ECE 454/750T10, Spring 2014 — Assignment 2
Due Fri, May 30, 11:59:59 PM

(Your submission must be typeset, and in pdf. Submit to the dropbox on Learn. If you are in 454, mention the names of both group members. Only one of the group members should make a submission.)

1.(5 points) Consider the Chord DHT with an m-bit ID and n peers, that uses Finger Tables for lookup$(\cdot)$. A lookup(k) is initiated at some peer with ID p, and the Finger Table at p says that a peer with ID q is the next peer to whom the query should be forwarded. Suppose we use the operator "$-$" between two IDs to indicate the number of peers between them in the ring.

Disprove the following claim: $q - p \geq (k - p)/2$.

2.(5 points) In a Chord DHT with m-bit identifiers:

(a) with Finger Tables as discussed in the lecture and textbook, suppose we have $n-1$ peers, and we insert a new, i.e., an n^{th}, peer. What is the worst-case number of Finger Tables that need to be updated?

(b) without Finger Tables, i.e., in which each peer p maintains only a lookup table of one entry, *succ*$(p+1)$, what is the worst-case number of lookup tables that need to be updated?

3.(5 points) Call a function $f(m)$ linear in its input m if there exists a positive constant c_1 and a constant c_0 such that $f(m) = c_1 m + c_0$. For example, $g(m) = 0.25m - 12$ is linear in m. Suppose we denote a function that is linear in m as $\Theta(m)$.

We have shown in the lecture that the number of hops to do lookup(k) for a key k in a Chord DHT which uses m-bit identifiers and finger tables is *at most* $\Theta(m)$.

Show that it is *at least* $\Theta(m)$, in the worst-case. That is, for any integer $m \geq 1$, there exists an instance of a Chord DHT with m-bit identifiers, a peer p and a key k such that the number of hops incurred by lookup(k) at peer p with finger tables in place is at least a linear function of m.

4.(5 points) Suppose we have a 2-dimensional CAN in which peers and content are organized into an $n \times n$ grid. Each peer/content has ID $\langle i, j \rangle$. You want the peers and content to also be part of a Chord DHT. Give an invertible function f: $[1, n] \times [1, n] \longrightarrow [0, n^2 - 1]$ to map IDs to and from the Chord DHT.

5.(5 points) Consider an unstructured overlay network with n nodes in which each node randomly chooses c neighbours. If P and Q are both neighbours of R, what is the probability that they are neighbours of one another?

6.(5 points) Suppose we quantify communication-cost in the context of the Bully and Ring election algorithms as follows. To send a constant-sized message from one process to another incurs a constant communication-cost.

In the Bully algorithm, assume that all the three messages, *ELECTION*, *OK* and *COORDINATOR* have constant-size. In the Ring algorithm, assume that an *ELECTION* message with one process ID is constant-size, every process ID is constant-size, and a *COORDINATOR* message is constant-size.

Suppose we have processes with IDs $1, 2, \ldots, n, n+1$, and the process with ID $n+1$ is the current coordinator. That coordinator process crashes. What is the worst-case communication-cost in the Bully algorithm? What is it for the Ring algorithm?

7.(5 points) In the Ring election algorithm, we could have multiple *ELECTION* messages going around the ring simultaneously. This is wasteful. Give an algorithm that kills off an unnecessary message as early as possible, but still maintains the correctness of the election.