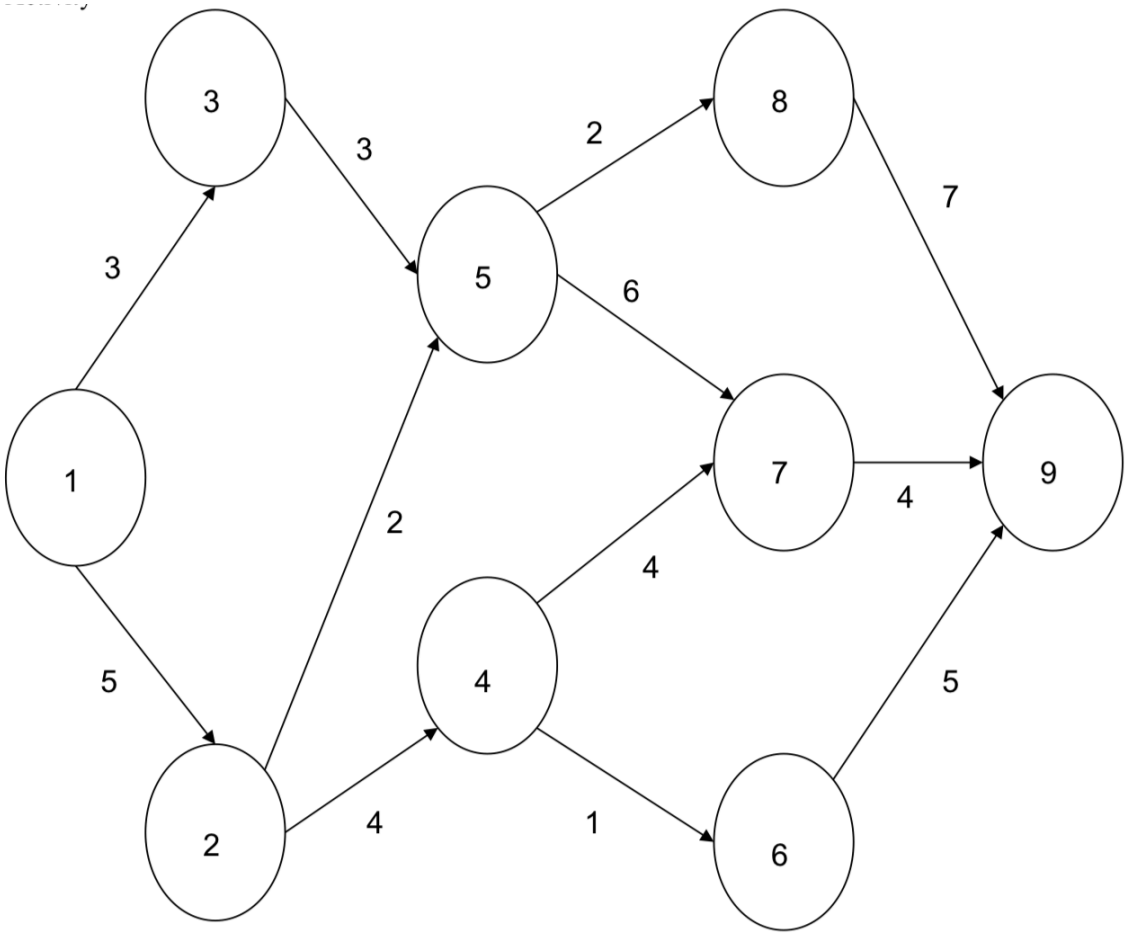
Assignment 5

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.



Formulate and solve the binary integer programming (BIP) model for this problem using library *lpsolve* or equivalent in R.

