

$termvar, x, y$ term variable

$index, i, j, k$

$term, t$

$::=$

term

	x	variable
	unit	unit
	contra	contradiction
	(t_1, t_2)	pair
	fst t	first projection
	snd t	second projection
	inj ₁ t	first injection
	inj ₂ t	second injection
	case t of $x.t_1, y.t_2$	sum case
	$\lambda x : T. t$	unary functions
	$t_1 t_2$	function application
	$\Box t$	past necessity functor
	$\Diamond t$	past possibility functor
	$\blacksquare t$	necessity functor
	$\blacklozenge t$	possibility functor
	let $\Box t_1 = t_2$ in t_3	past necessity elim
	let $\blacksquare t_1 = t_2$ in t_3	necessity elim
	let $\Diamond t_1 = t_2$ in t_3	past possibility elim
	let $\blacklozenge t_1 = t_2$ in t_3	possibility elim

$form, type, A, B, C, T$

$::=$

formula and type

	\top	true or the unit type
	\perp	false or the empty type
	$\Box A$	past necessity
	$\blacksquare A$	necessity
	$\Diamond A$	past possibility
	$\blacklozenge A$	possibility
	$A \wedge B$	conjunction
	$A \vee B$	disjunction
	$A \rightarrow B$	implication

Γ, Δ

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type context

	\emptyset	empty context
	A	formula el
	$x : T$	typed el
	Γ, Γ'	append

$\boxed{\Gamma; \Delta \vdash A}$

$\overline{\Gamma; \Delta, A \vdash A}$	L-AX
$\overline{\Gamma, A; \emptyset \vdash A}$	L-BAX
$\overline{\Gamma; \Delta \vdash \top}$	L-TRUE
$\overline{\Gamma; \Delta, \perp \vdash A}$	L-FALSE
$\frac{\Gamma; \Delta \vdash A \quad \Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \wedge B}$	L-CONJ

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash A} \quad \text{L_CONJE1} \\
\frac{\Gamma; \Delta \vdash A \wedge B}{\Gamma; \Delta \vdash B} \quad \text{L_CONJE2} \\
\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash A \vee B} \quad \text{L_DISJ1} \\
\frac{\Gamma; \Delta \vdash B}{\Gamma; \Delta \vdash A \vee B} \quad \text{L_DISJ2} \\
\frac{\Gamma; \Delta, A \vdash C \quad \Gamma; \Delta, B \vdash C \quad \Gamma; \Delta \vdash A \vee B}{\Gamma; \Delta \vdash C} \quad \text{L_DISJE} \\
\frac{\Gamma; \Delta, A \vdash B}{\Gamma; \Delta \vdash A \rightarrow B} \quad \text{L_IMPI} \\
\frac{\Gamma; \Delta \vdash A \rightarrow B \quad \Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash B} \quad \text{L_IMPE} \\
\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \Box A} \quad \text{L_BOXI} \\
\frac{\Gamma; \Delta \vdash \Box A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L_BOXE} \\
\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L_BDIAI} \\
\frac{\Gamma; \Delta \vdash \blacklozenge A \quad \Gamma; A \vdash \blacklozenge B}{\Gamma; \Delta \vdash \blacklozenge A} \quad \text{L_BDIAE} \\
\frac{\Gamma; \emptyset \vdash A}{\Gamma; \Delta \vdash \blacksquare A} \quad \text{L_BBOXI} \\
\frac{\Gamma; \Delta \vdash \blacksquare A \quad \Gamma, A; \Delta \vdash B}{\Gamma; \Delta \vdash B} \quad \text{L_BBOXE} \\
\frac{\Gamma; \Delta \vdash A}{\Gamma; \Delta \vdash \Diamond A} \quad \text{L_DIAI} \\
\frac{\Gamma; \Delta \vdash \Diamond A \quad \Gamma; A \vdash \Diamond B}{\Gamma; \Delta \vdash \Diamond B} \quad \text{L_DIAE}
\end{array}$$

