Logics of Context and Modal Type Theories

Valeria de Paiva
Systems and Practices Lab

parc
Palo Alto Research Center

Caveats

- Joint work with Bellin/Ritter and Mendler.
- Talk at CONTEXT'03, the fourth international multidisciplinary conference on context, in Stanford, June'03. Notions of context for a variety of applications
- Theme of my talk then: Proof Theory is essential for a logic of contexts.
- Today: what are AI contexts? why are they useful ? why should you care about them?

Motivating Example

- Consider a reliable, factual report about weapons proliferation:

  "Gulf ImpEx imported 5 shipments of medical goods in 1999. Their shipping records claim that none of the shipments contained any dual use materials. We have learned that at least one shipment contained 2 tons of fissile material. Most of this was of such low radioactivity as to prevent its dual use. We do not know whether the remainder of the fissile material could have been dual use."

- Question: according to this report, did Gulf ImpEx import any dual use materials?

Example continued

- To answer the question you needed to get inside several contexts, introduced by *claim, learn, prevent, know, could.*
- On an individual basis not hard. If

  “*Biopreparat claims that Russia’s stockpile of viruses has been destroyed.*”

  it doesn’t mean that the stockpile has been destroyed.

  (even my 10-year old can tell you that!)
- All obvious/old hat logical stuff, but we need to reason about these possibilities. Could use contexts.

Context analysis tasks

- To identify contextual structure and embedding, within sentences and texts
- To identify all the logical inferences that can be validly drawn from any given text (and only these).
- To enable reasoning across contexts e.g.:
  - If X claims that Y believes P, then X is not claiming that P is true
  - If X claims that Y knows P, then X claims that P is true

This talk focuses on logical machinery for the tasks of reasoning about and across contexts, in a formal way, using machinery that you're familiar with.

parc Palo Alto Research Center

Summing up

- Goal: *"Investigate existent logics of context for KR and design one for our project on symbolic text understanding"*
- There are a variety of these context logics
  - They capture slightly different intuitions
  - They are hard to compare logically
  - They lack important proof-theoretic properties
- Need to choose a methodology, design criteria for choosing & comparing logics, and apply it.

Overview

- Background
- Methodology: Proof theory counts
- Designing a Logic of Context &
- Simple Modal type theories
- Results: system, properties & comparisons
- Conclusions

The Importance of Proof Theory

The structure of proofs is as important to a logic as what it can prove

- A logic is defined as much by the patterns of inference it allows (proof theory) as by its semantics (model theory)
- Proof-theoretical properties (such as cut-elimination, strong normalization, sub-formula property etc.) lead to efficient, correct implementations
  - Soundness and completeness are not enough
- In AI: if you want to generate explanations from proofs, the structures of proofs matter
- Proof theory provides tools for looking at proof structure
- Main Tool here: Extended Curry-Howard isomorphism

Logic of Contexts & Modality (McCarthy'93)

- Goal: To provide AI programs with human capabilities, like reasoning in context.
- Engineering attitude: contexts as abstract mathematical entities with useful properties
  – Which kind of mathematical entities? Which properties? Useful for what?
- Basic ideas:
  – introduce syntactic modality ist(*c*,*p*) meaning proposition *p is true in context c.*
  – ist(*c*,*p*) always asserted within a context *c'*
  – Contexts can be entered and exited

parc
Palo Alto Research Center

Contexts, Modality, Natural Deduction

- McCarthy wanted a Natural Deduction system because of the natural correspondence between:
  - introducing/discharging hypotheses
  - entering/exiting contexts
- McCarthy's followers (Buvac/Mason) formalized context logic as Hilbert-style modal logic
- Modal logics are good because:
  - Modalities allow control over the context in which embedded expressions are evaluated.
  - Modalities avoid problems with self-referential paradoxes
- But most modal logics are not well-behaved proof-theoretically, they don't easily support natural deduction

How to get a modal logic of contexts

with Natural Deduction?

