The Ecumenical Perspective in Logic
Lecture 1

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General Plan for the minicourse

1. *Lecture 1: General Introduction (history, origins, motivation, the founding fathers)*
2. *Lecture 2: A tour through Ecumenical Systems*
    - Prawitz' system $\mathcal{NE}$;
    - Krauss' system;
    - Barroso-Nascimento's system ECI
    - Ecumenical Sequent Calculus
    - Ecumenical Tableaux
    - Ecumenical modal systems.

3. *Lecture 3: Ecumenical Semantics*
    - Ecumenical Kripke semantics;
    - Ecumenical PtS.
4. *Lecture 4: Additional topics*
    - Back to translations;
    - Glivenko from an Ecumenical Perspective;
    - Questions, Problems, The future.

It is a fact that for more than two millennia a single logic prevailed, Aristotelian logic, the logic that obeyed some fundamental principles, such :

The Big Five

- RCP (Rule of Contradictory Pairs) In a contradictory pair, one member is true and the other false
- BV (Bivalence) Every proposition is either true or false
- EM (Excluded Middle) Everything either holds or does not hold of any one thing at any one time
- CV (Contravalence) No proposition is both true and false
- NC (Non-Contradiction) Nothing both holds and does not hold of any one thing at any one time

General introduction - New Logics

Even with the fregean revolution of the late nineteenth century, this situation didn't change, at least till the beginning of the twentieth century: a single logic, classical logic.

General introduction - New Logics

However, it is also a fact that this state of affairs has changed drastically in the last hundred years: several logics have presented themselves as extensions or rivals of classical logic. For example, the addition of several operators of modal nature to the so-called classical propositional logic produced modal logics of various types (alethic modal logics, epistemic logics, deontic logics, temporal logics, computational logics).

Such logics can be considered as extensions of classical logic in the sense that they only widen the scope of logical-conceptual analysis. On the other hand, the revision of the unrestricted validity of certain fundamental principles produced a set of logics that presented themselves as real alternatives to classical logic, as rivals of classical logic.

Thus, for example, intuitionist logic questioned the unrestricted validity of principle of the excluded middle, some paraconsistent logics questioned the unrestricted validity of the principle of non-contradiction, and Gaps and Gluts questioned bivalence and contravalence, respectively.

General introduction - New Logics

The present scenario of a plurality of alternative logics (and it is a vast scenario!) opens up a wide range of new questions for Philosophy (these questions are taken from Romina Padró and Eduardo Barrio):

1. Are there or could there be good reasons to revise logic?
2. What logic should we use to revise our logic? Could there be more than one correct logic? Could there not be one correct logic?
3. What logic should we use to argue that there is more than one correct logic?
4. Do logical theories describe our inferential practices practices? Do they attempt to model them? The normativity of logic seems to make it difficult for theories to be mere reflections of what we do when we infer. Sometimes what we do, those courses of action we take when trying to give good reasons, is wrong. Simple research on inferential behavior shows that often what we do should not rationally have been done. And we should correct it.

General introduction - New Logics

5. But then, if theories not only describe inferential practices but are also criteria for correcting behavior, what criteria should we use to select logical theories?
6. Is there some kind of logical evidence that allows us to determine which theory we should adopt?
7. Are there pre-theoretical intuitions about validity that we should preserve?
8. Could a change in logic cause a mutilation in a scientific theory? In mathematics?
9. What it is that makes an inference valid? What gives a proof its epistemic power? (Dag Prawitz)

These and other questions are accompanied by several "isms" well known in the relevant literature:

General introduction - New Logics

1. *Revisionism/Anti-revisionism*
2. *Exceptionalism/Anti-exceptionalism*
3. *Logical Pluralism*
4. *Logical Ecletism*
5. *Logical Pragmatism*
6. *Logical nihilism*
7. *Maybe a new one: Logical Ecumenism!*

But it is undeniable that there seems to be a fundamental question underlying the very possibility of the emergence of alternative/rival logics:

> *But how can one question and propose a revision of fundamental principles such as the principle of the excluded middle and the principle of non-contradiction? How can such a conflict arise in the realm of logic? Are we facing an instance of Deep Disagreement in Logic?*

