

$termvar, x, y$ term variable

$index, i, j, k$

$term, t, r, s, n$

$::=$

x
 contra
 $\lambda x : T. t$
 $t_1 t_2$
 $\text{unbox}_{\square} t$
 $\text{unbox}_{\blacksquare} t$
 $\square t$
 $\diamond t$
 $\blacksquare t$
 $\blacklozenge t$
 $\text{adj}_R \blacklozenge y = x \text{ in } \square t$
 $\text{adj}_L \blacklozenge y = x \text{ in } t$
 $\text{let}_{\square} \Gamma \text{ be } t_1 \text{ in } t_2$
 $\text{let}_{\blacksquare} \Gamma \text{ be } t_1 \text{ in } t_2$
 $\text{let } \diamond x : A = s \mid \Gamma \text{ be } t_1 \text{ in } t_2$
 $\text{let } \blacklozenge x : A = s \mid \Gamma \text{ be } t_1 \text{ in } t_2$
 (t)

term

variable

unary functions

function application

past necessity functor

past possibility functor

necessity functor

possibility functor

past necessity elim

past necessity elim

S

$form, type, A, B, C, T$

$::=$

\perp
 $\square A$
 $\blacksquare A$
 $\diamond A$
 $\blacklozenge A$
 $A \rightarrow B$

formula and type

false or the empty type

past necessity

necessity

past possibility

possibility

implication

Γ, Δ

$::=$

\emptyset
 A
 $x : T$
 Γ, Γ'

type context

empty context

formula el

typed el

append

$\boxed{\Gamma \vdash t : A}$

$\frac{}{\Gamma, x : A \vdash x : A} \text{ TY_AX}$

$\frac{}{\Gamma, x : \perp \vdash \text{contra} : A} \text{ TY_FALSE}$

$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A. t : A \rightarrow B} \text{ TY_IMPI}$

$\frac{\Gamma \vdash t_1 : A \rightarrow B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1 t_2 : B} \text{ TY_IMPE}$

$\frac{\Gamma \vdash t : \square B}{\Gamma \vdash \text{unbox}_{\square} t : B} \text{ TY_BOXE}$

$\frac{\Gamma \vdash t_1 : \square A_1, \dots, \Gamma \vdash t_k : \square A_k \quad x_1 : \square A_1, \dots, x_k : \square A_k \vdash t : B}{\Gamma \vdash \text{let}_{\square} x_1 : \square A_1, \dots, x_k : \square A_k \text{ be } t_1, \dots, t_k \text{ in } t : \square B} \text{ TY_BOXI}$

$$\begin{array}{c}
\frac{\Gamma \vdash t : \blacksquare B}{\Gamma \vdash \text{unbox}_{\blacksquare} t : B} \quad \text{TY_BBOXE} \\
\\
\frac{\Gamma \vdash t_1 : \blacksquare A_1, \dots, \Gamma \vdash t_k : \blacksquare A_k \quad x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \vdash t : B}{\Gamma \vdash \text{let}_{\blacksquare} x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \text{ be } t_1, \dots, t_k \text{ in } t : \blacksquare B} \quad \text{TY_BBOXI} \\
\\
\boxed{\Gamma \vdash t_1 \approx t_2 : A} \\
\\
\frac{\Gamma, x : A \vdash t_2 \approx s_2 : B \quad \Gamma \vdash t_1 \approx s_1 : A}{\Gamma \vdash (\lambda x : A. t_2) t_1 \approx [s_1/x] s_2 : B} \quad \text{EQ_BETA} \\
\\
\frac{\Gamma \vdash t_1 \approx s_1 : \square A_1, \dots, \Gamma \vdash t_k \approx s_k : \square A_k \quad x_1 : \square A_1, \dots, x_k : \square A_k \vdash t \approx s : B}{\Gamma \vdash \text{unbox}_{\square} (\text{let}_{\square} x_1 : \square A_1, \dots, x_k : \square A_k \text{ be } t_1, \dots, t_k \text{ in } t) \approx [s_1/x_1] \dots [s_k/x_k] s : B} \quad \text{EQ_UNBOX} \\
\\
\frac{\Gamma \vdash t_1 \approx s_1 : \blacksquare A_1, \dots, \Gamma \vdash t_k \approx s_k : \blacksquare A_k \quad x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \vdash t \approx s : B}{\Gamma \vdash \text{unbox}_{\blacksquare} (\text{let}_{\blacksquare} x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \text{ be } t_1, \dots, t_k \text{ in } t) \approx [s_1/x_1] \dots [s_k/x_k] s : B} \quad \text{EQ_UNBBOX} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash t \approx t : A} \quad \text{EQ_REFL} \\
\\
\frac{\Gamma \vdash t_2 \approx t_1 : A}{\Gamma \vdash t_1 \approx t_2 : A} \quad \text{EQ_SYM} \\
\\
\frac{\Gamma \vdash t_1 \approx t_2 : A \quad \Gamma \vdash t_2 \approx t_3 : A}{\Gamma \vdash t_1 \approx t_3 : A} \quad \text{EQ_TRANS}
\end{array}$$

Definition rules: 14 good 0 bad
 Definition rule clauses: 26 good 0 bad