| | | | |
|---|---|---|---|
| *termvar*, $x$, $y$ | term variable | | |
| *index*, $i$, $j$, $k$ | | | |
| *term*, $t$, $r$, $s$, $n$ | ::= | | term |
| | \| | $x$ | variable |
| | \| | contra | |
| | \| | $\lambda x : T.t$ | unary functions |
| | \| | $t_1\ t_2$ | function application |
| | \| | $\Box\, t$ | past necessity functor |
| | \| | $\Diamond\, t$ | past possibility functor |
| | \| | $\blacksquare\, t$ | necessity functor |
| | \| | $\blacklozenge\, t$ | possibility functor |
| | \| | $\mathsf{let}\, \Box t_1 = t_2\, \mathsf{in}\, t_3$ | past necessity elim |
| | \| | $\mathsf{let}\, \blacksquare t_1 = t_2\, \mathsf{in}\, t_3$ | necessity elim |
| | \| | $\mathsf{let}\, \Diamond t_1 = t_2\, \mathsf{in}\, t_3$ | past possibility elim |
| | \| | $\mathsf{let}\, \blacklozenge t_1 = t_2\, \mathsf{in}\, t_3$ | possibility elim |
| | \| | $(t)$ S | |
| *form*, *type*, $A$, $B$, $C$, $T$ | ::= | | formula and type |
| | \| | $\bot$ | false or the empty type |
| | \| | $\Box\, A$ | past necessity |
| | \| | $\blacksquare\, A$ | necessity |
| | \| | $\Diamond\, A$ | past possibility |
| | \| | $\blacklozenge\, A$ | possibility |
| | \| | $A \to B$ | implication |
| $\Gamma$, $\Delta$ | ::= | | type context |
| | \| | $\emptyset$ | empty context |
| | \| | $A$ | formula el |
| | \| | $x : T$ | typed el |
| | \| | $\Gamma, \Gamma'$ | append |

$\boxed{\Gamma_1; \Gamma_2; \Delta \vdash A}$

$$\frac{}{\Gamma_1; \Gamma_2; \Delta, A \vdash A}\ \text{L\_AX}$$

$$\frac{}{\Gamma_1; \Gamma_2, A; \Delta \vdash A}\ \text{L\_BAX}$$

$$\frac{}{\Gamma_1, A; \Gamma_2; \Delta \vdash A}\ \text{L\_BBAX}$$

$$\frac{}{\Gamma_1; \Gamma_2; \Delta, \bot \vdash A}\ \text{L\_FALSE}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta, A \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash A \to B}\ \text{L\_IMPI}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta \vdash A \to B \quad \Gamma_1; \Gamma_2; \Delta \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash B}\ \text{L\_IMPE}$$

$$\frac{\Gamma_1; \Gamma_2; \emptyset \vdash A}{\Gamma_1; \Gamma_2; \Delta \vdash \Box\, A}\ \text{L\_BOXI}$$

$$\frac{\Gamma_1; \Gamma_2; \Delta \vdash \Box\, A \quad \Gamma_1; \Gamma_2, A; \Delta \vdash B}{\Gamma_1; \Gamma_2; \Delta \vdash B}\ \text{L\_BOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A}\ \text{L\_BDIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A \quad \Gamma_1;\Gamma_2;A \vdash \blacklozenge\, B}{\Gamma_1;\Gamma_2;\Delta \vdash \blacklozenge\, A}\ \text{L\_BDIAE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, A}\ \text{L\_BBOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, A \quad \Gamma_1, A;\Gamma_2;\Delta \vdash B}{\Gamma_1;\Gamma_2;\Delta \vdash B}\ \text{L\_BBOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash A}{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, A}\ \text{L\_DIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, A \quad \Gamma_1;\Gamma_2;A \vdash \Diamond\, B}{\Gamma_1;\Gamma_2;\Delta \vdash \Diamond\, B}\ \text{L\_DIAE}$$

