Artificial intelligence assignment 2; logic and logic reasoning.

1. Logic

Consider the following statements:

a. $((smoke \land heat \to fire) \leftrightarrow (smoke \to fire) \lor (heat \to fire))$

i. Show the truth table for this sentence.

S	H	F	((S	$\land$	H)	$\to$	F)	$\leftrightarrow$	((S	$\to$	F)	$\lor$	(H	$\to$	F))
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F	T	T	F	F	F	T	F	F
T	F	T	T	F	F	T	T	T	T	T	T	T	F	T	T
T	F	F	T	F	F	T	F	T	T	F	F	T	F	T	F
F	T	T	F	F	T	T	T	T	F	T	T	T	T	T	T
F	T	F	F	F	T	T	F	T	F	T	F	T	T	F	F
F	F	T	F	F	F	T	T	T	F	T	T	T	F	T	T
F	F	F	F	F	F	T	F	T	F	T	F	T	F	T	F

ii. Is this sentence Valid, Satisfiable, or neither?

This sentence is **valid**.

b. $(a_1 \lor a_3) \land (\neg a_1 \lor a_2) \land (\neg a_1 \lor a_4) \land (\neg a_1 \lor \neg a_4) \land (\neg a_3)$

i. Show the truth table for this sentence.

a_1	a_2	a_3	a_4	((a_1	$\lor$	a_3)	$\land$	($\neg$	a_1	$\lor$	a_2))	$\land$	(($\neg$	a_1	$\lor$	a_4)	$\land$	$\neg$	a_3)
T	T	T	T	T	T	T	T	F	T	T	T	F	F	T	T	T	F	F	T
T	T	T	F	T	T	T	T	F	T	T	T	F	F	T	F	F	F	F	T
T	T	F	T	T	T	F	T	F	T	T	T	T	F	T	T	T	T	T	F
T	T	F	F	T	T	F	T	F	T	T	T	F	F	T	F	F	F	T	F
T	F	T	T	T	T	T	F	F	T	F	F	F	F	T	T	T	F	F	T
T	F	T	F	T	T	T	F	F	T	F	F	F	F	T	F	F	F	F	T
T	F	F	T	T	T	F	F	F	T	F	F	F	F	T	T	T	T	T	F
T	F	F	F	T	T	F	F	F	T	F	F	F	F	T	F	F	F	T	F
F	T	T	T	F	T	T	T	T	F	T	T	F	T	F	T	T	F	F	T
F	T	T	F	F	T	T	T	T	F	T	T	F	T	F	T	F	F	F	T
F	T	F	T	F	F	F	F	T	F	T	T	F	T	F	T	T	T	T	F
F	T	F	F	F	F	F	F	T	F	T	T	F	T	F	T	F	T	T	F
F	F	T	T	F	T	T	T	T	F	T	F	F	T	F	T	T	F	F	T
F	F	T	F	F	T	T	T	T	F	T	F	F	T	F	T	F	F	F	T
F	F	F	T	F	F	F	F	T	F	T	F	F	T	F	T	T	T	T	F
F	F	F	F	F	F	F	F	T	F	T	F	F	T	F	T	F	T	T	F

ii. Is this sentence Valid, Satisfiable, or neither?

This sentence is **satisfiable**.

2. Vocabulary

First-order logic with consistent vocabulary.
Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define as well).

a. Define the vocabulary.

   i. Buys(x, p) = Person x buys Policy p.
   i. Expensive(p) = Policy p is expensive.
   iii. Barber(x) = Person x is a barber.
   iv. Shaves(x, y) = Person x shaves Person y.
   v. Born(x, UK) = Person x is born in the UK.
   vi. Parent(x, y) = Person x is the parent of Person y.
   vii. BornCitizen(x, UK) = Person x is born in the UK and is a citizen of the UK.
   viii. CitizenByDescent(x, UK) = Person x is a citizen of the UK by descent.
   ix. Politician(x) = Person x is a politician.
   x. canFool(x, y, t) = Person x fools Person y at time t.
   xi. Student(x) = Person x is a student.
   xii. Russian(x, y, z) = Person x took Russian in year y in semester z.
   xiii. German(x, y, z) = Person x took German in year y in semester z.
   xiv. BestScore(x, y) = Best Score x in course y.

