UNIVERSITÀ
POLITECNICA
DELLE MARCHE

Numerical Heat Transfer
for Applications

Finite Difference Method for Unsteady Heat Conduction

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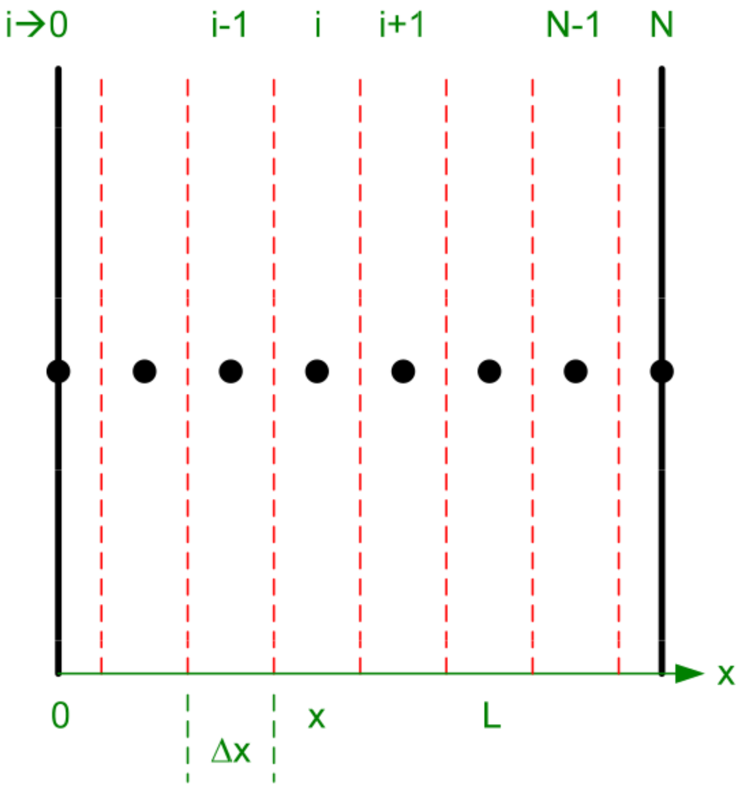
Unsteady heat conduction on 1-D plate



