

Artificial Intelligence

Naïve Bayes



Machine Learning

- Up until now: how use a model to make optimal decisions
- Machine learning: how to acquire a model from data / experience
 - Learning parameters (e.g. probabilities)
 - Learning structure (e.g. BN graphs)
 - Learning hidden concepts (e.g. neural nets)
- Today: model-based classification with Naive Bayes

Machine Learning: General Types

- Association
- Supervised Learning
 - Classification
 - Regression
 - Ranking
- Unsupervised Learning
- Semi-supervised Learning
- Reinforcement Learning

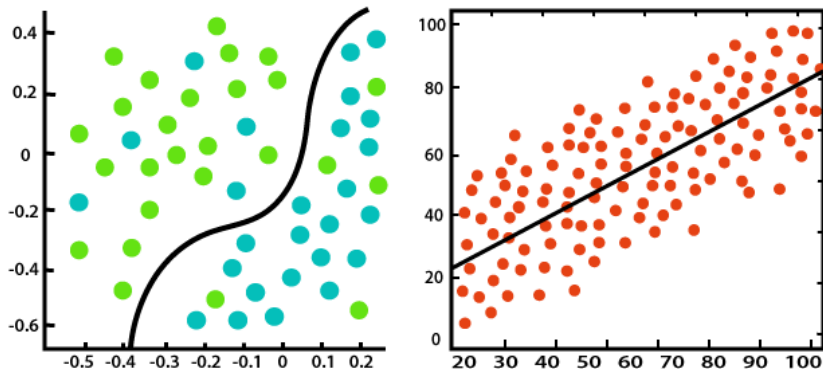
Machine Learning: Association

- Basket analysis: $P(Y|X)$: probability that somebody who buys X also buys Y where X and Y are products/services.
 - Example: $P(\text{chips}|\text{beer}) = 0.7$



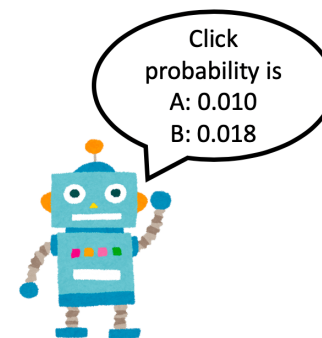
Machine Learning: Supervised Learning

- Scenario: Given a set of labeled training data $(x_1, y_1), \dots, (x_N, y_N)$, i.e., observations of the (input, output) measurement pair:
 - Classification: Y takes values in a finite, unordered set (e.g., spam or not, digit 0-9, object class in recognition)
 - Regression: Y is quantitative (e.g., 3D object pose, locations)
 - Ranking: Y is an ordering between (or equivalently, a set of ordering scores for) a finite set of predictors

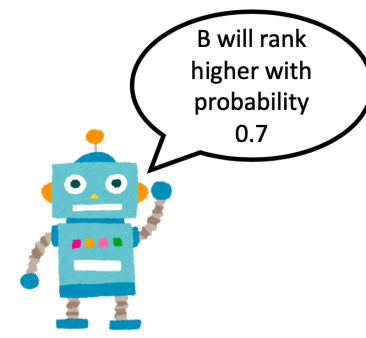
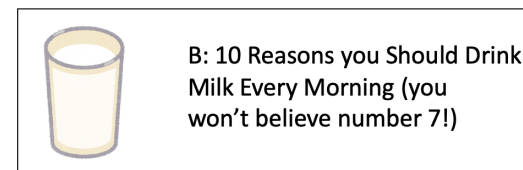
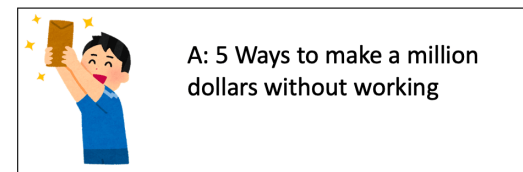


