$termvar,\ x,\ y$ term variable
$index,\ i,\ j,\ k$
$term,\ t$ ::= term
| x variable
| unit unit
| contra contradiction
| (t_1, t_2) pair
| fst t first projection
| snd t second projection
| $\text{inj}_1\ t$ first injection
| $\text{inj}_2\ t$ second injection
| case t of $x.t_1, y.t_2$ sum case
| $\lambda x : T.t$ unary functions
| $t_1\ t_2$ function application
| $\Box\, t$ past necessity functor
| $\Diamond\, t$ past possibility functor
| $\blacksquare\, t$ necessity functor
| $\blacklozenge\, t$ possibility functor
| let $\Box t_1 = t_2$ in t_3 past necessity elim
| let $\blacksquare t_1 = t_2$ in t_3 necessity elim
| let $\Diamond t_1 = t_2$ in t_3 past possibility elim
| let $\blacklozenge t_1 = t_2$ in t_3 possibility elim

$form,\ type,\ A,\ B,\ C,\ T$ ::= formula and type
| $\top$ true or the unit type
| $\bot$ false or the empty type
| $\Box\, A$ past necessity
| $\blacksquare\, A$ necessity
| $\Diamond\, A$ past possibility
| $\blacklozenge\, A$ possibility
| $A \wedge B$ conjunction
| $A \vee B$ disjunction
| $A \to B$ implication

$\Gamma,\ \Delta$ ::= type context
| $\emptyset$ empty context
| A formula el
| $x : T$ typed el
| Γ, Γ' append

$\boxed{\Gamma; \Delta \vdash A}$

$$\frac{}{\Gamma; \Delta, A \vdash A}\ \ \text{L\_AX}$$

$$\frac{}{\Gamma, A; \emptyset \vdash A}\ \ \text{L\_BAX}$$

$$\frac{}{\Gamma; \Delta \vdash \top}\ \ \text{L\_TRUE}$$

$$\frac{}{\Gamma; \Delta, \bot \vdash A}\ \ \text{L\_FALSE}$$

$$\frac{\Gamma; \Delta \vdash A \quad \Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \wedge B}\ \ \text{L\_CONJI}$$

$$\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash A} \quad \text{L\_CONJE1}$$

$$\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash B} \quad \text{L\_CONJE2}$$

$$\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash A \vee B} \quad \text{L\_DISJI1}$$

$$\frac{\Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \vee B} \quad \text{L\_DISJI2}$$

$$\frac{\Gamma; \Delta, A \vdash C \quad \Gamma; \Delta, B \vdash C \quad \Gamma; \Delta \vdash A \vee B}{\Gamma; \Delta \vdash C} \quad \text{L\_DISJE}$$

$$\frac{\Gamma; \Delta, A \vdash B}{\Gamma; \Delta \vdash A \rightarrow B} \quad \text{L\_IMPI}$$

$$\frac{\Gamma; \Delta \vdash A \rightarrow B \quad \Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash B} \quad \text{L\_IMPE}$$

$$\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \Box A} \quad \text{L\_BOXI}$$

$$\frac{\Gamma; \Delta \vdash \Box A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L\_BOXE}$$

$$\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L\_BDIAI}$$

$$\frac{\Gamma; \Delta \vdash \blacklozenge A \quad \Gamma; A \vdash \blacklozenge B}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L\_BDIAE}$$

$$\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \blacksquare A} \quad \text{L\_BBOXI}$$

$$\frac{\Gamma; \Delta \vdash \blacksquare A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L\_BBOXE}$$

$$\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \Diamond A} \quad \text{L\_DIAI}$$

$$\frac{\Gamma; \Delta \vdash \Diamond A \quad \Gamma; A \vdash \Diamond B}{\Gamma; \Delta \vdash \Diamond B} \quad \text{L\_DIAE}$$