b. Every person who buys a policy is clever.

$$\forall\ x\ \forall\ p\ \ Buys(x,p) \to Clever(x)$$

c. No person buys an expensive policy.

$$\forall\ x\ \forall\ p\ \ Expensive(p) \to \neg Buys(x,p)$$

d. There is a barber who shaves all men in town who do not shave themselves.

$$\exists\ x\ \ Barber(x) \wedge\ \ \forall\ y\ \ \neg Shaves(y,y) \to Shaves(x,y)$$

e. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

$$\forall\ x\ \ Born(x,UK) \wedge\ \ \forall\ y\ \ Parent(x,y) \to BornCitizen(y,UK) \to CitizenByBirth(x,UK)$$

f. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$$\forall\ x\ \ \neg Born(x,UK) \wedge\ \exists\ y\ \ Parent(x,y) \wedge BornCitizen(y,UK) \to CitizenByDescent(x,UK)$$

g. Politicians can fool some of the people all of the time and all of the people some of the time, but they cannot fool all of the of the people all of the time.

$$\forall\ x\ \ Politician(x) \wedge\ \ \exists\ y\ \forall\ t\ \ canFool(x, y, t)$$

$$\forall\ x\ \ Politician(x) \wedge\ \ \forall\ y\ \exists\ t\ \ canFool(x, y, t)$$

$$\forall\ x\ \ Politician(x) \wedge\ \ \forall\ y\ \forall\ t\ \neg canFool(x, y, t)$$

h. Some students took Russian in spring 2001.

$$\neg\forall\ x\ \ Student(x) \wedge Russian(x, 2001, Spring)$$

i. Only some students took German in spring 2001.

$$\neg\forall\ x\ \ Student(x) \wedge German(x, 2001, Spring)$$

j. The best score in German is always higher than the best score in Russian.

$$\forall\ x\ \forall\ y\ \ BestScore(x, German) > BestScore(y, Russian)$$

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3. Unifying Pairs of Expression

Attempt to unify the following pairs of expressions. Either show their most general unifiers, or explain why they will not unify. Use the a/b substitution form. Upper case letters are variables, lowercase are constants. Assume all variables are universally instantiated.

a. $p(X, a, Y)$ and $p(Z, Z, b)$

Substitution Set: $\{X/a, Y/b, Z/a\}$

Result Expression: $p(a, a, b)$

b. $p(X, X)$ and $p(a, b)$

It is not possible to unify these expressions because to unify them, the substitution set would have to contain two different values for variable X.

c. $p(X, Y, Z)$ and $p(c, d, d)$

Substitution Set: $\{X/c, Y/d, Z/d\}$

Result Expression: $p(c, d, d)$

d. $ancestor(X, father(X))$ and $ancestor(david, george)$

If we assume $father(david) = george$ then we can unify these expressions

par Substitution Set: $\{X/david\}$

Result Expression: $ancestor(david, father(david))$

e. $p(a, X)$ and $p(Y, Z)$

Substitution Set: $\{X/Z, Y/a\}$

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4. Knowledge Base

Consider the following knowledge base:

a. Prove that Q is true with:

1. $P \to Q$
2. $L \wedge M \to P$
3. $B \wedge L \to M$
4. $A \wedge P \to L$
5. $A \wedge B \to L$
6. A
7. B

i. Forward-Chaining
If we assume that A and B are true, then:

$$A \wedge B \to L$$

$$B \wedge L \to M$$

$$L \wedge M \to P$$

$$P \to Q$$

ii. Backward-Chaining

$$Q$$

$$P \to Q$$

$$L \wedge M \to P$$

$$B \wedge L \to M$$

$$A \wedge B \to L$$

$$A$$

$$B$$

iii. Resolution

b. Prove $t \to s$

1. $p \to q$
2. $[q \wedge r] \to s$
3. $[t \wedge u] \to r$
4. $u \to w$
5. $t \to y$
6. $y \to u$
7. $r \to p$
8. $p \to m$

i. Express in clause form

ii. Forward-Chaining

If we assume that t is true, then:

$$t \to y$$
$$y \to u$$
$$[t \wedge u] \to r$$
$$r \to p$$
$$p \to q$$
$$[q \wedge r] \to s$$

iii. Backward-Chaining

$$s$$
$$[q \wedge r] \to s$$
$$p \to q$$
$$r \to p$$
$$[t \wedge u] \to r$$
$$y \to u$$
$$t \to y$$
$$t$$

iv. Resolution

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5. Stories

a. "All dogs who are not tired and are smart are happy. Those dogs who do tricks are not stupid. Fido can do tricks and is full of energy. Happy dogs have exciting lives."

