

$termvar, x, y$  term variable

$index, i, j, k$

$term, t, r, s, n$

$::=$

$x$   
 $\text{contra}$   
 $\lambda x : T. t$   
 $t_1 t_2$   
 $\text{unbox}_{\square} t$   
 $\text{unbox}_{\blacksquare} t$   
 $\square t$   
 $\diamond t$   
 $\blacksquare t$   
 $\blacklozenge t$   
 $\text{adj}_R \blacklozenge y = x \text{ in } \square t$   
 $\text{adj}_L \blacklozenge y = x \text{ in } t$   
 $\text{let}_{\square} \Gamma \text{ be } t_1 \text{ in } t_2$   
 $\text{let}_{\blacksquare} \Gamma \text{ be } t_1 \text{ in } t_2$   
 $\text{let } \diamond x : A = s \mid \Gamma \text{ be } t_1 \text{ in } t_2$   
 $\text{let } \blacklozenge x : A = s \mid \Gamma \text{ be } t_1 \text{ in } t_2$   
 $(t)$

term

variable

unary functions

function application

past necessity functor

past possibility functor

necessity functor

possibility functor

past necessity elim

past necessity elim

S

$form, type, A, B, C, T$

$::=$

$\perp$   
 $\square A$   
 $\blacksquare A$   
 $\diamond A$   
 $\blacklozenge A$   
 $A \rightarrow B$

formula and type

false or the empty type

past necessity

necessity

past possibility

possibility

implication

$\Gamma, \Delta$

$::=$

$\emptyset$   
 $A$   
 $x : T$   
 $\Gamma, \Gamma'$

type context

empty context

formula el

typed el

append

$\boxed{\Gamma \vdash t : A}$

$\frac{}{\Gamma, x : A \vdash x : A} \text{ TY\_AX}$

$\frac{}{\Gamma, x : \perp \vdash \text{contra} : A} \text{ TY\_FALSE}$

$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A. t : A \rightarrow B} \text{ TY\_IMPI}$

$\frac{\Gamma \vdash t_1 : A \rightarrow B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1 t_2 : B} \text{ TY\_IMPE}$

$\frac{\Gamma \vdash t : \square B}{\Gamma \vdash \text{unbox}_{\square} t : B} \text{ TY\_BOXE}$

$\frac{\Gamma \vdash t_1 : \square A_1, \dots, \Gamma \vdash t_k : \square A_k \quad x_1 : \square A_1, \dots, x_k : \square A_k \vdash t : B}{\Gamma \vdash \text{let}_{\square} x_1 : \square A_1, \dots, x_k : \square A_k \text{ be } t_1, \dots, t_k \text{ in } t : \square B} \text{ TY\_BOXI}$

$$\begin{array}{c}
\frac{\Gamma \vdash t : \blacksquare B}{\Gamma \vdash \text{unbox}_{\blacksquare} t : B} \quad \text{TY\_BBOXE} \\
\\
\frac{\Gamma \vdash t_1 : \blacksquare A_1, \dots, \Gamma \vdash t_k : \blacksquare A_k \quad x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \vdash t : B}{\Gamma \vdash \text{let}_{\blacksquare} x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k \text{ be } t_1, \dots, t_k \text{ in } t : \blacksquare B} \quad \text{TY\_BBOXI} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash \Diamond t : \Diamond A} \quad \text{TY\_DIAI} \\
\\
\frac{\Gamma \vdash t_1 : \blacksquare A_1, \dots, \Gamma \vdash t_k : \blacksquare A_k \quad x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k, B \vdash t : \Diamond C \quad \Gamma \vdash s : \Diamond B}{\Gamma \vdash \text{let } \Diamond x : \Diamond B = s \mid x_1 : \blacksquare A_1, \dots, x_k : \blacksquare A_k, x : B \text{ be } t_1, \dots, t_k \text{ in } t : \Diamond C} \quad \text{TY\_DIAE} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash \blacklozenge t : \blacklozenge A} \quad \text{TY\_BDIAI} \\
\\
\frac{\Gamma \vdash t_1 : \Box A_1, \dots, \Gamma \vdash t_k : \Box A_k \quad \Gamma \vdash s : \blacklozenge B \quad x_1 : \Box A_1, \dots, x_k : \Box A_k, B \vdash t : \blacklozenge C}{\Gamma \vdash \text{let } \blacklozenge x : \blacklozenge B = s \mid x_1 : \Box A_1, \dots, x_k : \Box A_k, x : B \text{ be } t_1, \dots, t_k \text{ in } t : \blacklozenge C} \quad \text{TY\_BDIAE} \\
\\
\frac{x : A \vdash t : \Box B}{y : \blacklozenge A \vdash \text{adj}_L \blacklozenge x = y \text{ in } t : B} \quad \text{TY\_ADJL} \\
\\
\frac{x : \blacklozenge A \vdash t : B}{y : A \vdash \text{adj}_R \blacklozenge y = x \text{ in } \Box t : \Box B} \quad \text{TY\_ADJR}
\end{array}$$

$$\boxed{\Gamma \vdash t_1 \approx t_2 : A}$$

$$\begin{array}{c}
\frac{\Gamma, x : A \vdash t_2 \approx t'_2 : B \quad \Gamma \vdash t_1 \approx t'_1 : A}{\Gamma \vdash (\lambda x : A. t_2) t_1 \approx [t'_1/x] t'_2 : B} \quad \text{EQ\_BETA} \\
\\
\frac{\Gamma \vdash t : A}{\Gamma \vdash t \approx t : A} \quad \text{EQ\_REFL} \\
\\
\frac{\Gamma \vdash t_2 \approx t_1 : A}{\Gamma \vdash t_1 \approx t_2 : A} \quad \text{EQ\_SYM} \\
\\
\frac{\Gamma \vdash t_1 \approx t_2 : A \quad \Gamma \vdash t_2 \approx t_3 : A}{\Gamma \vdash t_1 \approx t_3 : A} \quad \text{EQ\_TRANS}
\end{array}$$

Definition rules: 18 good 0 bad  
 Definition rule clauses: 35 good 0 bad