Inferential Statistics Simulation

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Overview

In this simulation we are doing a quick overview investigation of the exponential distribution and comparison with the Central Limit Theorem. We will investigate the distribution of averages of 40 exponentials from a thousand simulations (i.e. 1000x40 matrix). The project is set in literate programming, allowing full research reproducibility.

Setup

```
knitr::opts_chunk$set(echo = TRUE, cache = TRUE)
require(ggplot2)
```

```
## Loading required package: ggplot2
```

```
require(knitr)
```

```
## Loading required package: knitr
```

```
set.seed(144)
```

Simulations

Params

```
n = 40
lambda = 0.2
simulations = 1000

distro <- matrix(
    rexp(n * simulations, lambda), simulations
    )
```

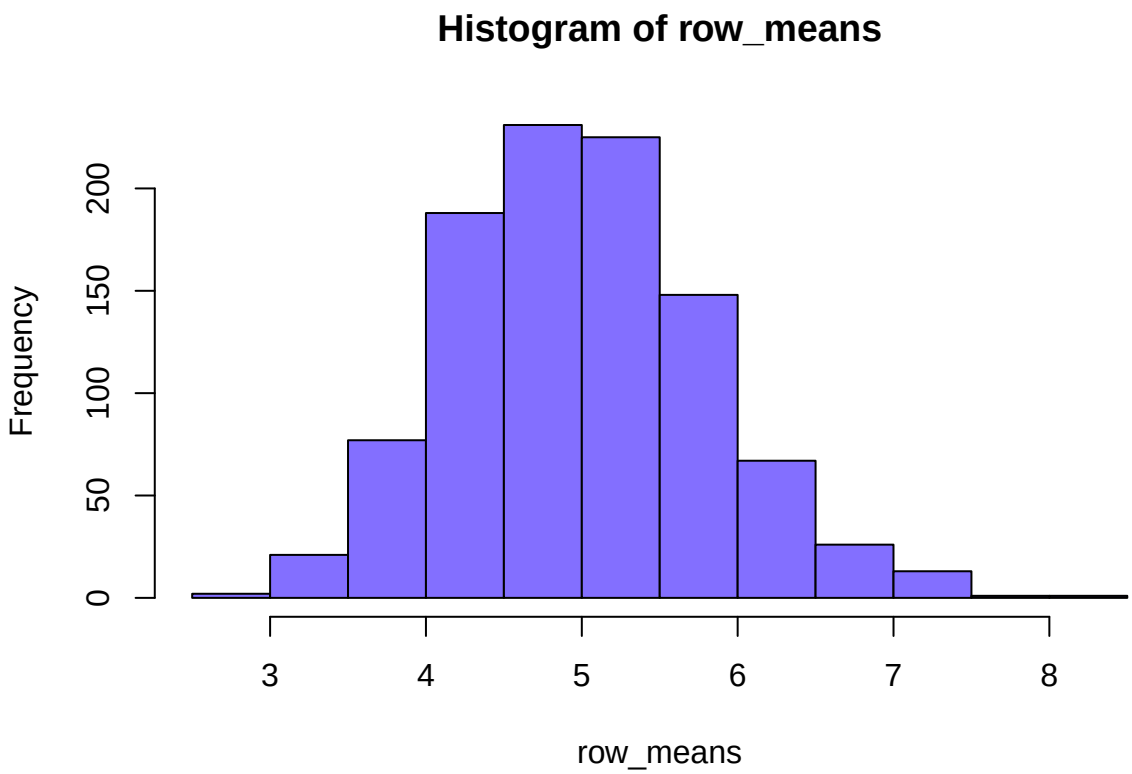
Stats

```
# Means
row_means <- apply(distro, 1, mean)
calc_mean <- mean(row_means)
exp_mean <- 1/lambda
# Standard deviation
calc_sd <- sd(row_means)
```

```r
exp_sd <- 1/lambda * 1/sqrt(n)
# Variance
calc_var <- calc_sd^2
exp_var <- exp_sd^2
```

The calculated mean is **4.998** and the expected mean is **5**, meaning the simulation result is approximately the same as E(x).

```r
hist(row_means, col = "slateblue1")
```



The simulation standard deviation is **0.807** and the expected sd is **0.791**.