A modal logic of contexts with natural deduction

- Multimodal (constructive) Box-K (Bellin, de Paiva,Ritter2001, Bellin1985), system Kn
- Basic problem: Given a sequent rule like

$$\frac{A1, A2, \ldots, An \mid\!- B}{ist(c,A1), ist(c,A2), \ldots, ist(c,An) \mid\!- ist(c,B)}$$

How to build a natural deduction tree with this rule?

- Solution: Modality rule "builds-in" substitutions

$$\frac{ist(c,A1), ist(c,A2), \ldots, ist(c,An) \qquad \begin{pmatrix} A1, A2, \ldots, An \\ \vdots \end{pmatrix} \atop B}{ist(c,B)}$$

parc
Palo Alto Research Center

A modal type theory too…

- Rule leads to term calculus, similar to own previous work on constructive S4 and linear logic
- Consider first one context c and write ist(c,A1) as Box A1
- Also write rules in sequent-style ND
- Modality rule written in FP-style

$$\frac{\Gamma \vdash M1{:}\,\mathrm{Box}\ A1,\ \Gamma \vdash M2{:}\ \mathrm{Box}\ A2,\ \ldots,\ \Gamma \vdash Mn{:}\ \mathrm{Box}\ An, \quad A1, A2, \ldots, An \vdash M{:}\ B}{\Gamma \vdash \mathrm{Box}\ M\ \mathrm{with}\ M1, M2, ..Mn\ \mathrm{for}\ x1,x2,..xn{:}\ \mathrm{Box}\ B}$$

parc
Palo Alto Research Center

Extended Curry-Howard

Logic	Programming Lang	Cat Semantics
Formula	Type	Object
Proof	Term	Morphism

Categorical Semantics of CK

- Model is a cartesian closed category with an endofunctor
- This gives sound and complete model for term calculus

System CK Properties

- Advantages: (Theorems proved in Bellin,dePaiva,Ritter'01)
  - Axiomatic, Sequent calculus and Natural Deduction presentations proved equivalent
  - Strong normalization/cut-elimination
  - Confluent and satisfies subject reduction
  - Corollary: Subformula property
  - Curry-Howard isomorphism (proofs-as-programs)
  - Categorical semantics
- Problems:
  - single rule: no introduction-elimination pair,
  - Only commuting conversions,
  - not a framework (can only do K, S4,…)
- Is it too weak? What kinds of embedded contexts can we have? Are they enough for the purpose of the application? (Not in this talk…)

Another modal type theory ?

- Separation of the type theory context into boxed and not boxed assumptions
- Judgements $\Gamma \mid \Delta \mid\!- M: A$ instead of Box $\Gamma , \Delta \mid\!- M: A$
- Approach works well for S4 and LL

  (Plotkin-Barber-Benton DILL calculus)
- Have good calculus plus categorical semantics
- Can adapt method for CK?

parc
Palo Alto Research Center

More modal type theory

- Adaptation gives “formal” introduction and elimination rules

$$\frac{\Gamma | - | \text{-} M1: \text{Box } A1,\ \Gamma | - | \text{-}\ M2: \text{Box } A2, \ldots, \Gamma | - | \text{-}\ Mn: \text{Box } An, \quad A1, A2, \ldots, An | \Delta | \text{-} M: B}{\Gamma | - | \text{-}\ \text{Box } M \text{ with } M1, M2, ..Mn \text{ for } x1,x2,..xn: \text{Box } B}$$

and “axiom” $A | - \ | \text{-}\ \text{Box } A$

- Obtain calculus with subject reduction, SN and confluence
- Problem: Categorical semantics unclear…