Classification

Regression



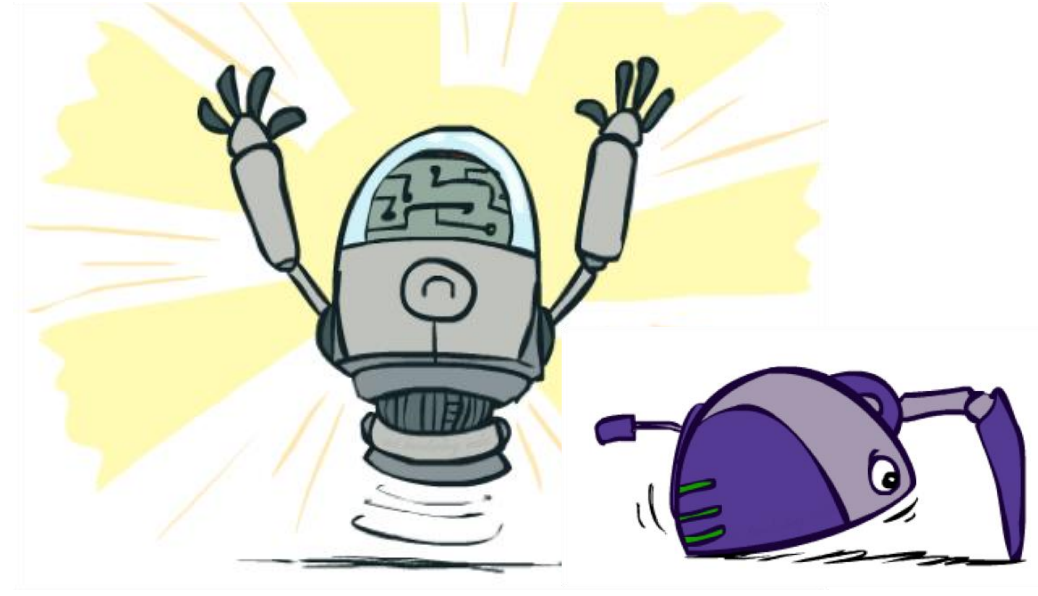
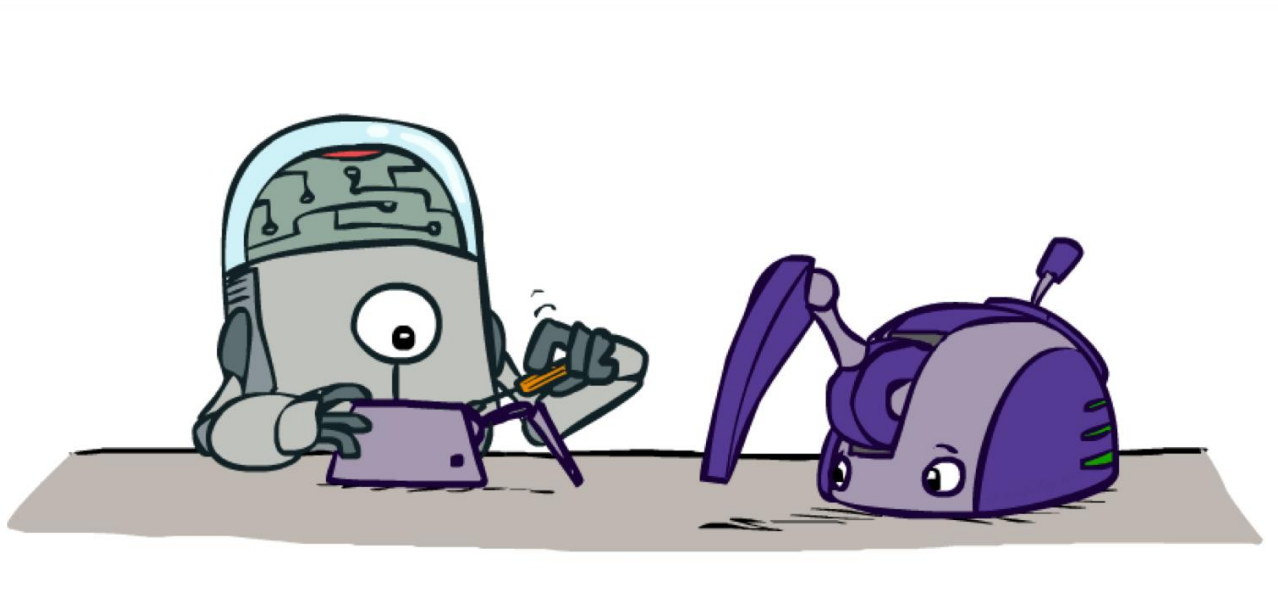
Classification