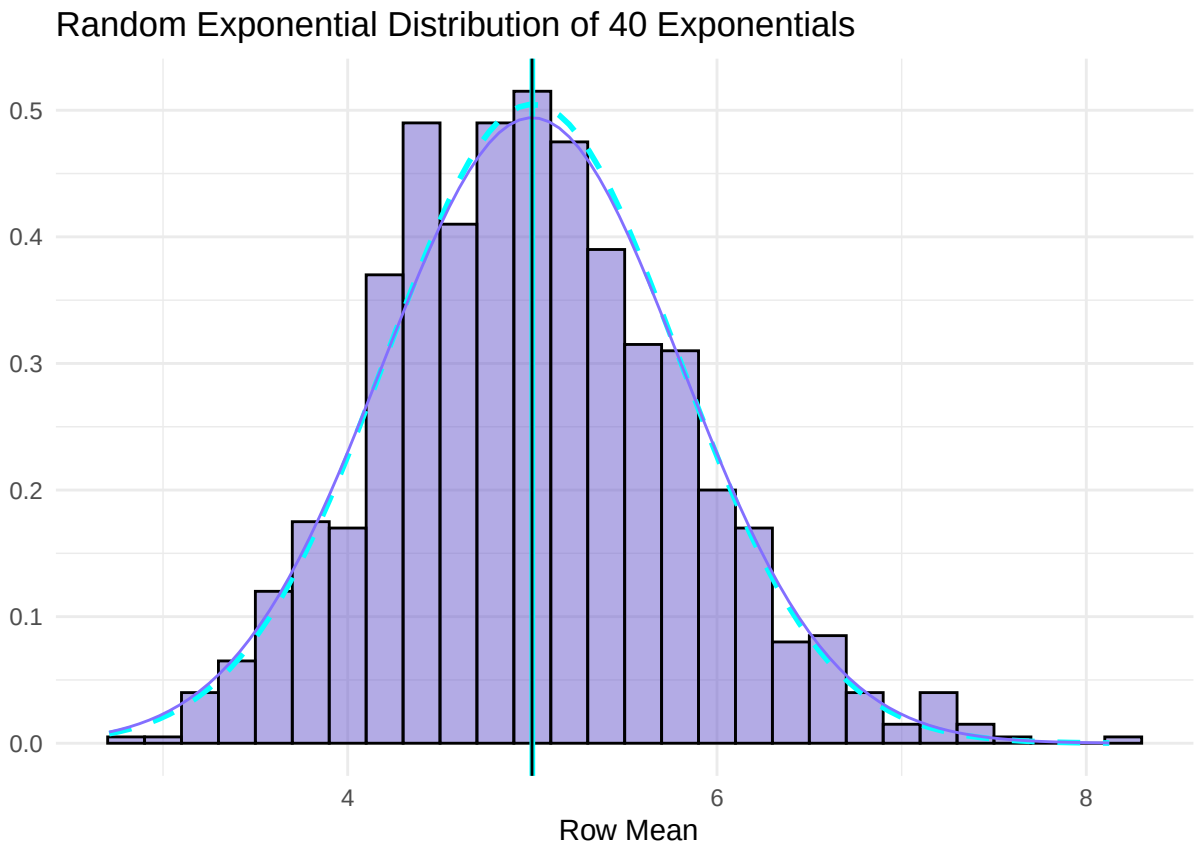
The calculated variance is **0.652** and the expected is **0.625**, meaning the variance of the simulation has approximately the same spread as expected by the CLT.