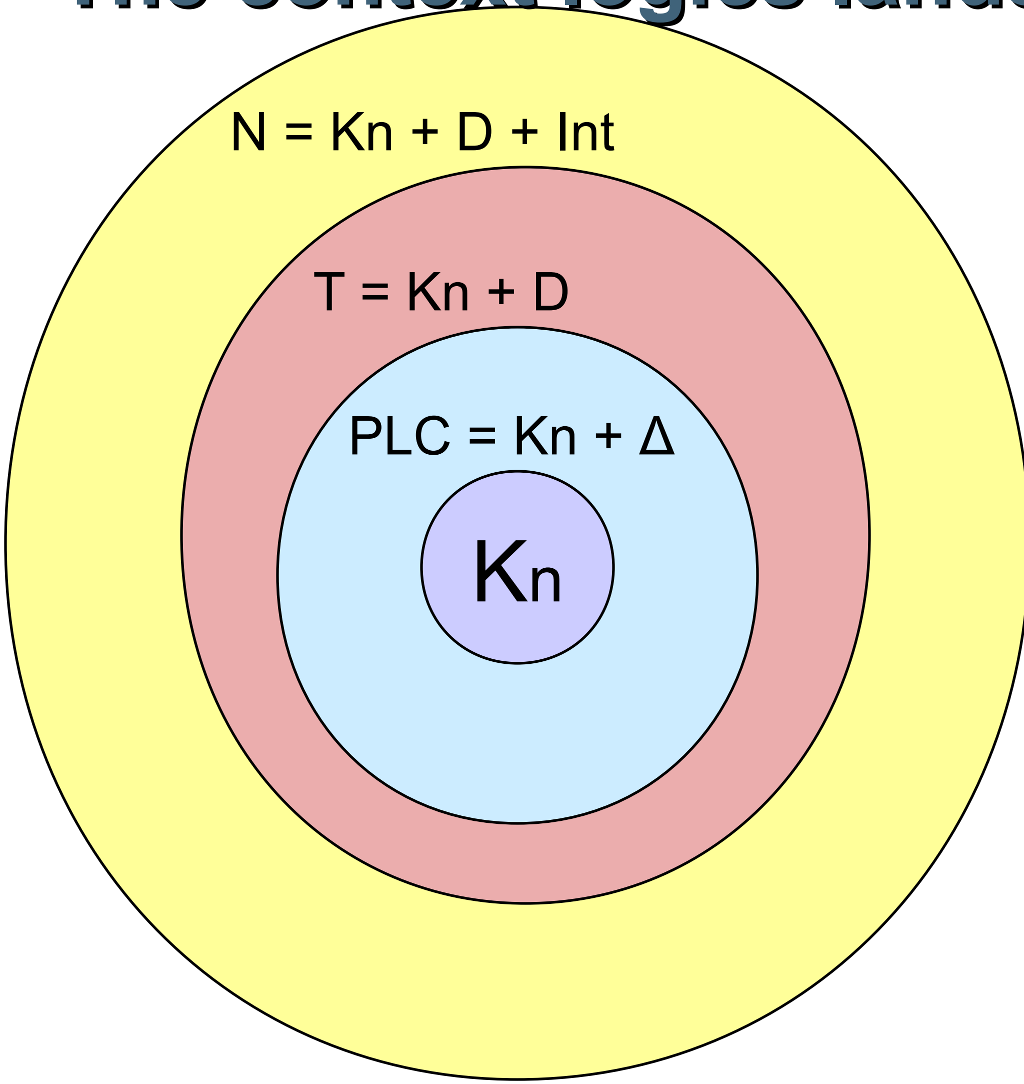
(monoidal adjunction not meaningful without T and 4)

parc
Palo Alto Research Center

Kripke Semantics for CK

- Joint with M. Mendler, based on our previous work with Alechina and Ritter
- Both necessity and possibility in CK
- Two accessibility relations, $\leq$ and R
- *fallible* worlds to deal with lack of axiom $\neg\Diamond f$
- Theorem: Soundness and completeness of possible world semantics with respect to AI logicians favorite semantics (draft only)

Comparing systems: The context logics landscape



PLC: Buvac/Mason

T: Massaci

N: Nayak

Conclusions & Further Work

- **Kn provides a basic modal logic with natural deduction, and a good basis for further extensions**
- **Kn type theory not brilliant: must choose between categorical semantics or inversion principle. Is there a way out?**
- Context logics using MCS/LMS? Simpson's framework?
- ND for constructive hybrid logics: use it for context logics ?
- ND for description logics?
- ND systems for the stronger context logics such as Massacci's T and Nayak's N
- For the NLU project, need quantifier version of Kn?

parc Palo Alto Research Center

Thank you!

http://www.cs.bham.ac.uk/~vdp/

KXDC…

- Knowledge eXtraction from Document Collections (KXDC)
- Goal: symbolic detection of entailment & inconsistency of content in medium-sized, domain-specific document collections
- Test domain: Eureka tips
- Meaning representation language needs contexts, which kind ?