Learning to Rank

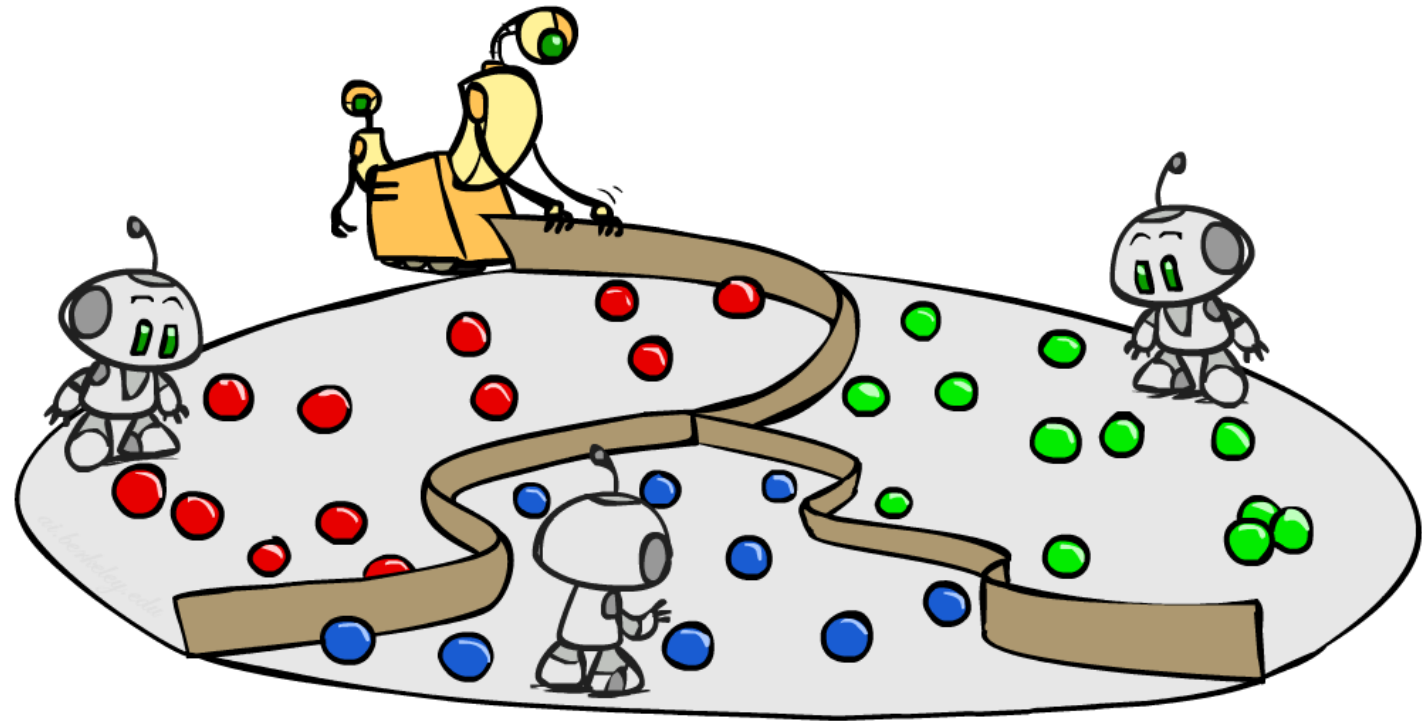
Machine Learning: Supervised Learning

- Objectives: Based on the training data, we aim to:
 - Correctly predict unseen test cases
 - Understand which inputs affect the outcome, and how
 - Assess the quality of our predictions and inferences



Machine Learning: Unsupervised Learning

- No output variable, just a set of features measured on a set of samples.
- Objectives: search for interesting/useful characteristics in the data, e.g.,
 - Find groups of samples that exhibit similarity in some sense
 - Find subset(s) of features that behave similarly
 - Find combinations of features with the most variation
- Different from supervised learning, but can be useful as a pre-processing step for supervised learning.