$$\boxed{\Gamma; \Delta \vdash t : A}$$

$$\begin{array}{c}
\overline{\Gamma; \Delta, x : A \vdash x : A} \quad \text{TY_AX} \\
\overline{\Gamma, x : A; \emptyset \vdash x : A} \quad \text{TY_BAX} \\
\overline{\Gamma; \Delta \vdash \text{unit} : \top} \quad \text{TY_TRUE} \\
\overline{\Gamma; \Delta, x : \perp \vdash \text{contra} : A} \quad \text{TY_FALSE} \\
\frac{\Gamma; \Delta \vdash t_1 : A \quad \Gamma; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash (t_1, t_2) : A \wedge B} \quad \text{TY_CONJI}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash t : A \wedge B}{\Gamma; \Delta \vdash \text{fst } t : A} \quad \text{TY_CONJE1} \\
\frac{\Gamma; \Delta \vdash t : A \wedge B}{\Gamma; \Delta \vdash \text{snd } t : B} \quad \text{TY_CONJE2} \\
\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \text{inj}_1 t : A \vee B} \quad \text{TY_DISJI1} \\
\frac{\Gamma; \Delta \vdash t : B}{\Gamma; \Delta \vdash \text{inj}_2 t : A \vee B} \quad \text{TY_DISJI2} \\
\frac{\Gamma; \Delta, x : A \vdash t_1 : C \quad \Gamma; \Delta, x : B \vdash t_2 : C \quad \Gamma; \Delta \vdash t : A \vee B}{\Gamma; \Delta \vdash \text{case } t \text{ of } x.t_1, x.t_2 : C} \quad \text{TY_DISJE} \\
\frac{\Gamma; \Delta, x : A \vdash t : B}{\Gamma; \Delta \vdash \lambda x : A. t : A \rightarrow B} \quad \text{TY_IMPI} \\
\frac{\Gamma; \Delta \vdash t_1 : A \rightarrow B \quad \Gamma; \Delta \vdash t_2 : A}{\Gamma; \Delta \vdash t_1 t_2 : B} \quad \text{TY_IMPE} \\
\frac{\Gamma; \emptyset \vdash t : A}{\Gamma; \Delta \vdash \Box t : \Box A} \quad \text{TY_BOXI} \\
\frac{\Gamma; \Delta \vdash t_1 : \Box A \quad \Gamma, x : A; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash \text{let } \Box x = t_1 \text{ in } t_2 : B} \quad \text{TY_BOXE} \\
\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \blacklozenge t : \blacklozenge A} \quad \text{TY_BDIAI} \\
\frac{\Gamma; \Delta \vdash t_1 : \blacklozenge A \quad \Gamma; x : A \vdash t_2 : \blacklozenge B}{\Gamma; \Delta \vdash \text{let } \blacklozenge x = t_1 \text{ in } t_2 : \blacklozenge B} \quad \text{TY_BDIAE} \\
\frac{\Gamma; \emptyset \vdash t : A}{\Gamma; \Delta \vdash \blacksquare t : \blacksquare A} \quad \text{TY_BBOXI} \\
\frac{\Gamma; \Delta \vdash t_1 : \blacksquare A \quad \Gamma, x : A; \Delta \vdash t_2 : B}{\Gamma; \Delta \vdash \text{let } \blacksquare x = t_1 \text{ in } t_2 : B} \quad \text{TY_BBOXE} \\
\frac{\Gamma; \Delta \vdash t : A}{\Gamma; \Delta \vdash \blacklozenge t : \blacklozenge A} \quad \text{TY_DIAI} \\
\frac{\Gamma; \Delta \vdash t_1 : \blacklozenge A \quad \Gamma; x : A \vdash t_2 : \blacklozenge B}{\Gamma; \Delta \vdash \text{let } \blacklozenge x = t_1 \text{ in } t_2 : \blacklozenge B} \quad \text{TY_DIAE}
\end{array}$$

Definition rules: 40 good 0 bad
 Definition rule clauses: 74 good 0 bad