot.com

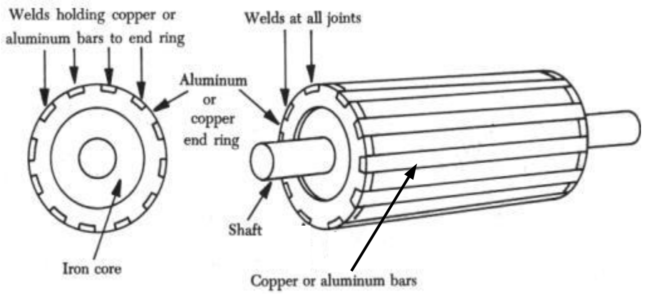


Cutaway views of squirrel-cage induction machines

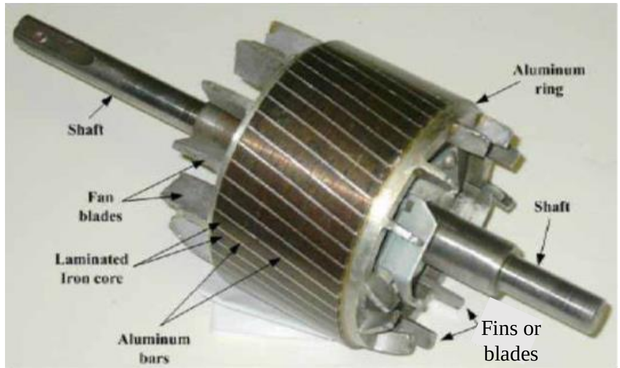
https://en.wikipedia.org/wiki/Induction_motor

Stator: Stationary part. It is made up of thin steel (iron-carbon alloy) laminations stacked together. The stator winding is placed in slots cut on the inner surface of the stator.

Rotor: Rotating part. The rotor also consists of laminated ferromagnetic material with slots cut on the outer surface. The rotor winding may be two types: 1) the *squirrel-cage type* or 2) the *wound-rotor type*. The squirrel-cage winding consists of aluminum or copper bars embedded in the rotor slots and shorted at both ends by aluminum or copper end rings as shown in the figures below. The wound-rotor winding has the same form as the stator winding.

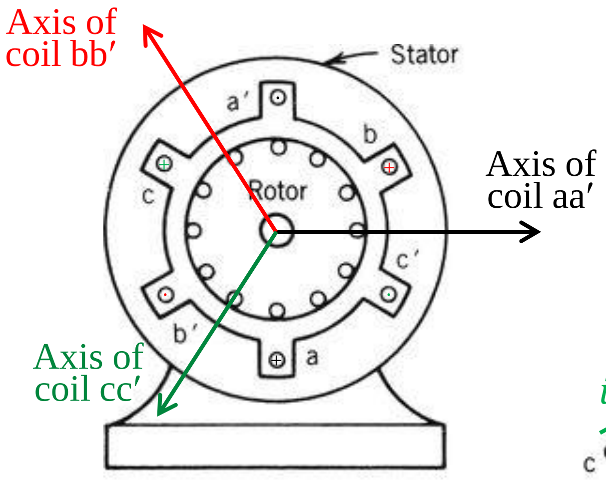


Graphical view of a squirrel-cage rotor http://avstop.com



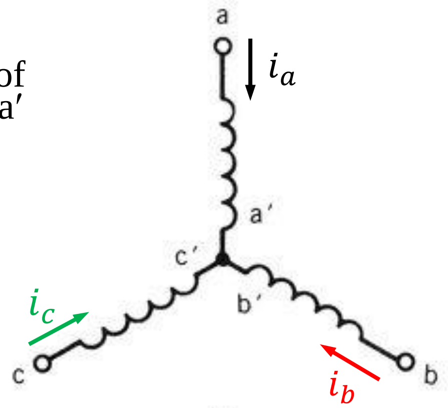
A squirrel-cage rotor https://automationforum.co

Three-Phase Squirrel-Cage Induction Machine



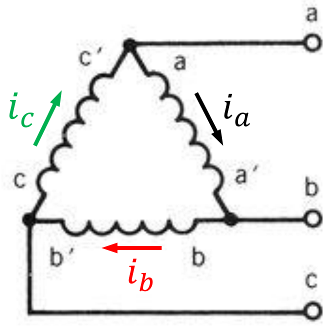
(a)

Cross-sectional view



(b)

Y-connected stator winding



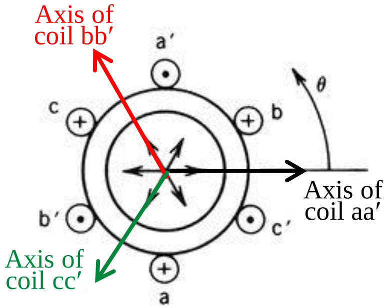
(c)

Δ-connected stator winding

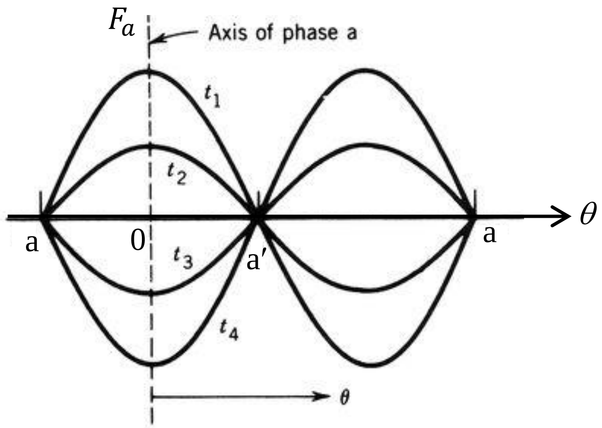
The three-phase stator winding is represented by three coils. Coil aa′ represents the phase-a winding for one pair of poles. Similarly, coil bb′ represents the phase-b winding, and coil cc′ represents the phase-c winding. Furthermore, the axes of these coils are 120 electrical degrees apart. The ends a′, b′ and c′ of the three phase windings can be connected in a wye or a delta configuration to form the three-phase connection.

Note that each stator phase winding is represented by a single coil (or, concentrated coils) placed in two stator slots for simplicity. In reality, the winding of each phase is distributed over several slots to produce mmf with sinusoidal space distribution when current flows.

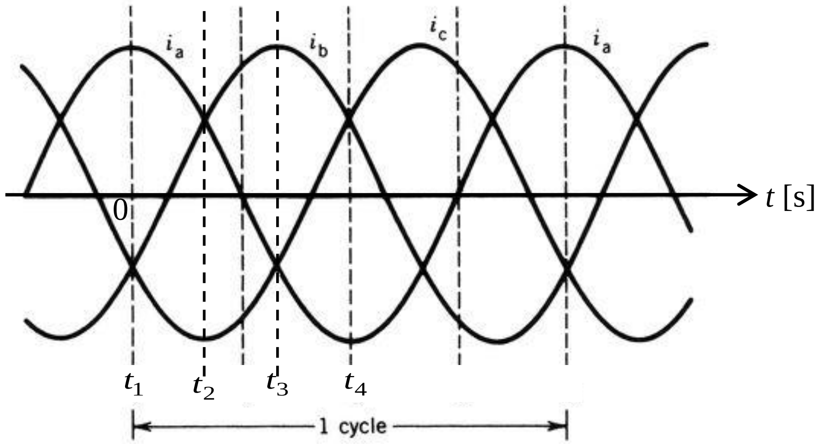
Sinusoidal Magnetomotive Force Produced by a Phase Winding



Three-phase windings are displaced from each other by 120 electrical degrees in space around the inner circumference of the stator.



Magnetomotive force F_a produced by phase-a current i_a at various time instants.



Three-phase currents

$$i_a = I_m \cdot \cos(\omega \cdot t)$$
$$i_b = I_m \cdot \cos(\omega \cdot t - 120°)$$
$$i_c = I_m \cdot \cos(\omega \cdot t + 120°)$$

The single turn (or, concentrated) coils represent the actual distributed windings. When a current flows through a phase coil, it produces a sinusoidally distributed mmf centered on the axis of coil representing the phase winding. The amplitude and direction of the mmf depend on the instantaneous value of the current.

Magnetomotive force produced by phase-a current i_a is

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a$$

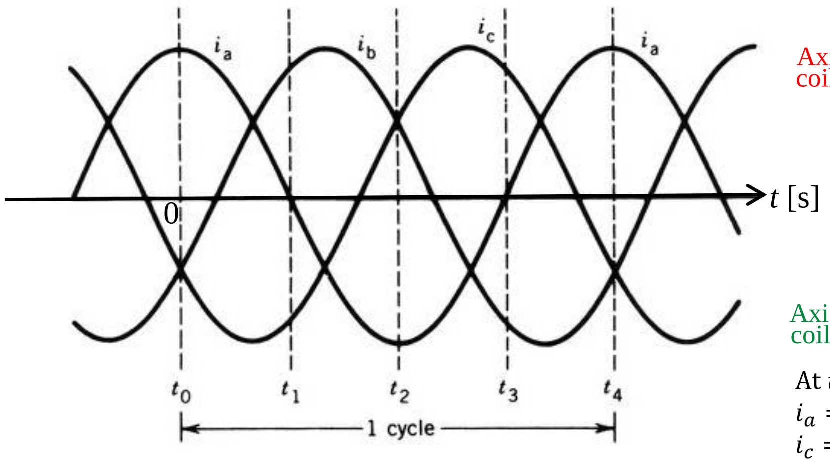
where N is the number of turns in phase-a.

Note that $N \cdot \cos(\theta) \cdot i_a$ is the algebraic sum of the currents inside a closed path that circles through the stator and rotor cores by crossing the air gap at θ and (θ+180) electrical degrees (Ampere's Law).

ROTATING MAGNETIC FIELD

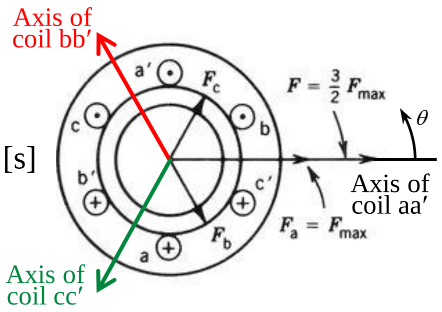
The rotating magnetic field (the resultant mmf wave due to the net effect of the all three phase mmf waves in a three-phase induction machine) can be calculated by i) the graphical method or ii) the analytical method.

Calculation of Rotating Magnetic Field by Graphical Method



The three-phase currents are

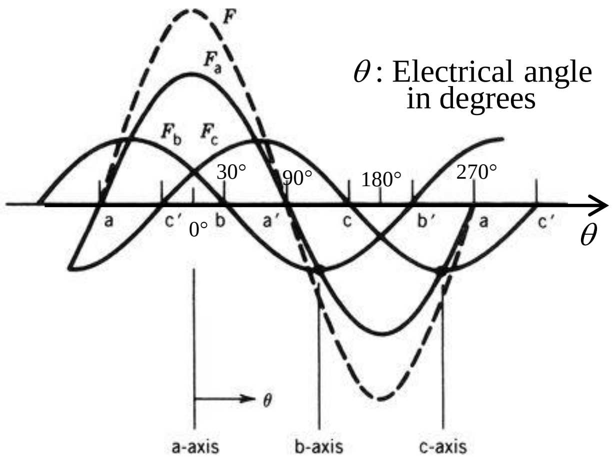
$$i_a = I_m \cdot \cos(\omega \cdot t)$$
$$i_b = I_m \cdot \cos(\omega \cdot t - 120^\circ)$$
$$i_c = I_m \cdot \cos(\omega \cdot t + 120^\circ)$$



At $t = t_0 = 0$ [s] or $t = t_4 = 2\pi/\omega$ [s],
$i_a = I_m > 0$, $i_b = -0.5 \cdot I_m < 0$, and
$i_c = -0.5 \cdot I_m < 0$

The three-phase mmfs are

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a$$
$$F_b(\theta) = N \cdot \cos(\theta - 120^\circ) \cdot i_b$$
$$F_c(\theta) = N \cdot \cos(\theta + 120^\circ) \cdot i_c$$

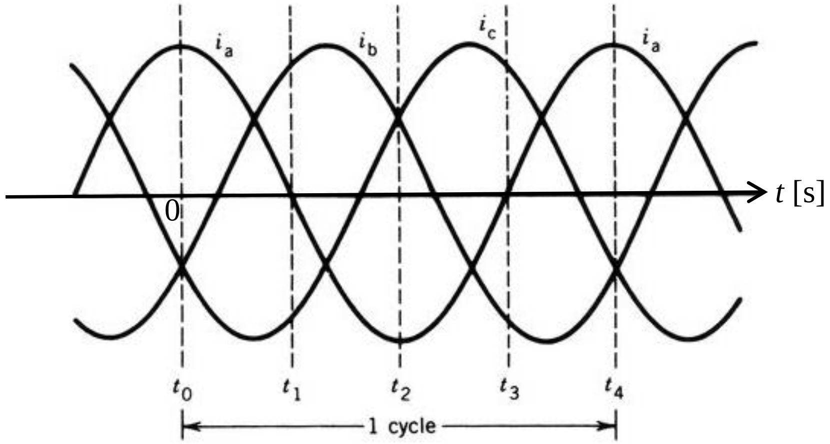


At $t = t_0 = 0$ [s] or $t = t_4 = 2\pi/\omega$ [s],
$i_a = I_m > 0$, $i_b = -0.5 \cdot I_m < 0$, and
$i_c = -0.5 \cdot I_m < 0$

The three-phase mmfs F_a, F_b, F_c, and the resultant mmf F at $t = t_0 = 0$ [s] or $t = t_4 = 2\pi/\omega$ [s].

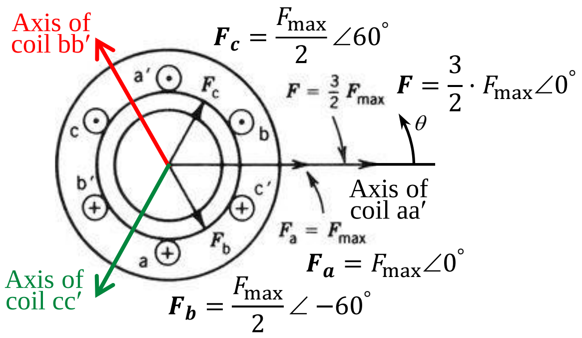
$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta)$$

Total Magnetomotive Force at $t = 0$ [s] ($\omega t = 0$ [rad] $= 0°$) by Graphical Method



The three-phase currents

$$i_a = I_m \cdot \cos(\omega \cdot t)$$
$$i_b = I_m \cdot \cos(\omega \cdot t - 120°)$$
$$i_c = I_m \cdot \cos(\omega \cdot t + 120°)$$



The three-phase mmfs $\boldsymbol{F_a}, \boldsymbol{F_b}, \boldsymbol{F_c}$, and the resultant mmf $\boldsymbol{F}$ at $t = t_0 = 0$ [s] or $t = t_4 = 2\pi/\omega$ [s].

The vector representation of three-phase mmfs

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a \Rightarrow \boldsymbol{F_a} = F_a(\theta = 0^\circ)\angle 0^\circ = N \cdot i_a \angle 0^\circ$$

$$F_b(\theta) = N \cdot \cos(\theta - 120^\circ) \cdot i_b \Rightarrow \boldsymbol{F_b} = F_b(\theta = 120^\circ)\angle 120^\circ = N \cdot i_b \angle 120^\circ$$

$$F_c(\theta) = N \cdot \cos(\theta + 120^\circ) \cdot i_c \Rightarrow \boldsymbol{F_c} = F_c(\theta = -120^\circ)\angle -120^\circ = N \cdot i_c \angle -120^\circ$$

For $t = t_0 = 0$ [s] ($\omega t = 0$ [rad] $= 0°$) and $F_{\max} = N \cdot I_m$,

$$i_a = I_m \ , \ \boldsymbol{F_a} = N \cdot i_a \angle 0^\circ = F_{\max}\angle 0^\circ = F_{\max} \cdot (1 + j \cdot 0)$$

$$i_b = -\frac{I_m}{2} \ , \ \boldsymbol{F_b} = N \cdot i_b \angle 120^\circ = -\frac{F_{\max}}{2}\angle 120^\circ = \frac{F_{\max}}{2}\angle -60^\circ = \frac{F_{\max}}{2} \cdot \left(0.5 - j \cdot \frac{\sqrt{3}}{2}\right)$$

$$i_c = -\frac{I_m}{2} \ , \ \boldsymbol{F_c} = N \cdot i_c \angle -120^\circ = -\frac{F_{\max}}{2}\angle -120^\circ = \frac{F_{\max}}{2}\angle 60^\circ = \frac{F_{\max}}{2} \cdot \left(0.5 + j \cdot \frac{\sqrt{3}}{2}\right)$$

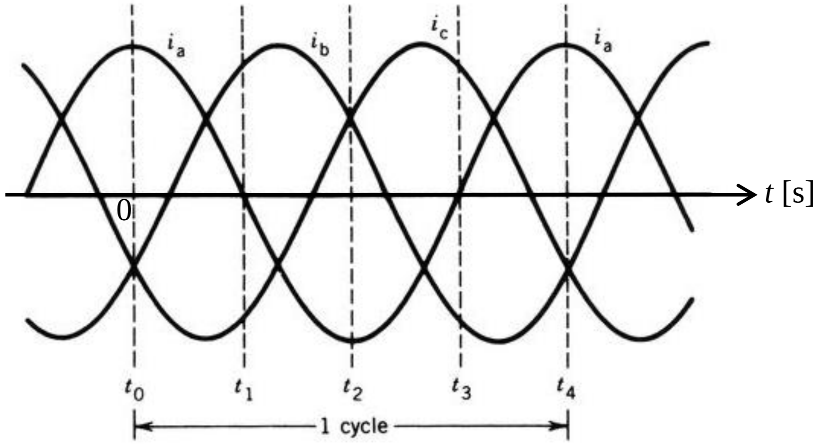
The total (resultant) mmf at $t = t_0 = 0$ [s] is

$$\boldsymbol{F} = \boldsymbol{F_a} + \boldsymbol{F_b} + \boldsymbol{F_c}$$

$$\boldsymbol{F} = F_{\max}\angle 0^\circ + \frac{F_{\max}}{2}\angle -60^\circ + \frac{F_{\max}}{2}\angle 60^\circ$$

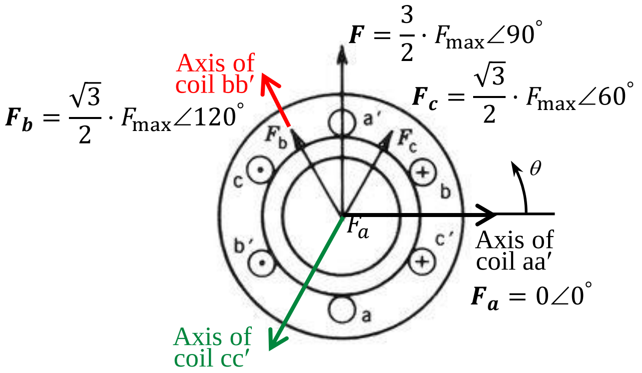
$$\boldsymbol{F} = \frac{3}{2} \cdot F_{\max}\angle 0^\circ \text{ (Total mmf vector)}$$

Total Magnetomotive Force at $t = t_1 = \pi/(2 \cdot \omega)$ [s] ($\omega t = \pi/2$ [rad] $= 90°$) by Graphical Method



The three-phase currents

$$i_a = I_m \cdot \cos(\omega \cdot t)$$
$$i_b = I_m \cdot \cos(\omega \cdot t - 120°)$$
$$i_c = I_m \cdot \cos(\omega \cdot t + 120°)$$



The three-phase mmfs $\boldsymbol{F_a}, \boldsymbol{F_b}, \boldsymbol{F_c}$, and the resultant mmf $\boldsymbol{F}$ at $t = t_1 = \pi/(2 \cdot \omega)$ [s].