General introduction - New Logics

Here is a possible (and traditional) answer for the appearence of intuitionistic logic:

> *With the discovery of paradoxes and inconsistencies in the early formalisation of set theory, mathematicians started to worry about the logical foundations of mathematics. Proofs by contradiction, which concluded the existence of a mathematical object without actually constructing it, were immediately thought by some to be the source of the problem.*
> *(Gilda Ferreira & Paulo Oliva)*

Mathematicians were then segregated between those who thought classical reasoning should be allowed as long as it was finitistically justified (e.g. Hilbert) and those who thought proofs in mathematics should avoid non-constructive arguments (e.g. Brouwer). Constructivism and intuitionistic logic were born.
(Gilda Ferreira & Paulo Oliva)

But,

> *It was soon discovered, however, that the consistency of arithmetic based on intuitionistic logic (Heyting arithmetic) is equivalent to the consistency of arithmetic based on classical logic (Peano arithmetic). Therefore, if one accepts that intuitionistic arithmetic is consistent, then one must also accept that classical arithmetic is consistent.*
> *(Gilda Ferreira & Paulo Oliva)*

General introduction - Translations

These foundational questions and background (like the consistency problem) was the initial motivation for the appearence of several translations/interpretations that would connect classical logic/arithmetic and intuitionistic logic/arithmetic, given that via a translation *"of classical into intuitionistic logic which preserves the statement 0= 1, any proof of 0= 1 in Peano arithmetic (if ever one is found) could be effectively translated into a proof of 0= 1 in Heyting arithmetic."*

In the late twenties and early thirties of last century several results were obtained concerning some relations between classical logic (CL) and intuitionistic logic (IL), and between classical arithmetic (PA) and intutionistic arithmetic (HA).

These translations/interpretations/embeddings took the form of functions from the language of CL (PA) into some fragment of the language of the IL (HA) that preserve some important properties, like derivability and theoremhood.

In 1925 Kolmogorov proved that CPL could be translated into IPL.

Definition

Define a translation $A \mapsto k[A]$ by replacing all subformulas B by $\neg\neg B$ (defined by induction on the complexity of B)

Example: $k[(p \to \neg q)] = \neg\neg(\neg\neg p \to \neg\neg\neg q)$

General introduction - Translations

In 1927 Glivenko proved two important results relating classical propositional logic (CPL) to intuitionistic propositional logic (IPL). Glivenko's first result shows that A is a theorem of CPL iff $\neg\neg A$ is a theorem of IPL.

Theorem

$\vdash_{CPL} A \Leftrightarrow \vdash_{IPL} \neg\neg A$

Glivenko's second result establishes that we cannot distinguish CPL from IPL with respect to theorems of the form $\neg A$.

Theorem

$\vdash_{CPL} \neg A \Leftrightarrow \vdash_{IPL} \neg A$

In 1933 Gödel and Gentzen defined interpretations of PA into HA.

Gentzen's translation

Definition

The function Ge from the language of PA into the language of HA is defined as:

1. $Ge[A] = \neg\neg A$, for A atomic $\neq \bot$;
2. $Ge[\neg A] = \neg Ge[A]$,
3. $Ge[(A \circledast B)] = (Ge[A] \circledast [B])$, where $\circledast \in \{\wedge, \rightarrow, \forall\}$;
4. $Ge[(A \vee B)] = \neg(\neg Ge[A] \wedge \neg Ge[B])$;
5. $Ge[\exists x A(x)] = \neg \forall x \neg Ge[A(x)]$.

In fact, Gentzen's and Gödel's results encapsulate a stronger result. Let us call a formula A stable iff $\vdash_{IL} A \leftrightarrow \neg\neg A$, and let us call a theory T atomically stable iff every atomic formula in T is stable.

Theorem

Theorem: Let T be any classical first order theory formulated in the fragment $\{\neg, \forall, \wedge\}$. If T is atomically stable then every theorem of T is also an intuitionistic theorem.

