First-Order Logic

AIMA Ch.8

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

PL: Pros and Cons

- Propositional logic is declarative
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases)
- Propositional logic is compositional:
- Meaning in propositional logic is context-independent
- Propositional logic has very limited expressive power
  - E.g., cannot say "pits cause breezes in adjacent squares“ except by writing one sentence for each square

First-order Logic

Whereas propositional logic assumes the world contains facts,

First-order logic (like natural language) assumes the world contains

- **Objects**: people, houses, numbers, colors, baseball games, wars, …
- **Relations**: red, round, prime, brother of, bigger than, part of, comes between, …
- **Functions**: father of, best friend, one more than, plus, …

FOL: Syntax Basic Elements

- Constants KingJohn, 2, NUS,...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg \wedge \vee \Rightarrow \Leftrightarrow$
- Equality =
- Quantifiers $\forall \; \exists$

Atomic sentences

- Atomic sentence:
  predicate (term1,...,termn)
  term1 = term2

- Term
  function (term1,...,termn)
  constant or *variable*
- Examples
  Brother(KingJohn,RichardTheLionheart)
  Longer (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))

Complex sentences

- Complex sentences are made from atomic sentences using connectives

$$\neg S,\ S1 \land S2,\ S1 \lor S2,\ S1 \Rightarrow S2,\ S1 \Leftrightarrow S2$$

- E.g.
  - *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*
  - *Greater(1,2)* $\lor$ *LessorEqual(1,2)*
  - *Greater(1,2)* $\land$ $\neg$*Greater(1,2)*

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

e.g, Father(John) = Henry

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} & \rightarrow & (\textit{Sentence}) \mid [\textit{Sentence}] \\
 & \mid & \neg\ \textit{Sentence} \\
 & \mid & \textit{Sentence} \land \textit{Sentence} \\
 & \mid & \textit{Sentence} \lor \textit{Sentence} \\
 & \mid & \textit{Sentence} \Rightarrow \textit{Sentence} \\
 & \mid & \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
 & \mid & \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\textit{Term} & \rightarrow & \textit{Function}(\textit{Term}, \ldots) \\
 & \mid & \textit{Constant} \\
 & \mid & \textit{Variable} \\
\textit{Quantifier} & \rightarrow & \forall \mid \exists \\
\textit{Constant} & \rightarrow & A \mid X_1 \mid \textit{John} \mid \ldots \\
\textit{Variable} & \rightarrow & a \mid x \mid s \mid \ldots \\
\textit{Predicate} & \rightarrow & \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \ldots \\
\textit{Function} & \rightarrow & \textit{Mother} \mid \textit{Left leg} \mid \ldots \\
\text{Operator Precedence}^{3} & : & \neg, \land, \lor, \Rightarrow, \leftrightarrow
\end{array}
$$

Order of operations

- $\neg$ is evaluated first
- $\wedge$ $\vee$ are evaluated next
- Quantifiers are evaluated next
- $\Rightarrow$ is evaluated last