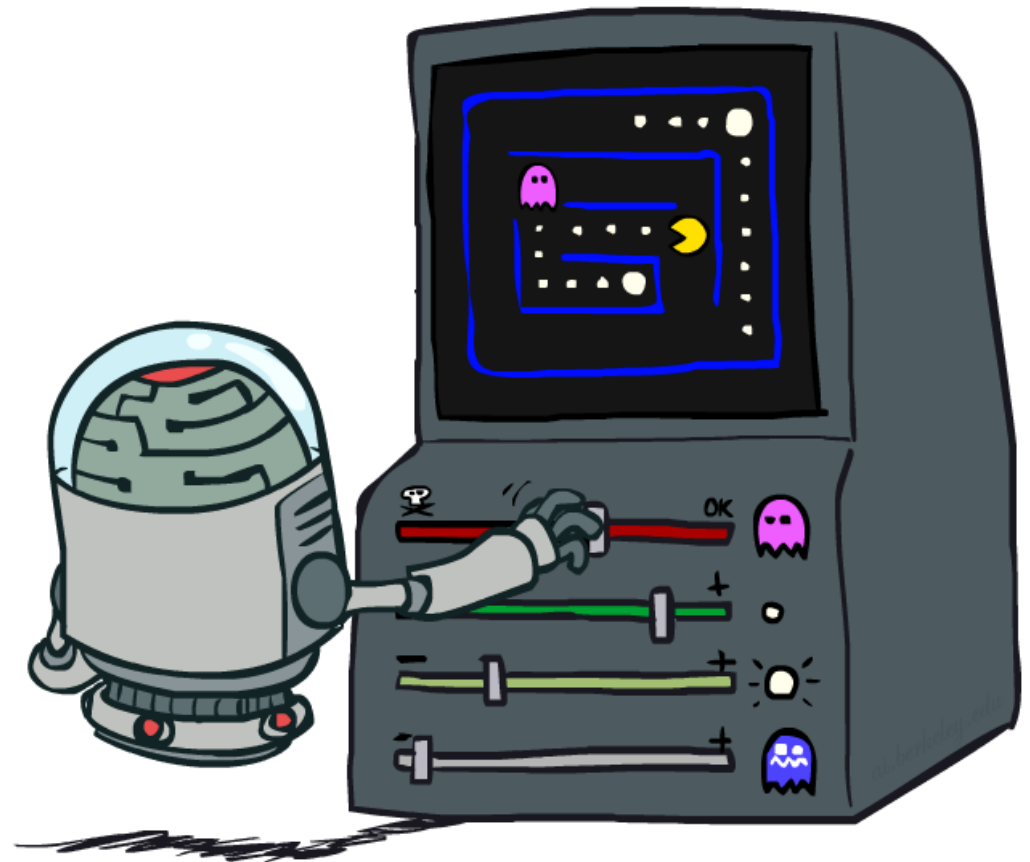


Machine Learning: Semi-supervised Learning

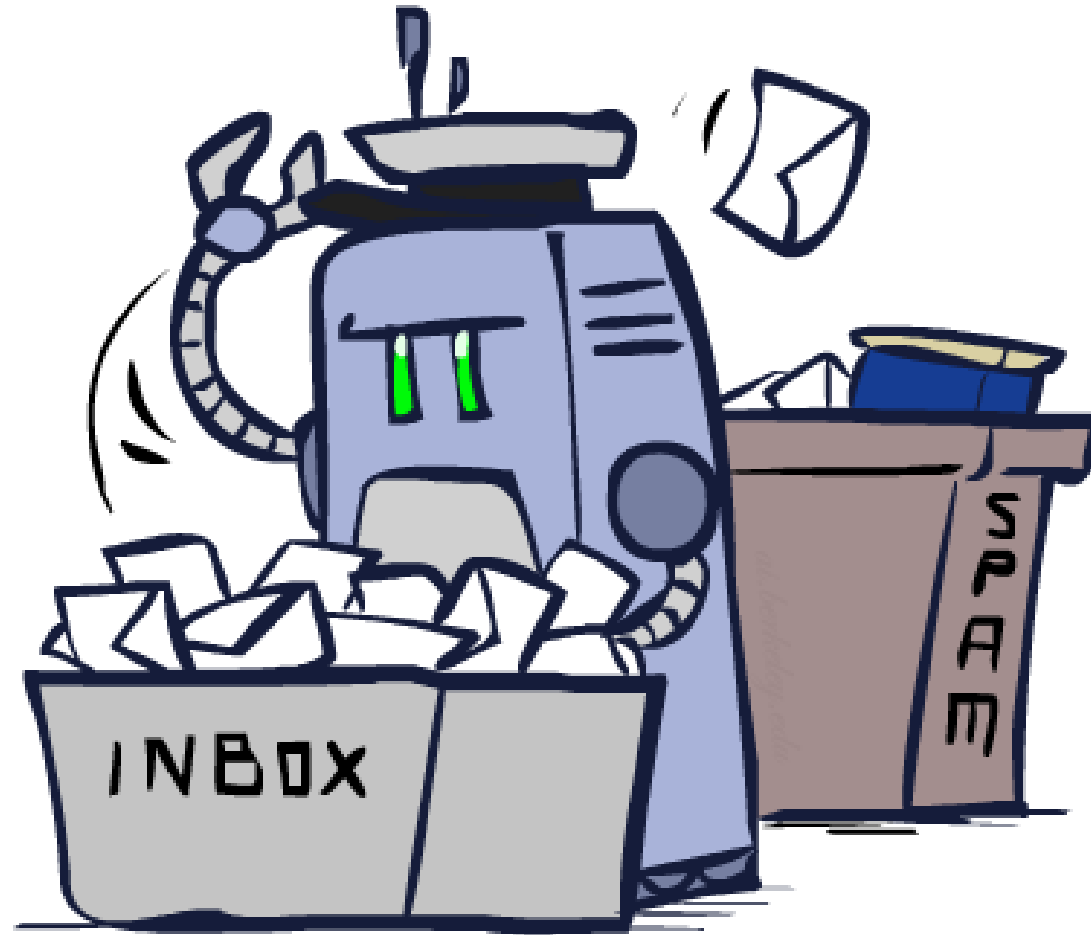
- Traditional classifiers learn only from labeled data.
- Label data can be expensive / difficult to collect.
 - Human annotation is slow, boring!
 - Labels can require experts, or special devices to acquire.
- We prefer to get better performance for free: Unlabeled data!
- Goal: build a model from both labeled and unlabeled data, or partially labeled data

Machine Learning: Reinforcement Learning

- No supervised output but delayed reward
 - (Usually) A sequence of outputs
- Learning a policy
- Typical applications:
 - Game playing
 - Credit assignment problem
 - Robot in a maze

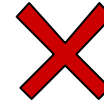


Classification



Example: Spam Filter

- Input: an email
- Output: spam/ham
- Setup:
 - Get a large collection of example emails, each labeled “spam” or “ham”
 - Note: someone has to hand label all this data!
