$termvar,\ x,\ y$ term variable
$index,\ i,\ j,\ k$
$term,\ t,\ r,\ s,\ n$ ::= term
| x variable
| contra
| $\lambda x : T.t$ unary functions
| $t_1\ t_2$ function application
| $\mathsf{unbox}_\square\ t$
| $\mathsf{unbox}_\blacksquare\ t$
| $\square\ t$ past necessity functor
| $\lozenge\ t$ past possibility functor
| $\blacksquare\ t$ necessity functor
| $\blacklozenge\ t$ possibility functor
| $\mathsf{adj}_\mathsf{R}\ \blacklozenge y = x\ \mathsf{in}\ \square t$
| $\mathsf{adj}_\mathsf{L}\ \blacklozenge y = x\ \mathsf{in}\ t$
| $\mathsf{let}_\square\ \Gamma\ \mathsf{be}\ t_1\ \mathsf{in}\ t_2$ past necessity elim
| $\mathsf{let}_\blacksquare\ \Gamma\ \mathsf{be}\ t_1\ \mathsf{in}\ t_2$ past necessity elim
| $\mathsf{let}\ \lozenge x : A = s\ |\ \Gamma\ \mathsf{be}\ t_1\ \mathsf{in}\ t_2$
| $\mathsf{let}\ \blacklozenge x : A = s\ |\ \Gamma\ \mathsf{be}\ t_1\ \mathsf{in}\ t_2$
| (t) S

$form,\ type,\ A,\ B,\ C,\ T$ ::= formula and type
| $\bot$ false or the empty type
| $\square\ A$ past necessity
| $\blacksquare\ A$ necessity
| $\lozenge\ A$ past possibility
| $\blacklozenge\ A$ possibility
| $A \to B$ implication

$\Gamma,\ \Delta$ ::= type context
| $\emptyset$ empty context
| A formula el
| $x : T$ typed el
| Γ, Γ' append

$\boxed{\Gamma \vdash t : A}$

$$\frac{}{\Gamma, x : A \vdash x : A}\ \text{TY\_AX}$$

$$\frac{}{\Gamma, x :\bot\vdash \mathsf{contra} : A}\ \text{TY\_FALSE}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A.t : A \to B}\ \text{TY\_IMP}\text{I}$$

$$\frac{\Gamma \vdash t_1 : A \to B \quad \Gamma \vdash t_2 : A}{\Gamma \vdash t_1\ t_2 : B}\ \text{TY\_IMP}\text{E}$$

$$\frac{\Gamma \vdash t : \square\ B}{\Gamma \vdash \mathsf{unbox}_\square\ t : B}\ \text{TY\_BOX}\text{E}$$

$$\frac{\Gamma \vdash t_1 : \square\ A_1, ..., \Gamma \vdash t_k : \square\ A_k \quad x_1 : \square\ A_1, ..., x_k : \square\ A_k \vdash t : B}{\Gamma \vdash \mathsf{let}_\square\ x_1 : \square\ A_1, ..., x_k : \square\ A_k\ \mathsf{be}\ t_1, ..., t_k\ \mathsf{in}\ t : \square\ B}\ \text{TY\_BOX}\text{I}$$

1

$$\frac{\Gamma \vdash t : \blacksquare B}{\Gamma \vdash \mathsf{unbox}_\blacksquare\ t : B} \quad \text{TY\_BBOX}\text{E}$$

$$\frac{\Gamma \vdash t_1 : \blacksquare A_1, \, ..., \Gamma \vdash t_k : \blacksquare A_k \quad x_1 : \blacksquare A_1, \, ..., x_k : \blacksquare A_k \vdash t : B}{\Gamma \vdash \mathsf{let}_\blacksquare\ x_1 : \blacksquare A_1, \, ..., x_k : \blacksquare A_k\ \mathsf{be}\ t_1, \, ..., t_k\ \mathsf{in}\ t : \blacksquare B} \quad \text{TY\_BBOX}\text{I}$$

$$\boxed{\Gamma \vdash t_1 \approx t_2 : A}$$

$$\frac{\Gamma, x : A \vdash t_2 \approx t_2' : B \quad \Gamma \vdash t_1 \approx t_1' : A}{\Gamma \vdash (\lambda x : A.t_2)\,t_1 \approx [t_1'/x]t_2' : B} \quad \text{EQ\_BETA}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash t \approx t : A} \quad \text{EQ\_REFL}$$

$$\frac{\Gamma \vdash t_2 \approx t_1 : A}{\Gamma \vdash t_1 \approx t_2 : A} \quad \text{EQ\_SYM}$$

$$\frac{\Gamma \vdash t_1 \approx t_2 : A \quad \Gamma \vdash t_2 \approx t_3 : A}{\Gamma \vdash t_1 \approx t_3 : A} \quad \text{EQ\_TRANS}$$

```
Definition rules:        12 good     0 bad
Definition rule clauses: 22 good     0 bad
```