General introduction - Translations

Kuroda's negative translation is somehow simpler (1951):

Definition

1. $P_{Ku} = P$, for P atomic;
2. $(A \wedge B)_{Ku} = (A_{Ku} \wedge B_{Ku})$;
3. $(A \to B)_{Ku} = (A_{Ku} \to B_{Ku})$;
4. $(A \vee B)_{Ku} = (A_{Ku} \vee B_{Ku})$
5. $(\forall x A(x))_{Ku} = \forall x \neg\neg A(x)_{Ku}$;
6. $(\exists x A(x))_{Ku} = \exists x A(x)_{Ku}$.

Definition

$Ku[A] = \neg\neg A_{Ku}$.

Back to the foundational problem

As we have already said, the idea was to find a "translation of classical into intuitionistic logic which preserves the statement 0= 1. So any proof of 0= 1 in Peano arithmetic (if ever one is found) can be effectively translated into a proof of 0= 1 in Heyting arithmetic".

Main result: PA is consistent if and only if HA is consistent!

General Introduction - Constructivization

The interpretations defined by Gödel and Gentzen differ only with respect to the fragment of the language of HA: Gödel interprets the implication sign $\rightarrow$ in terms of $\neg$ and $\wedge$, while Gentzen keeps it in the image language. This small syntactical difference has an important consequence, since the interpretation of implication in terms of negation and conjunction allows Gödel to obtain the following nice result as a preparatory step for the definition of his interpretation function:

General Introduction - Constructivization

Theorem

Theorem: Let A be a formula in the fragment $\{\neg, \wedge\}$*. Then* $\vdash_{CL} A \Leftrightarrow \vdash_{IL} A$*.*

Proof: Every theorem A in the fragment $\{\neg, \wedge\}$ has a cannonical form: $\exists B_1, ..., B_k$ such that $A \leftrightarrow \neg B_1 \wedge \ldots \wedge \neg B_k$. The result then follows directly from Glivenko's theorem for classical propositional logic.

The immediate effect of this result is that the fragment $\{\neg, \wedge\}$ is insufficient to distinguish the class of classical propositional theorems from the class of intuitionistic propositional theorems. A nice way to put Gödel's result is:

We can do classical propositional logic without classical logic!

Theorem (Involution)

If $\neg\neg A$ is an intuitionistic theorem in the fragment $\{\neg, \wedge, \perp, \forall\}$, then A is also a theorem in the same fragment.

Theorem (Externalization)

If $\forall x \neg\neg A(x)$ is an intuitionistic theorem in the fragment $\{\neg, \wedge, \perp, \forall\}$, then $\neg\neg\forall x A(x)$ is also a theorem in the same fragment.

Theorem (Weak-Glivenko)

Let $A(x)$ be a formula in the fragment $\{\neg, \wedge\}$. If $\vdash_{CL} \neg\forall x A(x)$, then $\vdash_{IL} \neg\forall x A(x)$.

General Introduction - Constructivization

Theorem

Let $A(x)$ be quantifier free in the fragment $\{\neg, \wedge, \bot, \exists\}$. Then, if $\vdash_{CL} \exists x A(x)$, then $\vdash_{IL} \exists x A(x)$.

Theorem

Let A be a sentence in the fragment $\{\neg, \wedge, \bot, \exists\}$ such that no quantifier occurs in the scope of any quantifier. Then, if $\vdash_{CL} A$, then $\vdash_{IL} A$.

The moral behind this result is: no classical logic without the iteration of quantifiers!

A different (and interesting) approach to the relation between classical logic and non-classical logics proposed by Quine in 1972 is based on the reasonable idea that the litigants are talking about distinct things (or speaking different things), and that if they are talking about different things, there is not "the same thing" - a rule or a principle - on which they diverge and dispute.

According to this position, it is as if the participants of the conflict were speaking different languages and did not realize it.
This approach can be expressed in this (robust) argument against the possibility of revisionism in logic.

General Introduction - Revisionism questioned

1. If the deviant/revisionist logician thinks he not accepting the general validity of a classical principle of reasoning, then he gives new meanings to the concepts used in the formulation of the principle.
2. If the deviant logician gives new meanings to the concepts used in the formulation of the principle, then the deviant logician and the classical logician are not talking about the same thing (principle).
3. If they are are talking about different things, they cannot disagree!!!

