

$termvar, x, y$ term variable

$index, i, j, k$

$term, t, r, s, n$

$::=$

term

	x	variable
	contra	
	$\lambda x : T. t$	unary functions
	$t_1 t_2$	function application
	$\Box t$	past necessity functor
	$\Diamond t$	past possibility functor
	$\blacksquare t$	necessity functor
	$\blacklozenge t$	possibility functor
	$\text{let } \Box t_1 = t_2 \text{ in } t_3$	past necessity elim
	$\text{let } \blacksquare t_1 = t_2 \text{ in } t_3$	necessity elim
	$\text{let } \Diamond t_1 = t_2 \text{ in } t_3$	past possibility elim
	$\text{let } \blacklozenge t_1 = t_2 \text{ in } t_3$	possibility elim
	(t)	S

$form, type, A, B, C, T$

$::=$

formula and type

	\perp	false or the empty type
	$\Box A$	past necessity
	$\blacksquare A$	necessity
	$\Diamond A$	past possibility
	$\blacklozenge A$	possibility
	$A \rightarrow B$	implication

Γ, Δ

$::=$

type context

	\emptyset	empty context
	A	formula el
	$x : T$	typed el
	Γ, Γ'	append

$\boxed{\Gamma_1; \Gamma_2; \Delta \vdash A}$

$\frac{}{\Gamma_1; \Gamma_2; \Delta, A \vdash A}$	L_AX
$\frac{}{\Gamma_1; \Gamma_2, A; \Delta \vdash A}$	L_BAX
$\frac{}{\Gamma_1, A; \Gamma_2; \Delta \vdash A}$	L_BBAX
$\frac{}{\Gamma_1; \Gamma_2; \Delta, \perp \vdash A}$	L_FALSE
$\frac{\Gamma_1; \Gamma_2; \Delta, A \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash A \rightarrow B}$	L_IMPI
$\frac{\Gamma_1; \Gamma_2; \Delta \vdash A \rightarrow B \quad \Gamma_1; \Gamma_2; \Delta \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash B}$	L_IMPE
$\frac{\Gamma_1; \Gamma_2; \emptyset \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \Box A}$	L_BOXI
$\frac{\Gamma_1; \Gamma_2; \Delta \vdash \Box A \quad \Gamma_1; \Gamma_2, A; \Delta \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash B}$	L_BOXE

$$\begin{array}{c}
\frac{\Gamma_1; \Gamma_2; \Delta \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \blacklozenge A} \text{ L_BDIAI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash \blacklozenge A \quad \Gamma_1; \Gamma_2; A \vdash \blacklozenge B}{\Gamma_1; \Gamma_2; \Delta \vdash \blacklozenge A} \text{ L_BDIAE} \\
\frac{\Gamma_1; \Gamma_2; \emptyset \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \blacksquare A} \text{ L_BBOXI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash \blacksquare A \quad \Gamma_1, A; \Gamma_2; \Delta \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash B} \text{ L_BBOXE} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \Diamond A} \text{ L_DIAI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash \Diamond A \quad \Gamma_1; \Gamma_2; A \vdash \Diamond B}{\Gamma_1; \Gamma_2; \Delta \vdash \Diamond B} \text{ L_DIAE}
\end{array}$$

$$\boxed{\Gamma_1; \Gamma_2; \Delta \vdash t : A}$$

$$\begin{array}{c}
\frac{}{\Gamma_1; \Gamma_2; \Delta, x : A \vdash x : A} \text{ TY_AX} \\
\frac{}{\Gamma_1; \Gamma_2, x : A; \Delta \vdash x : A} \text{ TY_BAX} \\
\frac{}{\Gamma_1, x : A; \Gamma_2; \Delta \vdash x : A} \text{ TY_BBAX} \\
\frac{}{\Gamma_1; \Gamma_2; \Delta, x : \perp \vdash \text{contra} : A} \text{ TY_FALSE} \\
\frac{\Gamma_1; \Gamma_2; \Delta, x : A \vdash t : B}{\Gamma_1; \Gamma_2; \Delta \vdash \lambda x : A. t : A \rightarrow B} \text{ TY_IMPI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t_1 : A \rightarrow B \quad \Gamma_1; \Gamma_2; \Delta \vdash t_2 : A}{\Gamma_1; \Gamma_2; \Delta \vdash t_1 t_2 : B} \text{ TY_IMPE} \\
\frac{\Gamma_1; \Gamma_2; \emptyset \vdash t : A}{\Gamma_1; \Gamma_2; \Delta \vdash \Box t : \Box A} \text{ TY_BOXI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t_1 : \Box A \quad \Gamma_1; \Gamma_2, x : A; \Delta \vdash t_2 : B}{\Gamma_1; \Gamma_2; \Delta \vdash \text{let } \Box x = t_1 \text{ in } t_2 : B} \text{ TY_BOXE} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t : A}{\Gamma_1; \Gamma_2; \Delta \vdash \blacklozenge t : \blacklozenge A} \text{ TY_BDIAI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t_1 : \blacklozenge A \quad \Gamma_1; \Gamma_2; x : A \vdash t_2 : \blacklozenge B}{\Gamma_1; \Gamma_2; \Delta \vdash \text{let } \blacklozenge x = t_1 \text{ in } t_2 : \blacklozenge B} \text{ TY_BDIAE} \\
\frac{\Gamma_1; \Gamma_2; \emptyset \vdash t : A}{\Gamma_1; \Gamma_2; \Delta \vdash \blacksquare t : \blacksquare A} \text{ TY_BBOXI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t_1 : \blacksquare A \quad \Gamma_1, x : A; \Gamma_2; \Delta \vdash t_2 : B}{\Gamma_1; \Gamma_2; \Delta \vdash \text{let } \blacksquare x = t_1 \text{ in } t_2 : B} \text{ TY_BBOXE} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t : A}{\Gamma_1; \Gamma_2; \Delta \vdash \Diamond t : \Diamond A} \text{ TY_DIAI} \\
\frac{\Gamma_1; \Gamma_2; \Delta \vdash t_1 : \Diamond A \quad \Gamma_1; \Gamma_2; x : A \vdash t_2 : \Diamond B}{\Gamma_1; \Gamma_2; \Delta \vdash \text{let } \Diamond x = t_1 \text{ in } t_2 : \Diamond B} \text{ TY_DIAE}
\end{array}$$

Definition rules: 28 good 0 bad

Definition rule clauses: 48 good 0 bad