termvar, x, y	term variable		
index, i, j, k			
term, t, r, s, n	::=		term
	|	x	variable
	|	contra	
	|	$\lambda x : T.t$	unary functions
	|	$t_1\ t_2$	function application
	|	$\Box\, t$	past necessity functor
	|	$\Diamond\, t$	past possibility functor
	|	$\blacksquare\, t$	necessity functor
	|	$\blacklozenge\, t$	possibility functor
	|	$\mathsf{let}\, \Box t_1 : T = t_2\, \mathsf{in}\, t_3$	past necessity elim
	|	$\mathsf{let}\, \blacksquare t_1 : T = t_2\, \mathsf{in}\, t_3$	necessity elim
	|	$\mathsf{let}\, \Diamond t_1 : T = t_2\, \mathsf{in}\, t_3$	past possibility elim
	|	$\mathsf{let}\, \blacklozenge t_1 : T = t_2\, \mathsf{in}\, t_3$	possibility elim
	|	(t) S	
form, *type*, A, B, C, T	::=		formula and type
	|	$\bot$	false or the empty type
	|	$\Box\, A$	past necessity
	|	$\blacksquare\, A$	necessity
	|	$\Diamond\, A$	past possibility
	|	$\blacklozenge\, A$	possibility
	|	$A \to B$	implication
Γ, Δ	::=		type context
	|	$\emptyset$	empty context
	|	A	formula el
	|	$x : T$	typed el
	|	Γ, Γ'	append

$\boxed{\Gamma_1; \Gamma_2; \Delta \vdash A}$

$$\frac{}{\Gamma_1; \Gamma_2; \Delta, A \vdash A}\ \text{L\_AX}$$

$$\frac{}{\Gamma_1; \Gamma_2, A; \Delta \vdash A}\ \text{L\_BAX}$$

$$\frac{}{\Gamma_1, A; \Gamma_2; \Delta \vdash A}\ \text{L\_BBAX}$$

$$\frac{}{\Gamma_1; \Gamma_2; \Delta, \bot \vdash A}\ \text{L\_FALSE}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta, A \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash A \to B}\ \text{L\_IMPI}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta \vdash A \to B \quad \Gamma_1; \Gamma_2; \Delta \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash B}\ \text{L\_IMPE}$$

$$\frac{\Gamma_1; \Gamma_2; \emptyset \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \Box\, A}\ \text{L\_BOXI}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta \vdash \Box\, A \quad \Gamma_1; \Gamma_2, A; \Delta \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash B}\ \text{L\_BOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A}\ \text{L\_BDIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A \quad \Gamma_1;\Gamma_2;A \vdash \blacklozenge\, B}{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A}\ \text{L\_BDIAE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, A}\ \text{L\_BBOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, A \quad \Gamma_1, A;\Gamma_2;\Delta \vdash B}{\Gamma_1;\Gamma_2;\Delta \vdash B}\ \text{L\_BBOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, A}\ \text{L\_DIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, A \quad \Gamma_1;\Gamma_2;A \vdash \Diamond\, B}{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, B}\ \text{L\_DIAE}$$

$\boxed{\Gamma_1;\Gamma_2;\Delta \vdash t : A}$

$$\frac{}{\Gamma_1;\Gamma_2;\Delta, x : A \vdash x : A}\ \text{TY\_AX}$$

$$\frac{}{\Gamma_1;\Gamma_2, x : A;\Delta \vdash x : A}\ \text{TY\_BAX}$$

$$\frac{}{\Gamma_1, x : A;\Gamma_2;\Delta \vdash x : A}\ \text{TY\_BBAX}$$

$$\frac{}{\Gamma_1;\Gamma_2;\Delta, x :\bot \vdash \mathsf{contra} : A}\ \text{TY\_FALSE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta, x : A \vdash t : B}{\Gamma_1;\Gamma_2;\Delta \vdash \lambda x : A.t : A \to B}\ \text{TY\_IMPI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : A \to B \quad \Gamma_1;\Gamma_2;\Delta \vdash t_2 : A}{\Gamma_1;\Gamma_2;\Delta \vdash t_1\ t_2 : B}\ \text{TY\_IMPE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t : A}{\Gamma_1;\Gamma_2;\Delta \vdash \Box\, t : \Box\, A}\ \text{TY\_BOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : \Box\, A \quad \Gamma_1;\Gamma_2, x : A;\Delta \vdash t_2 : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \Box x : \Box\, A = t_1\ \mathsf{in}\ t_2 : B}\ \text{TY\_BOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t : A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, t : \blacksquare\, A}\ \text{TY\_BBOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : \blacksquare\, A \quad \Gamma_1, x : A;\Gamma_2;\Delta \vdash t_2 : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \blacksquare x : \blacksquare\, A = t_1\ \mathsf{in}\ t_2 : B}\ \text{TY\_BBOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t : A \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash Mt : MA}\ \text{TY\_DIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : MA \quad \Gamma_1;\Gamma_2;x : A \vdash t_2 : MB \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ M\, x : MA = t_1\ \mathsf{in}\ t_2 : MB}\ \text{TY\_DIAE}$$