Thus, they do not really disagree!!!!

General Introduction - Revisionism questioned

Dag Prawitz formulates Quine's view in the following way in 2015:

> *Quine's main view of the deviant logician is that "he only changes the subject" - he is not opposing the laws of orthodox logic, but is talking about something else. For instance, if one stops to regard contradictions* $(A \wedge \neg A)$ *as implying all other sentences, then according to Quine , one is not "talking about negation, '*$\neg$*', 'not'".*

Two different logics can therefore never be in conflict in the sense of making contrary assertions - they are simply talking about different things. In common (non-Quinean) parlance one may state this thesis by saying that two logics that come out with different sets of logical truths must attach different meanings to the logical constants.

Introduction – What is ecumenism?

Prawitz agrees partially with Quine:

> *The view voiced by Quine that the different codifications shall not be seen as being in conflict with each other is supported here, but in a way quite different from how he was thinking. The classical logician is not asserting what the intuitionistic logician denies. For instance, the classical logician asserts* $(A \vee_c \neg A)$ *to which the intuitionist does not object; he objects to the universal validity of* $(A \vee_i \neg A)$*, which is not asserted by the classical logician.*

Introduction – What is ecumenism?

The same idea was used by Gilles Dowek also in 2015:

> *An alternative is to use the idea of Hilbert and Poincaré that axioms and deduction rules define the meaning of the symbols of the language and it is then possible to explain that some judge the proposition* $(P \vee \neg P)$ *true and others do not because they do not assign the same meaning to the symbols* $\vee$, $\neg$, *etc.*
> *(Dowek [2015])*

Introduction – What is ecumenism?

Taking this idea seriously, we should not say that the proposition $(P \vee \neg P)$ has a classical proof but no constructive proof, but we should say that the proposition $(P \vee_c \neg_c P)$ has a proof and the proposition $(P \vee \neg P)$ does not, that is we should introduce two symbols for each connective and quantifier, for instance a symbol $\vee$ for the constructive disjunction and a symbol $\vee_c$ for the classical one, instead of introducing two judgments: "has a classical proof" and "has a constructive proof"".

(Dowek [2015])

And Prawitz continues:

> *If they are sufficiently* ***ecumenical*** *and can use the other's vocabulary in their own speech, a classical logician and an intuitionist can both adopt the present mixed system, and the intuitionist must then agree that* $(A \vee_c \neg A)$ *is trivially provable for any sentence A, even when it contains intuitionistic constants, and the classical logician must admit that he has no ground for universally asserting* $(A \vee_i \neg A)$*, even when* A *contains only classical constants.*

Remark: as far as we known, this is the first appearence of the expression ecumenical in the relevant literature!!

Introduction – What is ecumenism?

When the classical and intuitionistic codifications attach different meanings to a constant, we need to use different symbols, and I shall use a subscript c for the classical i for the intuitionistic. The classical and intuitionistic constants can then have a ***peaceful coexistence*** *in a language that contains both.*
(Prawitz, 2015)

Introduction – What is ecumenism?

But what does it mean to be ecumenical?

But what does it mean to coexist in peace?

Introduction – What is ecumenism?

The terms ecumenism and ecumenical come from the Greek *oikoumene*, which means “the whole inhabited world”.

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Ecumenism: the search process for unicity, where different thoughts, ideas or points of view can harmonically co-exist.

Introduction – What is ecumenism?

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Ecumenism: the search process for unicity, where different thoughts, ideas or points of view can harmonically co-exist.

- What (really) are ecumenical systems?
- What are they good for?
- Why should anyone be interested in ecumenical systems?
- What is the real motivation behind the definition and development of ecumenical systems?

Logical-Philosophical motivation

- *It is possible to understand classical reasoning as based coherently on some other explanation of meaning.*
- *Classical reasoning can be shown to cohere fully with meaning explanations just as intuitionistic reasoning.*
- *We can provide classical reasoning with a meaning explanation based on proofs/ rules rather than on truth-conditions.*

Logical-Philosophical motivation

Logical inferentialism:

- the meaning of the logical constants can be specified by the rules that determine their correct use;
- proof-theoretical requirements on admissible logical rules: harmony and separability;
- pure logical systems: no other operator is pressuposed in the explanation of a given operator.