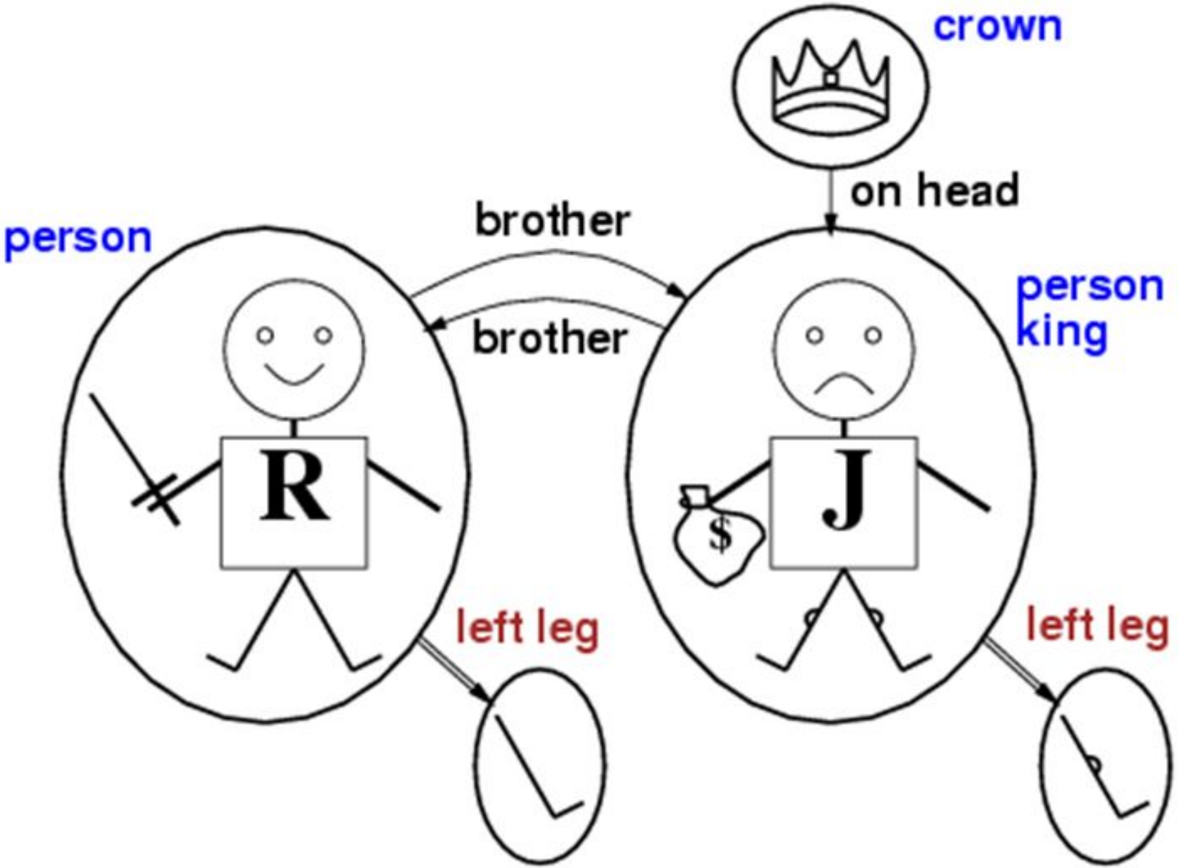
E.g., $\forall x\ At(x, SKKU) \wedge Smart(x)$

FOL syntax review

- Three kinds of symbols
  - Constant: objects
  - Predicate: relations
  - Functions can return values other than truth values
- Terms: *LeftLeg(John)*
- Atomic Sentences state facts: *Brother(Richard, John)*
- Complex Sentence: *Brother(R, J)* $\wedge$ *Brother(J, R) or* $\neg$*King(Richard)* $\Rightarrow$ *King(John)*
- Universal Quantifiers: $\forall x King(x) \Rightarrow Person(x)$
- Existential Quantifiers: $\exists x Crown(x) \wedge OnHead(x, John)$

Models for FOL

- Objects:
- Relation: a set of tuples of objects that are related
- Function



Examples

What is the interpretation for:

- $King(Richard) \lor King(John)$
- $\neg Brother(LeftLeg(Richard), John)$
- $In(Paris, France) \land In(Marseilles, France)$

Truth in FOL

- Sentences are true with respect to a model and an interpretation
- Model contains objects and relations among them
- *predicate*$(term_1,\ldots,term_n)$ is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by predicate
- Consider the interpretation
  - Richard → Richard the Lionheart
  - John → the evil King John
  - Brother → the brotherhood relation
  - *Brother(Richard, John)* is true just in case Richard the Lionheart and the evil King John are in the brotherhood relation in the model

Universal quantification

$\forall$ x <variables> <sentence>

- Everyone at SKKU is smart: $\forall x\, At(x, SKKU) \implies Smart(x)$
- $\forall x\, P$ is true in a model iff P is true with x being each possible object in the model
- Equivalent to the conjunction of instantions of P

$$At(John, SKKU) \implies Smart(John)$$
$$\wedge\, At(Richard, SKKU) \implies Smart(Richard)$$
$$\wedge\, At(Richard, crown) \implies Smart(crown)$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x, SKKU) \wedge Smart(x)$

  means "Everyone is at SKKU and everyone is smart"

Examples

What is the interpretation for:

- $\forall x \forall y\ Brother(x, y) \Rightarrow Sibling(x, y)$
- $\forall c\ Country(c) \land Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$
- $\exists Country(c) \land Border(c, Spain) \land Border(c, Italy)$
- $\forall s\ Breezy(s) \Rightarrow \exists x\ Adjacent(r, s) \land Pit(r)$

Existential quantification

$\exists$ <variables> <sentence>

- Someone at SKKU is smart: $\exists x\ At(x, SKKU) \land Smart(x)$
- $\exists x\ P$ is true in a model iff P is true with x being some possible object in the model
- Equivalent to the disjunctions of instantions of P

$$(At(John, SKKU) \land Smart(John))$$
$$\lor\ (At(Richard, SKKU) \land Smart(Richard))$$
$$\lor\ (At(Richard, crown) \land Smart(crown))$$

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x\ At(x, SKKU) \Rightarrow Smart(x)$$

is true if there is anyone who is not at SKKU!