$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0$$

$$T(x=0,t) = 1$$

$$T(x=1,t) = 0$$

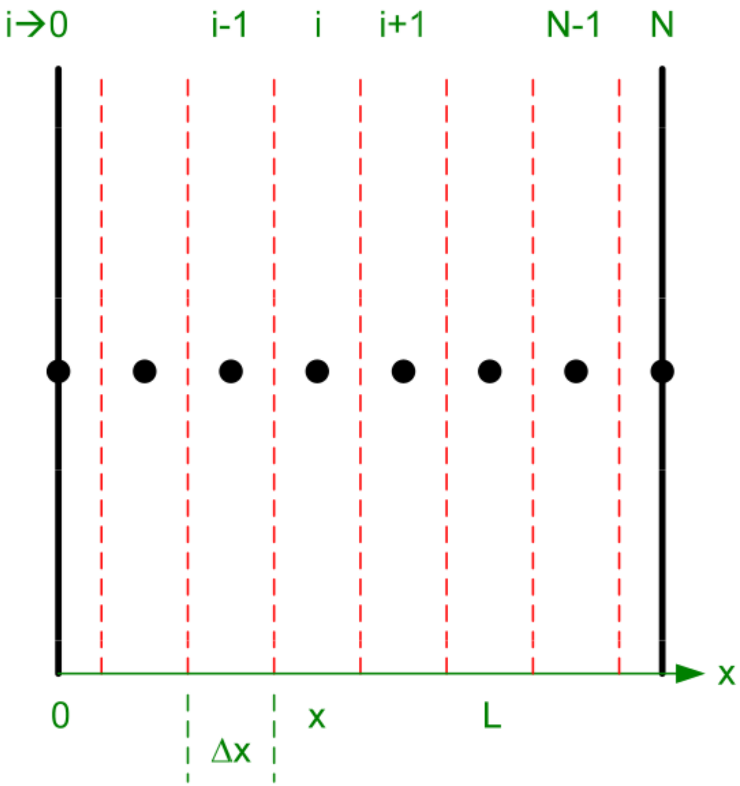
$$\partial_n T\,(y=0,t) = 0$$

$$\partial_n T\,(y=1,t) = 0$$

$$T(x,t=0) = 1$$

$$\Rightarrow \quad T(x, t >> 1) = 1 - \frac{x}{L}$$

Unsteady heat conduction on 1-D plate



$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0$$

$$\frac{T_i^{(n+1)} - T_i^{(n)}}{\Delta t} = \alpha \underbrace{\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}}$$

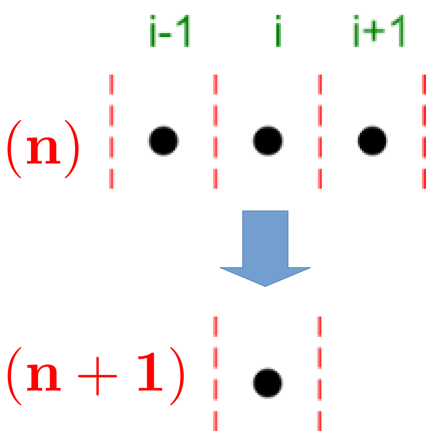
Implicit or explicit ?

Unsteady heat conduction on 1-D plate – Explicit method

$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0 \quad \Rightarrow \quad \frac{T_i^{(n+1)} - T_i^{(n)}}{\Delta t} = \alpha \frac{T_{i+1}^{(n)} - 2T_i^{(n)} + T_{i-1}^{(n)}}{\Delta x^2}$$

A similar formulation is highly attractive since starting from the solution at the time (n) is possible to compute directly the solution at the new time step (n+1)

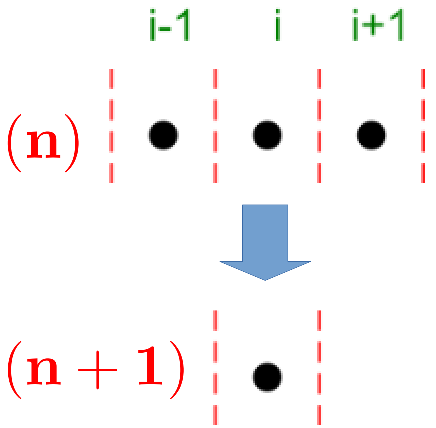
$$\boxed{T_i^{(n+1)} = \left(1 - \frac{2\alpha \Delta t}{\Delta x^2}\right) T_i^{(n)} + \frac{\alpha \Delta t}{\Delta x^2}\left[T_{i-1}^{(n)} + T_{i+1}^{(n)}\right]}$$



$$T_i = T_0 \qquad T_{i+1} = 0 \qquad T_{i-1} = 0$$

$$T_i^{(n+1)} = \left(1 - \frac{2\alpha \Delta t}{\Delta x^2}\right) T_0$$

Unsteady heat conduction on 1-D plate – Explicit method



$$T_i = T_0 \qquad T_{i+1} = 0 \qquad T_{i-1} = 0$$

$$T_i^{(n+1)} = \left(1 - \frac{2\alpha\Delta t}{\Delta x^2}\right) T_0$$

In order to mantain the physical meaning of the solution is important to satisfy the following conditions:

$$T_i^{(n+1)} < T_i^{(n)} \qquad T_i^{(n+1)} > 0$$

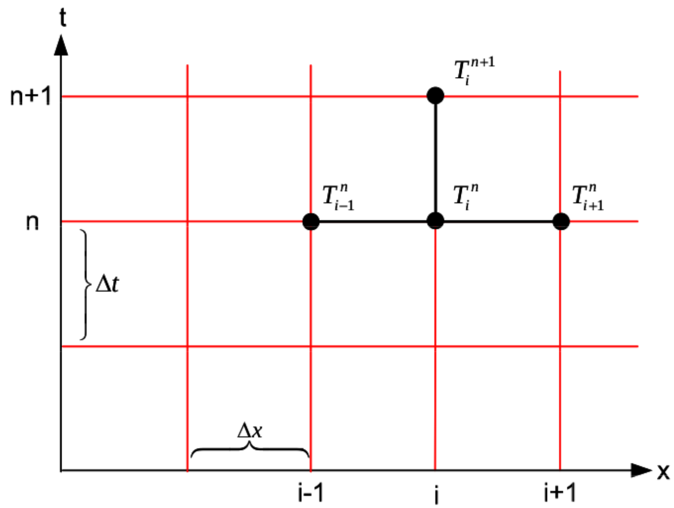
$$\boxed{0 < \frac{\alpha\Delta t}{\Delta x^2} < \frac{1}{2}}$$

The time-step size has an upper limit in explicit methods. Implicit techinques does not suffer of similar issue.

Unsteady heat conduction on 1-D plate – Implicit method

$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0 \quad \Rightarrow \quad \frac{T_i^{(n+1)} - T_i^{(n)}}{\Delta t} = \alpha \frac{T_{i+1}^{(n+1)} - 2T_i^{(n+1)} + T_{i-1}^{(n+1)}}{\Delta x^2}$$

$$\boxed{\left(1 + \frac{2\alpha\Delta t}{\Delta x^2}\right) T_i^{(n+1)} + \frac{\alpha\Delta t}{\Delta x^2}\left[T_{i-1}^{(n+1)} + T_{i+1}^{(n+1)}\right] = -T_i^{(n)}}$$



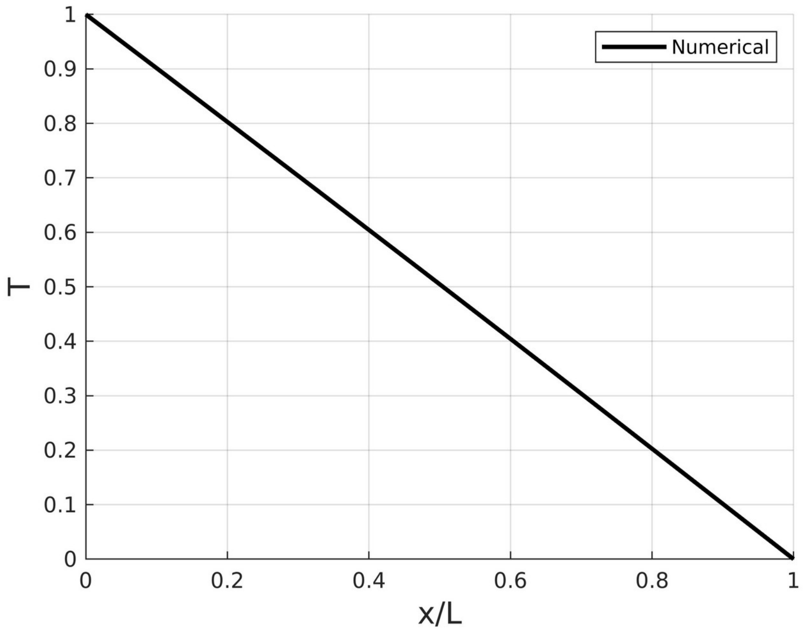
The adoption of the implicit approach requires a solution of linear system in each time-step since the unknowns are coupled (for neighbours nodes). However, in this case solution techniques does not suffer of the stability limit as in the explicit fashion.

Ex4_FD - Finite difference solution 1-D unsteady plate– Explicit method

Matlab code

```
clc
clear all

%--------------------------------
%
L      = 1;
N      = 31;  % nodes' number
%--------------------------------
alpha = 0.01;
dt     = 0.01;
%
dx = L/(N-1);
Fo = alpha*dt/dx^2;
endTime = 50;

N_time = endTime/dt;
%-------------------------------
T       = zeros(N,1);
T(1)    = 1;
T(end) = 0;
%
T0     = ones(N,1);

for i=1:N_time
    for j=2:N-1
        T(j) = (1-2*Fo)*T0(j) + Fo*(T0(j-1) +
T0(j+1)) ;
    end
        T0   = T;
end
```

