0.1 Statement of problem

Suppose we are solving the Dirichlet problem for the Laplace equation in three dimensions on a surface of revolution such that:

$$\Delta u = 0 \qquad \text{in } \Omega \tag{1}$$

$$u = f \qquad \text{on } \Gamma \tag{2}$$

$$u \to 0 \qquad \text{as } \mathbf{x} \to \infty \tag{3}$$

Recall that the kernel-independent fast multipole method uses a continuous distribution of an equivalent density on a surface enclosing a box in the quad- or octree to represent the potential generated by sources in that box, rather than using analytic multipole expansions like the original FMM. This allowed us to construct an FMM that only requires kernel evaluations without sacrificing efficiency. It is also relatively easier to implement, since it requires very few changes to apply to different kernels.

The original KIFMM explored three data sets for the 3D case: densities distributed on the unit sphere, densities distributed uniformly on the unit cube, and densities distributed at the eight corners of the unit cube [Ying, Biros, Zorin]. This project examines the case of another non-uniform distribution, surfaces of revolution. In particular, surfaces of revolution that are rotationally symmetric with respect to the azimuthal angle, or axisymmetric. In trying to construct a KIFMM for this case, there are several issues. I propose that we use [Yao, Martinsson, and Young]'s technique of

The KIFMM is similar to the FMM, apart from how the equivalent densities are represented efficiently, and how the translation operators are computed.

There are two steps, a potential evaluation, and the solve of an integral equation. In particular, the evaluation of the check potential using the original sources, and the inversion of the integral equation to obtain equivalent density. Both steps require discretization. For the integral equation solve, [YBZ] uses Tikhonov regularization.

In the original KIFMM, translation operators are a pre-computation that only differ based on relative position and level in the hierarchical tree. Here, however, the kernel is not so simple. We must look for some other constant relationship between translation operators.

Requirements for KIFMM: smoothness and uniqueness, satisfied if and only if the equivalent surfaces do not intersect, etc...

There are problems that arise for this case.

Left to do: implement actual Green's function, accelerate using SVD in 2D or FFT in 3D.

$$\text{M2L: } \int_{\mathbf{y}^{B,d}} G(\mathbf{x},\mathbf{y})\phi^{B,d}(\mathbf{y})d\mathbf{y} = \int_{\mathbf{y}^{A,u}} G(\mathbf{x},\mathbf{y})\phi^{A,u}(\mathbf{y})d\mathbf{y} \text{ for all } \mathbf{x} \in \mathbf{x}^{B,d}$$

0.2 The Riemann Hypothesis

0.3 Another section

Bibliography

[1] J. B. Conway, *Functions of One Complex Variable I*. Second edition. Springer-Verlag, Graduate Texts in Mathematics **11**, 1991.