$\boxed{\Gamma_1;\Gamma_2;\Delta \vdash t : A}$

$$\frac{}{\Gamma_1;\Gamma_2;\Delta, x : A \vdash x : A}\ \text{TY\_AX}$$

$$\frac{}{\Gamma_1;\Gamma_2, x : A;\Delta \vdash x : A}\ \text{TY\_BAX}$$

$$\frac{}{\Gamma_1, x : A;\Gamma_2;\Delta \vdash x : A}\ \text{TY\_BBAX}$$

$$\frac{}{\Gamma_1;\Gamma_2;\Delta, x :\bot \vdash \mathsf{contra} : A}\ \text{TY\_FALSE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta, x : A \vdash t : B}{\Gamma_1;\Gamma_2;\Delta \vdash \lambda x : A.t : A \to B}\ \text{TY\_IMPI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : A \to B \quad \Gamma_1;\Gamma_2;\Delta \vdash t_2 : A}{\Gamma_1;\Gamma_2;\Delta \vdash t_1\ t_2 : B}\ \text{TY\_IMPE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t : A}{\Gamma_1;\Gamma_2;\Delta \vdash \Box\, t : \Box\, A}\ \text{TY\_BOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : \Box\, A \quad \Gamma_1;\Gamma_2, x : A;\Delta \vdash t_2 : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \Box x = t_1\, \mathsf{in}\, t_2 : B}\ \text{TY\_BOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t : A}{\Gamma_1;\Gamma_2;\Delta \vdash \blacksquare\, t : \blacksquare\, A}\ \text{TY\_BBOXI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : \blacksquare\, A \quad \Gamma_1, x : A;\Gamma_2;\Delta \vdash t_2 : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \blacksquare x = t_1\, \mathsf{in}\, t_2 : B}\ \text{TY\_BBOXE}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t : A \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash Mt : MA}\ \text{TY\_DIAI}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : MA \quad \Gamma_1;\Gamma_2;x : A \vdash t_2 : MB \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ M\, x = t_1\, \mathsf{in}\, t_2 : MB}\ \text{TY\_DIAE}$$

$\boxed{\Gamma_1;\Gamma_2;\Delta \vdash t_1 = t_2 : A}$

$$\frac{\Gamma_1;\Gamma_2;\Delta, x : A \vdash t_2 = t_2' : B \quad \Gamma_1;\Gamma_2;\Delta \vdash t_1 = t_1' : A}{\Gamma_1;\Gamma_2;\Delta \vdash (\lambda x : A.t_2)\, t_1 = [t_1'/x]t_2' : B}\ \text{EQ\_BETA}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t_1 = t_1' : A \quad \Gamma_1;\Gamma_2, x : A;\Delta \vdash t_2 = t_2' : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\, \Box x = \Box\, t_1\, \mathsf{in}\, t_2 = [t_1'/x]t_2' : B}\ \text{EQ\_BOX}$$

$$\frac{\Gamma_1;\Gamma_2;\emptyset \vdash t_1 = t_1' : A \quad \Gamma_1, x : A;\Gamma_2;\Delta \vdash t_2 = t_2' : B}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ \blacksquare x = \blacksquare\, t_1\ \mathsf{in}\ t_2 = [t_1'/x]t_2' : B}\ \text{EQ\_BBOX}$$

$$\frac{\Gamma_1;\Gamma_2;\Delta \vdash t_1 : A \quad \Gamma_1;\Gamma_2;x : A \vdash t_2 : MB \quad M \in \{\Diamond, \blacklozenge\}}{\Gamma_1;\Gamma_2;\Delta \vdash \mathsf{let}\ M\ x = M t_1\ \mathsf{in}\ t_2 = [t_1'/x]t_2' : \Diamond\, B}\ \text{EQ\_DIA}$$

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Definition rules:         30 good    0 bad
Definition rule clauses:  52 good    0 bad
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