Sol.-

Decision variable:

X_{ij} = 1, the arc from node i to node j is chosen in the optimal (longest) path otherwise X_{ij} = 0

Objective: Maximize the total time required from node 1 to node 9:

Max Z. = $5X_{12} + 3X_{13} + 3X_{35} + 2X_{25} + 4X_{24} + 4X_{47} + 1X_{46} + 2X_{58} + 6X_{57} + 6X_{57} + 5X_{69} + 4X_{79} + 7X_{89}$

Constraint:

For longest route problem, following constraint are to be satisfied,

For origin Node 1, outgoing arc is equal to 1,

$X_{12} + X_{13} = 1$
For intermediate nodes,

Arc in = arc out

For node 2: $X_{12} – X_{25} - X_{24} = 0$
For node 3: $X_{13} – X_{35} = 0$
For node 4: $X_{24} – X_{46} – X_{47} = 0$
For node 5: $X_{25} + X_{35} - X_{57} - X_{58} = 0$
For node 6: $X_{46} – X_{69} = 0$
For node 7: $X_{47} + X_{57} – X_{79} = 0$
For node 8: $X_{58} – X_{89} = 0$

For destination node: Arc in = 1

Therefore, For node 9, $X_{69} + X_{79} + X_{89} = 1$

The critical path (longest path) is 17

By running this problem on lp solver, the following are the results:

“X_{12}” “1”
“X_{25}” “1”
“X_{57}” “1”
“X_{79}” “1”

2. **Selecting an Investment Portfolio** An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

	Stock							
	S1	S2	S3	H1	H2	H3	C1	C2
Price per share	$40	$50	$80	$60	$45	$60	$30	$25
Growth rate	0.05	0.10	0.03	0.04	0.07	0.15	0.22	0.25
Dividend	$2.00	$1.50	$3.50	$3.00	$2.00	$1.00	$1.80	$0.00

1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?
2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

Sol.-

Expected return of stock= D1 /P0 + g

where, D1 = Future dividend per share

P0 = Current price per share

g = growth rate

S1 = 2(1+0.05) / 40 + 0.05 = 10.25

S2=1.50 (1+0.10) /50 + 0.10 = 13.3

S3= 3.50 (1+0.03)/ 80 + 0.03 = 7.51

H1=3. (1+0.04)/ 60 + 0.04 = 9.2

H2 =2 (1+0.07) /45 + 0.07 = 11.76

H3= 1 (1+0.15) /60 + 0.15 = 16.92

C1 = 1.8 (1+0.22)/ 30 + 0.22 = 29.32

C2 = 0 (1+0.25) /25 = 0

Maximum amount invested in 1 sector = 2.5 million * 40% = 1 million

Minimum investment in each stock = .1 million

Return on the portfolio if invested equally = 10.25 + 13.3 + 7.51 + 9.2 + 11.76 + 16.92 +29.32 + 0 / 8 = 12.28

40% investment in sector 3 (c1, c2) = C1 = 900000 , C2 = 100000

40% investment in sector 2 (H1 , H2, H3) = H1 =100000, H2 = 100000, H3 = 800000

Balance in sector 1 (S1, S2, S3) = S1 = 100000, S2= 300000 , S3 = 100000

Maximum return in portfolio = = 10.25*(.1/2.5) + 13.3 *(.3/2.5) + 7.51 *(.1/2.5) + 9.2* (.1/2.5) + 11.76 * (0.1/2.5)+ 16.92 * (.8/2.5) +29.32 * (.9/2.5)+ 0 * (.1/2.5) = 19.11 %

Optimal number of share to buy each of the stock =

S1 = 100000/ 40 = 2500

S2= 100000/ 50 = 2000

S3= 300000/ 80 = 3750

H1= 100000/ 60 = 1666.67

H2 = 100000/ 45 = 2222.22

H3= 800000/ 60 = 13333.33

C1 = 900000/ 30 = 30000

C2 = 100000/ 25 = 4000

Dollar amount invested in each stock-

S1 = 2500 * $40 = $100,000

S2 = 2000 * $50 = $100,000
S3 = 3750 * $80 = $300,000
H1 = 1667 * $60 = $100,020
H2 = 2222 * $45 = $99,990
H3 = 13333 * $60 = $799,980
C1 = 30000 * $30 = $900,000
C2 = 4000 * $25 = $100,000