$\boxed{\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_2 : A}$

$$\frac{\Gamma_1;\Gamma_2;\Delta, x : A \vdash t_2 \approx t_2' : B \quad \Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_1' : A}{\Gamma_1;\Gamma_2;\Delta \vdash (\lambda x : A.t_2)\ t_1 \approx [t_1'/x]t_2' : B}\ \text{EQ\_BETA}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t_1 \approx t_1' : A \quad \Gamma_1;\Gamma_2, x : A;\Delta \vdash t_2 \approx t_2' : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \Box x : \Box\, A = \Box\, t_1\ \mathsf{in}\ t_2 \approx [t_1'/x]t_2' : B}\ \text{EQ\_BOX}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t_1 \approx t_1' : A \quad \Gamma_1, x : A;\Gamma_2;\Delta \vdash t_2 \approx t_2' : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\,\blacksquare x : \blacksquare\, A = \blacksquare\, t_1\,\mathsf{in}\,t_2 \approx [t_1'/x]t_2' : B}\ \text{EQ\_BBOX}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_1' : A \quad \Gamma_1;\Gamma_2;x : A \vdash t_2 \approx t_2' : MB \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ M\,x : MA = Mt_1\,\mathsf{in}\,t_2 \approx [t_1'/x]t_2' : MB}\ \text{EQ\_DIA}$$

$$\frac{\begin{array}{l}\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_1' : \Box\, C \\ \Gamma_1;\Gamma_2, y : C;\Delta \vdash t_2 \approx t_2' : \Box\, A \quad \Gamma_1;\Gamma_2, x : A;\Delta \vdash t_3 \approx t_3' : A\end{array}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\,\Box x : \Box\, A = \mathsf{let}\,\Box y : \Box\, C = t_1\,\mathsf{in}\,t_2\,\mathsf{in}\,t_3 \approx \mathsf{let}\,\Box y : \Box\, C = t_1'\,\mathsf{in}\,\mathsf{let}\,\Box x : \Box\, A = t_2'\,\mathsf{in}\,t_3' : B}\ \text{EQ\_CBOX}$$

$$\frac{\begin{array}{l}\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_1' : \blacksquare\, C \\ \Gamma_1, y : C;\Gamma_2;\Delta \vdash t_2 \approx t_2' : \blacksquare\, A \quad \Gamma_1, x : A;\Gamma_2;\Delta \vdash t_3 \approx t_3' : A\end{array}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\,\blacksquare x : \blacksquare\, A = \mathsf{let}\,\blacksquare y : \blacksquare\, C = t_1\,\mathsf{in}\,t_2\,\mathsf{in}\,t_3 \approx \mathsf{let}\,\blacksquare y : \blacksquare\, C = t_1'\,\mathsf{in}\,\mathsf{let}\,\blacksquare x : \blacksquare\, A = t_2'\,\mathsf{in}\,t_3' : B}\ \text{EQ\_CBBOX}$$

$$\frac{\begin{array}{l}\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_1' : MC \\ \Gamma_1;\Gamma_2;y : C \vdash t_2 \approx t_2' : MA \quad \Gamma_1;\Gamma_2;x : A \vdash t_3 \approx t_3' : MB \quad M \in \{\Diamond, \blacklozenge\}\end{array}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ M\,x : MA = \mathsf{let}\ M\,y : MC = t_1\,\mathsf{in}\,t_2\,\mathsf{in}\,t_3 \approx \mathsf{let}\ M\,y : MC = t_1\,\mathsf{in}\,\mathsf{let}\ M\,x : MA = t_2\,\mathsf{in}\,t_3 : MB}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t : A}{\Gamma_1;\Gamma_2;\Delta \vdash t \approx t : A}\ \text{EQ\_REFL}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_2 \approx t_1 : A}{\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_2 : A}\ \text{EQ\_SYM}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_2 : A \quad \Gamma_1;\Gamma_2;\Delta \vdash t_2 \approx t_3 : A}{\Gamma_1;\Gamma_2;\Delta \vdash t_1 \approx t_3 : A}\ \text{EQ\_TRANS}$$

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Definition rules:         36 good    0 bad
Definition rule clauses:  67 good    0 bad
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