Properties of quantifiers

- $\forall x \forall y$ is same as $\forall y \forall x$
- $\exists x\ \exists y$ is same as $\exists y \exists x$
- $\exists x\ \forall y$ is not same as $\forall y\ \exists x$
    - $\exists x\ \forall y$ Loves(x, y): there is a person who loves everyone in the world
    - $\forall y\ \exists x$ Loves(x, y): Everyone in the world is loved by at least one person
- Quantifier duality: each can be expressed using the other
    - $\forall x$ Likes(x, IceCream) is same as $\neg \exists x \neg$ Likes(x, IceCream)
    - $\exists x$ Likes(x, Broccoli) is same as $\neg \forall x \neg$Likes(x, Broccoli)

Sentence Examples

Brothers are siblings

$\forall x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

$\forall x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \wedge Parent(x, y)).$

Questions

- “Some real numbers are rational”

  (A) $\exists x\ (\text{real}(x) \vee \text{rational}(x))$
  (B) $\forall x\ (\text{real}(x) \rightarrow \text{rational}(x))$
  (C) $\exists x\ (\text{real}(x) \wedge \text{rational}(x))$
  (D) $\exists x\ (\text{rational}(x) \rightarrow \text{real}(x))$

- "None of my friends are perfect."

  (A) $\exists x(F(x) \wedge \neg P(x))$
  (B) $\exists x(\neg F(x) \wedge P(x))$
  (C) $\exists x(\neg F(x) \wedge \neg P(x))$
  (D) $\neg \exists x(F(x) \wedge P(x))$

https://www.geeksforgeeks.org/propositional-and-first-order-logic-gq/

FOL in NLP

- FOL can represent the meaning of a sentence
  - ‘Maharani is a restaurant’: *Restaurant(Maharani)*
  - ‘Angus walks’: *walk(angus)*
  - ‘Angus sees Bertie’: *see(angus, bertie)*
- With existential quantifier
  - “a restaurant that serves Mexican food near ICSI”:
  - $\exists x\ Restaurant(x) \land Serves(x,MexicanFood) \land Near((LocationOf(x),LocationOf(ICSI))$
  - $\text{Restaurant(AyCaramba)} \land \text{Serves(AyCaramba,MexicanFood)}$
    $\land \text{Near((LocationOf(AyCaramba),LocationOf(ICSI))}$

FOL in NLP

- With universal quantifier
  - "All vegetarian restaurants serve vegetarian food"
  - $\forall x\ VegetarianRestaurant(x) \Rightarrow Serves(x, VegetarianFood)$
  - $VegetarianRestaurant(Maharani) \Rightarrow Serves(Maharani, VegetarianFood)$
  - $VegetarianRestaurant(SteakHouse) \Rightarrow Serves(SteakHouse, VegetarianFood)$

FOL Example

“The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American”

“The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles , and all of its missiles were sold to it by Colonel West, who is American”

- R1: American(x) ∧ Weapon(y) ∧ Sells(x, y.z) ∧ Hostile(z) ⇒ Criminal(x)
- R2: Owns(Nono, M1) Nono has some missiles
- R3: Missile(M1)
- R4: Missile(x) ⇒ Weapon(x) # A missile is a weapon
- R5: Missile(x) ∧ Owns(Nono, x) ⇒ Sells(West, x, Nono) #All missiles sold by West
- R6: Enemy(x, America) ⇒ Hostile(x) Enemies of America are hostile
- R7: American(West) West is american
- R8: Enemy(Nono, America)

Wumpus: Time domain included

Can be represented more concisely

- at time step
  - 3: *Percept([Stench, Breeze, Glitter, None, None], 3)*
- at time step
  - 6: *Percept([None, Breeze, None, None, Scream], 6)*
- Actions can be:
  - *Turn(Right), Turn(Left), Forward, Shoot, Grab*
- Reflex actions:
  - $\forall t\ Glitter(t) \Rightarrow BestAction(Grab, t)$

Encoding complex rules

Instead of encoding stuff like:

- Adjacent(Square1,2, Square1,1)
- Adjacent(Square3,4, Square4,4)

Encode: $\forall x, y, a, b\ Adjacent([x, y], [a, b]) \Leftrightarrow (x = a \wedge (y = b-1 \vee y = b+1)) \vee (y = b \wedge (x = a-1 \vee x = a+1))$

Creating a Knowledge Base

- Identify the Task
- Assemble the relevant knowledge
- Decide on a vocabulary of predicates, functions and constants
- Encode general knowledge about the domain (rules)
- Encode a description of the problem
- Pose queries to the inference procedure and get answers
- Debug the KB

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world
- Situation calculus:
  - conventions for describing actions and change in FOL
  - can formulate planning as inference on a situation calculus KB