The vector representation of three-phase mmfs

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a \Rightarrow \boldsymbol{F_a} = F_a(\theta = 0^{\circ})\angle 0^{\circ} = N \cdot i_a \angle 0^{\circ}$$

$$F_b(\theta) = N \cdot \cos(\theta - 120^{\circ}) \cdot i_b \Rightarrow \boldsymbol{F_b} = F_b(\theta = 120^{\circ})\angle 120^{\circ} = N \cdot i_b \angle 120^{\circ}$$

$$F_c(\theta) = N \cdot \cos(\theta + 120^{\circ}) \cdot i_c \Rightarrow \boldsymbol{F_c} = F_c(\theta = -120^{\circ})\angle -120^{\circ} = N \cdot i_c \angle -120^{\circ}$$

For $t = t_1 = \frac{\pi}{2 \cdot \omega}$ [s] $\left(\omega \cdot t = \frac{\pi}{2}\right.$ [rad] $\left.= 90°\right)$ and $F_{\max} = N \cdot I_m$,

$$i_a = 0, \quad \boldsymbol{F_a} = N \cdot i_a \angle 0^{\circ} = 0\angle 0^{\circ} = 0$$

$$i_b = \frac{\sqrt{3}}{2} \cdot I_m, \quad \boldsymbol{F_b} = N \cdot i_b \angle 120^{\circ} = \frac{\sqrt{3} \cdot F_{\max}}{2} \angle 120^{\circ} = \frac{\sqrt{3} \cdot F_{\max}}{2} \cdot \left(-0.5 + j \cdot \frac{\sqrt{3}}{2}\right)$$

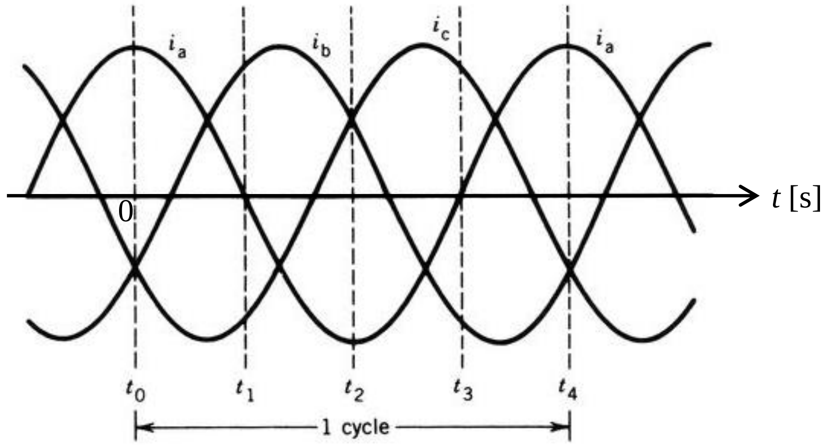
$$i_c = -\frac{\sqrt{3}}{2} \cdot I_m, \quad \boldsymbol{F_c} = N \cdot i_c \angle -120^{\circ} = -\frac{\sqrt{3} \cdot F_{\max}}{2} \angle -120^{\circ} = \frac{\sqrt{3} \cdot F_{\max}}{2} \angle 60^{\circ} = \frac{\sqrt{3} \cdot F_{\max}}{2} \cdot \left(0.5 + j \cdot \frac{\sqrt{3}}{2}\right)$$

The total (resultant) mmf at $t = t_1 = \frac{\pi}{2 \cdot \omega}$ is

$$\boldsymbol{F} = \boldsymbol{F_a} + \boldsymbol{F_b} + \boldsymbol{F_c}$$

$$\boldsymbol{F} = 0 - \frac{\sqrt{3} \cdot F_{\max}}{4} + j \cdot \frac{3 \cdot F_{\max}}{4} + \frac{\sqrt{3} \cdot F_{\max}}{4} + j \cdot \frac{3 \cdot F_{\max}}{4}$$

$$\boldsymbol{F} = j \cdot \frac{3 \cdot F_{\max}}{2} = \frac{3}{2} \cdot F_{\max} \angle 90^{\circ} \text{ (Total mmf vector)}$$



The three-phase currents

$i_a = I_m \cdot \cos(\omega \cdot t)$
$i_b = I_m \cdot \cos(\omega \cdot t - 120°)$
$i_c = I_m \cdot \cos(\omega \cdot t + 120°)$

The vector representation of three-phase mmfs

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a$$

$$\Rightarrow \boldsymbol{F_a} = F_a(\theta = 0^\circ)\angle 0^\circ = N \cdot i_a \angle 0^\circ$$

$$F_b(\theta) = N \cdot \cos(\theta - 120^\circ) \cdot i_b$$

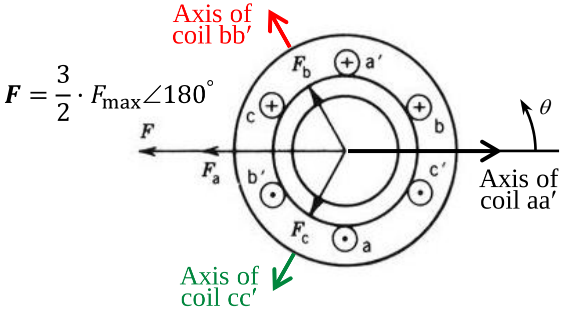
$$\Rightarrow \boldsymbol{F_b} = F_b(\theta = 120^\circ)\angle 120^\circ = N \cdot i_b \angle 120^\circ$$

$$F_c(\theta) = N \cdot \cos(\theta + 120^\circ) \cdot i_c$$

$$\Rightarrow \boldsymbol{F_c} = F_c(\theta = -120^\circ)\angle -120^\circ = N \cdot i_c \angle -120^\circ$$

Total Magnetomotive Force at $\omega \cdot t = 180°$ by Graphical Method

At $t = t_2 = \frac{\pi}{\omega}$ [s] or, $\omega \cdot t = \pi$ [rad] $= 180°$

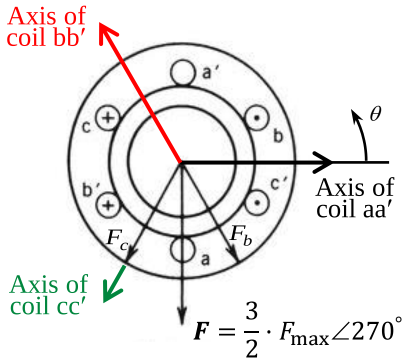


The three-phase mmfs, and the resultant mmf at $t = t_2 = \frac{\pi}{\omega}$ [s].

At $t = t_2 = \frac{\pi}{\omega}$ [s], $\boldsymbol{F} = \frac{3}{2} \cdot F_{\max}\angle 180^\circ$ where $F_{\max} = N \cdot I_m$.

Total Magnetomotive Force at $\omega \cdot t = 270°$ by Graphical Method

At $t = t_3 = \frac{3}{2} \cdot \frac{\pi}{\omega}$ [s] or, $\omega \cdot t = 3 \cdot \frac{\pi}{2}$ [rad] $= 270°$



The three-phase mmfs, and the resultant mmf at $t = t_3 = \frac{3}{2} \cdot \frac{\pi}{\omega}$ [s].

At $t = t_3 = \frac{3}{2} \cdot \frac{\pi}{\omega}$, $\boldsymbol{F} = \frac{3}{2} \cdot F_{\max}\angle 270^\circ$ where $F_{\max} = N \cdot I_m$.

Remarks on Resultant MMF Produced by Three-Phase Currents

1. As time passes, the resultant mmf wave maintains its sinusoidal distribution in space with constant amplitude, but moves around the air gap.

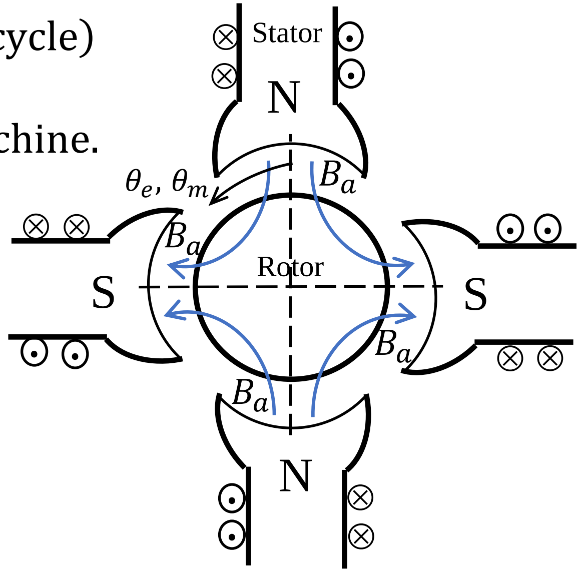
2. In a *p*-pole machine, one-cycle of variation of the current (one electrical cycle) will make the mmf rotate by 2/*p* revolution (2/*p* mechanical cycle). The revolutions per minute *n* (rpm) of the traveling mmf wave is related to the frequency f (Hz) of the three-phase currents by

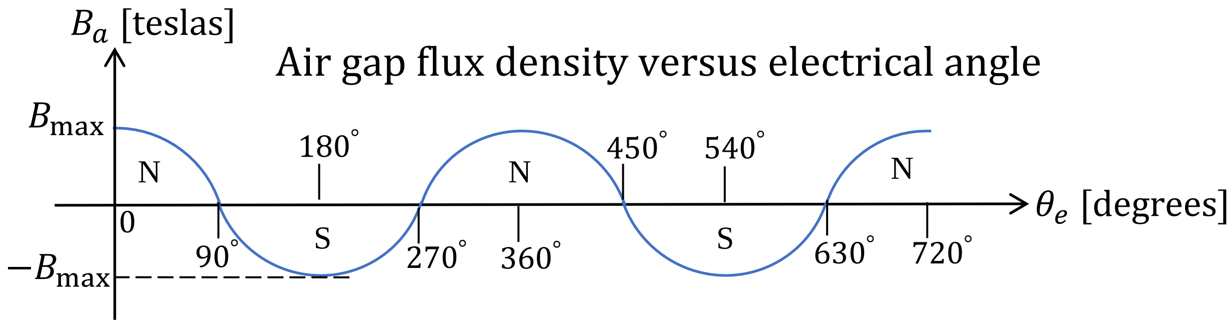
$$n = \frac{2}{p} \cdot f \cdot 60 = \frac{120}{p} \cdot f$$

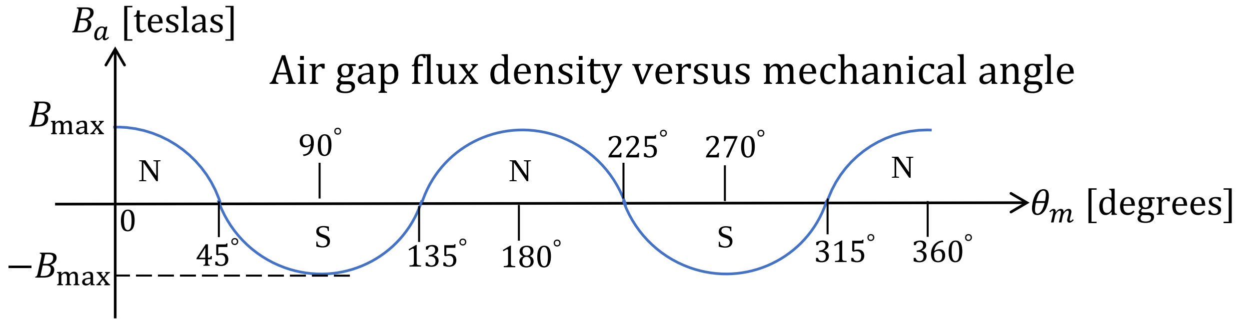
3. Swapping (interchanging) any two phase leads of the three-phase induction machine makes the rotating mmf rotate in the opposite direction. For example, swapping phase b and phase c currents (that is, $i_b = I_m \cdot \cos(\omega \cdot t + 120°)$ and $i_c = I_m \cdot \cos(\omega \cdot t - 120°)$) while maintaining the phase a current at $i_a = I_m \cdot \cos(\omega \cdot t)$ causes the travelling mmf F to rotate in the clockwise direction.

Electrical Angle θ_e versus Mechanical Angle θ_m for a Four-Pole Machine

One full rotation (one mechanical cycle) around the air gap is equal to two electrical cycles for a four–pole machine.







UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 18

Date: April 08, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 18 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Calculation of Rotating Magnetic Field by Analytical Method

Consider a two-pole machine with three-phase windings on the stator. At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding and the amplitude proportional to the instantaneous value of the phase current.

Magnetomotive forces produced by the three-phase windings along θ are

$$F_a(\theta) = N \cdot \cos(\theta) \cdot i_a$$
$$F_b(\theta) = N \cdot \cos(\theta - 120°) \cdot i_b$$
$$F_c(\theta) = N \cdot \cos(\theta + 120°) \cdot i_c$$

where N is the number of turns in each phase winding. Note that the mmfs are shifted from each other by 120° since the phase axes are shifted from each other by 120°.

The three-phase currents are

$$i_a(t) = I_m \cdot \cos(\omega \cdot t)$$
$$i_b(t) = I_m \cdot \cos(\omega \cdot t - 120°)$$
$$i_c(t) = I_m \cdot \cos(\omega \cdot t + 120°)$$

The resultant (total) mmf at point θ is

$$F = F_a + F_b + F_c$$

$$F(\theta, t) = N \cdot I_m \cdot \cos(\omega \cdot t) \cdot \cos(\theta) + N \cdot I_m \cdot \cos(\omega \cdot t - 120°) \cdot \cos(\theta - 120°) + N \cdot I_m \cdot \cos(\omega \cdot t + 120°) \cdot \cos(\theta + 120°)$$

Using the trigonometric identity

$$\cos(A) \cdot \cos(B) = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B)$$

the resultant mmf becomes

$$F(\theta, t) = \frac{1}{2} \cdot N \cdot I_m \cdot (\cos(\omega \cdot t - \theta) + \cos(\omega \cdot \cancel{t + \theta}) + \cos(\omega \cdot t - \theta) + \cos(\omega \cdot t + \cancel{\theta - 240°}) + \cos(\omega \cdot t - \theta) + \cos(\omega \cdot t + \cancel{\theta + 240°}))$$

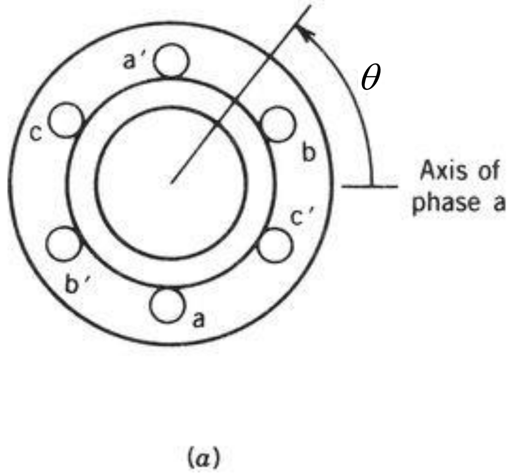
$$F(\theta, t) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\omega \cdot t - \theta)$$

Illustration of Rotating Magnetic Field

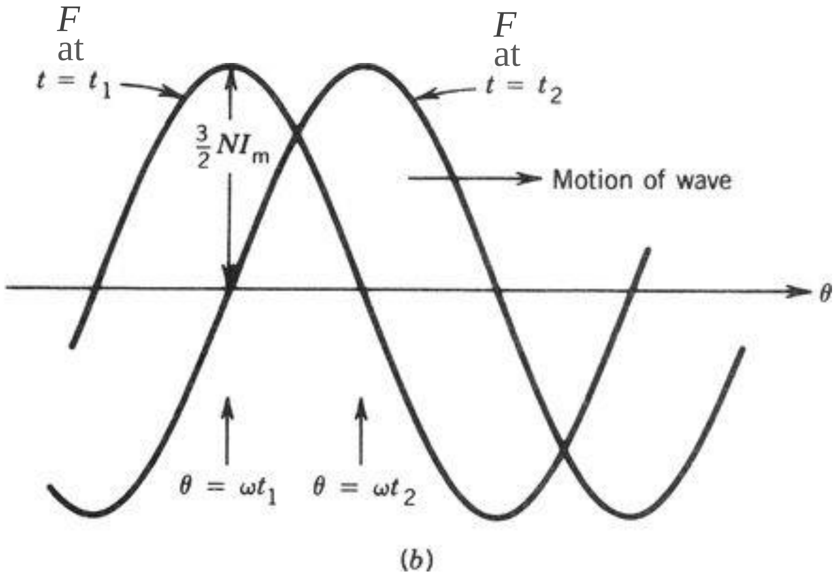
The resultant mmf

$$F(\theta, t) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\omega \cdot t - \theta)$$

rotates at angular velocity $\omega = 2 \cdot \pi \cdot f$ in radians per second. That is, at any instant of time, say t_1, the mmf wave is distributed sinusoidally around the air gap with the positive peak acting along $\theta = \omega \cdot t_1$ as shown in the figure below. At a later instant, say t_2, the positive peak of the sinusoidally distributed wave is along $\theta = \omega \cdot t_2$. This means that the mmf wave has moved by $\omega \cdot (t_2 - t_1)$ around the air gap.



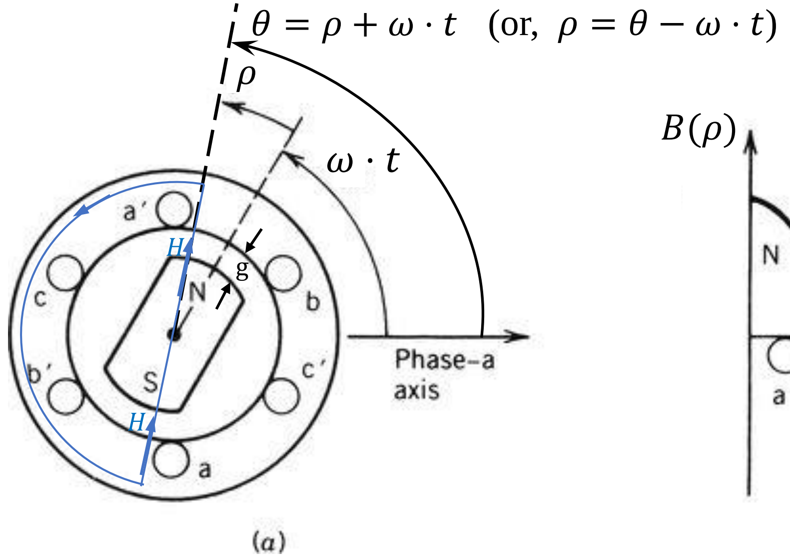
The origin of θ is the axis of phase a.



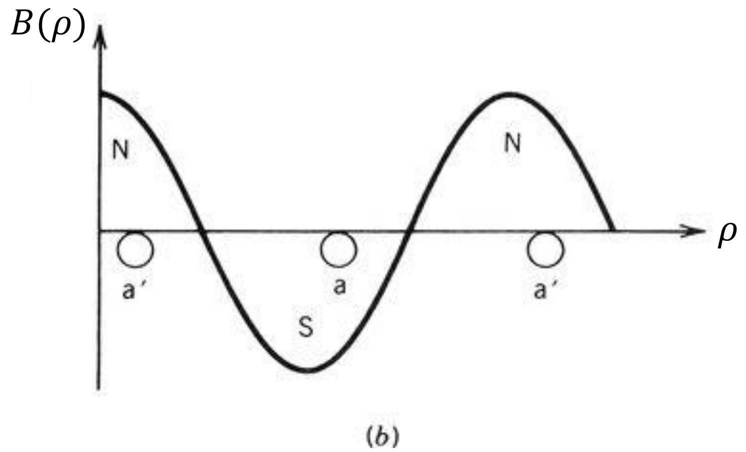
Motion of the resultant mmf F.

Air Gap Flux due to Rotating Magnetic Field

The rotating magnetic field produced by the three-phase stator windings for a 2-pole machine can be visualized by a pair of magnets rotating in the air gap.



Representation of rotating mmf by a pair of magnets rotating in the air gap.



Air gap flux density distribution.

$$F(\theta,t) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\omega \cdot t - \theta) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\theta - \omega \cdot t) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\rho)\ , \quad \rho = \theta - \omega \cdot t$$

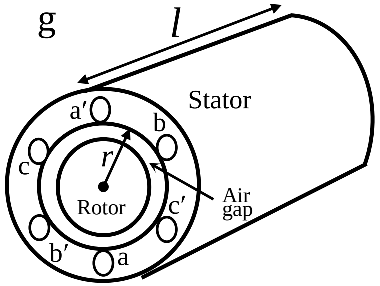
Then, applying the Ampere's circuit law to the closed loop (blue trace) in the above figure, using the symmetry, and assuming that the stator and rotor cores has infinite permeability, the air gap field intensity H and in turn the air gap flux density B at angle ρ can be found from

$$2 \cdot H(\rho) \cdot g \cong F(\theta,t) = \frac{3}{2} \cdot N \cdot I_m \cdot \cos(\rho) \Longrightarrow B(\rho) = \mu_0 \cdot H(\rho) = B_{max}\cos(\rho) \quad \text{where } B_{max} = \frac{3}{4} \cdot \frac{\mu_0 \cdot N \cdot I_m}{g}$$

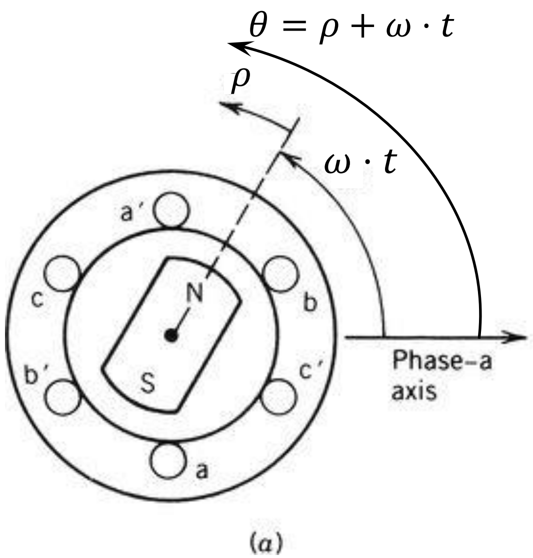
The air gap flux per pole is

$$\Phi_p = \int_{-\pi/2}^{\pi/2} B(\rho) \cdot l \cdot r \cdot d\rho = 2 \cdot B_{\max} \cdot l \cdot r$$

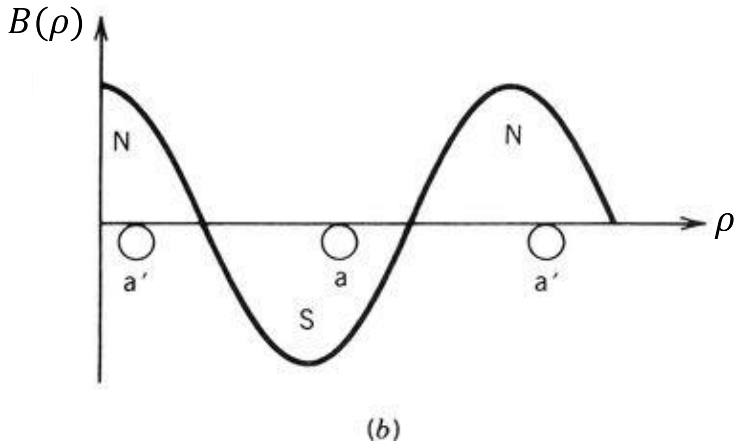
where l is the axial length and r is the radius of the stator at the air gap as shown on the right.



Induced Voltages by Rotating Magnetic Field



Representation of rotating mmf by a pair of magnets rotating in the air gap.



Air gap flux density distribution.

Let us assume that the stator phase coils have N turns, and the coil sides of each phase are 180 electrical degrees apart (full-pitch coil). The flux linkage for stator phase coil aa′ is given by

$$\lambda_a(\omega \cdot t) = N \cdot \Phi_p \cdot \cos(\omega \cdot t)$$

From Faraday's law, the voltage induced in stator phase coil aa′ is obtained as

$$e_a = \frac{d\lambda_a}{dt} = -N \cdot \Phi_p \cdot \omega \cdot \sin(\omega \cdot t) = -E_{\max} \cdot \sin(\omega \cdot t) \text{ where } E_{\max} = N \cdot \Phi_p \cdot \omega$$

Note that the polarity of this induced voltage is such that if the stator coil were short-circuited, the induced voltage would cause a current to flow in the direction that would oppose any change in the flux linkage of the stator coil (*Electric Machinery, by A.E. Fitzgerald, C. Kingsley Jr., S.D. Umans, 6th Edition, 2003*).

Three-Phase Induced Voltages

Since the three-phase coils are shifted from each other by 120 electrical degrees, the flux linkages for stator phase coils bb′ and cc′ are

$$\lambda_b(\omega \cdot t) = N \cdot \Phi_p \cdot \cos(\omega \cdot t - 120°)$$
$$\lambda_c(\omega \cdot t) = N \cdot \Phi_p \cdot \cos(\omega \cdot t + 120°)$$

and, the induced voltages in the stator phase coils bb′ and cc′ are

$$e_b = \frac{d\lambda_b}{dt} = -N \cdot \Phi_p \cdot \omega \cdot \sin(\omega \cdot t - 120°) = -E_{\max} \cdot \sin(\omega \cdot t - 120°)$$
$$e_c = \frac{d\lambda_c}{dt} = -N \cdot \Phi_p \cdot \omega \cdot \sin(\omega \cdot t + 120°) = -E_{\max} \cdot \sin(\omega \cdot t + 120°)$$

In brief, the stator three-phase induced voltages by the rotating magnetic field are

$$e_a = -E_{\max} \cdot \sin(\omega \cdot t)$$
$$e_b = -E_{\max} \cdot \sin(\omega \cdot t - 120°)$$
$$e_c = -E_{\max} \cdot \sin(\omega \cdot t + 120°)$$

where $E_{\max} = N \cdot \Phi_p \cdot \omega$. The rms value of the induced voltages is

$$E_{rms} = \frac{E_{\max}}{\sqrt{2}} = \frac{N \cdot \Phi_p \cdot \omega}{\sqrt{2}} = \frac{2 \cdot \pi \cdot f \cdot N \cdot \Phi_p}{\sqrt{2}} = 4.44 \cdot f \cdot N \cdot \Phi_p$$

where f is the stator frequency in Hz, N is the total number of series turns per phase, and Φ_p is the flux per pole in webers.