$$T(x, t >> 1) = 1 - \frac{x}{L}$$



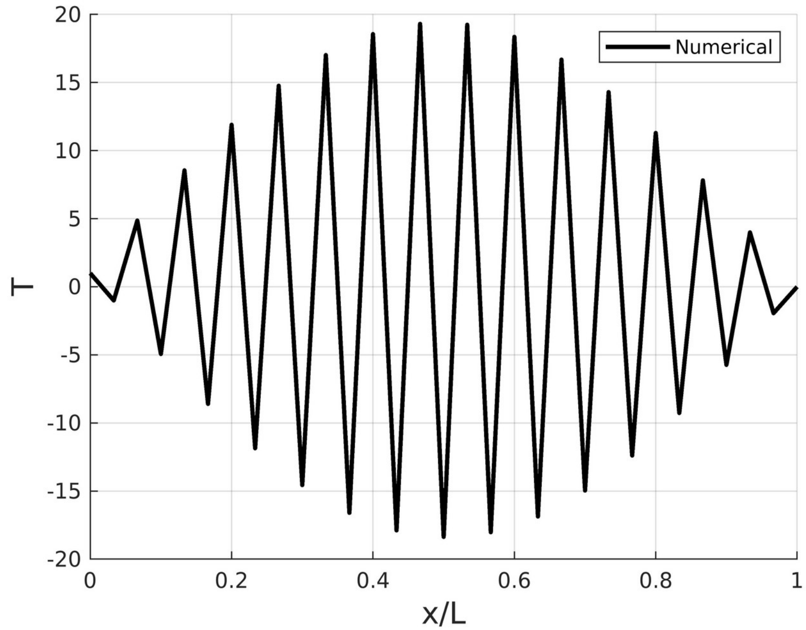
$$\mathrm{Fo} = 0.09$$

Ex4_FD - Finite difference solution 1-D unsteady plate – Explicit method

Matlab code

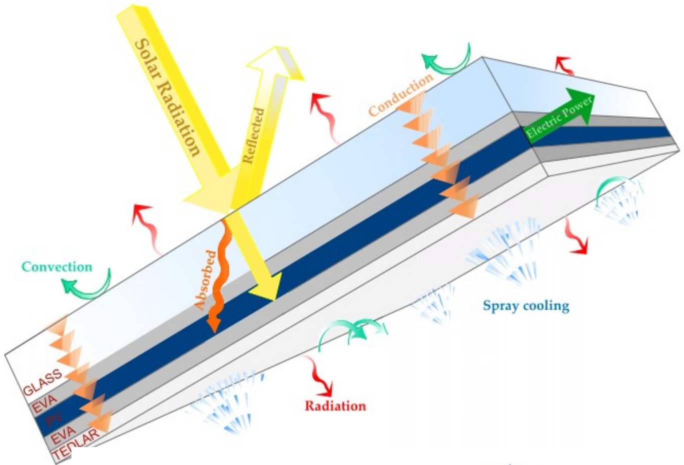
```
clc
clear all

%--------------------------------
%
L      = 1;
N      = 31;  % nodes' number
%--------------------------------
alpha = 0.01;
dt     = 0.056;
%
dx = L/(N-1);
Fo = alpha*dt/dx^2;
endTime = 50;

N_time = endTime/dt;
%-------------------------------
T       = zeros(N,1);
T(1)    = 1;
T(end) = 0;
%
T0     = ones(N,1);

for i=1:N_time
    for j=2:N-1
        T(j) = (1-2*Fo)*T0(j) + Fo*(T0(j-1) +
T0(j+1)) ;
    end
        T0   = T;
end
```

