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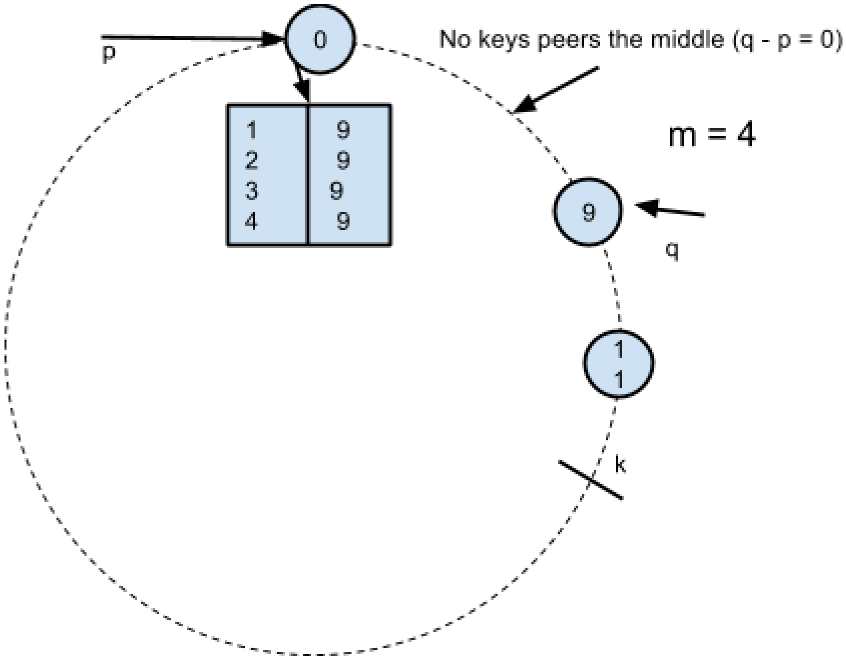
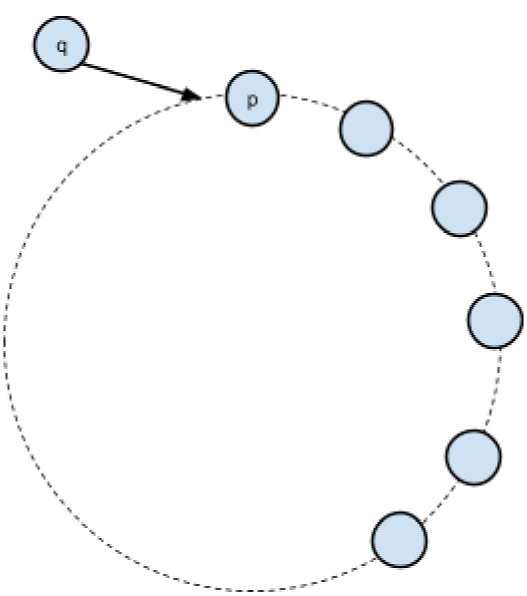
Assignment 2

1. Disprove $q - p \geq (k - p) / 2$



$q - p \geq (k - p) / 2$ (*given*)
$0 \geq (k - p) / 2$, *because no peers between q and p*
$k - p = 2$, *as* 2 *peers between k and p, thus*
$(k - p) / 2 > 0$, *thus proven.*

2.
(a) The worst case number of finger tables that need to be updated is n. Consider the following scenario:



Consider a Chord DHT where all the peers have keys less than 2^(m-1) i.e. the peers are on right half of the circle. In this case, the last entry in the finger table for all the peers point to p as the last entry is succ(peer + 2^(m-1)).

Let the nth peer (q) be inserted right before p. In this case, the finger tables for all n-1 nodes have to be updated because their last entry is changed to point to q from p.

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(b) In this case, each peer keeps track of only one peer that is succ(peer + 1). Hence, when we add an nth peer, only the (n-1)th peer's entry will have to be updated i.e. only 1 finger table will have to be updated.

3. For an m-bit identifier, consider number of peers = 2^m. Take a peer p and look for p's predecessor i.e. k = p - 1. As proved in the lecture slides, the number of jumps to peer k will be log(n) = log(2^m) = m. Hence, for every m >= 1 we can find a case where the worst case is atleast a linear function of m.

4. Consider an nxn matrix where the coordinates go from (1,1) to (n,n). We have to map this using an invertible function to a Chord DHT of size n^2 - 1. Let x and y denote the coordinates from the matrix. An invertible function that can be used is f(n) = (x-1)n + y-1.

For ex: (1,1) would map to Chord DHT of (1-1)n + 1-1 = 0
(n,n) would map to Chord DHT of (n-1)n + n-1 = n^2 - 1

You can used this same function to get back the original coordinates from Chord DHT by using 1 <= x <= n and 1 <= y <= n as the constraint and solving for an x and y.

5.
Incomplete

6.
Bully
Considering the situation where there are $n = 2^m$ nodes in the Chord DHT system. To practice the worst case scenario, say that the peer with the smallest priority initiates an election. Then, the total cost of messages in the first round of election is $(n - 1) + (n - 1)$. Here the first term comes from the node with the lowest priority sending the ELECTION message to all other peers with higher priority, and all of them responding with OK (all are of higher priority). Then, the peer with the second smallest priority sends the ELECTION message to n-2 peers and all n-2 peers respond with OK. In the end, the peer with the highest priority sends a COORDINATOR message to n-1 peers. Using this, we get the following terms:

$$Total\ Messages = [(n-1)+(n-1)] + [(n-2)+(n-2)] + [(n-3)+(n-3)] + ... + [(n-n)+(n-n)] + n-1$$
$$= 2(n)(\tfrac{n(n-1)}{2}) + n-1$$
$$= n(n-1) + (n-1)$$
$$= (n+1)(n-1)$$
$$= (n^2 - 1)$$

$Cost = Total\ Messages * c\ (where\ c\ is\ the\ cost\ of\ one\ message) = c(n^2 - 1)$

This, in a situation where $n = 2^m$, the worst case cost is quadratic to the number of peers = O(n)

Ring

The worst case in Ring Election is when all the n peers send a message at the same time. Each peer attaches its ID to the message increasing the cost by c. When the message returns back to the initiator the total cost for each message is c*n. Since this is for n initiators, the cost is c*n^2.

7.

Incomplete - This is BRANDON'S. To be done in our own way.

We can solve the multiple ELECTION messages problem in the Ring election algorithm by keeping a variable that stores the longest ELECTION message seen by peer p. If there is a new ELECTION message at peer p whose length is less than the longest, we should not forward to the next peer.

```
longestMessageLength = 0 (initial value)
messageLength = incoming message length
if (messageLength > longestMessageLength) {
        add my process ID
        longestMessageLength = messageLength
        forward to next peer
}
else {
        do nothing, do not forward
}
```