STANDSTILL OPERATION

Consider a three-phase wound-rotor induction machine with the rotor circuit left open-circuited. When the three-phase stator windings are connected to a three-phase supply, a rotating magnetic field will be produced in the air gap which rotates at

$$n_s = \frac{f_1}{(p/2)} \cdot 60 = 120 \cdot \frac{f_1}{p}$$

where n_s is the synchronous speed in rpm, f_1 is the stator electrical frequency in Hz, and p is the number of poles. This rotating field induces voltages in both stator and rotor windings at the same frequency f_1 since it cuts both the stator and rotor at the same speed (the rotor is not moving). The magnitudes of these voltages are

$$E_{1\max} = 2 \cdot \pi \cdot f_1 \cdot N_1 \cdot \Phi_p$$

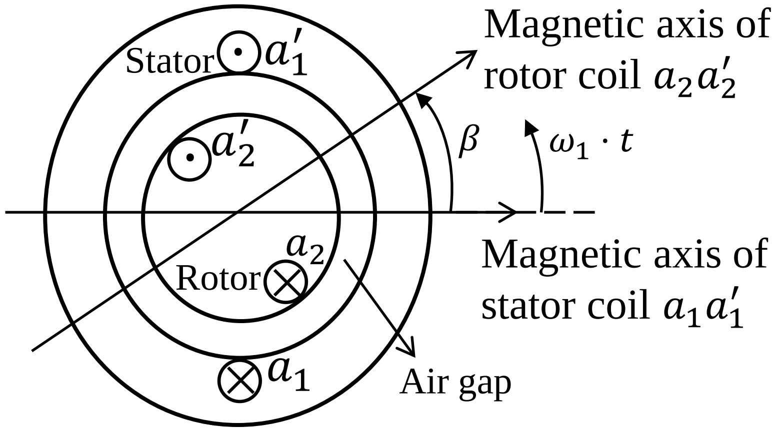
$$E_{2\max} = 2 \cdot \pi \cdot f_1 \cdot N_2 \cdot \Phi_p$$

where N_1 is the number of turns in each stator phase winding and N_2 is the number of turns in each rotor phase winding. Then,

$$\frac{E_{1\max}}{E_{2\max}} = \frac{E_{1rms}}{E_{2rms}} = \frac{N_1}{N_2}$$

Phase Shifter at Standstill Operation

At standstill operation, the rotor can be held in such a position that the magnetic axes of the corresponding phase windings in the stator and the rotor make an angle of β as shown in the figure on the right. Then, the flux linkages for stator phase coil a_1a_1' and rotor phase coil a_2a_2' that are due to the stator rotating magnetic field are given, respectively, by



$$\lambda_{1a}(\omega_1 \cdot t) = N_1 \cdot \Phi_p \cdot \cos(\omega_1 \cdot t)$$

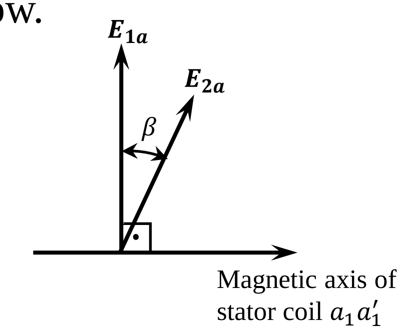
$$\lambda_{2a}(\omega_1 \cdot t) = N_2 \cdot \Phi_p \cdot \cos(\omega_1 \cdot t - \beta)$$

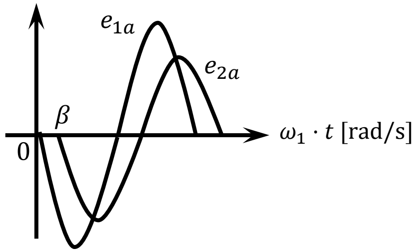
where $\omega_1 = 2 \cdot \pi \cdot f_1$ and f_1 is the stator electrical frequency in Hz. As a result,

$$e_{1a} = \frac{d\lambda_{1a}}{dt} = -N_1 \cdot \Phi_p \cdot \omega_1 \cdot \sin(\omega_1 \cdot t) = -E_{1\max} \cdot \sin(\omega_1 \cdot t) = E_{1\max} \cdot \cos(\omega_1 \cdot t + 90^\circ) \text{ where } E_{1\max} = N_1 \cdot \Phi_p \cdot \omega_1$$

$$e_{2a} = \frac{d\lambda_{2a}}{dt} = -N_2 \cdot \Phi_p \cdot \omega_1 \cdot \sin(\omega_1 \cdot t - \beta) = -E_{2\max} \cdot \sin(\omega_1 \cdot t - \beta) = E_{2\max} \cdot \cos(\omega_1 \cdot t - \beta + 90^\circ) \text{where } E_{2\max} = N_2 \cdot \Phi_p \cdot \omega_1$$

In brief, the induced voltage in the rotor winding will be phase-shifted from that of the stator winding by the angle of β as shown below.





RUNNING OPERATION

If the stator windings are connected to a three-phase supply and the rotor circuit is closed, the induced voltages in the rotor windings produce rotor currents that interact with the air gap field (the stator rotating magnetic field) to produce torque. The rotor, if free to do so, will then start rotating. The rotor will eventually reach a steady-state speed n that is less than the synchronous speed n_s (the speed of the stator rotating field). It is clear that at $n = n_s$ there is no induced voltage and current, hence no torque.

The *slip s* is defined as

$$s = \frac{n_s - n}{n_s}$$

The term $s \cdot n_s$ is called *slip rpm.*

The frequency f_2 of the induced voltage and current in the rotor circuit is due to the relative speed of the stator rotating field with respect to the rotor. That is,

$$f_2 = \frac{p}{2} \cdot \frac{(n_s - n)}{60} = \frac{p}{2} \cdot \frac{s \cdot n_s}{60} = s \cdot f_1$$

The rotor circuit frequency f_2 is called *slip frequency*. The rms value of the voltage induced in the rotor circuit at slip s is

$$E_{2s} = \frac{2 \cdot \pi}{\sqrt{2}} \cdot f_2 \cdot N_2 \cdot \Phi_p = \frac{2 \cdot \pi}{\sqrt{2}} \cdot s \cdot f_1 \cdot N_2 \cdot \Phi_p = s \cdot E_2$$

where $E_2 = E_{2rms}$ is the rms value of the induced voltage in the rotor circuit at standstill.

The induced currents in the three-phase rotor windings also produce a rotating field whose speed (rpm) n_2 with respect to the rotor is

$$n_2 = \frac{120}{p} \cdot f_2 = \frac{120}{p} \cdot s \cdot f_1 = s \cdot n_s$$

Because the rotor itself is rotating at n rpm, the induced rotor field rotates in the air gap at speed

$$n + n_2 = (1 - s) \cdot n_s + s \cdot n_s = n_s$$

in rpm. In brief, both the stator field and the induced rotor field rotate in the air gap at the same synchronous speed n_s.

EXAMPLE 5.1 from the textbook

A 3ϕ, 460 V (Line-to-Line), 100 hp, 60 Hz, four-pole induction machine delivers rated output power at a slip of 0.05. Determine the following: (a) Synchronous speed and motor speed. (b) Speed of the rotating air gap field. (c) Frequency of the rotor circuit. (d) Slip rpm. (e) Speed of the rotor field relative to the (i) rotor structure, (ii) stator structure and (iii) stator rotating field. (f) Rotor induced voltage at the operating speed if the stator-to-rotor turns ratio is 1:0.5. Assume that the stator phase induced voltage by the total rotating field is equal to the rated line-to-neutral voltage applied to the stator.

Solution

(a) $n_s = 120 \cdot \frac{f_1}{p} = 120 \times \frac{60}{4} = 1800 \text{ rpm}, \quad n = (1-s)\cdot n_s = (1-0.05) \times 1800 = 1710 \text{ rpm}$

(b) $1800 \text{ rpm (same as the synchorous speed)}$

(c) $f_2 = s \cdot f_1 = 0.05 \times 60 = 3 \text{ Hz}$

(d) $\text{slip rpm} = s \cdot n_s = 0.05 \times 1800 = 90 \text{ rpm}$

(e) (i) $n_2 = s \cdot n_s = 0.05 \times 1800 = 90 \text{ rpm}$

(ii) $n + n_2 = n_s = 1800 \text{ rpm}$

(iii) $n_s - n_s = 0 \text{ rpm}$

(f) $E_{2s} = s \cdot E_2 = s \cdot \frac{N_2}{N_1} E_{1rms} = 0.05 \times \frac{0.5}{1} \times \frac{460 \text{ V}}{\sqrt{3}} = 6.64 \text{ V/phase}$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 19

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by

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P.S. The pictures, notations, formulas, examples, and statements in these lecture 19 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

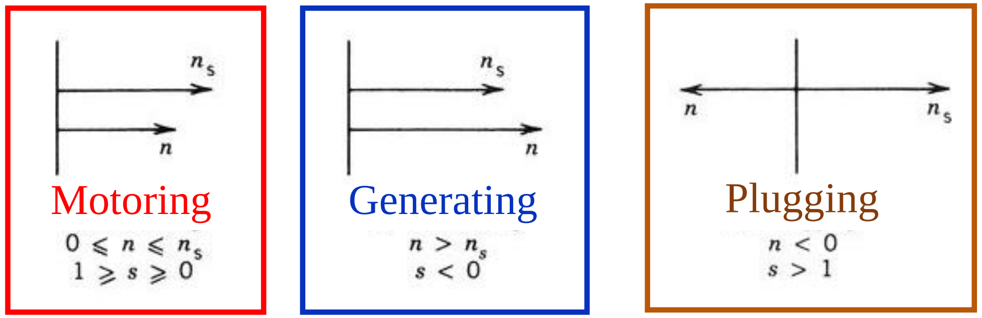
MODES of OPERATION for INDUCTION MACHINES

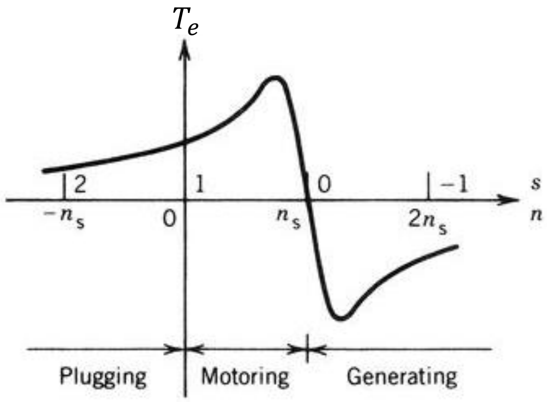
The induction machine can be operated in three modes. These are, 1) motoring, 2) generating, and 3) plugging (braking).

1) Motoring: When the stator terminals are connected to a three-phase supply, the rotor will rotate in the direction of the stator rotating magnetic field, and the steady-state speed n is less than the synchronous speed n_s. The torque produced by the machine is in the same direction as the speed.

2) Generating: As the speed of the induction machine is increased above the synchronous speed by an external prime mover such as a gas turbine, the stator rotating field cuts the rotor windings in the opposite direction. As a result, the induction machine produces a torque in the opposite direction of the speed (opposing torque). That is, the induction machine receives mechanical power and generates electrical power. In generating mode, both the stator rotating field and the rotor rotate in the same direction with respect to the fixed stator.

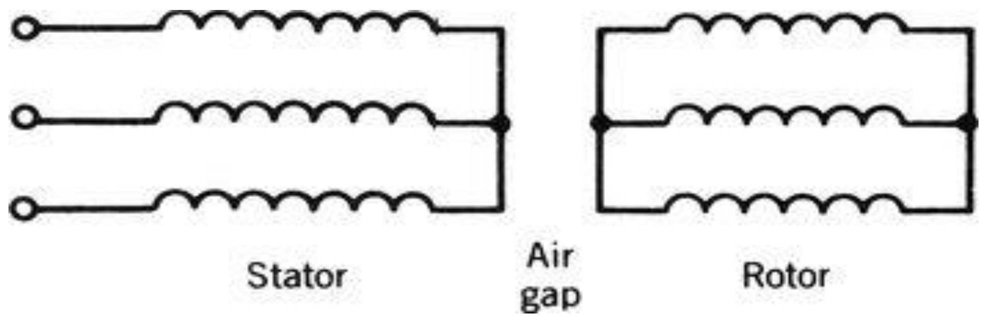
3) Plugging (Braking): In this mode, the stator rotating field and the rotor rotate in the opposite directions with respect to the fixed stator. As a result, the torque produced by the induction machine opposes the rotation.





EQUIVALENT CIRCUIT MODEL OF INDUCTION MACHINE

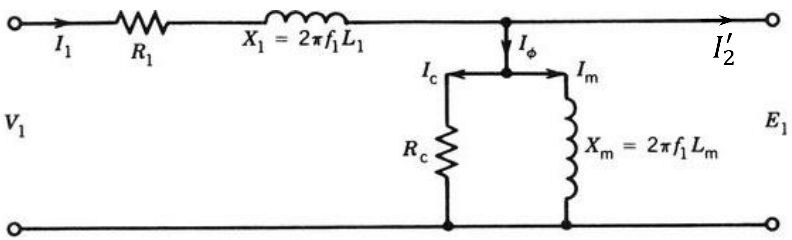
A steady-state (constant speed) per-phase equivalent circuit of a three-phase induction machine will be derived. Assumption: a three-phase wound-rotor induction machine with the rotor terminals shorted.



Three-phase wound-rotor induction motor.

Both stator and rotor rotating magnetic fields rotate at synchronous speed in the air gap. The resultant (total) air gap field rotates at the synchronous speed as well, and it induces voltages in stator windings at the supply frequency f_1 and rotor windings at the slip frequency $f_2 = s \cdot f_1$.

STATOR WINDING



Stator equivalent circuit at supply frequency f_1.

R_1 = Per-phase stator winding resistance.

L_1 = Per-phase stator leakage inductance.

V_1 = Per-phase terminal voltage.

I_1 = Stator current.

I_ϕ = Exciting current.

I_m = Magnetizing current.

I_c = Core loss current.

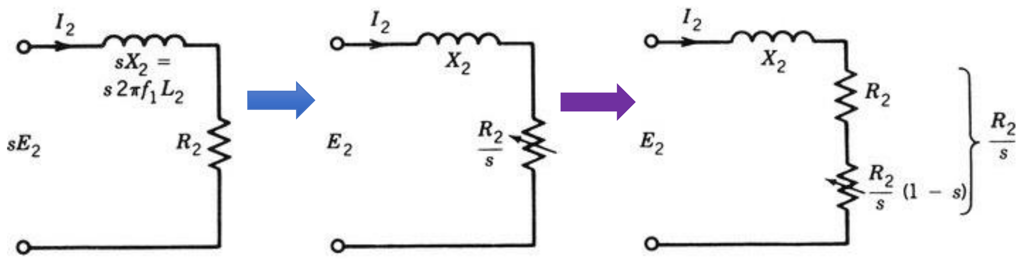
E_1 = Per-phase induced voltage in the stator winding.

I_2' = Rotor current referred to stator.

L_m = Per-phase stator magnetizing inductance.

R_c = Per-phase stator core loss resistance.

ROTOR WINDING



Rotor equivalent circuit at slip s. The rotor circuit frequency is $f_2 = s \cdot f_1$.

E_2 = Per-phase induced voltage in rotor at standstill.

R_2 = Per-phase rotor circuit resistance.

L_2 = Per-phase rotor leakage inductance.

Per-phase induced voltage in rotor at slip s:

$$s \cdot E_2 = s \cdot \frac{N_2}{N_1} E_1$$

The rotor current I_2 is

$$\boldsymbol{I_2} = \frac{s \cdot E_2}{R_2 + j \cdot s \cdot X_2} \quad \text{where } X_2 = 2 \cdot \pi \cdot f_1 \cdot L_2$$ (The circuit on the left in the figure above)

$$\boldsymbol{I_2} = \frac{E_2}{(R_2/s) + j \cdot X_2}$$ (The circuit at the middle in the figure above)

$$\frac{R_2}{s} = R_2 + \frac{R_2}{s} \cdot (1-s)$$

$$\boldsymbol{I_2} = \frac{E_2}{\left[R_2 + \frac{R_2}{s} \cdot (1-s)\right] + j \cdot X_2}$$ (The circuit on the right in the figure above)

From the circuit on the right in the figure above, the power transferred from the stator to the rotor via the air gap is

$$P = P_{ag} = I_2^2 \cdot R_2 + I_2^2 \cdot \frac{R_2}{s} \cdot (1-s)\,.$$

Power Transferred from Stator to Rotor in Induction machine

The power transferred from the stator to the rotor via the air gap (the power crossing the air gap) is given by

$$P = P_{ag} = \underbrace{I_2^2 \cdot R_2}_{P_2} + \underbrace{I_2^2 \cdot \frac{R_2}{s} \cdot (1-s)}_{P_{mech}}$$

where

$$P_2 = I_2^2 \cdot R_2 = s \cdot P_{ag}$$

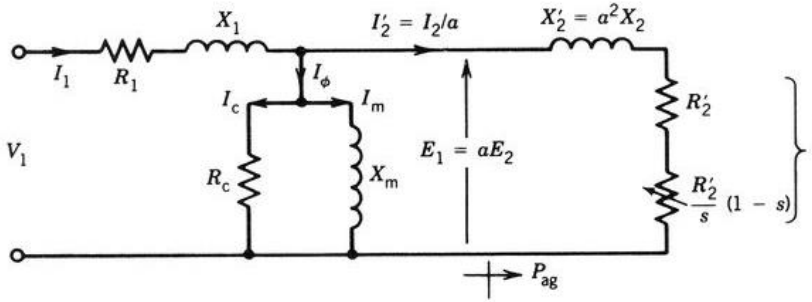
is the rotor copper (ohmic) loss per phase and

$$P_{mech} = I_2^2 \cdot \frac{R_2}{s} \cdot (1-s) = (1-s) \cdot P_{ag}$$

is the mechanical power developed by induction machine. Note that more of the air gap power is converted into mechanical power at low slip. In brief, the induction machine is operated at low slips for a good power efficiency.

Part of the mechanical power developed by the induction machine is lost to overcome the bearing friction and windage (air drag due to the spinning rotor and cooling fan). The remainder of the mechanical power is available as output shaft power to drive a mechanical load.

Complete Equivalent Circuit of Induction Machine



$R_2' = a^2R_2$

$\frac{R_2'}{s}(1-s) = \frac{a^2R_2}{s}(1-s)$

Induction machine equivalent circuit with the rotor quantities referred to the stator.

Note that E_1 in the equivalent stator circuit and E_2 in the equivalent rotor circuit are at same line frequency f_1. However, E_1 and E_2 may be different if the turns of the stator and rotor windings are different ($N_1 \neq N_2$). Then, the equivalent rotor circuit is incorporated into the equivalent stator circuit by taking into account the turns ratio $a = N_1/N_2$ as shown in the figure above. The form of the induction machine equivalent circuit is identical to that of a two-winding transformer. Furthermore, the preservation of the air gap power P_{ag} implies that the impedance of the rotor referred to the stator can be used when combining the stator and rotor circuits to obtain the complete equivalent circuit. That is,

$$P_{ag} = I_2'^2 \cdot R_2' + I_2'^2 \cdot \frac{R_2'}{s} \cdot (1-s) = I_2^2 \cdot R_2 + I_2^2 \cdot \frac{R_2}{s} \cdot (1-s)$$

EXAMPLE 5.2 from the textbook

A 3ϕ, 15 hp, 460 V, four-pole, 60 Hz, 1728 rpm induction machine delivers full output power to a load connected to its shaft. The windage and friction loss of the motor is 750 W. Determine (a) the mechanical power developed, (b) the air gap power, and (c) the rotor copper loss.

Solution

(a) $1 \text{ hp} = 746 \text{ W}$

Full load power $= 15 \text{ hp} \times 746 = 11{,}190 \text{ W}$

Mechanical power developed = Shaft power + Windage and friction loss

$$P_{mech} = 11{,}190 + 750 = 11940 \text{ W}$$

(b)

$$n_s = 120 \cdot \frac{f_1}{p} = 120 \times \frac{60}{4} = 1800 \text{ rpm},$$

$$s = \frac{n_s - n}{n_s} = \frac{1800 - 1728}{1800} = 0.04$$

Air gap power:

$$P_{ag} = \frac{P_{mech}}{1 - s} = \frac{11{,}940}{1 - 0.04} = 12{,}437.5 \text{ W}$$

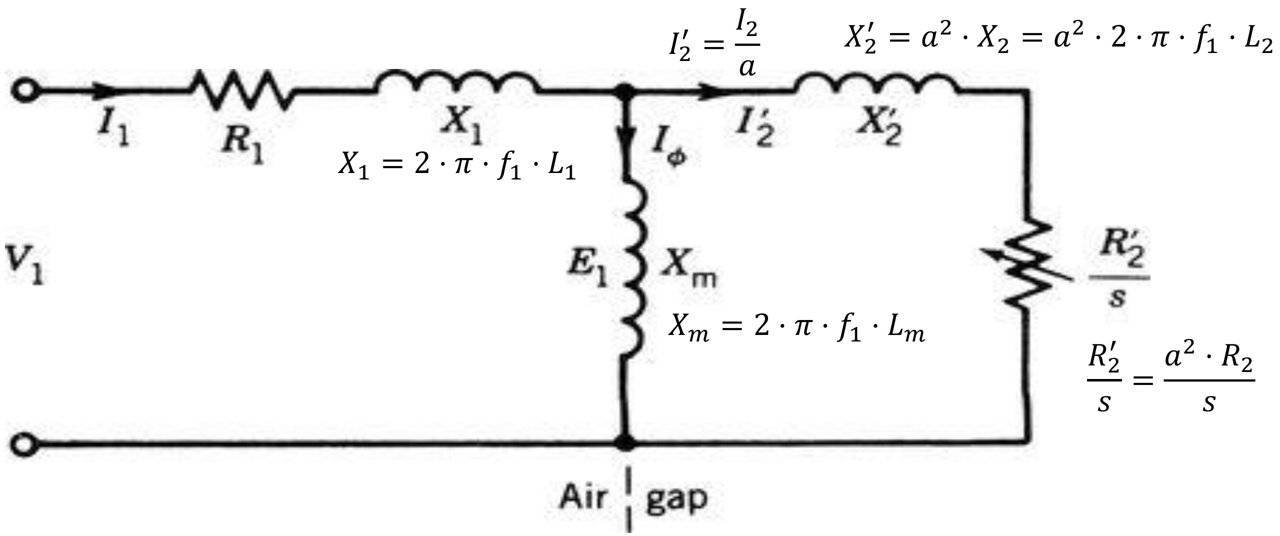
(c) Rotor copper loss:

$$P_2 = s \cdot P_{ag} = 0.04 \times 12{,}437.5 = 497.5 \text{ W}$$

VARIOUS EQUIVALENT CIRCUIT CONFIGURATIONS OF INDUCTION MACHINE

There are several simplified versions of the equivalent circuit to predict the performance of the induction machine. In the most common versions, the motor core loss is lumped with windage and friction loss and the core loss resistance R_c is removed from the equivalent circuit.