Want to know more?
http://www2.parc.com/istl/groups/nltt/kxdc.html

parc Palo Alto Research Center

What makes a good logic ? natural deduction + normalization

- A **normalization** theorem
  - Any proof can be made detour free
  - Detours are introduction of a connective followed immediately by its elimination.
- Normalization guarantees
  - Ability to translate into other styles
  - Properties like cut-elimination
    - » Means that proofs that use lemmas can be proved without them.
  - Subformula property
    - » Means that proof search will not lead to infinite regress

parc
Palo Alto Research Center

Comparison to other systems

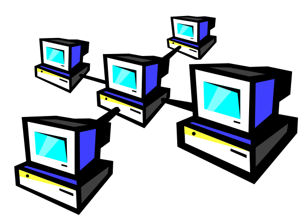
- PLC has nice first-order like semantics
  - But controversial Δ axiom ist(c1,ist(c2,A)) OR ist(c1, NOT(ist(c2,A))
- System N (Nayak) seems too strong
  - Positive (negative) introspection ist(c1,A) → ist(c2, ist(c1,A)), hard to deal with proof theoretically and
  - Debatable from the application viewpoint
- Massacci's system T seems designed to embed PLC
  - Interesting implementation, but mixes syntax and semantics
- MCS/LMS systems, generalization of Prawitz ND
  - No explicit modality (needs translation)
  - Labels/contexts are essential,
  - Need more work to compare labeled systems,
  - No Curry-Howard Iso, no categorical semantics. Yet?

parc
Palo Alto Research Center

Comparison: Propositional Logic of Contexts (PLC)

- Buvac, Buvac and Mason, Fundamenta Informatica 1995
- Axiomatic system, plus first-order like semantics
  - Comparison to possible worlds semantics
  - Some correspondence theory
- Context as a set of truth assignments, the possible states of affairs in that context
- Modality interpreted as validity, “ist(c,A)” is true iff “A” is true in all truth assignments associated with “c”
  - Different contexts can have different vocabularies
  - Truth assignments are partial (Kleene 3-valued logic)
- PLC is basically multimodal K plus controversial axiom Delta: ist(c1,ist(c2,A)) OR ist(c1, NOT(ist(c2,A)))

if all contexts have the same vocabulary.



Comparison: Massacci's Tableaux System

- A tableaux version of a logic of contexts would be useful
  - Massacci's system T is stronger than PLC, axiom D is valid
  - (As in PLC) Sequences of basic contexts and
  - Each context has its own vocabulary
  - Worked out only for propositional case
- Semantics in terms of superficial valuations & labeled deduction rules
  - Complexity results
  - Implementation

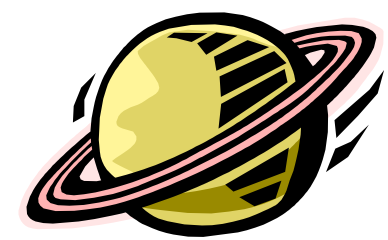
- Discussion
  - Fitting-style system, mixes syntax/semantics
  - axiom D justification?
  - Logic tailored to embed PLC?
  - Cannot provide Natural Deduction with good properties

Comparison: Nayak's Multiple Theories

- Nayak AAAI'94
- Representing and reasoning with multiple theories
  - Using modal logic directly, notation
  - "ist(c, A)" intuitively means 'in context "c" A holds'.
  - Each context has its own vocabulary
  - Worked out only for propositional case

- Traditional (off the shelf) Axiomatic system
  - Awkward "meaningful formulas"
  - Stronger logical properties of modality

  Multimodal K plus ist(c1,A) → ist(c2, ist(c1,A), D
  - Complexity results



- Discussion
  - Generalized introspection (positive and negative) plus axiom D

  ist(c,A) → NOT ist(c, NOT A)
  - Too strong a logic?
  - Cannot provide Natural Deduction with good properties

parc
Palo Alto Research Center