

$termvar, x$ term variable

$index, i, j, k$

$term, t$::= x variable

$form, type, A, B, C, T$::= formula and type

- | \top true or the unit type
- | \perp false or the empty type
- | $\Box A$ past necessity
- | $\blacksquare A$ necessity
- | $\Diamond A$ past possibility
- | $\blacklozenge A$ possibility
- | $A \wedge B$ conjunction
- | $A \vee B$ disjunction
- | $A \rightarrow B$ implication

Γ, Δ ::= type context

- | \emptyset empty context
- | A formula el
- | $x : T$ typed el
- | Γ, Γ' append

$\boxed{\Gamma; \Delta \vdash A}$

$$\frac{}{\Gamma; \Delta, A \vdash A} \text{ L_AX}$$

$$\frac{}{\Gamma, A; \emptyset \vdash A} \text{ L_BAX}$$

$$\frac{}{\Gamma; \Delta \vdash \top} \text{ L_TRUE}$$

$$\frac{}{\Gamma; \Delta, \perp \vdash A} \text{ L_FALSE}$$

$$\frac{\Gamma; \Delta \vdash A \quad \Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \wedge B} \text{ L_CONJ I}$$

$$\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash A} \text{ L_CONJ E1}$$

$$\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash B} \text{ L_CONJ E2}$$

$$\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash A \vee B} \text{ L_DISJ I1}$$

$$\frac{\Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \vee B} \text{ L_DISJ I2}$$

$$\frac{\Gamma; \Delta, A \vdash C \quad \Gamma; \Delta, B \vdash C \quad \Gamma; \Delta \vdash A \vee B}{\Gamma; \Delta \vdash C} \text{ L_DISJ E}$$

$$\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \Box A} \text{ L_BOX I}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash \Box A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L_BOXE} \\
\\
\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L_BDIAI} \\
\\
\frac{\Gamma; \Delta \vdash \blacklozenge A \quad \Gamma; A \vdash \blacklozenge B}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L_BDIAE} \\
\\
\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \blacksquare A} \quad \text{L_BBOXI} \\
\\
\frac{\Gamma; \Delta \vdash \blacksquare A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L_BBOXE} \\
\\
\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \Diamond A} \quad \text{L_DIAI} \\
\\
\frac{\Gamma; \Delta \vdash \Diamond A \quad \Gamma; A \vdash \Diamond B}{\Gamma; \Delta \vdash \Diamond A} \quad \text{L_DIAE}
\end{array}$$

$$\boxed{\Gamma \vdash t : T}$$

Definition rules: 18 good 0 bad
 Definition rule clauses: 33 good 0 bad