IEEE Recommended Equivalent Circuit



V_1 = Per-phase stator terminal voltage.

I_1 = Stator current.

E_1 = Per-phase induced voltage in the stator winding.

I_ϕ = Exciting (or, excitation) current.

I_2' = Rotor current referred to stator.

s = Slip

$a = \frac{N_1}{N_2}$ =Turns ratio between stator and rotor.

f_1 = Supply (or, line, or stator) frequency in Hz.

R_1 = Per-phase stator winding resistance.

X_1 = Per-phase stator leakage reactance.

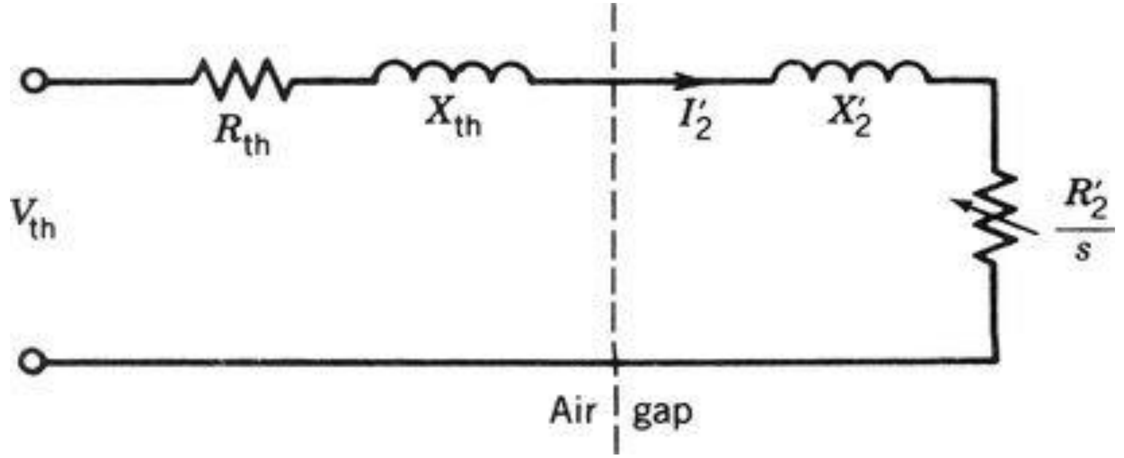
X_m = Per-phase stator magnetizing reactance.

R_2' = Per-phase rotor resistance referred to stator.

X_2' = Per-phase rotor leakage reactance referred to stator.

THEVENIN EQUIVALENT CIRCUIT OF INDUCTION MACHINE

In order to simplify the calculations, V_1, R_1, X_1, and X_m in the IEEE recommended equivalent circuit can be replaced by the Thevenin equivalent circuit values V_{th}, R_{th}, and X_{th} as shown below.



Thevenin equivalent circuit of induction machine.

The Thevenin voltage is

$$V_{th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} \cdot V_1 \quad \text{and} \quad V_{th} \approx \frac{X_m}{X_1 + X_m} \cdot V_1 = K_{th} \cdot V_1 \quad \text{for} \quad R_1^2 << (X_1 + X_m)^2 \quad \text{and} \quad K_{th} = \frac{X_m}{X_1 + X_m}$$

The Thevenin impedance is

$$\boldsymbol{Z_{th}} = \frac{j \cdot X_m \cdot (R_1 + j \cdot X_1)}{R_1 + j \cdot (X_1 + X_m)} = \frac{R_1 \cdot X_m^2}{R_1^2 + (X_1 + X_m)^2} + j \cdot \frac{X_m \cdot (R_1^2 + X_1^2 + X_1 \cdot X_m)}{R_1^2 + (X_1 + X_m)^2} = R_{th} + j \cdot X_{th}$$

where

$$R_{th} \approx \left(\frac{X_m}{X_1 + X_m}\right)^2 \cdot R_1 = K_{th}^2 \cdot R_1 \quad \text{for} \quad R_1^2 << (X_1 + X_m)^2$$

and

$$X_{th} \approx X_1 \quad \text{for} \quad X_1 << X_m \quad \text{and} \quad R_1^2 << X_1 \cdot X_m$$

No-Load Test and Blocked-Rotor Test for Obtaining Equivalent Circuit Parameters of Induction Machine

The parameters of the equivalent circuit, R_c, X_m, R_1, X_1, R_2 and X_2 can be determined from a no-load test, a blocked-rotor test, and measurement of the stator winding dc resistance.

1- No-Load Test: Gives information about i) the magnetizing reactance X_m, ii) the exciting current I_ϕ and iii) the rotational losses (the core loss resistance R_c, and the friction and windage loss). This test is performed by applying the rated (nominal) voltage to the stator windings at the rated frequency. The rotor is kept uncoupled from any mechanical load.

2- Blocked-Rotor Test: Gives information about i) the rotor winding resistances R_2 and ii) the stator and rotor leakage reactances X_1 and X_2. This test is performed at a reduced stator voltage and rated current. Furthermore, the IEEE recommends a frequency of 25 percent of the rated frequency for the blocked-rotor test. This is to take into account the skin effect for the cage rotors with deep-bars, which causes the rotor resistance to increase with the increasing frequency (e.g., the rotor resistance at 60 Hz can be three times higher than the dc value).

PERFORMANCE CHARACTERISTICS OF INDUCTION MACHINE

The equivalent circuits are used to predict the performance characteristics of the induction machine at steady-state operation. The performance characteristics are the efficiency, power factor, current, starting torque, maximum (or, pull-out) torque, and so forth.

The mechanical power developed by the per phase of the induction machine is

$$P_{mech1\phi} = I_2^2 \cdot \frac{R_2}{s} \cdot (1-s)$$

Then, the total mechanical power developed by a three-phase induction machine is

$$P_{mech} = 3 \cdot P_{mech1\phi} = 3 \cdot I_2^2 \cdot \frac{R_2}{s} \cdot (1-s) = T_e \cdot \omega_{mech}$$

where T_e is the electromagnetic torque developed by the induction machine in N,

$$\omega_{mech} = \frac{2 \cdot \pi \cdot n}{60} = (1-s) \cdot \frac{2 \cdot \pi \cdot n_s}{60} = (1-s) \cdot \omega_{syn}$$

is the motor speed in rad/s, and

$$\omega_{syn} = 120 \cdot \frac{f_1}{p} \cdot \frac{2 \cdot \pi}{60} = \frac{4 \cdot \pi \cdot f_1}{p}$$

is the synchronous speed in rad/s.

Note that

$$T_e \cdot \omega_{syn} = T_e \cdot \frac{\omega_{mech}}{(1-s)} = 3 \cdot I_2^2 \cdot \frac{R_2}{s}$$

On the other hand, the total air gap power of a three-phase induction machine is

$$P_{ag} = 3 \cdot P_{ag1\phi} = 3 \cdot I_2^2 \cdot \frac{R_2}{s}$$

Then, the following is valid for a three-phase induction machine:

$$T_e \cdot \omega_{syn} = P_{ag}$$

In brief, the torque developed by a three-phase induction machine is

$$T_e = \frac{P_{ag}}{\omega_{syn}} = \frac{3}{\omega_{syn}} \cdot I_2^2 \cdot \frac{R_2}{s} = \frac{3}{\omega_{syn}} \cdot I_2'^2 \cdot \frac{R_2'}{s}$$

From the Thevenin equivalent circuit,

$$I_2' = \frac{V_{th}}{\sqrt{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2}}$$

Then,

$$T_e = \frac{3}{\omega_{syn}} \cdot \frac{V_{th}^2}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2} \cdot \frac{R_2'}{s}$$

The total rotor copper losses of a three-phase induction machine is

$$P_2 = 3 \cdot I_2^2 \cdot R_2 = s \cdot P_{ag}$$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 20

Date: April 15, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 20 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

Torque Developed by Induction Machine

The torque produced by the per phase of the induction machine is

$$T_{e1\phi} = \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2} \cdot \frac{R_2'}{s}$$

where $V_{th} \approx K_{th} \cdot V_1,\ R_{th} \approx K_{th}^2 \cdot R_1,\ X_{th} \approx X_1,$ and $K_{th} = \frac{X_m}{X_1+X_m}$.

For a three-phase machine, the above torque expression is multiplied by 3 to obtain the total torque developed by the machine.

For small slip values (or, high speeds),

$(R_{th}+\frac{R_2'}{s}) >> (X_{th}+X_2')$ and $\frac{R_2'}{s} >> R_{th}$

Therefore,

$$T_{e1\phi} \approx \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{R_2'} \cdot s$$

(The torque-speed relationship is almost linear near the synchronous speed.)

At large values of slip (or, low speeds),

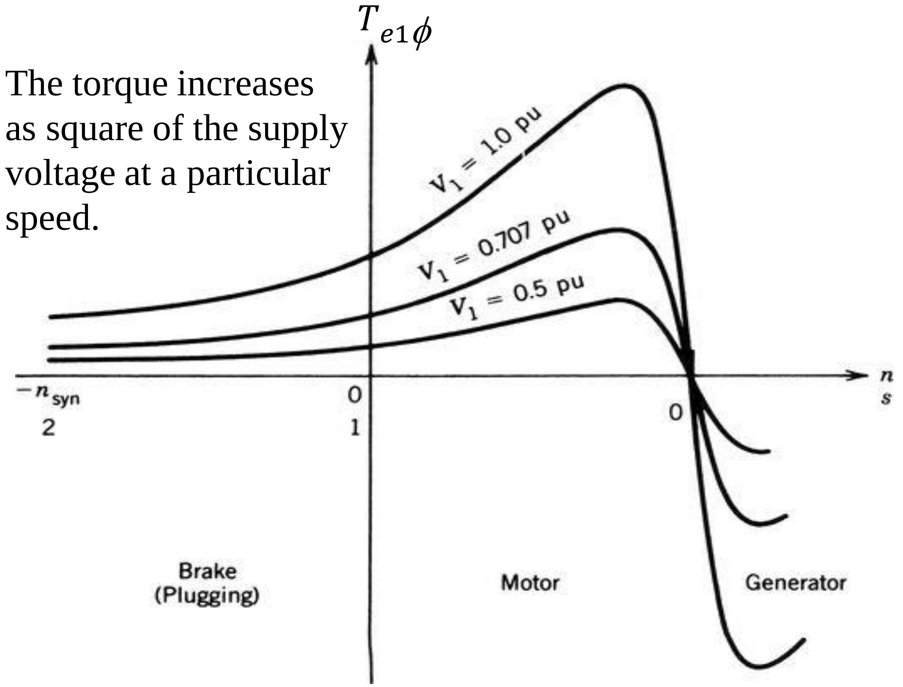
$(R_{th}+\frac{R_2'}{s}) << (X_{th}+X_2')$

and

$$T_{e1\phi} \approx \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{(X_{th} + X_2')^2} \cdot \frac{R_2'}{s}$$

(The torque varies almost inversely with slip near s = 1.)

The torque increases as square of the supply voltage at a particular speed.



Torque-speed (or, torque-slip) profile of induction machine at different stator voltages.

Maximum (Pull-Out) Torque Developed by Induction Machine

The torque developed per phase by induction machine is given by

$$T_{e1\phi} = \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2} \cdot \frac{R_2'}{s}$$

The maximum torque is obtained by differentiating the above torque expression with respect to the slip:

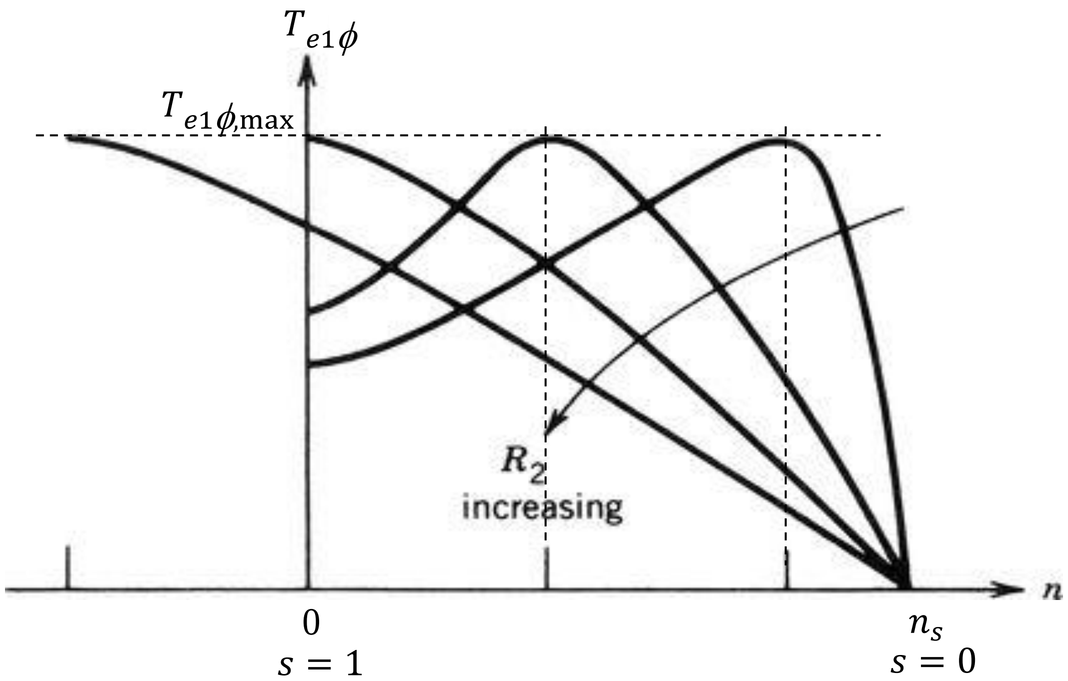
$$\frac{dT_{e1\phi}}{ds} = 0 \quad \Rightarrow \quad \frac{R_2'}{s} = \sqrt{R_{th}^2 + (X_{th} + X_2')^2} \quad \Rightarrow \quad s_{T\max} = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}} = \frac{a^2 \cdot R_2}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$$

The maximum torque per phase is

$$T_{e1\phi,\max} = T_{e1\phi}\Big|_{s = s_{T\max}}$$

$$T_{e1\phi,\max} = \frac{1}{2 \cdot \omega_{syn}} \cdot \frac{V_{th}^2}{R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2')^2}}$$

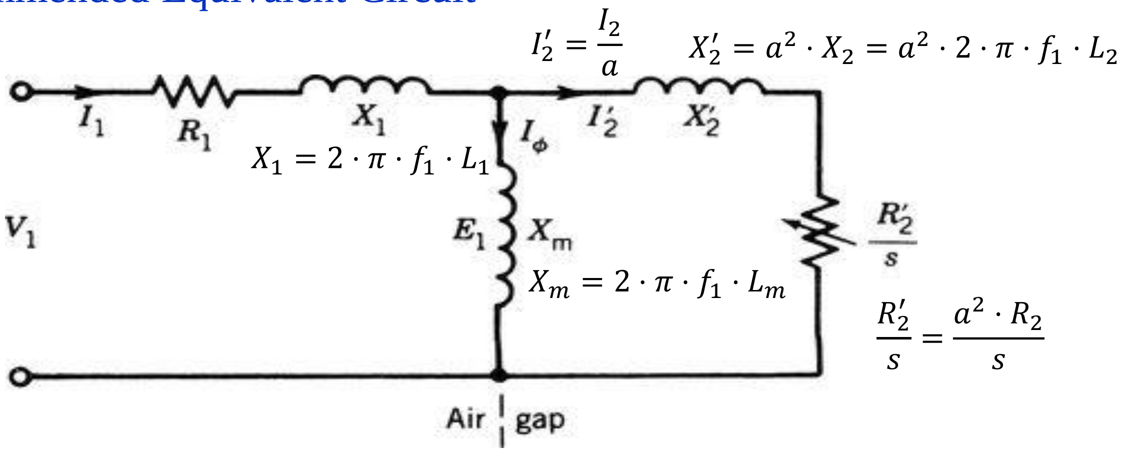
Note that the rotor resistance R_2 determines the speed (slip) at which the maximum torque occurs. The larger R_2 yields the maximum torque at a lower speed. Furthermore, the maximum value of the torque remains the same as R_2 is varied.



Torque-speed characteristics for varying R_2.

Stator Current of Induction Machine

IEEE Recommended Equivalent Circuit



From the IEEE recommended equivalent circuit above, the input impedance is

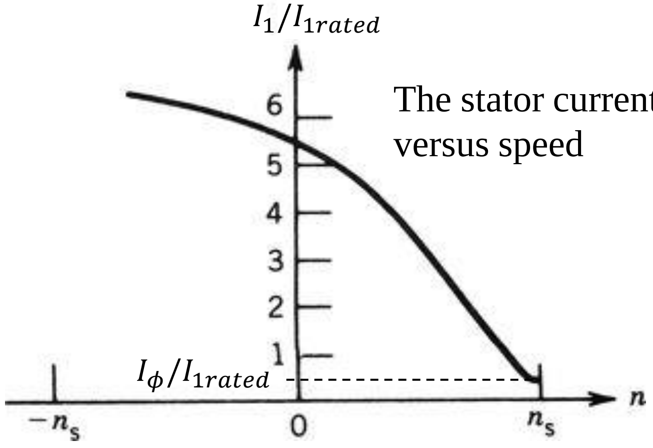
$$\boldsymbol{Z_1} = R_1 + j \cdot X_1 + j \cdot X_m // \boldsymbol{Z'_2} = R_1 + j \cdot X_1 + j \cdot X_m // \left(\frac{R'_2}{s} + j \cdot X'_2\right) = R_1 + j \cdot X_1 + \frac{j \cdot X_m \cdot \left(\frac{R'_2}{s} + j \cdot X'_2\right)}{\frac{R'_2}{s} + j \cdot (X_m + X'_2)} = |\boldsymbol{Z_1}| \angle \theta_1$$

The stator current is

$$\boldsymbol{I_1} = \frac{\boldsymbol{V_1}}{\boldsymbol{Z_1}} = \boldsymbol{I_\phi} + \boldsymbol{I'_2}$$

At synchronous speed, $s = 0$, $\frac{R'_2}{s} \to \infty$, $\boldsymbol{I'_2} \to 0$ and $\boldsymbol{I_1} = \boldsymbol{I_\phi}$.

At low speeds (larger values of slip s), $\boldsymbol{Z'_2} = R'_2/s + j \cdot X'_2$ is small and therefore $\boldsymbol{I'_2}$ and hence $\boldsymbol{I_1}$ is large. In fact, the typical starting current (at s = 1) is five to eight times the rated current.



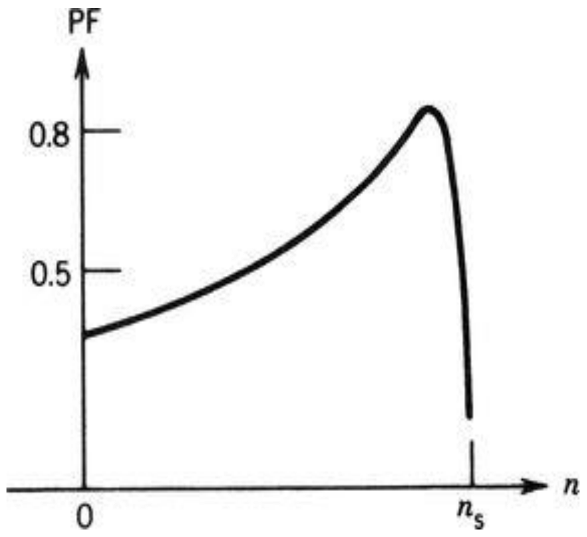
The stator current versus speed

Input Power Factor of Induction Machine

The input power factor of induction machine is given

$$\text{PF} = \cos(\theta_1)$$

where θ_1 is the angle of the input impedance in the IEEE equivalent circuit, and $-\theta_1$ is the phase angle of the stator input current $\boldsymbol{I_1}$. The typical power factor variation with speed is shown in the figure below.



Input power factor of induction machine as a function of speed.

Efficiency of Induction Machine

The efficiency of the 3ϕ induction motor is

$$\text{Eff} = \frac{P_{out}}{P_{in}}$$

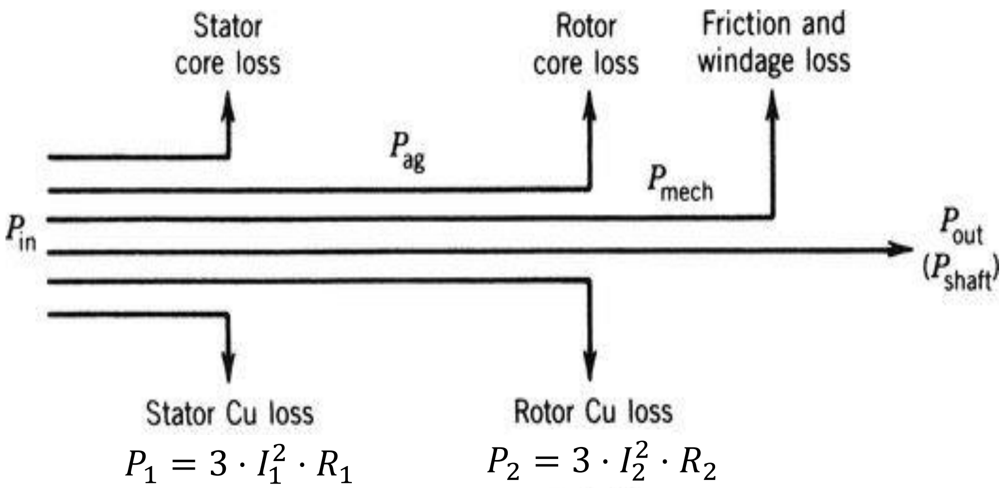
where

$$P_{in} = 3 \cdot V_1 \cdot I_1 \cdot \cos(\theta_1)$$

is the input power (power input to the stator),

$$P_{out} = P_{shaft} = T_{load} \cdot \omega_{mech}$$

is the output (shaft) power that is delivered to the load, and T_{load} is the load torque. Note that the torque developed by the motor (T_e) must overcome the load torque T_{load} and the opposing friction and windage torques. That is, $T_e > T_{load}$ for motor operation. The difference between P_{in} and P_{out} is the various losses in the machine as illustrated by the power flow diagram below.



The power flow in a three-phase induction motor.

In the power flow diagram,

$$P_1 = 3 \cdot I_1^2 \cdot R_1$$

is the ohmic (copper) loss in the stator three-phase windings where R_1 is the resistance value of each stator phase winding, and

$$P_2 = 3 \cdot I_2^2 \cdot R_2$$

is the ohmic loss in the rotor three-phase windings where R_2 is the resistance value of each rotor phase winding.