Problem

Why do we need different operators if we can constructively understand classical operators by means of translations?

Logical-Philosophical motivation

But why can't we take $(A \vee_c B)$ *as an abreviation for* $\neg(\neg A \wedge \neg B)$*?*

Logical-Philosophical motivation

- IL: if what you mean by $(A \vee B)$ is $\neg(\neg A \wedge \neg B)$, then I can accept the validity of $(A \vee \neg A)$!

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- CL: but I do not mean $\neg(\neg A \wedge \neg\neg A)$ by $(A \vee \neg A)$. One must distinguish the excluded-middle from the the principle of non-contradiction. When I say that Goldbach's conjecture is either true or false, I am not saying that it would be contradictory to assert that it is not true and that it is not the case that it is not true!

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- E.g.:

Quinne dagger

A	B	$A \downarrow B$
1	1	0
1	0	0
0	1	0
0	0	1

Sheffer stroke

A	B	$A \uparrow B$
1	1	0
1	0	1
0	1	1
0	0	1

Logical-Philosophical motivation

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- IL: but you must realize that, at the end of the day, you just have one logical operator!!! (can you guess one?)
- CL: But this is not at all true! The fact that we can define one operator in terms of other operators does not imply that we don't have different operators!
  It is true that we can prove $\vdash (A \vee_c B) \leftrightarrow \neg(\neg A \wedge \neg B)$ in the ecumenical system, but this does not mean that we don't have three different operators: $\neg$, $\vee_c$ and $\wedge$.

A taste of Ecumenism

$\vee_c$-introduction and $\rightarrow_c$-introduction:

$$\dfrac{\begin{matrix}[\neg A] & & [\neg B] \\ & \Pi & \\ & \bot & \end{matrix}}{(A \vee_c B)} \qquad\qquad \dfrac{\begin{matrix}[A] & & [\neg B] \\ & \Pi & \\ & \bot & \end{matrix}}{(A \rightarrow_c B)}$$

Logical-Philosophical motivation

We get a codification of classical reasoning based on meaning explanations of the same kind as we got for intuitionistic reasoning, by adding classical predicates with introduction rules in the way just exemplified, by adopting the above introduction rules for $\vee_c$ and $\rightarrow_c$, and by taking the introduction rules for the other logical constants to be the ones stated by Gentzen. Classical reasoning is in this way shown to cohere fully with meaning explanations just as intuitionistic reasoning, which contradicts Dummett's negative thesis.

(Dag Prawitz)

Mathematical motivation

Theorem 1. There exist $x, y \notin \mathbb{Q}$ such that $x^y \in \mathbb{Q}$.

Mathematical motivation

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Proof. Consider $a = \sqrt{2}^{\sqrt{2}}$.
If $a \in \mathbb{Q}$, then take $x = y = \sqrt{2}$.
If $a \notin \mathbb{Q}$, then take $x = a$ and $y = \sqrt{2}$. Then

$$x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$$

$\square$

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Classical mathematician: cool!!! ☺

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$\square$

Classical mathematician: cool!!! ☺

Intuitionistic mathematician: but $\sqrt{2}^{\sqrt{2}} \in \mathbb{Q}$ or $\sqrt{2}^{\sqrt{2}} \notin \mathbb{Q}$??? ☹

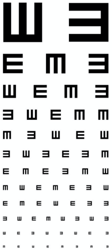
Mathematical motivation (example by Emerson Sales)

if $x + y = 2z$ then $x \geq z$ or $y \geq z$.

How should we interpret "if then" and "or" in this sentence, so that it will be valid? The answer is: it depends!

Mathematical motivation (example by Emerson Sales)

if $x + y = 2z$ then$_i$ $x \geq z$ or$_i$ $y \geq z$.