Plotting the Comparison

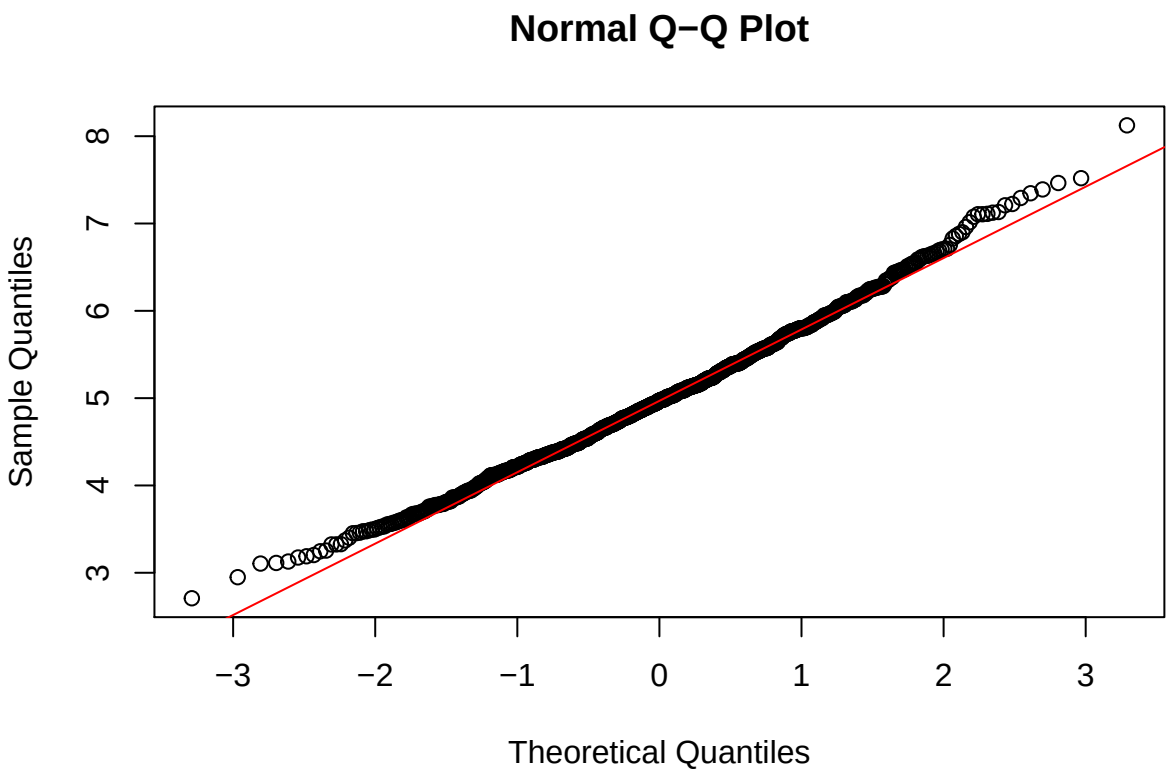
```r
df_rmeans <- data.frame(row_means)

ggplot(df_rmeans, aes(x = row_means)) +
    geom_histogram(
        binwidth = lambda,
        fill = "slateblue3",
        color = "black",
        alpha = .5,
        aes(y = ..density..)) +
    stat_function(
        fun = dnorm,
        args = list(mean = exp_mean, sd = exp_sd),
```

```
        color = "cyan",
        linetype = "dashed",
        size = 1) +
    geom_vline(
        xintercept = exp_mean,
        color = "cyan",
        size = 1) +
    stat_function(
        fun = dnorm,
        args = list(mean = calc_mean, sd = calc_sd),
        color = "slateblue1") +
    geom_vline(xintercept = calc_mean) +
    labs(
        title = "Random Exponential Distribution of 40 Exponentials",
        x = " Row Mean",
        y = ""
    ) +
    theme_minimal()
```




```
qqnorm(row_means)
qqline(row_means, col = "red")
```



By the comparison, the sample distribution (darker) is iid distributed, being almost identical to the theoretical distribution (cyan), therefore agreeing with the Central Limit Theorem.