Efficiency versus Speed of Induction Machine

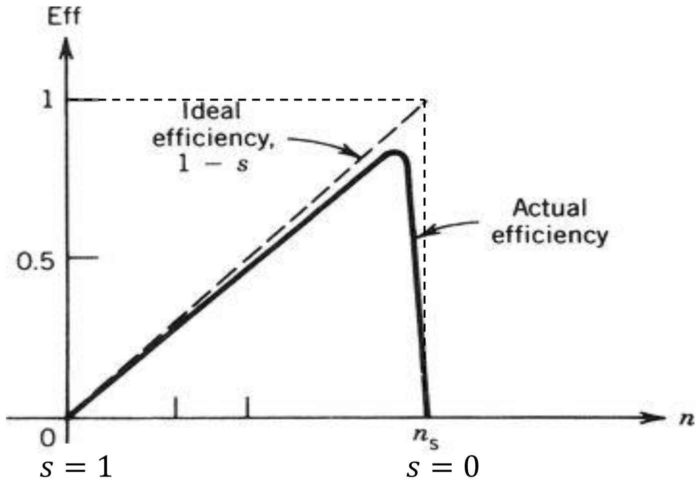
If all losses are neglected except the rotor ohmic loss,

$$P_{ag} = P_{in}$$
$$P_2 = s \cdot P_{ag}$$
$$P_{out} = P_{mech} = (1 - s) \cdot P_{ag}$$

and the *ideal efficiency* is

$$\mathrm{Eff}_{(\mathrm{ideal})} = \frac{P_{out}}{P_{in}} = 1 - s = \frac{n}{n_s}$$

$\mathrm{Eff}_{(\mathrm{ideal})}$ is also called the *internal efficiency* since it represents the ratio of the output power to the air gap power. Note that the ideal efficiency is the function of slip (speed). The plot of the ideal efficiency versus speed in the figure below indicates that the induction machine must operate near its synchronous speed (near zero slip) for high power efficiency. The actual efficiency (Eff), which also takes into account the other losses, is lower than the ideal efficiency $\mathrm{Eff}_{(\mathrm{ideal})}$.



Power efficiency of induction machine as a function of speed.

EXAMPLE 5.4 from the textbook

A 3ϕ, 460 V, 1740 rpm, 60 Hz, four-pole wound-rotor induction motor has the following parameters per phase: $R_1 = 0.25\ \Omega$, $R_2' = 0.2\ \Omega$, $X_1 = X_2' = 0.5\ \Omega$, $X_m = 30\ \Omega$. The rotational losses are $P_{rot} = 1700$ watts (the core, the friction and the windage losses). With the rotor terminals short-circuited, find

(a) (i) Starting current when started direct on full voltage. (ii) Starting torque.

(b) (i) Full-load slip. (ii) Full-load current. (iii) Ratio of starting current to full-load current. (iv) Full-load power factor. (v) Full-load torque. (vi) Internal efficiency and motor efficiency at full load.

(c) (i) Slip at which maximum torque is developed. (ii) Maximum torque developed.

Solution

(a) $V_1 = \dfrac{460}{\sqrt{3}} = 265.6\ \mathrm{V/phase}$

(i) At start, $s = 1$. From the IEEE-recommended equivalent circuit, the input impedance is

$$\boldsymbol{Z_1} = R_1 + j\cdot X_1 + \frac{j\cdot X_m\cdot\left(\frac{R_2'}{s} + j\cdot X_2'\right)}{\frac{R_2'}{s} + j\cdot(X_m + X_2')} = 0.25 + j\cdot 0.5 + \frac{j\cdot 30\cdot(0.2 + j\cdot 0.5)}{0.2 + j\cdot 30.5} = 0.443 + j\cdot 0.993 = 1.08\angle 66^\circ\ \Omega$$

$$\boldsymbol{I_{1,start}} = \frac{\boldsymbol{V_1}}{\boldsymbol{Z_1}} = \frac{265.6\angle 0^\circ}{1.08\angle 66^\circ} = 245.9\angle -66^\circ\ \mathrm{A}$$

(ii) $\omega_{syn} = \dfrac{4\cdot\pi\cdot f_1}{p} = \dfrac{4\cdot\pi\cdot 60}{4} = 188.5\ \mathrm{rad/s}$

$$V_{th} \approx \frac{X_m}{X_1 + X_m}\cdot V_1 = \frac{30}{0.5 + 30}\cdot 265.6 = 261.2\ \mathrm{V} \quad \text{for} \quad R_1^2 << (X_1 + X_m)^2$$

$$R_{th} \approx \left(\frac{X_m}{X_1 + X_m}\right)^2 \cdot R_1 = \left(\frac{30}{0.5 + 30}\right)^2 \cdot 0.25 = 0.24\ \Omega \ \text{ for } \ R_1^2 << (X_1 + X_m)^2$$

$$X_{th} \approx X_1 = 0.5\ \Omega \ \text{ for } \ X_1 << X_m \ \text{ and } \ R_1^2 << X_1 \cdot X_m$$

$$I_2' = \frac{V_{th}}{\sqrt{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2}} = \frac{261.3}{\sqrt{(0.24 + 0.2)^2 + (0.5 + 0.5)^2}} = 239.2\ \text{A}$$

$$T_e = \frac{3}{\omega_{syn}} \cdot I_2'^2 \cdot \frac{R_2'}{s} = \frac{3}{188.5} \cdot (239.2)^2 \cdot \frac{0.2}{1} = 182.1\ \text{N} \cdot \text{m}$$

(b)

(i) $n_{syn} = \frac{2 \cdot f_1}{p} \cdot 60 = \frac{2 \cdot 60}{4} \cdot 60 = 1800\ \text{rpm},$ and $s = \frac{1800 - 1740}{1800} = 0.0333$

(ii) $\frac{R_2'}{s} = \frac{0.2}{0.0333} = 6.01\ \Omega$

$$\boldsymbol{Z_1} = R_1 + j \cdot X_1 + \frac{j \cdot X_m \cdot \left(\frac{R_2'}{s} + j \cdot X_2'\right)}{\frac{R_2'}{s} + j \cdot (X_m + X_2')} = 0.25 + j \cdot 0.5 + \frac{j \cdot 30 \cdot (6.01 + j \cdot 0.5)}{6.01 + j \cdot 30.5} = 5.847 + j \cdot 2.095 = 6.211\angle 19.7°$$

$$\boldsymbol{I_{1,FL}} = \frac{\boldsymbol{V_1}}{\boldsymbol{Z_1}} = \frac{265.6\angle 0°}{6.211\angle 19.7°} = 42.76\angle -19.7°\ \text{A}$$

(iii) $\frac{I_{1,start}}{I_{1,FL}} = \frac{245.9}{42.76} = 5.75$

(iv) $\mathrm{PF} = \cos(\theta_1) = \cos(19.7°) = 0.941$

(v) $$I_2' = \frac{V_{th}}{\sqrt{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2}} = \frac{261.3}{\sqrt{(0.24 + 6.01)^2 + (0.5 + 0.5)^2}} = 41.28 \text{ A}$$

$$T_e = \frac{3}{\omega_{syn}} \cdot I_2'^2 \cdot \frac{R_2'}{s} = \frac{3}{188.5} \cdot (41.28)^2 \cdot 6.01 = 162.99 \text{ N} \cdot \text{m}$$

(vi) $\mathrm{Eff}_{(\mathrm{internal})} = 1 - s = 1 - 0.0333 = 0.967 \rightarrow 96.7\%$

$P_{ag} = T_e \cdot \omega_{syn} = 162.99 \cdot 188.5 = 30723.6 \text{ W}$

$P_2 = s \cdot P_{ag} = 0.0333 \cdot 30723.6 = 1023.1 \text{ W}$

$P_{mech} = (1 - s) \cdot P_{ag} = (1 - 0.0333) \cdot 30723.6 = 29700.5 \text{ W}$

$P_{out} = P_{mech} - P_{rot} = 29700.5 - 1700 = 28000.5 \text{ W}$

$P_{in} = 3 \cdot V_1 \cdot I_1 \cdot \cos(\theta_1) = 3 \cdot 265.6 \cdot 42.76 \cdot 0.941 = 32061.0 \text{ W}$

$$\mathrm{Eff} = \frac{P_{out}}{P_{in}} = \frac{28000.5}{32061.0} = 0.873 \rightarrow 87.3\%$$

(c)

(i) $$s_{T\max} = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}} = \frac{0.2}{\sqrt{(0.24)^2 + (0.5 + 0.5)^2}} = 0.1945$$

(ii) $$T_{e,\max} = \frac{3}{2 \cdot \omega_{syn}} \cdot \frac{V_{th}^2}{R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2')^2}} = \frac{3}{2 \cdot 188.5} \cdot \frac{(261.2)^2}{0.24 + \sqrt{(0.24)^2 + (0.5 + 0.5)^2}} = 428.03 \text{ N} \cdot \text{m}$$

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 21

Date: April 20, 2021

by

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Spring 2021

P.S. The pictures, notations, formulas, examples, and statements in these lecture 21 notes have been adopted and adapted mainly from the course textbook "Principles of Electric Machines and Power Electronics" by P. C. Sen, Third Edition, 2013.

Variable Speed Control of Induction Motor

The operating (rated) speed of induction motor is very close to the synchronous speed and the speed varies (rises/drops) by a very small amount as the load torque changes (decreases/increases). On the other hand, many applications, such as transportation systems, pumps, fans, compressors, require several speeds or a continuously adjustable wide range of speeds. One of the techniques for achieving a variable speed control of the induction motor is the Volts per Hertz (V/Hz) control.

Volts per Hertz Control

For small slip values (speeds close to the synchronous speed), the torque developed by per phase of induction motor can be approximated by

$$T_{e1\phi} \approx \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{R_2'} \cdot s = \frac{1}{\omega_{syn}} \cdot \frac{V_{th}^2}{R_2'} \cdot \frac{\omega_{syn} - \omega_{mech}}{\omega_{syn}}$$

where

$$V_{th} \approx K_{th} \cdot V_1 \;,\; K_{th} = \frac{X_m}{X_1 + X_m} = \frac{L_m}{L_1 + L_m} \;, \quad \omega_{syn} = 2\pi \cdot \frac{n_s}{60} = 4\pi \frac{f_1}{p} \;, \quad \omega_{mech} = 2\pi \cdot \frac{n}{60} \;,$$

$$R_2' = a^2 \cdot R_2 \;, \qquad a = \frac{N_1}{N_2} \text{ is the turns ratio between stator and rotor phase windings ,}$$

and p is the number of motor poles. Then,

$$\boxed{T_{e1\phi} \approx \left(\frac{p \cdot K_{th}}{4\pi}\right)^2 \cdot \frac{1}{R_2'} \cdot \left(\frac{V_1}{f_1}\right)^2 \cdot (\omega_{syn} - \omega_{mech})}$$

Or,

$$\boxed{\omega_{mech} \approx \omega_{syn} - \left(\frac{4\pi}{p \cdot K_{th}}\right)^2 \cdot R_2' \cdot \left(\frac{f_1}{V_1}\right)^2 \cdot T_{e1\phi}}$$

By maintaining V_1/f_1 constant at its nominal value V_{1n}/f_{1n} (Volts per Hertz Control, or constant V/Hz Operation) while changing the stator frequency f_1, the slope of the linear characteristic between the mechanical speed and the produced torque at the small values of the slip is preserved. In V_{1n}/f_{1n}, V_{1n} and f_{1n} are the nominal (rated) values of the stator voltage and stator frequency, respectively.

Furthermore, the maximum torque per phase is

$$T_{e1\phi,\max} = \frac{1}{2 \cdot \omega_{syn}} \cdot \frac{V_{th}^2}{R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2')^2}}$$

where

$$R_{th} \approx K_{th} \cdot R_1 \ , X_{th} \approx X_1 \ , X_2' = a^2 \cdot X_2$$

If the stator resistance R_1 is small (hence, $R_{th} \ll (X_{th}+X_2')$),

$$T_{e1\phi,\max} \approx \frac{1}{2 \cdot \omega_{syn}} \cdot \frac{V_{th}^2}{(X_{th} + X_2')} \approx p \cdot \left(\frac{K_{th}}{4\pi}\right)^2 \frac{1}{(L_1 + L_2')} \cdot \left(\frac{V_1}{f_1}\right)^2$$

In brief, the maximum torque per phase remains constant under constant V/Hz operation for negligible R_{th} .

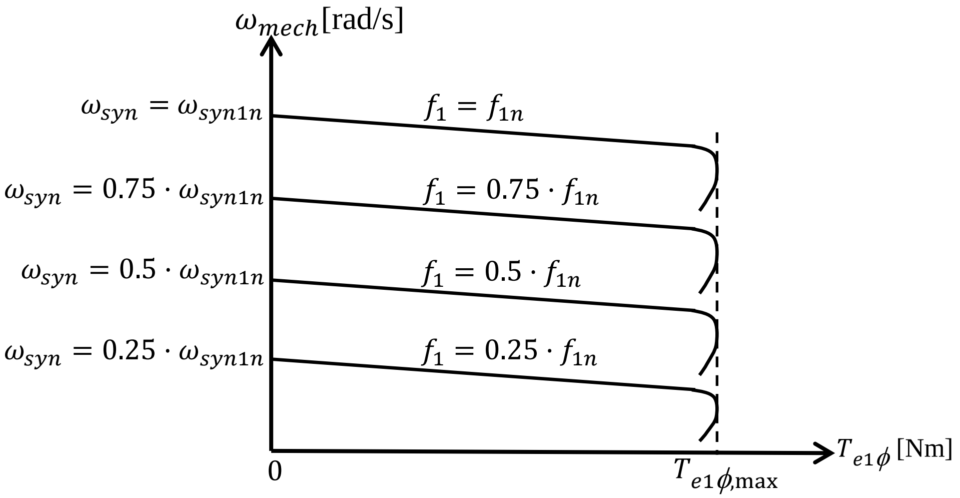
Speed versus Torque Characteristics under Constant Volts per Hertz Control

$$\omega_{mech} \approx \omega_{syn} - \underbrace{\left(\frac{4\pi}{p \cdot K_{th}}\right)^2 \cdot R_2' \cdot \left(\frac{f_1}{V_1}\right)^2} \cdot T_{e1\phi}$$

The slope remains the same for $V_1/f_1 = V_{1n}/f_{1n}$ = constant.

$$T_{e1\phi,\max} \approx p \cdot \left(\frac{K_{th}}{4\pi}\right)^2 \frac{1}{(L_1 + L_2')} \cdot \left(\frac{V_1}{f_1}\right)^2$$

The maximum torque remains the same for $V_1/f_1 = V_{1n}/f_{1n}$ = constant.



Speed versus torque characteristics under V/Hz control where $V_1/f_1 = V_{1n}/f_{1n}$ = constant and $\omega_{syn1n} = 4\pi \cdot f_{1n}/p$ is the synchronous speed in rad/s at nominal stator frequency f_{1n} in Hz.

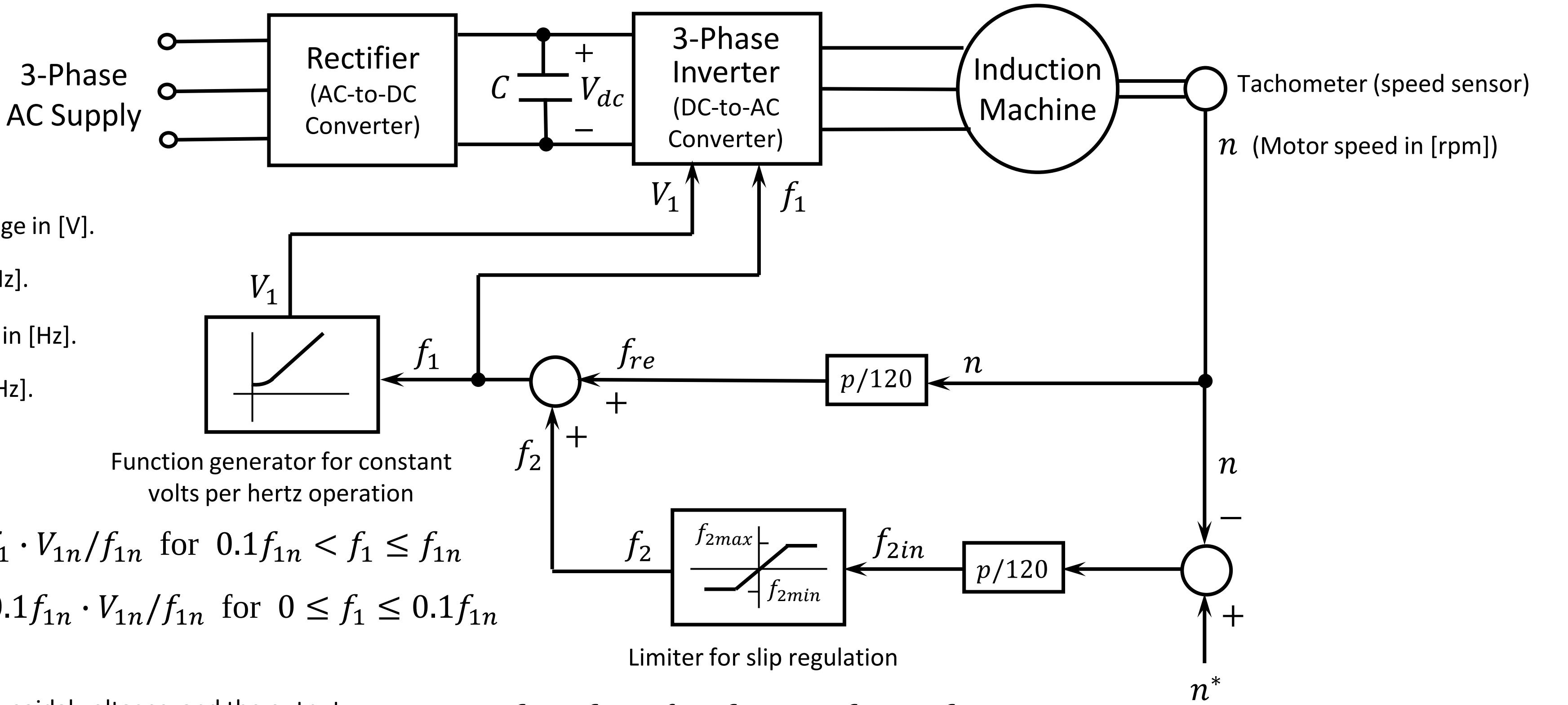
Constant Flux Operation

If the voltage drop across the stator resistance R_1 and stator leakage reactance X_1 in the IEEE-equivalent circuit is small compared to the terminal voltage V_1, then, $V_1 \approx E_1$ where $E_1 = E_{1rms}$ is the rms value of the induced voltage in the stator. As a result,

$$\frac{V_1}{f_1} \approx \frac{E_1}{f_1} = 4.44 \cdot N_1 \cdot \Phi_p$$

where Φ_p is the motor total air gap flux per pole in webers. In brief, the constant V/Hz operation becomes constant flux (constant E_1/f_1) operation.

AC Drive for Slip Regulation and Constant Volts per Hertz Operation of Induction Machine



V_{dc} : DC bus voltage in [V].

V_1 : RMS value of stator phase voltage in [V].

V_{1n} : Nominal (rated) RMS value of stator voltage in [V].

f_1 : Stator frequency of induction machine in [Hz].

f_{1n} : Nominal (rated) value of stator frequency in [Hz].

f_2 : Slip frequency (Rotor circuit frequency) in [Hz].

f_{re} : Rotor electrical speed in [Hz].

p : Number of motor poles.

C : DC bus capacitor in [F].

$$V_1 = f_1 \cdot V_{1n}/f_{1n} \quad \text{for} \quad 0.1f_{1n} < f_1 \le f_{1n}$$

$$V_1 \cong 0.1f_{1n} \cdot V_{1n}/f_{1n} \quad \text{for} \quad 0 \le f_1 \le 0.1f_{1n}$$

Rectifier: The inputs are 3-phase balanced sinusoidal voltages, and the output is DC voltage V_{dc}.

3-Phase Inverter: The input is DC voltage V_{dc}, and the outputs are 3-phase balanced sinusoidal voltages whose rms value and frequency are V_1 and f_1, respectively.

$$f_2 = f_{2in} \quad \text{for} \quad f_{2min} < f_{2in} < f_{2max}$$

$$f_2 = f_{2min} \quad \text{for} \quad f_{2in} \le f_{2min}$$

$$f_2 = f_{2max} \quad \text{for} \quad f_{2in} \ge f_{2max}$$

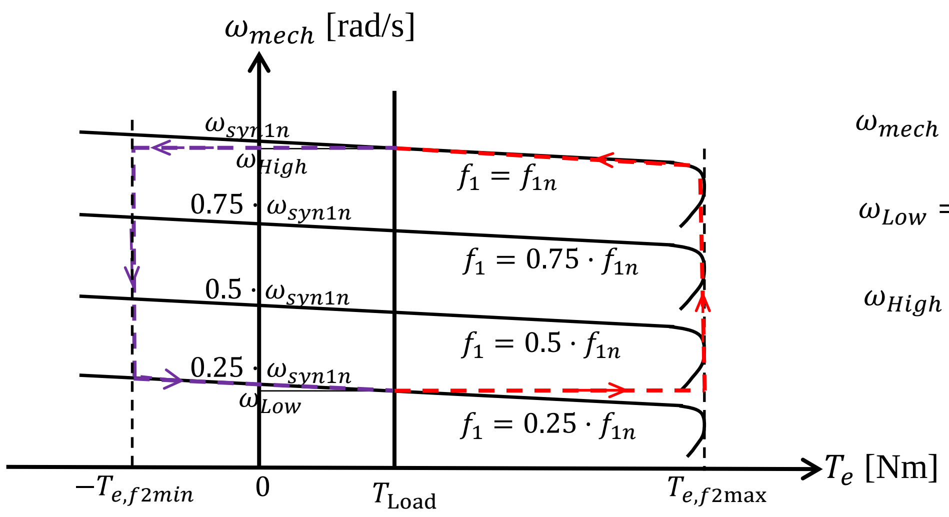
$$0 < f_{2max} \le f_{2b}$$

$$-f_{2b} \le f_{2min} \le 0$$

$f_{2b} = s_{T\max} \cdot f_{1n}$ (Breakdown frequency in [Hz] at which the maximum torque is developed).

High efficiency and high power factors are obtained for $f_2 \le f_{2b}$ in motor mode.

Speed versus Torque Trajectories of Induction Machine under Slip Regulation and Constant Volts per Hertz Operation



$$\omega_{mech} = \frac{2\pi}{60} \cdot n$$

$$\omega_{Low} = \frac{2\pi}{60} \cdot n_{Low}$$

$$\omega_{High} = \frac{2\pi}{60} \cdot n_{High}$$

Speed versus torque trajectories for (i) acceleration from ω_{Low} to ω_{High} at constant load torque T_{Load} under slip frequency regulation and constant V_1/f_1 operation (red dashed line), and (ii) deceleration from ω_{High} to ω_{Low} at constant load torque T_{Load} under slip frequency regulation and constant V_1/f_1 operation (purple dashed line).

The slip frequency f_2 (rotor circuit frequency) is regulated as following:

$f_{2min} \leq f_2 \leq f_{2max}$

$f_{2max} = f_{2b} = s_{T\max} \cdot f_{1n}$ (f_{2max} is set to breakdown frequency f_{2b} at which the maximum positive torque is developed).

