Topic 2: Rocket landing control

The SpaceX company achieved the first successful propulsive vertical landing of an orbital-class rocket stage in December 2015[1]. The rocket in question, Falcon 9, is equipped with Merlin 1D rocket engines, capable of vectored thrust, and grid fins which deploy from the stage-1 fuselage following separation; these actuators allow sufficient *controllability* of the rocket to permit a safe vertical landing. From a technical point of view, the successful landings were also enabled by theoretical advances in how the kind of nonlinear optimal control problem associated with safe rocket landing can be modelled and solved[2].

A simplified model of the rocket landing problem—assuming that "nose-up" stabilization is handled separately—views the rocket as a point mass, m, with position $\mathbf{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^\top \in \mathbb{R}^3$ and velocity $\mathbf{v} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^\top \in \mathbb{R}^3$. The coordinates $(x, y, z) \in \mathbb{R}^3$ are defined with z measured vertically upwards from ground (so $z = 0$ is sea level) and so that (x, y) is the lateral plane parallel to ground. The net thrust vector emerging from the engine is $\mathbf{f} = \begin{bmatrix} f_x & f_y & (f_z - mg) \end{bmatrix}^\top$—where the z component is explicitly accounting for gravity—which causes acceleration of the mass according to the (discretized) dynamics

$$\begin{bmatrix} \mathbf{r}(k+1) \\ \mathbf{v}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & T\,\mathbf{I} \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{r}(k) \\ \mathbf{v}(k) \end{bmatrix} + \frac{1}{m} \begin{bmatrix} (1/2)T^2\,\mathbf{I} \\ T\,\mathbf{I} \end{bmatrix} \mathbf{f},$$

where $\mathbf{I}$ is the 3×3 identity matrix, 0 is the 3×3 matrix of zeros, and T is the sampling time, which you may assume to be 0.5 s.

The aim is to steer the rocket from an initial position $\mathbf{r}(0)$ and velocity $\mathbf{v}(0)$ to the target ground position $\mathbf{r}_t = 0$ at rest ($\mathbf{v}_t = 0$). This should be done *safely* and at *minimum fuel cost*; that is, a number of constraints should be met during the mission:

- The engines are capable of exerting a thrust satisfying:
  - limits on vertical thrust: $0 \le f_z/m \le 12\,\mathrm{N\,kg^{-1}}$.
  - limits on lateral thrust:

    $$|f_x| \le f_z \tan\theta \quad \text{and} \quad |f_y| \le f_z \tan\theta$$

    where $\theta = 10$ degrees is the maximum angle for thrust vectoring.
- The vertical speed shall not exceed $15\,\mathrm{m\,s^{-1}}$ in descent.
- The lateral speeds, $|v_x|$ and $|v_y|$, shall not exceed $20\,\mathrm{m\,s^{-1}}$.
- In order to avoid premature ground collision, the positional trajectory shall respect a glide-slope constraint

  $$|r_x| \le \frac{r_z}{\tan\phi} \quad \text{and} \quad |r_y| \le \frac{r_z}{\tan\phi}$$

  where ϕ is the glide-slope angle, and is 30 degrees.

Your task is to design, implement and tune an MPC controller for this system in order to achieve safe landing from initial altitudes up to 500 m and initial lateral distances up to 600 m from the target. You should investigate the feasibility of the mission for a range of different initial positions and also non-zero initial velocities, representing the real situation of the rocket already having downward and lateral speed at the commencement of landing control. You should consider that, although it is desired to minimize fuel, the mission should complete in finite time.

(40 marks)

[1] https://youtu.be/glEvogjdEVY

[2] Behçet Açıkme e, Lars Blackmore (2011), Lossless convexification of a class of optimal control problems with non-convex control constraints, *Automatica* **47**(2), 341–347. https://doi.org/10.1016/j.automatica.2010.10.037