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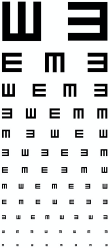

classical mathematician ☹

intuitionistic mathematician ☹

Mathematical motivation (example by Emerson Sales)

$$\text{not } \big(\text{not } \big(\text{if } x + y = 2z \text{ then}_i\ x \geq z \text{ or}_i\ y \geq z\big)\big).$$

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classical mathematician ☺
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classical mathematician	☺
intuitionistic mathematician	☺ ☺

Mathematical motivation

The basic idea rests on the observation that classical mathematics uses the logical operators

and, or, if-then, for all, there exists

with two different meanings. One meaning is constructive and is determined by the rules of intuitionistic logic. The other meaning involves proofs by contradiction and becomes non-constructive through the elimination of double negation. Such proofs are usually called indirect. These two meanings can only be separated as long as one abstains from eliminating double negations. With elimination of double negation the two meanings fuse and the proof loses its constructive validity.

(Peter Krauss)

Mathematical motivation

> *We should rather like to persuade classical mathematicians to carry out their proofs distinguishing between inuitionistic and classical logic operators depending on what they actually prove. This way they may abstain from eliminating double negations without leaving their familiar traditional tracks of reasoning. Moreover, this way their reasoning stays constructively valid and therefore preserves the possibility of a computational interpretation.*

(Peter Krauss)

Mathematical motivation

> *Familiar classical reasoning just amounts to abolishing this distinction by permitting elimination of double negation. The price for this notational simplification is to abandon the possibility of a computational interpretation. In "pure abstract" mathematics this might not be of any concern. However, in computational mathematics attempting to take advantage of such "pure abstract" results it is of considerable interest to trace their origin in search of a computational interpretation.*

(Peter Krauss)

Mathematical motivation

It should not come as a surprise that for this form of classical reasoning we cannot give a uniform method of describing the places in familiar classical assertions where intuitionistic logical operators are being used and where classical logical operators are being used, because this depends on how the assertion under consideration is actually being proved. There are several situations that may arise. Sometimes various options are available to be proved which all are logically equivalent, however the form of the resulting assertions may look quite different. We give some simple examples:

(Peter Krauss)

Mathematical motivation

- $\forall_i x(A \to_i \exists_c y B)$
- $\forall_i x(A \to_c \exists_i y B)$
- $\forall_c x(A \to_i \exists_i y B)$

Mathematical motivation

They are logically equivalent in Krauss' system, "but which one will be used in an assertion of classical mathematics will depend on the proof we have produced for it".

Mathematical motivation

The indiscriminate use of elimination of double negation extinguishes all finer distinctions in classical proofs. This makes it impossible to trace constructively valid parts of classical reasoning. For most pursuits in classical mathematics this appears to be irrelevant. However, as soon as the classical mathematician is interested in an algorithmic interpretation of his results, he ought to locate the places where he uses elimination of double negation because at those places the possibility of an algorithmic interpretation definitely gets lost.

(Peter Krauss)

We don't have to be classical everywhere and all the time!

We can clearly see that we also have a computational motivation via the constructivization of classical proofs!

Summation
(and many questions!)

1. *Different operators, different meanings, but the same semantical framework;*
2. *Two rival logics can co-exist in peace: accept and reject the same things.*
3. *We don't have to be classical everywhere and all the time!*
4. *Ecumenism: a new way to extract computational conten from classical proofs?*

5. *But why can't we take the classical operators as being explicitely defined by (as an abreviation) constructive operators?*
6. *There is a connection between translations and the ecumenical perspective, but what is the real nature of this connection?*
7. *Can we define an ecumenical semantics based on truth-conditions?*

8. *Can we develop a true proof-theoretical semantics for ecumenical systems?*
9. *Can we have the usual proof-theoretical results for Prawitz' ecumenical system? Normalization? Unicity of Normal Form? Strong Normalization?*
10. *And what about other ecumenical proposals? Combining other logics?*

11. *Would it be possible to develop mathematics from using ecumenical systems?*
12. *The rules we saw for $\rightarrow_c$ and $\vee_c$ uses negation ($\neg$) in their formulation? Can we get rid of these occurrences of negation? Can we have "pure" systems?*
13. *As we saw in Krauss' proposal we don't have a classical negation and an intuitionistic negation. Given that we have two implications and the $\bot$, shouldn't we have two negations?*