$-f_{2b} \leq f_{2min} \leq 0$ (f_{2min} is specified to a value at which a desired value of minimum (maximum negative) torque is developed).

In the above operation, the steady-state (constant) value of the motor actual speed n is less than or equal to the set speed n^*.
The speed error $(n^* - n)$ depends on the load torque; that is, the larger the load torque, the higher the speed error is.

EXAMPLE

The speed of a 3ϕ, 460 V, 1740 rpm, 60 Hz, four-pole induction motor is regulated by constant volts per hertz (V_1/f_1) control. The parameters of the induction motor are $R_1 = 0.25\ \Omega$, $R_2' = 0.2\ \Omega$, $X_1 = X_2' = 0.5\ \Omega$, $X_m = 30\ \Omega$. Determine (a) the stator voltage at stator frequency 20 Hz, and (b) the maximum value of the slip frequency.

Solution

(a) Under constant volts per hertz control,

$$\frac{V_1}{f_1} = \frac{V_{1n}}{f_{1n}} \quad \Longrightarrow \quad V_1 = \frac{f_1}{f_{1n}} V_{1n} = \frac{20}{60} \times \frac{460}{\sqrt{3}} = 88.5 \text{ V}$$

(b) Under constant volts per hertz control, the maximum value of the slip frequency is set to the breakdown frequency at which the maximum positive torque is developed.

$$f_{2max} = f_{2b} = s_{T\max} \cdot f_{1n}$$

where

$$s_{T\max} = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}} = \frac{0.2}{\sqrt{(0.24)^2 + (0.5 + 0.5)^2}} = 0.1945$$

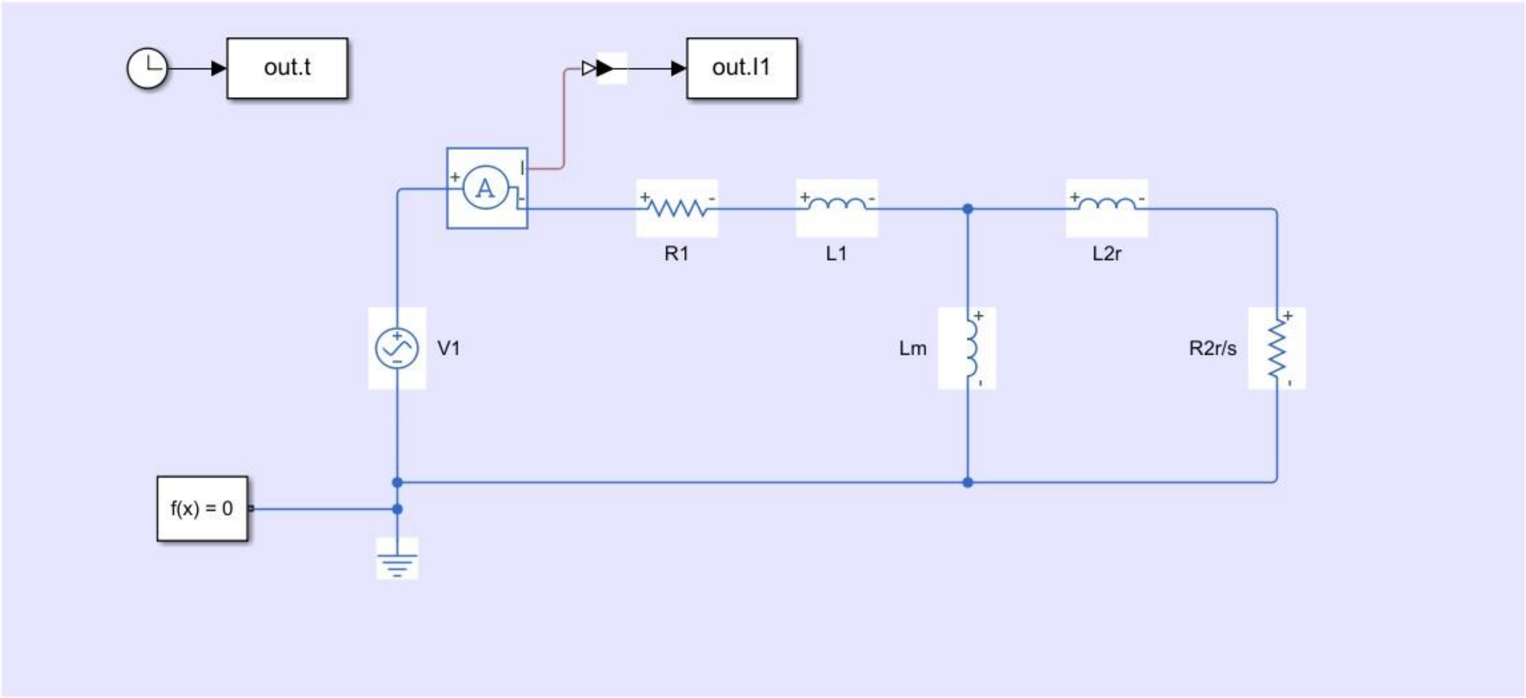
Then,

$$f_{2max} = 0.1945 \cdot 60 = 11.67 \text{ Hz.}$$

Matlab/Simulink Demo
for
Simulation of Induction Machine
at Steady-State Operation

Simulink Model of Induction Machine Based On IEEE Recommended Equivalent Circuit

IEEE Equivalent Circuit of Induction Machine



UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 22

Date: April 22, 2021

by

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Electrical and Computer Engineering Department

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Spring 2021

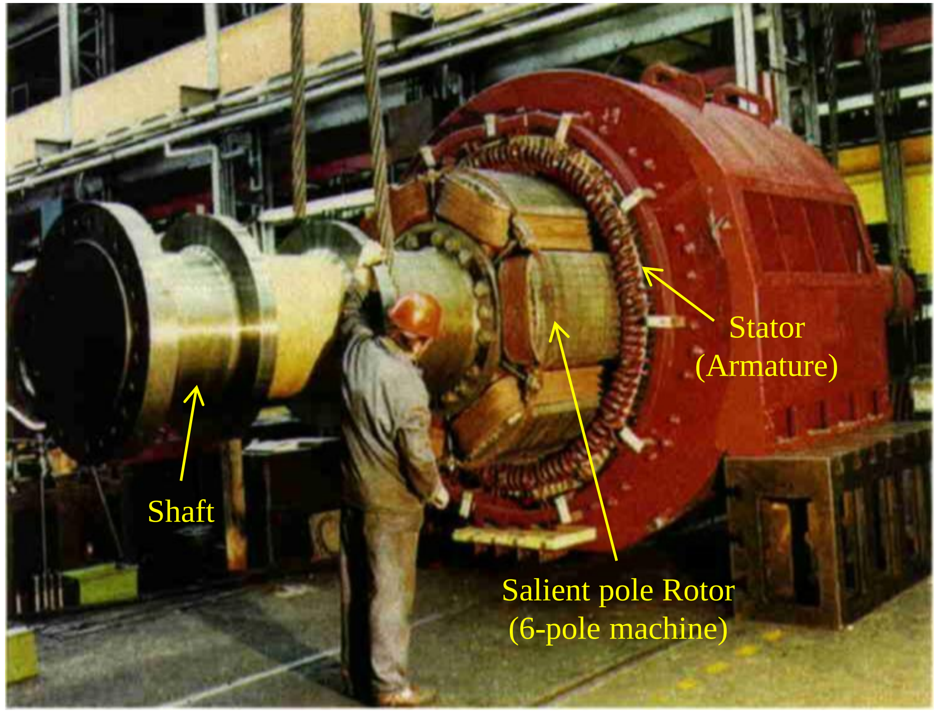
P.S. The pictures, notations, formulas, examples, and statements in these lecture 22 notes have been adopted and adapted mainly from the course textbook "Principles of Electric Machines and Power Electronics" by P. C. Sen, Third Edition, 2013.

SYNCHRONOUS MACHINES

Unlike induction machines, the rotating air gap field and rotor in the synchronous machine rotate at the same speed, which is the synchronous speed ($n_s = 120 \cdot f/p$). Synchronous machines are mainly used as generators. Its rotor poles are excited by a dc current and its stator windings are connected to the ac power supply system.

Construction of Synchronous Machines

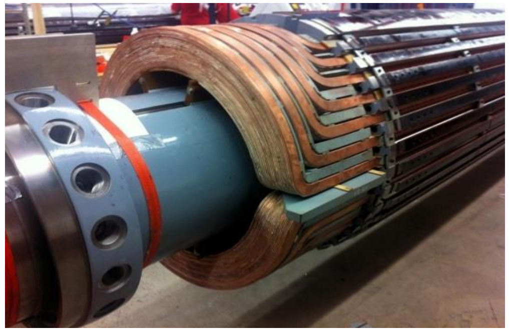
http://engineeringteach.blogspot.com



http://http://emadrlc.blogspot.com

Salient pole Rotor

http://www.starcraft.pro



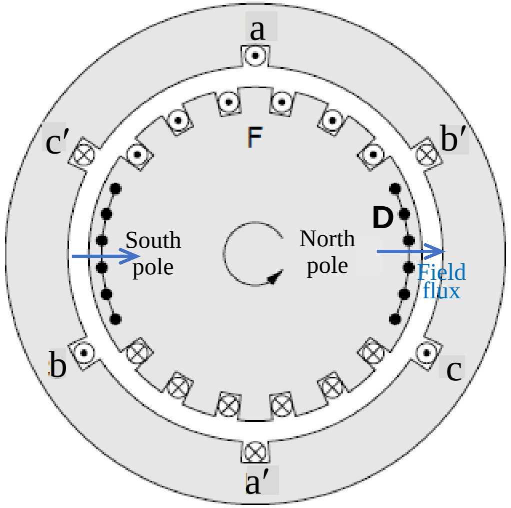
Cylindrical (or nonsalient pole, or round) rotor

Basic Structure of Three-Phase Synchronous Machine

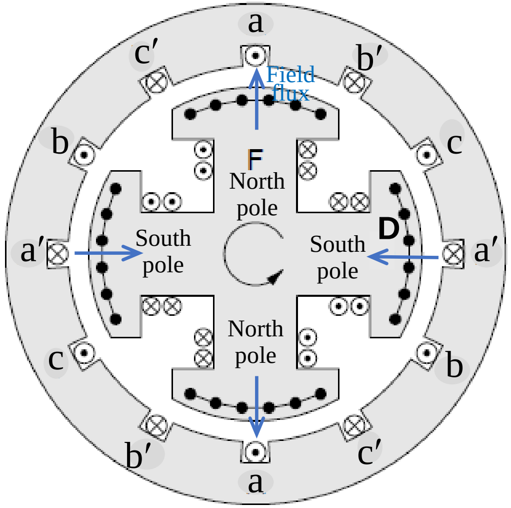
http://top10electrical.blogspot.com

F: Field winding
D: Damper winding

High speed synchronous generators like the ones driven by steam turbines have round (nonsalient pole, or cylindrical) rotor.



Round rotor
p = 2, n = 3600 rpm for $f = 60$ Hz.

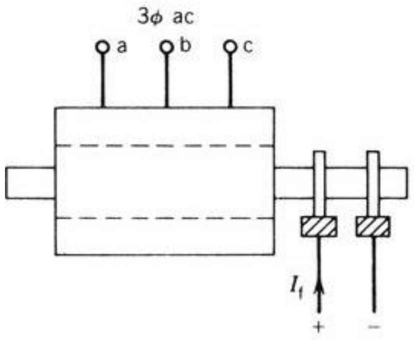


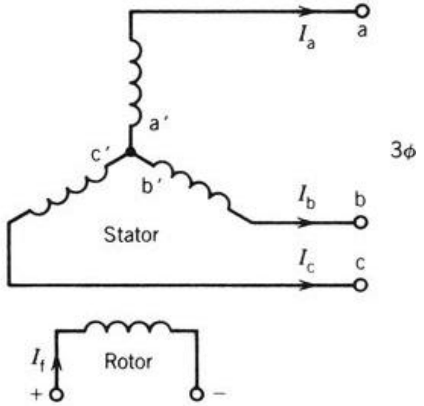
Salient pole rotor
p = 4, n = 1800 rpm for $f = 60$ Hz.

F: Field winding
D: Damper winding

The lower speed synchronous generators like the ones driven by water turbines have salient pole rotor.

I_f : Field current, which is a dc current

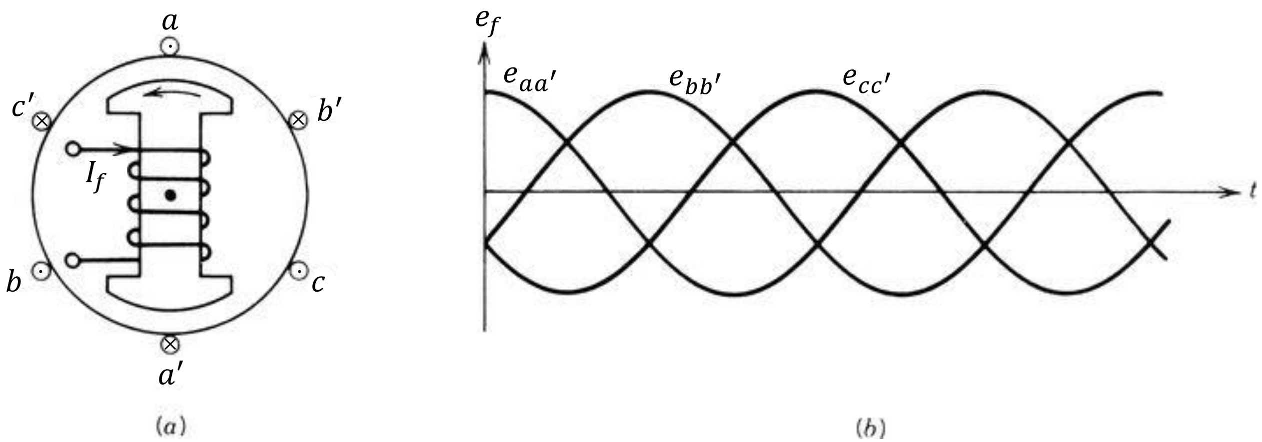




Rotor: Has field winding, which carries dc current. The field winding is fed from an external dc source through slip rings and brushes. The rotor has also cage-type damper winding which is used to start the machine or damp out the transient oscillations.

Stator: Has three-phase distributed windings connected to the ac supply system. The stator winding is also called armature winding, in which the voltage is induced.

Synchronous Generators



Excitation voltages in synchronous machine.

When the field current I_f flows through the rotor field winding, it establishes a sinusoidally distributed flux density in the air gap. If the rotor is rotated by a prime mover such as a turbine, a rotating field is produced in the air gap. This field is called the excitation field which induces excitation voltages in the three-phase stator (armature) windings aa', bb', and cc'. The excitation voltages $e_{aa'}$, $e_{bb'}$ and $e_{cc'}$ as shown in the figure above, have the same magnitudes but are phase shifted by 120 electrical degrees.

The rotor speed n and the frequency f of the induced excitation voltages are related by $n = \frac{120 \cdot f}{p}$

where p is the number of poles.

The excitation voltage in rms is

$$E_f = 4.44 \cdot f \cdot \Phi_f \cdot N$$

$$E_f \propto n \cdot \Phi_f$$

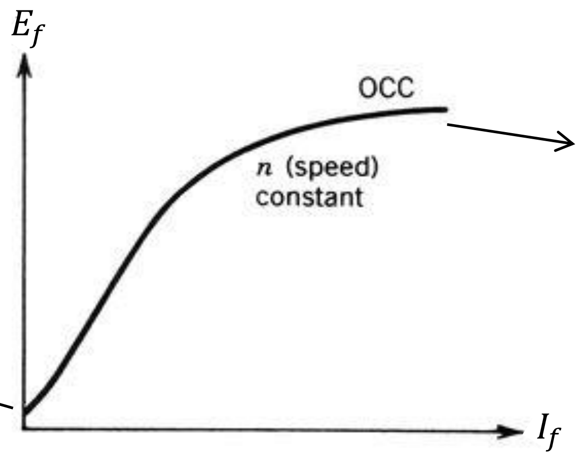
where Φ_f is the air gap flux per pole due to the excitation (field) current I_f, and N is the number of turns in each stator phase winding.

Magnetization Characteristic of a Synchronous Machine

$$E_f \propto \; n \cdot \Phi_f$$

The excitation voltage is proportional to the machine speed and excitation flux, and the excitation flux depends on the excitation current I_f.

The induced voltage at $I_f = 0$ is due to the residual magnetism.



E_f levels off as I_f is further increased since Φ_f does not increase linearly with I_f anymore due to the saturation of the magnetic circuit.

Magnetization characteristic or open circuit characteristic (OCC) of a synchronous machine.

Space Phasor Diagram of a Synchronous Machine

The stator current $\boldsymbol{I_a}$ will flow when the stator terminals are connected to a three-phase load.

$\boldsymbol{\Phi_f}$: Field flux due to I_f.

$\boldsymbol{\Phi_{ar}}$: Armature reaction flux due to $\boldsymbol{I_a}$.

$\boldsymbol{\Phi_r}$: The net (resultant) air gap flux. The rotating air gap field.

$$\boldsymbol{\Phi_r} = \boldsymbol{\Phi_f} + \boldsymbol{\Phi_{ar}}$$

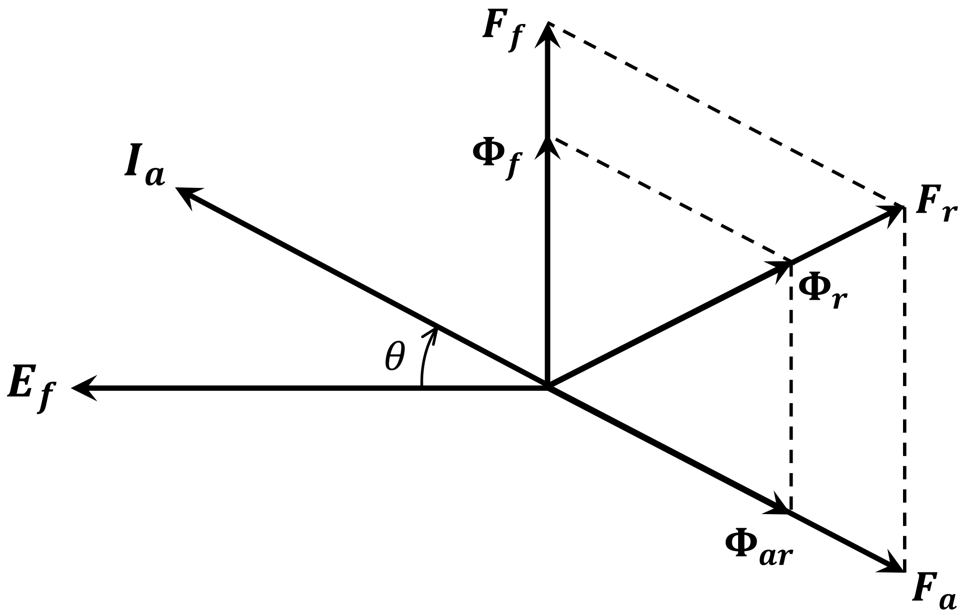
Note that the resultant and the component fluxes rotate in the air gap at the same speed given by

$$n = 120 \cdot f/p$$

$\boldsymbol{F_f}$: Rotor field mmf due to I_f.

$\boldsymbol{F_a}$: Armature reaction mmf due to $\boldsymbol{I_a}$.

$\boldsymbol{F_r}$: Resultant mmf. $\boldsymbol{F_r} = \boldsymbol{F_f} + \boldsymbol{F_a}$



The excitation voltage is the time derivative of the field flux, and thus, its phasor $\boldsymbol{E_f}$ leads field flux phasor $\boldsymbol{\Phi_f}$ by 90°.

Assumption: $\boldsymbol{I_a}$ lags $\boldsymbol{E_f}$ by angle θ.

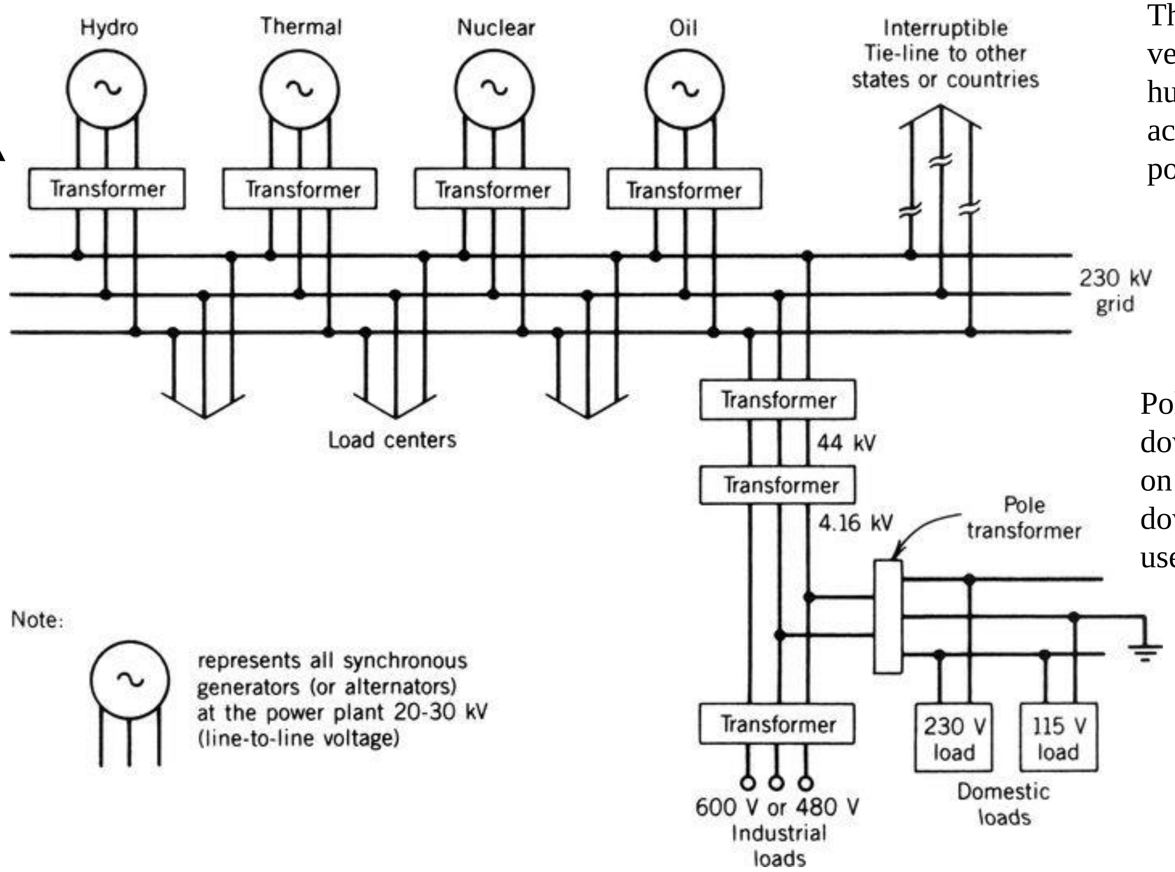
The armature reaction mmf $\boldsymbol{F_a}$ is phase shifted from the armature (stator) current $\boldsymbol{I_{as}}$ by 180° since $\boldsymbol{F_a}$ has tendency to oppose the field mmf $\boldsymbol{F_f}$.

