

Programming Languages

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Assignment - 2

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Lambda HW

For the following forms apply β -reduction and α -substitution to reduce to lowest form. Indicate at each step the rule that you are applying.

$$1) (\lambda x. x) (\lambda x. x)$$

$$\Rightarrow (\lambda x. x) (\lambda z. z)$$

α -substitution
rename x with z

$$\Rightarrow [(\lambda z. z) / x] x$$

β reduction

$$\Rightarrow (\lambda z. z) = I$$

replace λx with $\lambda z. z$

$$2) (\lambda x. x x) (\lambda x. \lambda y. x x)$$

$$\Rightarrow (\lambda x. x x) (\lambda z. \lambda y. z z)$$

α -sub replace
 x with z

$$\Rightarrow (\lambda z. \lambda y. z z) (\lambda z. \lambda y. z z)$$

β red replace x
with $\lambda z. \lambda y. z z$

$$\Rightarrow (\lambda z. \lambda y. z z) (\lambda x. \lambda y. x x)$$

α sub replace
 z with x

β red replace z
with $\lambda x. \lambda y. x x$

$$\Rightarrow \lambda y. (\lambda x. \lambda y. x x) (\lambda x. \lambda y. x x)$$

$$\Rightarrow (\lambda x. \lambda y. x x) (\lambda x. \lambda y. x x) \text{ same as 3rd step.}$$

It is recursive.

This example is nonterminating
as β reduction is not possible

$$(3) ((\lambda x. (x y)) (\lambda z z))$$

β red replace x
with $\lambda z. z$

$$\Rightarrow \left[\left(\frac{\lambda z. z}{x} \right) y \right]$$

$$\Rightarrow (\lambda z. z) y$$

β red replace z
with y

$$\Rightarrow [y/z] z$$

$$\Rightarrow y$$

$$(4) (\lambda z. z) (\lambda y. y y) (\lambda x. x a)$$

$$\Rightarrow \left[\frac{(\lambda y. y y)}{z} \right] z. (\lambda x. x a)$$

β red replace z
with $\lambda y. y y$

$$\Rightarrow (\lambda y. y y). (\lambda x. x a)$$

β red replace y
with $\lambda x. x a$

$$\Rightarrow (\lambda x. x a) (\lambda x. x a)$$

β red replace
 x with a

$$\Rightarrow (\lambda x. x a) a$$

$$\Rightarrow a a$$

$$(5) (\lambda z. z) (\lambda z. z z) (\lambda z. z y)$$

β red z with $\lambda z. z z$

$$\Rightarrow (\lambda z. z z) (\lambda z. z y)$$

β red z with $\lambda z. z y$

$$\Rightarrow (\lambda z. z y) (\lambda z. z y)$$

β red z with $\lambda z. z y$

$$\Rightarrow (\lambda z. z y) y$$

β red z with y

$$\Rightarrow y y$$

$$(6) (\lambda x. \lambda y. x y y) (\lambda a. a) b$$

β red - x with $\lambda a. a$

$$\Rightarrow (\lambda y (\lambda a. a) y y) b$$

β red - y with b

$$\Rightarrow (\lambda a. a) b b$$

β red - a with b

$$\Rightarrow b b$$

$$(7) (\lambda x. x x) (\lambda y. y x) z$$

$$\Rightarrow (\lambda y. y x) (\lambda y. y x) z$$

$$\Rightarrow ((\lambda y. y x) x) z$$

$$\Rightarrow x x z$$

β red - x with $\lambda y. y x$

β red - y with $\lambda y. y x$

β red - y with x

$$(8) (\lambda x. (\lambda y. (x y)) y) z$$

$$\Rightarrow (\lambda x. (\lambda a. (x a)) y) z$$

$$\Rightarrow \lambda a (z a) y$$

$$\Rightarrow z y$$

α conversion y to a

β red - x with z

β red - a with y

$$(9) ((\lambda x. x x) (\lambda y. y)) (\lambda y. y)$$

β red - ~~with~~ x with $\lambda y. y$

$$\Rightarrow ((\lambda y. y) (\lambda y. y)) (\lambda y. y)$$

β red - y with $\lambda y. y$

$$\Rightarrow (\lambda y. y) (\lambda y. y)$$

β red - y with $\lambda y. y$

$$\Rightarrow (\lambda y. y)$$

$$\Rightarrow I \text{ (Identity element)}$$

$$(10) (((\lambda x. \lambda y (x y)) (\lambda y. y)) w)$$

α conversion - y to a

$$\Rightarrow (((\lambda x. \lambda a. (x a)) (\lambda y. y)) w)$$

β red - replace x with $\lambda y. y$

$$\Rightarrow ((\lambda a. (\lambda y. y) a) w)$$

β red - a with w

$$\Rightarrow (\lambda y. y) w$$

β red - y with w

$$\Rightarrow w$$