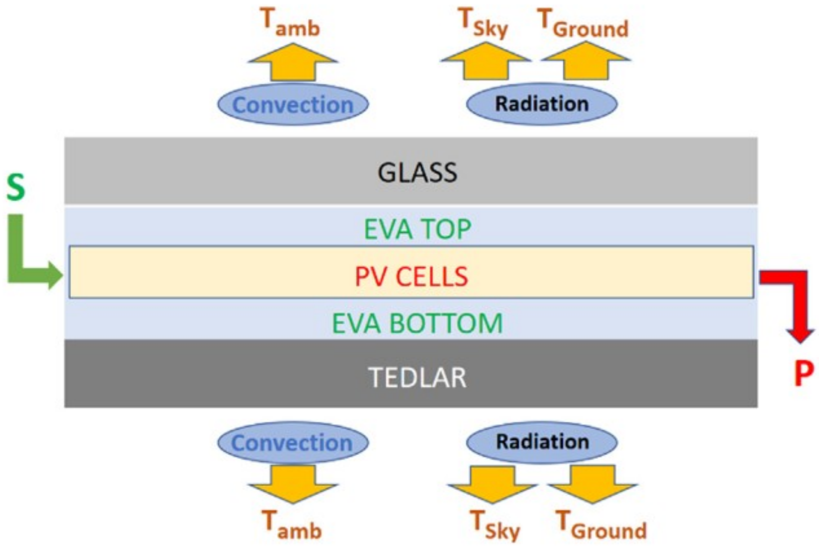
$$T(x, t >> 1) = 1 - \frac{x}{L}$$

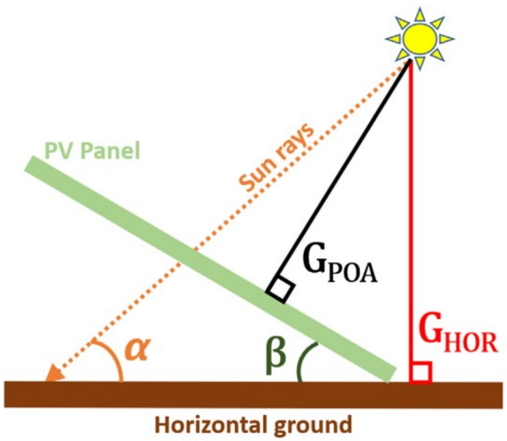


$$\mathrm{Fo} = 0.504$$

Thermal model of PV panel in a nutshell





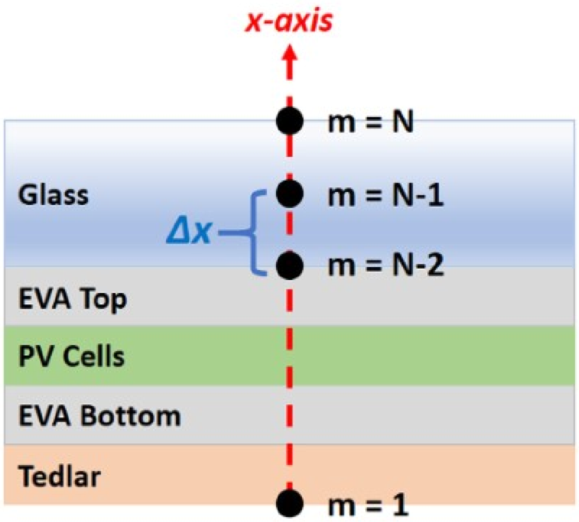


$$G_{POA} = \frac{G_{HOR} \sin(\alpha + \beta)}{\sin(\alpha)}$$

$$S = \widetilde{\alpha_{PV}} \times \tau(\theta_b) \times G_{POA}$$

<u>Total adsorbed radtion from PV cells</u>

Thermal model of PV panel in a nutshell



A portion of total incident irradiation, which is absorbed by different layers of the PV panel and the portion of total absorbed solar irradiation (S), which is not converted to electric current by the PV cells, builds up to produce heat within the PV.
In this model only front glass (fg) and PV cells are able to adsorbe radiation.

$$\rho c \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + G\left(t\right)$$

$$G_{fg} = \frac{1}{V_{fg}} \alpha_{fg} \times G_{POA} \times A_{PV,p}$$

$$G_{PV} = \frac{1}{V_{PV}} \left[S \times A_{PV} \times \left(1 - \eta_{PV}\right)\right]$$