The Infinite Bus

Transformers are used to step up the generator voltages (20 – 30 kV) to the infinite bus voltage (e.g., 230 kV).

The power transmission is at very high voltage levels (in hundreds of kilovolts) to achieve higher efficiency of power transmission.

Pole transformer is a step down transformer mounted on a utility pole, and it steps down the voltage to the level used by the customer.



Infinite bus (or grid) system.

The synchronous generators, in general, are connected to a power supply system known as an *infinite bus*, or *grid*. Because a large number of synchronous generators of large sizes are connected together, the voltage and frequency of the infinite bus hardly change.

Equivalent Circuit Model of Synchronous Machine

$\boldsymbol{\Phi_f}$: Excitation (field) flux due to I_f.

$\boldsymbol{\Phi_a}$: Armature flux due to $\boldsymbol{I_a}$.

$$\boldsymbol{\Phi_a = \Phi_{al} + \Phi_{ar}}$$

$\boldsymbol{\Phi_{al}}$: Leakage flux, which links with the stator winding only.

$\boldsymbol{\Phi_{ar}}$: Armature reaction flux, which links with both the stator and field windings.

$\boldsymbol{\Phi_r}$: The net (resultant) air gap flux. It is the rotating air gap field.

$$\boldsymbol{\Phi_r = \Phi_f + \Phi_{ar}}$$

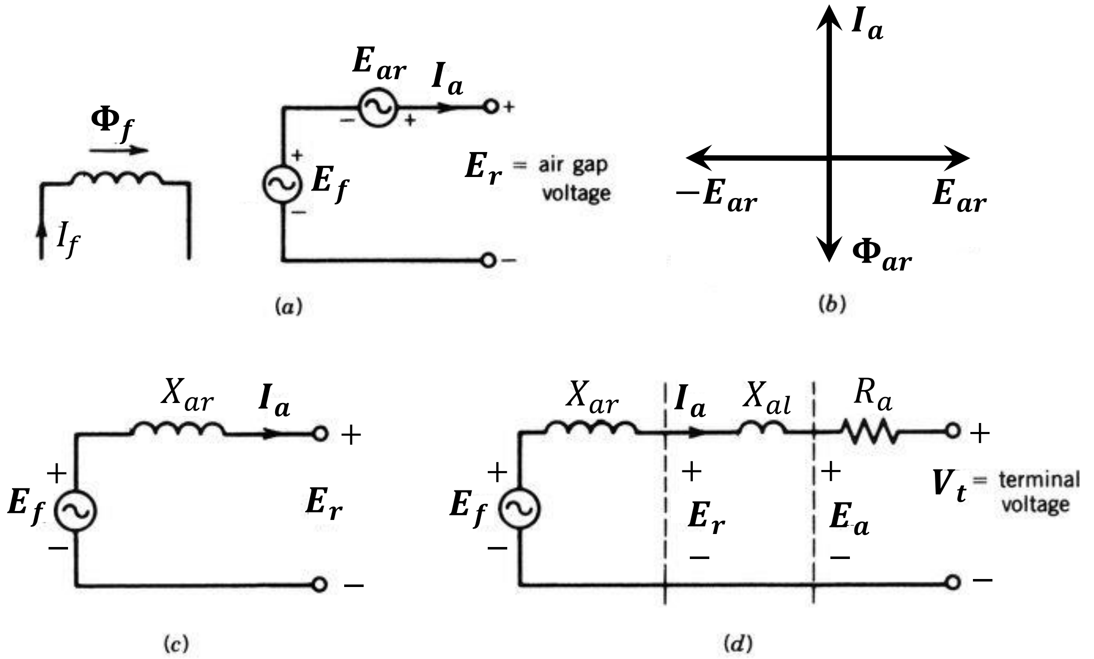
The resultant voltage (air gap voltage) $\boldsymbol{E_r}$ induced in the stator winding is the time derivative of the resultant air gap flux.

$$\boldsymbol{E_r = E_f + E_{ar} \Rightarrow E_f = -E_{ar} + E_r}$$

where the excitation voltage $\boldsymbol{E_f}$ can be found from the open-circuit curve, and the armature reaction voltage $\boldsymbol{E_{ar}}$ depends on $\boldsymbol{\Phi_{ar}}$ and hence on $\boldsymbol{I_a}$.

$$\boldsymbol{-E_{ar}} = \boldsymbol{I_a} \cdot j \cdot X_{ar}$$
$$\boldsymbol{E_f} = \boldsymbol{I_a} \cdot j \cdot X_{ar} + \boldsymbol{E_r}$$

X_{ar} : Reactance of armature reaction (or, magnetizing reactance).

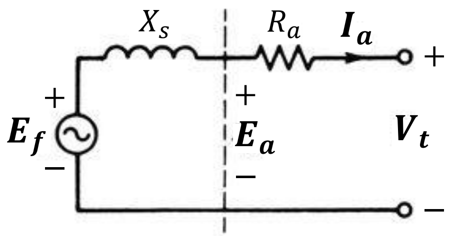


Equivalent circuit of a synchronous machine.

If the stator winding resistance R_a and the stator leakage reactance X_{al} (which accounts for the leakage flux $\boldsymbol{\Phi_{al}}$) are included, the stator phase winding terminal voltage $\boldsymbol{V_t}$ becomes

$$\boldsymbol{V_t} = \boldsymbol{E_r} - \boldsymbol{I_a} \cdot j \cdot X_{al} - \boldsymbol{I_a} \cdot R_a$$

$$\boldsymbol{V_t} = \boldsymbol{E_f} - \boldsymbol{I_a} \cdot j \cdot X_{ar} - \boldsymbol{I_a} \cdot j \cdot X_{al} - \boldsymbol{I_a} \cdot R_a$$



The equivalent circuit of synchronous machine.

$X_s = X_{ar} + X_{al}$, X_s: *Synchronous reactance*

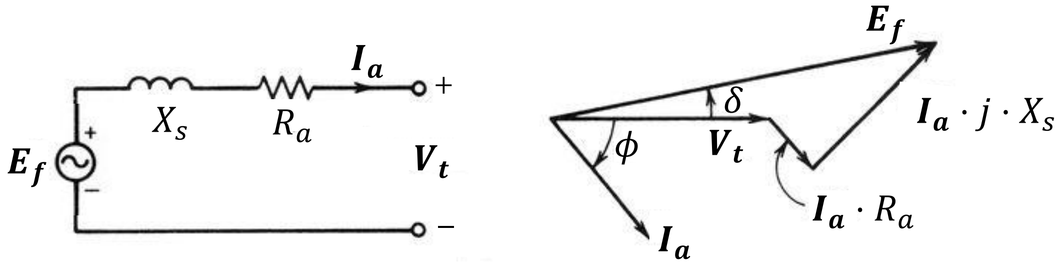
$\boldsymbol{Z_s} = R_a + j \cdot X_s$, $\boldsymbol{Z_s}$: *Synchronous impedance*

The synchronous reactance X_s takes into account all the flux, magnetizing as well as leakage, produced by the armature (stator) current I_a.

A remark: At steady-state operation when the rotor speed is constant at synchronous speed, the relative speed of the resultant air gap flux with respect to the rotor is zero. As a result, there is no voltage induced in the rotor field winding due to the air gap flux. In brief, the circuit time constant of the rotor field winding is not considered at steady-state operation.

Phasor Diagram of Synchronous Machine

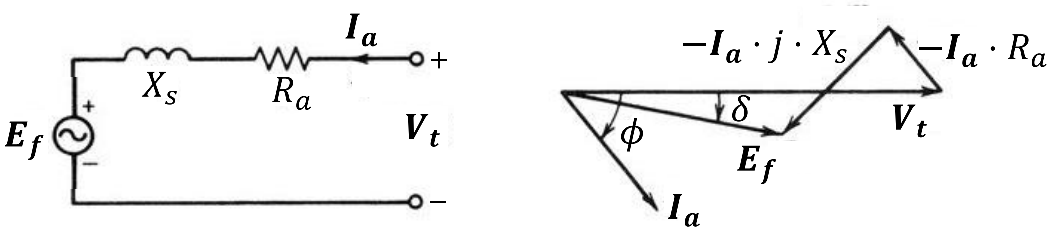
The phasor diagram is based on the per-phase equivalent circuit of the synchronous machine. The terminal voltage is taken as the reference phasor in constructing the phasor diagram. That is, $\boldsymbol{V_t} = V_t\angle 0^\circ = V_t$.



The per-phase equivalent circuit and the phasor diagram of synchronous generator.

$\boldsymbol{E_f} = \boldsymbol{V_t} + \boldsymbol{I_a} \cdot R_a + \boldsymbol{I_a} \cdot j \cdot X_s = E_f\angle\delta\,,\ \ \delta > 0^\circ$ Equation for the synchronous generator.

The synchronous generator is considered to deliver a lagging current $(\phi < 0^\circ)$ to the load or infinite bus represented by $\boldsymbol{V_t}$.



The per-phase equivalent circuit and the phasor diagram of synchronous motor.

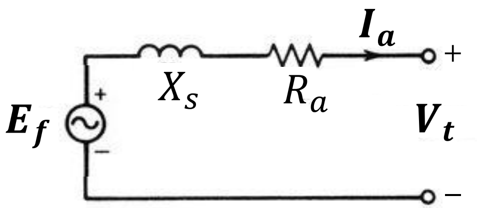
$\boldsymbol{E_f} = \boldsymbol{V_t} - \boldsymbol{I_a} \cdot R_a - \boldsymbol{I_a} \cdot j \cdot X_s = E_f\angle\delta\,,\ \ \delta < 0^\circ$ Equation for the synchronous motor.

The synchronous motor is considered to draw a lagging current $(\phi < 0^\circ)$ from the infinite bus.

Note that the angle δ between $\boldsymbol{V_t}$ and $\boldsymbol{E_f}$ is positive for the generating action and negative for the motoring action. The angle δ is called as the *power angle* or *torque angle*.

Power and Torque Characteristics of Synchronous Machine

Analytical expressions at steady-state operation are derived for i) the power transfer between the machine and the infinite bus and ii) the torque developed by the machine.



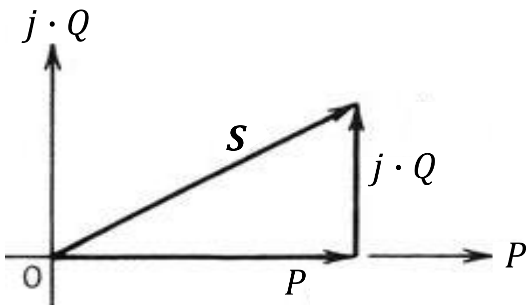
The per-phase equivalent circuit of synchronous generator.

$$\boldsymbol{V_t} = V_t\angle 0^\circ\,,\quad \boldsymbol{E_f} = E_f\angle\delta\,,\quad \boldsymbol{I_a} = I_a\angle\phi \quad \text{and} \quad \boldsymbol{Z_s} = R_a + j\cdot X_s = |Z_s|\angle\theta_s$$

The per-phase complex power $\boldsymbol{S}$ in [VA] at the terminals is

$$\boldsymbol{S} = \boldsymbol{V_t}\cdot\boldsymbol{I_a^*} = V_t\cdot I_a\cdot\cos(-\phi) + j\cdot V_t\cdot I_a\cdot\sin(-\phi) = P + j\cdot Q$$

where $\boldsymbol{I_a^*}$ is the conjugate of the current phasor $\boldsymbol{I_a}$.



Complex power phasor for a lagging reactive power. That is, $\phi < 0^\circ$ and $Q > 0$.

$$\boldsymbol{I_a^*} = \left(\frac{\boldsymbol{E_f} - \boldsymbol{V_t}}{\boldsymbol{Z_s}}\right)^* = \frac{\boldsymbol{E_f^*}}{\boldsymbol{Z_s^*}} - \frac{\boldsymbol{V_t^*}}{\boldsymbol{Z_s^*}} = \frac{E_f\angle-\delta}{|Z_s|\angle-\theta_s} - \frac{V_t\angle 0^\circ}{|Z_s|\angle-\theta_s} = \frac{E_f}{|Z_s|}\angle(\theta_s - \delta) - \frac{V_t}{|Z_s|}\angle\theta_s$$

$$\boldsymbol{S} = \frac{V_t\cdot E_f}{|Z_s|}\angle(\theta_s - \delta) - \frac{{V_t}^2}{|Z_s|}\angle\theta_s \ \text{ VA/phase}$$

The real (active) power P and the reactive power Q per phase are

$$P = \frac{V_t\cdot E_f}{|Z_s|}\cos(\theta_s - \delta) - \frac{{V_t}^2}{|Z_s|}\cos(\theta_s) \ \text{ W/phase} \quad \text{and} \quad Q = \frac{V_t\cdot E_f}{|Z_s|}\sin(\theta_s - \delta) - \frac{{V_t}^2}{|Z_s|}\sin(\theta_s) \ \text{ VAR/phase}$$

To simplify the derivation of expressions for the power and torque developed by a synchronous machine, the armature resistance R_a and the core losses are neglected. Then, $|Z_s| = X_s$ and $\theta_s = 90°$. As a result, for a three-phase machine,

$$P_{3phase} = \frac{3 \cdot V_t \cdot E_f}{X_s} \sin(\delta) = P_{\max} \sin(\delta) \text{ W} \quad \text{where} \quad P_{\max} = \frac{3 \cdot V_t \cdot E_f}{X_s}$$

$$Q_{3phase} = \frac{3 \cdot V_t \cdot E_f}{X_s} \cos(\delta) - \frac{3 \cdot {V_t}^2}{X_s} \text{ VAR}$$

Because the stator losses are neglected, the power developed at the terminals is also the air gap power. The torque developed by the machine is

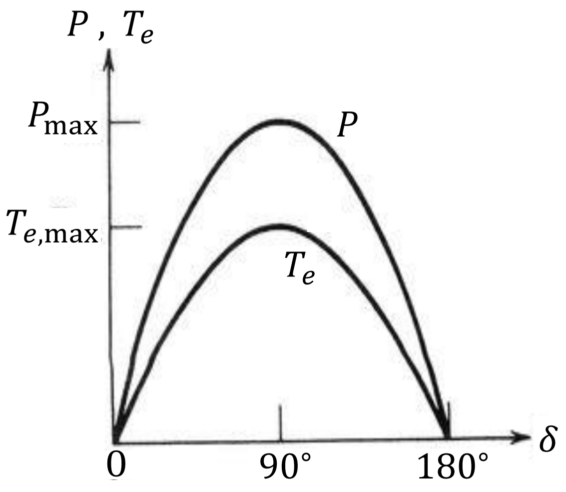
$$T_e = \frac{P_{3phase}}{\omega_{syn}} = \frac{3}{\omega_{syn}} \cdot \frac{V_t \cdot E_f}{X_s} \sin(\delta) = T_{e,\max} \cdot \sin(\delta) \text{ Nm} \quad \text{for} \quad T_{e,\max} = \frac{3}{\omega_{syn}} \cdot \frac{V_t \cdot E_f}{X_s} = \frac{P_{\max}}{\omega_{syn}}$$

where $\omega_{syn} = (4\pi/p) \cdot f$ is the synchronous speed in [rad/s], f is the stator electrical frequency in [Hz], and p is the number of machine poles. The synchronous speed in rpm is $n_s = (60/2\pi) \cdot \omega_{syn}$.

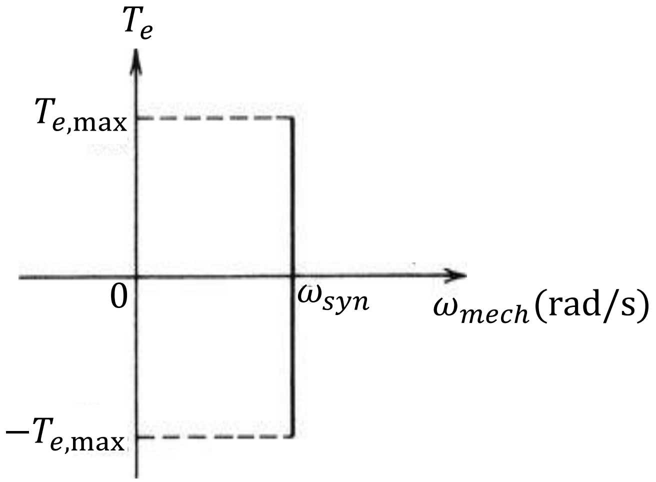
Note that both power and torque vary sinusoidally with the angle δ, which is called the *power angle* or *torque angle*.

$P_{\max}$ and $T_{e,\max}$ are known as the *static stability limits*.

The maximum torque $T_{e,\max}$ is also known as the *pull-out torque*.



Power and torque versus angle.



Torque versus speed.

UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ECE 4363 – Electromechanical Energy Conversion

Lecture 23

Date: April 27, 2021

by

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University of Houston, Houston, TX

Spring 2021

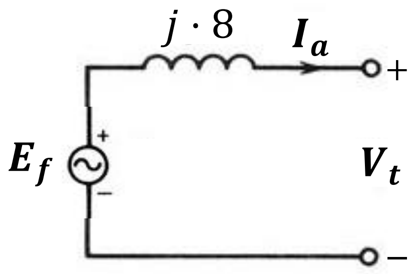
P.S. The pictures, notations, formulas, examples, and statements in these lecture 23 notes have been adopted and adapted mainly from the course textbook “Principles of Electric Machines and Power Electronics” by P. C. Sen, Third Edition, 2013.

EXAMPLE 6.3 from the textbook

A 3ϕ, 5 kVA, 208 V (line-to-line), four-pole, 60 Hz, star-connected synchronous generator has negligible winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage. The machine is operated in parallel with a 3ϕ, 208 V (line-to-line), 60 Hz power supply. (a) Determine the excitation voltage and the power angle when the generator is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition. (b) If the field excitation current is now increased by 20 percent (without changing the input prime mover power and the output real power at the terminals), find the stator current, power factor, and reactive power in kVAR supplied by the generator. Assume that the excitation voltage change linearly with the excitation current. (c) With the field current as in (a), the prime mover power is slowly increased. What is the steady-state (or static) stability limit? What are the corresponding values of the stator (or armature) current, power factor, and reactive power at this maximum power transfer condition ?

Solution

(a)



The per-phase equivalent circuit for the generator.

$$V_t = \frac{208}{\sqrt{3}} = 120 \ \text{V/phase}, \ \ \boldsymbol{V_t} = V_t\angle 0°$$

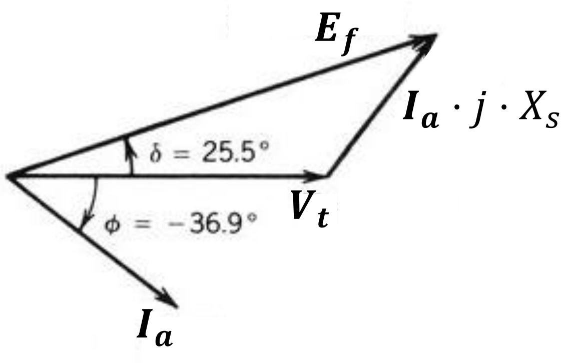
$$I_a = \frac{5000}{\sqrt{3}\cdot 208} = 13.9 \ \text{A}, \ \ \text{PF} = \cos(\phi) = 0.8 \ \text{lagging} \ \Rightarrow \phi = -36.9°, \ \ \boldsymbol{I_a} = I_a\angle\phi$$

$$\boldsymbol{E_f} = V_t\angle 0° + \boldsymbol{I_a}\cdot j\cdot X_s$$

$$\boldsymbol{E_f} = 120\angle 0° + 13.9\angle -36.9° \cdot 8\angle 90° = 206.9\angle 25.5°$$

Excitation voltage: $E_f = 206.9$ V/phase and Power angle: $\delta = +25.5°$

$$\boldsymbol{E_f} = E_f\angle\delta$$



$$\boldsymbol{V_t} = V_t\angle 0^\circ = 120\ \mathrm{V}$$

$$\boldsymbol{I_a} = I_a\angle\phi = 13.9\angle - 36.9^\circ\ \mathrm{A}$$

$$\boldsymbol{E_f} = E_f\angle\delta = 206.9\angle 25.5^\circ\ \mathrm{V}$$

The phasor diagram of the generator.