   i. translate the sentences of into predicate form

      1. $Dog(x)$ = x is a dog
      2. $Tired(x)$ = x is tired
      3. $Smart(x)$ = x is smart
      4. $Happy(x)$ = x is happy
      5. $Trick(x)$ = x can do tricks
      6. $Stupid(x)$ = x is Stupid
      7. $Exciting(x)$ = x has an exciting life
      8. $Energy(x)$ = x is full of energy

   ii. transform the predicate sentences into clause form

      1. $\forall\ x\ \ (Dog(x) \wedge \neg Tired(x) \wedge Smart(x) \rightarrow Happy(x))$
      2. $\forall\ x\ \ (Dog(x) \wedge Trick(x) \rightarrow \neg Stupid(x))$
      3. $Dog(fido) \wedge Trick(fido) \wedge Energy(fido)$
      4. $Happy(x) \rightarrow Exciting(x)$

   iii. prove via forward chaining that fido has an exciting life

$$Dog(fido) \wedge Trick(fido) \wedge Energy(fido)$$

$$Dog(fido) \wedge Energy(fido) \rightarrow \neg Tired(fido)$$

$$Dog(fido) \wedge Trick(fido) \rightarrow Smart(fido)$$

$$Dog(fido) \wedge \neg Tired(fido) \wedge Smart(fido) \rightarrow Happy(fido)$$

$$Happy(fido) \rightarrow Exciting(fido)$$

   iv. prove via backward chaining that fido has an exciting life

$$Exciting(fido)$$

$$Happy(fido) \rightarrow Exciting(fido)$$

$$Dog(fido) \wedge \neg Tired(fido) \wedge Smart(fido) \rightarrow Happy(fido)$$

$$Dog(fido) \wedge Trick(fido) \rightarrow Smart(fido)$$

$$Dog(fido) \wedge Energy(fido) \rightarrow \neg Tired(fido)$$

$$Dog(fido) \wedge Trick(fido) \wedge Energy(fido)$$

   v. prove via resolution that fido has an exciting life

b. "Anyone passing the history exams and winning the lottery is happy. But anyone who studies or is lucky can pass all the exams. John did not study, but he is lucky. Anyone who is lucky wins the lottery."

i. translate the sentences of into predicate form

1. $History(x)$ = x passed the history exams
2. $Lottery(x)$ = x won the lottery
3. $Happy(x)$ = x is happy
4. $Study(x)$ = x studied
5. $Lucky(x)$ = x is lucky
6. $Exams(x)$ = x passed all the exams

ii. transform the predicate sentences into clause form

1. $\forall\ x\ \ (History(x) \wedge Lottery(x) \rightarrow Happy(x))$
2. $\forall\ x\ \ (Exams(x) \rightarrow (History(x))$
3. $\forall\ x\ \ (Study(x) \vee Lucky(x) \rightarrow Exams(x))$
4. $\forall\ x\ \ (Lucky(x) \rightarrow Lottery(x))$

iii. prove via forward chaining that John is happy

$$\neg Study(john) \wedge Lucky(john)$$

$$Lucky(john) \rightarrow Exams(john)$$

$$Lucky(john) \rightarrow Lottery(john)$$

$$Exams(john) \rightarrow History(john)$$

$$History(john) \wedge Lottery(john) \rightarrow Happy(john)$$

iv. prove via backward chaining that John is happy

$$Happy(john)$$

$$History(john) \wedge Lottery(john) \rightarrow Happy(john)$$

$$Exams(john) \rightarrow History(john)$$

$$Lucky(john) \rightarrow Lottery(john)$$

$$Lucky(john) \rightarrow Exams(john)$$

$$Lucky(john)$$

v. prove via resolution that John is happy