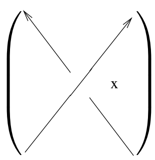
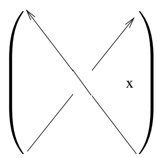
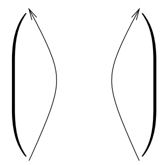
ENIGE FORMULES

Skein relatie voor Jones polynoom:

A^4X  $-\,A^{-4}X$  $+\,(A^2-A^{-2})X$  $=0$

Relatie tussen sporen:

$$\mathrm{Tr}((\alpha\otimes \mathrm{Id}_W)(\mathrm{Id}_U\otimes\beta)) = \mathrm{Tr}(\mathrm{Tr}_1(\alpha)\,\mathrm{Tr}_2(\beta))$$

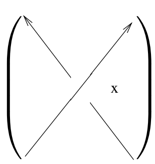
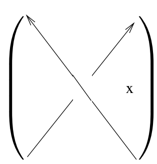
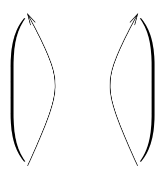
Condities voor linkinvariant:

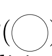
$$(\eta\otimes\eta)R = R(\eta\otimes\eta)$$
$$\mathrm{Tr}_2((\mathrm{Id}_V\otimes\eta)R^{\pm1}) = \mathrm{Id}_V$$

Standaard oplossing van YB-vgl:

$$\begin{pmatrix} q & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & q-q^{-1} & 0\\ 0 & 0 & 0 & q\end{pmatrix}$$

Skein relatie voor de HOMFLY polynoom:

xP  $-\,x^{-1}P$  $-\,yP$  $=0$

en $P($$)=1$.

Conditie voor een quasi-cocommutatieve bialgebra:

$$\Delta^{\mathrm{op}}(h) = P\Delta(h)P^{-1}$$

Conditie voor een braided bialgebra:

$$(\Delta\otimes\mathrm{Id}_H)(P) = P_{13}P_{23}$$
$$(\mathrm{Id}_H\otimes\Delta)(P) = P_{13}P_{12}$$

Vermenigvuldigingsregel in de Drinfeld double:

$$(a\otimes b)(a'\otimes b') = \sum_{b,a'} \langle a'_{(1)}, S^{-1}b_{(3)}\rangle\langle a'_{(3)}, b_{(1)}\rangle a a'_{(2)}\otimes b_{(2)}b'$$

De impliciete karakterisatie van de vermenigvuldiging:

$$(a\otimes 1)(a'\otimes 1) = aa'\otimes 1$$
$$(1\otimes b)(1\otimes b') = 1\otimes bb'$$
$$(a\otimes 1)(1\otimes b) = a\otimes b$$
$$\sum_{a,b}\langle a_{(1)}, b_{(2)}\rangle(1\otimes b_{(1)})(a_{(2)}\otimes 1) = \sum_{a,b}\langle a_{(2)}, b_{(1)}\rangle a_{(1)}\otimes b_{(2)}$$

De universele R-matrix:

$$P = \sum_i 1 \otimes b_i \otimes a_i \otimes 1$$

met $(a_i)_i$, $(b_i)_i$ duale basissen voor de bilineaire vorm.

Hopf algebra structuur op $U_q(\mathfrak{sl}_2(\mathbb{C}))$.

$$\begin{aligned}
KK^{-1} &= K^{-1}K = 1 \\
KEK^{-1} &= q^2 E \\
KFK^{-1} &= q^{-2} F \\
EF - FE &= \frac{K - K^{-1}}{q - q^{-1}} \\
\Delta(E) &= 1 \otimes E + E \otimes K \\
\Delta(F) &= K^{-1} \otimes F + F \otimes 1 \\
\Delta(K) &= K \otimes K \\
\Delta(K^{-1}) &= K^{-1} \otimes K^{-1} \\
\epsilon(E) &= \epsilon(F) = 0 \\
\epsilon(K) &= \epsilon(K^{-1}) = 1 \\
S(E) &= -EK^{-1} \\
S(F) &= -KF \\
SK &= K^{-1} \\
SK^{-1} &= K
\end{aligned} \tag{0.1}$$

Hopf bilineaire vorm op U^+:

$$\langle K^a E^b, K^c E^d \rangle = q^{-2ac} \frac{(b)!_{q^{-2}}}{(q - q^{-1})^b} \delta_{b,d}$$

Braiding op $\bar{U}_q(\mathfrak{sl}(\mathbb{C}))$:

$$\frac{1}{d} \left(\sum_{a,b=1}^{d-1} q^{2ab} K^a \otimes K^{-b} \right) \left(\sum_{c=0}^{d-1} \frac{E^c \otimes E^c}{\langle E^c, E^c \rangle} \right)$$

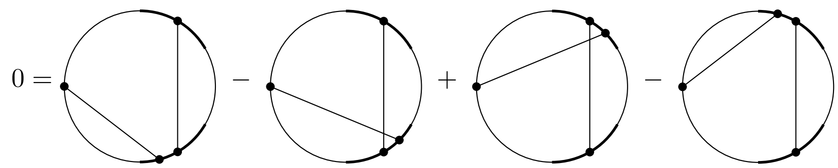
Irreduciebele representaties van $U_q(\mathfrak{sl}_2)$.

$$W_m^{\pm} = \bigoplus_{u=0}^{m} \mathbb{C} w_u$$

$$\begin{aligned}
Kw_u &= \pm q^{m-2u} w_u \\
Fw_u &= w_{u+1} \qquad u = 0, \ldots, m-1 \\
Fw_m &= 0 \\
Ew_u &= \sigma_u w_{u-1} \qquad u = 1, \ldots, m \\
Ew_0 &= 0
\end{aligned}$$

$$\sigma_u = \pm \frac{q^u - q^{-u}}{q - q^{-1}} \cdot \frac{q^{m-u+1} - q^{-(m-u+1)}}{q - q^{-1}}$$

De 4-term relatie (in A en A^{fr})

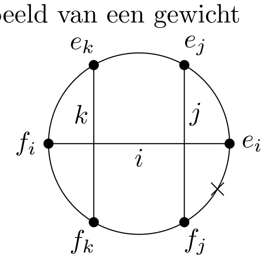


Antipode voor A

$$S(\bar{D}) = \sum_{\substack{S_1 \coprod \cdots \coprod S_n = K \\ S_i \neq \emptyset}} (-1)^n \bar{D}_{S_1} \cdots \bar{D}_{S_n}$$

K is de verzameling koorden in D.

Een voorbeeld van een gewicht



$$\rightarrow \qquad \textstyle\sum_{ijk} e_i e_j e_k f_i f_k f_j \in U\mathfrak{g}$$

De deframing formule voor koorddiagrammen

$$\overline{D}^{\mathrm{defr}} = \sum_{J \subset \mathrm{koorden}(D)} (-\Theta)^{|K|-|J|} \overline{D}_J$$

met $K = \mathrm{koorden}(D)$.