(b) The new excitation voltage is

$$E_f' = 1.2 \cdot 206.9 = 248.28\ \mathrm{V}$$

Because the real (active) power at the output terminals remains the same

$$\frac{V_t \cdot E_f}{X_s}\sin(\delta) = \frac{V_t \cdot E_f'}{X_s}\sin(\delta') \Rightarrow E_f \cdot \sin(\delta) = E_f' \cdot \sin(\delta') \Rightarrow \sin(\delta') = \frac{E_{af}}{E_{af}'} \cdot \sin(\delta) = \frac{\sin(25.5^\circ)}{1.2} \Rightarrow \delta' = 21^\circ$$

The stator current is

$$\boldsymbol{I_a} = \frac{\boldsymbol{E_f'} - \boldsymbol{V_t}}{j \cdot X_s} = \frac{248.28\angle 21^\circ - 120\angle 0^\circ}{8\angle 90^\circ} = \frac{142.87\angle 38.52^\circ}{8\angle 90^\circ} = 17.86\angle - 51.5^\circ\ \mathrm{A}$$

Power factor $= \cos(51.5^\circ) = 0.62$ lagging

Reactive power $Q_{3phase} = 3 \cdot V_t \cdot I_a \cdot \sin(51.5^\circ) \times 10^{-3}\ \mathrm{kVA} = 3 \cdot 120 \cdot 17.86 \cdot 0.78 \times 10^{-3} = 5.03\ \mathrm{kVAR}$

Or,

$$Q_{3phase} = \frac{3 \cdot V_t \cdot E_f'}{X_s}\cos(\delta') - \frac{3 \cdot {V_t}^2}{X_s}$$

$$Q_{3phase} = 3 \cdot \left(\frac{120 \times 248.28}{8} \cdot \cos(21°) - \frac{120^2}{8}\right) \times 10^{-3}\ \text{kVAR}$$

$$Q_{3phase} = 3 \cdot (3476.86 - 1800) \times 10^{-3}\ \text{kVAR} = 5.03\ \text{kVAR}$$

(c) The maximum power transfer occurs at $\delta = 90°$

$$P_{\max} = \frac{3 \cdot V_t \cdot E_f}{X_s} = \frac{3 \cdot 120 \cdot 206.9}{8} = 9.32\ \text{kW}$$

$$\boldsymbol{I_a} = \frac{\boldsymbol{E_f} - \boldsymbol{V_t}}{j \cdot X_s} = \frac{206.9\angle + 90° - 120\angle 0°}{8\angle 90°} = 29.9\angle 30.1°\ \text{A}$$

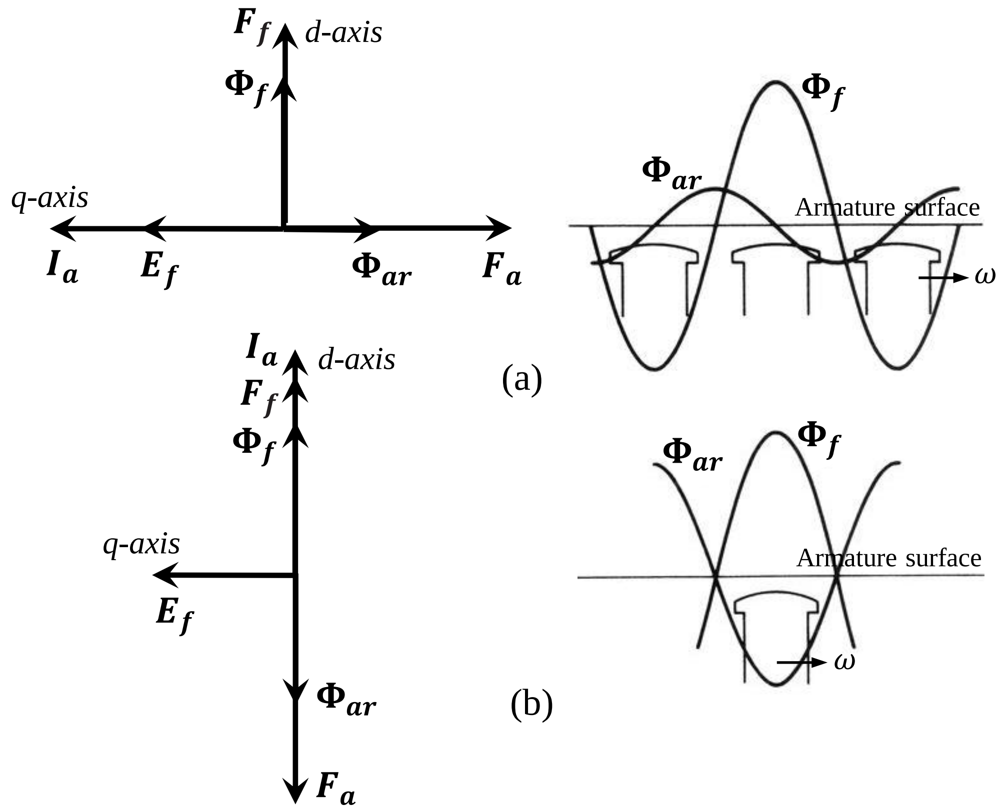
Stator current = $I_s = 29.9$ A, Power factor = $\cos(30.1°) = 0.865$ leading

The reactive power at $\delta = 90°$ is

$$Q_{3phase} = -\frac{3 \cdot {V_t}^2}{X_s} = -\frac{3 \cdot (120)^2}{8} = -5.4\ \text{kVAR}$$

Salient-Pole Synchronous Machines

Due to the nonuniform air gaps, the magnetic reluctance is low along the field (rotor) poles and high between the field poles. The armature reaction mmf $\boldsymbol{F_a}$ (which is due to the armature current $\boldsymbol{I_a}$) produces more flux if it is acting along the pole axis, called the *d-axis* (*direct-axis*), and less flux if it is acting along the interpolar axis, called the *q-axis* (*quadrature-axis*). By convention, the *q-axis* leads the *d-axis* by 90° in the counter-clockwise direction.



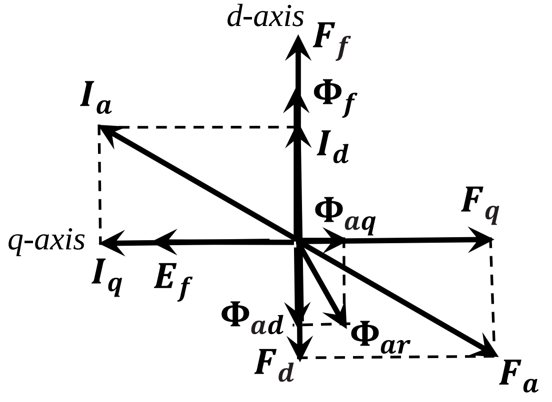
(a)

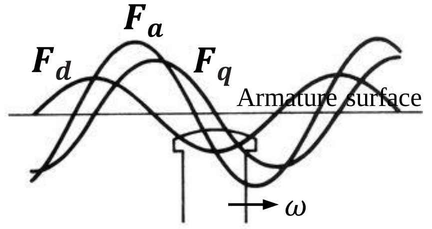
(b)

In Figure (a) on the left, the stator current $\boldsymbol{I_a}$ is in phase with the excitation voltage $\boldsymbol{E_f}$. That is, $\boldsymbol{I_a}$ and $\boldsymbol{F_a}$ are acting along the *q-axis* and negative *q-axis*, respectively. As a result, the armature reaction flux $\boldsymbol{\Phi_{ar}}$ has a smaller amplitude.

In Figure (b) on the left, the stator current $\boldsymbol{I_a}$ lags the excitation voltage $\boldsymbol{E_f}$ by 90°. That is, $\boldsymbol{I_a}$ and $\boldsymbol{F_a}$ are acting along the *d-axis* and negative *d-axis*, respectively. As a result, the armature reaction flux $\boldsymbol{\Phi_{ar}}$ has a larger amplitude.

d-q Currents and Reactances in a Salient-Pole Synchronous Machine





In the figure on the left, the stator current $\boldsymbol{I_a}$ lags the excitation voltage $\boldsymbol{E_f}$ by less than 90°. That is, $\boldsymbol{I_a}$ and $\boldsymbol{F_a}$ are not acting along either the *q-axis* or the *d-axis*. The analysis is then done by resolving $\boldsymbol{I_a}$, $\boldsymbol{F_a}$ and $\boldsymbol{\Phi_{ar}}$ into the respective *d-axis* and *q-axis* components. Note that $\boldsymbol{\Phi_{ar}}$ lags $\boldsymbol{F_a}$ since the reactance along *d-axis* is larger than the reactance along the *q-axis*.

The armature current $\boldsymbol{I_a}$ can be resolved into two components: 1) the *d* component $\boldsymbol{I_d}$ that is acting along the *d-axis* and 2) the *q* component $\boldsymbol{I_q}$ that is acting along the *q-axis*. Similarly, the armature reaction mmf $\boldsymbol{F_a}$ can be resolved into *d* and *q* components $\boldsymbol{F_d}$ and $\boldsymbol{F_q}$. The current components $\boldsymbol{I_d}, \boldsymbol{I_q}$ or the mmf components $\boldsymbol{F_d}$, $\boldsymbol{F_q}$ produce armature reaction flux components $\boldsymbol{\Phi_{ad}}$ and $\boldsymbol{\Phi_{aq}}$ along the *d-* and *q-* axes, respectively. These two fluxes are represented by the following reactances:

X_{ad}: *d-axis* armature reaction (or, magnetizing) reactance to account for the flux $\boldsymbol{\Phi_{ad}}$ produced by the *d-axis* current $\boldsymbol{I_d}$.

X_{aq}: *q-axis* armature reaction (or, magnetizing) reactance to account for the flux $\boldsymbol{\Phi_{aq}}$ produced by the *q-axis* current $\boldsymbol{I_q}$.

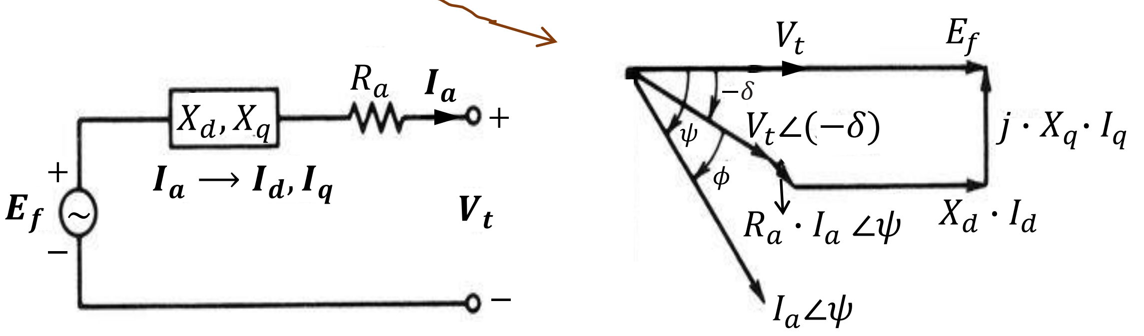
The *d-axis* and the *q-axis* synchronous reactances are obtained by

$X_d = X_{ad} + X_{al}$: *d-axis* synchronous reactance
$X_q = X_{aq} + X_{al}$: *q-axis* synchronous reactance

where X_{al} is the armature leakage reactance which takes into account the leakage flux produced by the armature current. Note that X_{al} is assumed to be the same for both *d-axis* and *q-axis* currents since the leakage fluxes are mainly confined to the stator frame. Note also that $X_d > X_q$ since the air gap along the *d-axis* is smaller.

Phasor Diagram and Equivalent Circuit of a Salient-Pole Synchronous Machine

The phasor diagram is based on the per-phase equivalent circuit of the salient pole synchronous machine. The terminal voltage is taken as the reference phasor in constructing the phasor diagram. That is, $\boldsymbol{V_t} = V_t\angle 0^\circ = V_t$.



Assumption:
$\delta > 0^\circ$, $\phi < 0^\circ$.

The per-phase equivalent circuit and the phasor diagram of a salient-pole synchronous generator. Assumption: $\delta > 0^\circ$, $\phi < 0^\circ$.

$$\boxed{\boldsymbol{E_f} = \boldsymbol{V_t} + R_a \cdot \boldsymbol{I_a} + j \cdot X_d \cdot \boldsymbol{I_d} + j \cdot X_q \cdot \boldsymbol{I_q}} \quad \text{and} \quad \boldsymbol{I_a} = \boldsymbol{I_d} + \boldsymbol{I_q}$$

$\boldsymbol{V_t} = V_t\angle 0^\circ$, $\boldsymbol{E_f} = E_f\angle\delta$, $\boldsymbol{I_a} = I_a\angle\phi$, $\boldsymbol{I_d} = I_d\angle(\delta - 90^\circ)$ and $\boldsymbol{I_q} = I_q\angle\delta$.

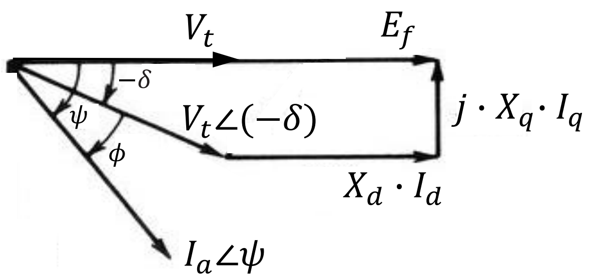
Note that the angles of $\boldsymbol{I_q}$ and $\boldsymbol{E_f}$ are the same.

$$\boxed{E_f = V_t\angle(-\delta) + R_a \cdot I_a \angle\psi + X_d \cdot I_d + j \cdot X_q \cdot I_q}$$

Equations for generator action

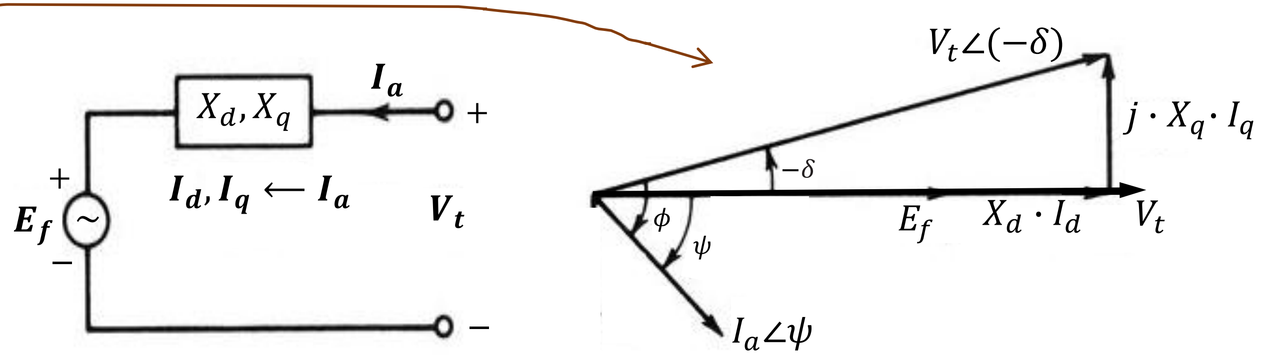
$$\psi = \phi - \delta$$

where the angle ψ between $\boldsymbol{E_f}$ and $\boldsymbol{I_a}$ is the internal power factor angle, and the angle ϕ between $\boldsymbol{V_t}$ and $\boldsymbol{I_a}$ is the terminal power factor angle.



Assumption:
$\delta > 0°, \phi < 0°.$

The phasor diagram of a salient-pole synchronous generator by neglecting the armature resistance R_a. Assumption: $\delta > 0°$, $\phi < 0°$.



Assumption:
$\delta < 0°, \phi < 0°.$

The per-phase equivalent circuit and the phasor diagram of a salient-pole synchronous motor by neglecting the armature resistance R_a. Assumption: $\delta < 0°$, $\phi < 0°$.

$$\boxed{\boldsymbol{V_t} = R_a \cdot \boldsymbol{I_a} + j \cdot X_d \cdot \boldsymbol{I_d} + j \cdot X_q \cdot \boldsymbol{I_q} + \boldsymbol{E_f}}$$

$$\boldsymbol{V_t} = V_t\angle 0°, \quad \boldsymbol{E_f} = E_f\angle\delta, \quad \boldsymbol{I_a} = I_a\angle\phi, \quad \boldsymbol{I_d} = I_d\angle\left(\delta - 90°\right) \text{ and } \boldsymbol{I_q} = I_q\angle\delta.$$

$$\boxed{V_t\angle(-\delta) = E_f + R_a \cdot I_a\angle\psi + X_d \cdot I_d + j \cdot X_q \cdot I_q}$$

$$\psi = \phi - \delta$$

Equations for motor action

Note that

$$\boldsymbol{I_a} = \boldsymbol{I_d} + \boldsymbol{I_q} \;\; \Longrightarrow \;\; I_a\angle\phi = I_d\angle(\delta - 90^\circ) + I_q\angle\delta \;\; \Longrightarrow \;\; I_a\angle(\phi - \delta) = I_d\angle(-90^\circ) + I_q\angle 0^\circ.$$

Then, for $\psi = \phi - \delta$,

$$I_a\angle\psi = I_d\angle(-90^\circ) + I_q\angle 0^\circ = I_q - j \cdot I_d$$

which means

$$I_d = -I_a \cdot \sin(\psi)$$

$$I_q = I_a \cdot \cos(\psi)$$

In brief, the currents I_d and I_q can be calculated by the above two equations if the internal power factor angle ψ is known.

However, the terminal power factor angle ϕ is normally known, and the angle δ is calculated to obtain ψ as following. The below calculations are for generator mode where the armature resistance R_a is neglected and it is assumed that $\delta > 0°$ and $\phi < 0°$:

$$I_d = -I_a \cdot \sin(\psi) = -I_a \cdot \sin(\phi - \delta)$$

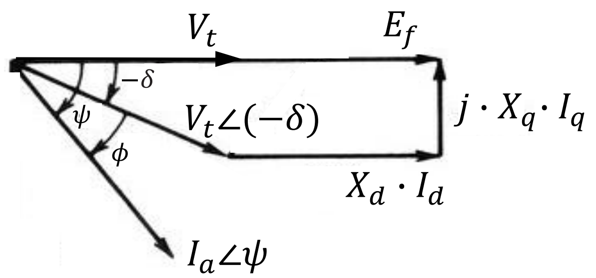
$$I_q = I_a \cdot \cos(\psi) = I_a \cdot \cos(\phi - \delta)$$

From the phasor diagram on the right,

$$V_t \cdot \sin(\delta) = X_q \cdot I_q = I_a \cdot X_q \cdot \cos(\phi - \delta)$$

Or,

$$V_t \cdot \sin(\delta) = I_a \cdot X_q \cdot \cos(\phi) \cdot \cos(\delta) + I_a \cdot X_q \cdot \sin(\phi) \cdot \sin(\delta)$$



The phasor diagram of a salient-pole synchronous generator by neglecting the armature resistance R_a. Assumption: $\delta > 0°$, $\phi < 0°$.

Then, dividing both sides of the previous equation by $\sin(\delta)$, one can obtain the following:

$$\tan(\delta) = \frac{I_a \cdot X_q \cdot \cos(\phi)}{V_t - I_a \cdot X_q \cdot \sin(\phi)}$$

for $\delta > 0°$ and $\phi < 0°$.

Furthermore, from the phasor diagram with negligible R_a,

$$E_f = V_t \cdot \cos(-\delta) + I_d \cdot X_d$$

Power and Torque Characteristics of Salient-Pole Synchronous Machine

To simplify the derivation of expressions for the power and torque developed by a salient-pole synchronous machine, the armature resistance R_a and the core losses are neglected.

$$\boldsymbol{V_t} = V_t\angle 0^\circ, \ \ \boldsymbol{I_a} = \boldsymbol{I_d} + \boldsymbol{I_q}, \ \ \boldsymbol{I_d} = I_d\angle\left(\delta - 90^\circ\right), \ \ \boldsymbol{I_q} = I_q\angle\delta$$

The per-phase complex power $\boldsymbol{S}$ at the terminals is

$$\boldsymbol{S} = \boldsymbol{V_t}\cdot\boldsymbol{I_a^*} = V_t\angle 0^\circ\cdot\left(\boldsymbol{I_d}+\boldsymbol{I_q}\right)^* = V_t\angle 0^\circ\cdot\left(I_d\angle\left(\delta-90^\circ\right)+I_q\angle\delta\right)^* = V_t\angle(-\delta)\cdot\left(I_q + j\cdot I_d\right)$$

From the phasor diagram for the generator action ($\delta > 0^\circ$),

$$I_d = \frac{E_f - V_t\cdot\cos(-\delta)}{X_d} \quad \text{and} \quad I_q = \frac{V_t\cdot\sin(\delta)}{X_q}$$

Then,

$$\boldsymbol{S} = \frac{({V_t}^2\cdot\sin(\delta))\angle(-\delta)}{X_q} + \frac{(V_t\cdot E_f)\angle(-\delta+90^\circ)}{X_d} - \frac{({V_t}^2\cdot\cos(-\delta))\angle(-\delta+90^\circ)}{X_d} \ \text{VA/phase}$$

Since $\boldsymbol{S} = P + j\cdot Q$, the real (active) power P and the reactive power Q per phase are

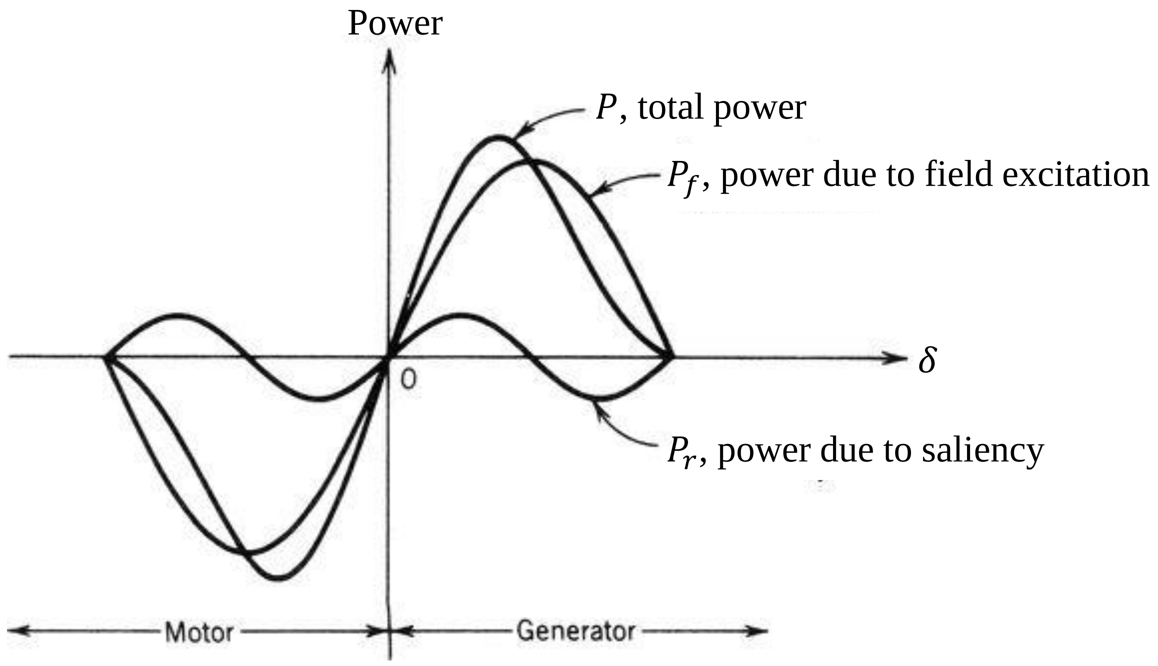
$$P = \frac{V_t\cdot E_f}{X_d}\cdot\sin(\delta) + \frac{{V_t}^2\cdot(X_d - X_q)}{2\cdot X_d\cdot X_q}\cdot\sin(2\delta) \ \text{W/phase}$$

$$Q = \frac{V_t\cdot E_f}{X_d}\cdot\cos(\delta) - {V_t}^2\cdot\left(\frac{\sin^2(\delta)}{X_q}+\frac{\cos^2(\delta)}{X_d}\right) \ \text{VAR/phase}$$

Note that the real power P has two terms:

$$P = \underbrace{\frac{V_t \cdot E_f}{X_d} \cdot \sin(\delta)}_{P_f} + \underbrace{\frac{{V_t}^2 \cdot (X_d - X_q)}{2 \cdot X_d \cdot X_q} \cdot \sin(2\delta)}_{P_r} \text{ W/phase}$$

The first term P_f represents power due to the excitation voltage E_f. The second term P_r represents the effects of salient poles and produces the *reluctance torque*. Note that the expressions for the real and reactive power of the salient pole machine reduce to those of cylindrical rotor machine for $X_d = X_q = X_s$. The real power versus angle characteristic of the salient-pole synchronous machine is shown in the below figure.



Power-angle characteristic of a salient-pole synchronous machine.

The torque developed by the salient-pole synchronous machine is given by

$$T_e = \frac{P_{3phase}}{\omega_{syn}} = \frac{3}{\omega_{syn}} \cdot \left(\frac{V_t \cdot E_f}{X_d} \sin(\delta) + \frac{{V_t}^2 \cdot (X_d - X_q)}{2 \cdot X_d \cdot X_q} \sin(2\delta) \right)$$

where the power (or, torque) angle $\delta > 0°$ for generator action and $\delta \leq 0°$ for motor action. Note again that the developed torque has two terms. The first term is the torque due to the excitation voltage E_f. The second term is the reluctance torque that is due to the saliency $(X_d \neq X_q)$.