$$\boxed{\Gamma; \Delta \vdash t : A}$$

$$\frac{}{\Gamma; \Delta, x : A \vdash x : A} \quad \text{TY\_AX}$$

$$\frac{}{\Gamma, x : A; \emptyset \vdash x : A} \quad \text{TY\_BAX}$$

$$\frac{}{\Gamma; \Delta \vdash \mathsf{unit} : \top} \quad \text{TY\_TRUE}$$

$$\frac{}{\Gamma; \Delta, x : \bot \vdash \mathsf{contra} : A} \quad \text{TY\_FALSE}$$

$$\frac{\Gamma; \Delta \vdash t_1 : A \quad \Gamma; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash (t_1, t_2) : A \wedge B} \quad \text{TY\_CONJI}$$

$$\frac{\Gamma; \Delta \vdash t : A \wedge B}{\Gamma; \Delta \vdash \mathsf{fst}\ t : A} \quad \text{TY\_CONJE1}$$

$$\frac{\Gamma; \Delta \vdash t : A \wedge B}{\Gamma; \Delta \vdash \mathsf{snd}\ t : B} \quad \text{TY\_CONJE2}$$

$$\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \mathsf{inj}_1\ t : A \vee B} \quad \text{TY\_DISJI1}$$

$$\frac{\Gamma; \Delta \vdash t : B}{\Gamma; \Delta \vdash \mathsf{inj}_2\ t : A \vee B} \quad \text{TY\_DISJI2}$$

$$\frac{\Gamma; \Delta, x : A \vdash t_1 : C \quad \Gamma; \Delta, x : B \vdash t_2 : C \quad \Gamma; \Delta \vdash t : A \vee B}{\Gamma; \Delta \vdash \mathsf{case}\ t\ \mathsf{of}\ x.t_1, x.t_2 : C} \quad \text{TY\_DISJE}$$

$$\frac{\Gamma; \Delta, x : A \vdash t : B}{\Gamma; \Delta \vdash \lambda x : A.t : A \to B} \quad \text{TY\_IMPI}$$

$$\frac{\Gamma; \Delta \vdash t_1 : A \to B \quad \Gamma; \Delta \vdash t_2 : A}{\Gamma; \Delta \vdash t_2\ t_2 : B} \quad \text{TY\_IMPE}$$

$$\frac{\Gamma; \emptyset \vdash t : A}{\Gamma; \Delta \vdash \square\ t : \square\ A} \quad \text{TY\_BOXI}$$

$$\frac{\Gamma; \Delta \vdash t_1 : \square\ A \quad \Gamma, x : A; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash \mathsf{let}\ \square x = t_1\ \mathsf{in}\ t_2 : B} \quad \text{TY\_BOXE}$$

$$\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \blacklozenge\ t : \blacklozenge\ A} \quad \text{TY\_BDIAI}$$

$$\frac{\Gamma; \Delta \vdash t_1 : \blacklozenge\ A \quad \Gamma; x : A \vdash t_2 : \blacklozenge\ B}{\Gamma; \Delta \vdash \mathsf{let}\ \blacklozenge x = t_1\ \mathsf{in}\ t_2 : \blacklozenge\ B} \quad \text{TY\_BDIAE}$$

$$\frac{\Gamma; \emptyset \vdash t : A}{\Gamma; \Delta \vdash \blacksquare\ t : \blacksquare\ A} \quad \text{TY\_BBOXI}$$

$$\frac{\Gamma; \Delta \vdash t_1 : \blacksquare\ A \quad \Gamma, x : A; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash \mathsf{let}\ \blacksquare x = t_1\ \mathsf{in}\ t_2 : B} \quad \text{TY\_BBOXE}$$

$$\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \lozenge\ t : \lozenge\ A} \quad \text{TY\_DIAI}$$

$$\frac{\Gamma; \Delta \vdash t_1 : \lozenge\ A \quad \Gamma; x : A \vdash t_2 : \lozenge\ B}{\Gamma; \Delta \vdash \mathsf{let}\ \lozenge x = t_1\ \mathsf{in}\ t_2 : \lozenge\ B} \quad \text{TY\_DIAE}$$

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Definition rules:        40 good    0 bad
Definition rule clauses: 74 good    0 bad
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