 - Want to learn to predict labels of new, future emails
- Features: The attributes used to make the ham / spam decision
 - Words: FREE!
 - Text Patterns: \$dd, CAPS
 - Non-text: SenderInContacts, WidelyBroadcast
 - ...



Dear Sir.

First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidential and top secret. ...



TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

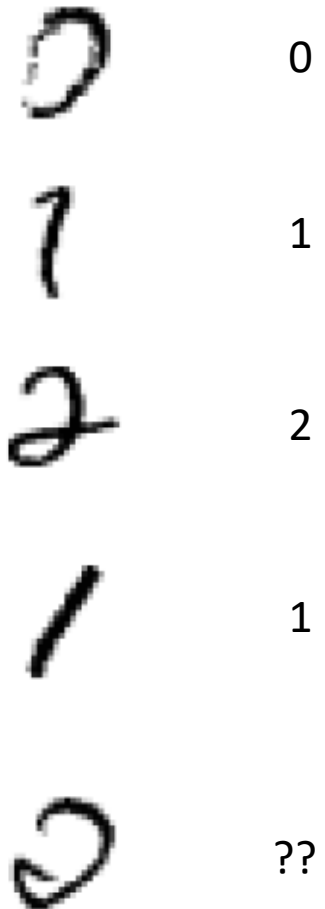
99 MILLION EMAIL ADDRESSES FOR ONLY \$99



Ok, I know this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

Example: Digit Recognition

- Input: images / pixel grids
- Output: a digit 0-9
- Setup:
 - Get a large collection of example images, each labeled with a digit
 - Note: someone has to hand label all this data!
 - Want to learn to predict labels of new, future digit images
- Features: The attributes used to make the digit decision
 - Pixels: (6,8)=ON
 - Shape Patterns: NumComponents, AspectRatio, NumLoops
 - ...
 - Features are increasingly induced rather than crafted

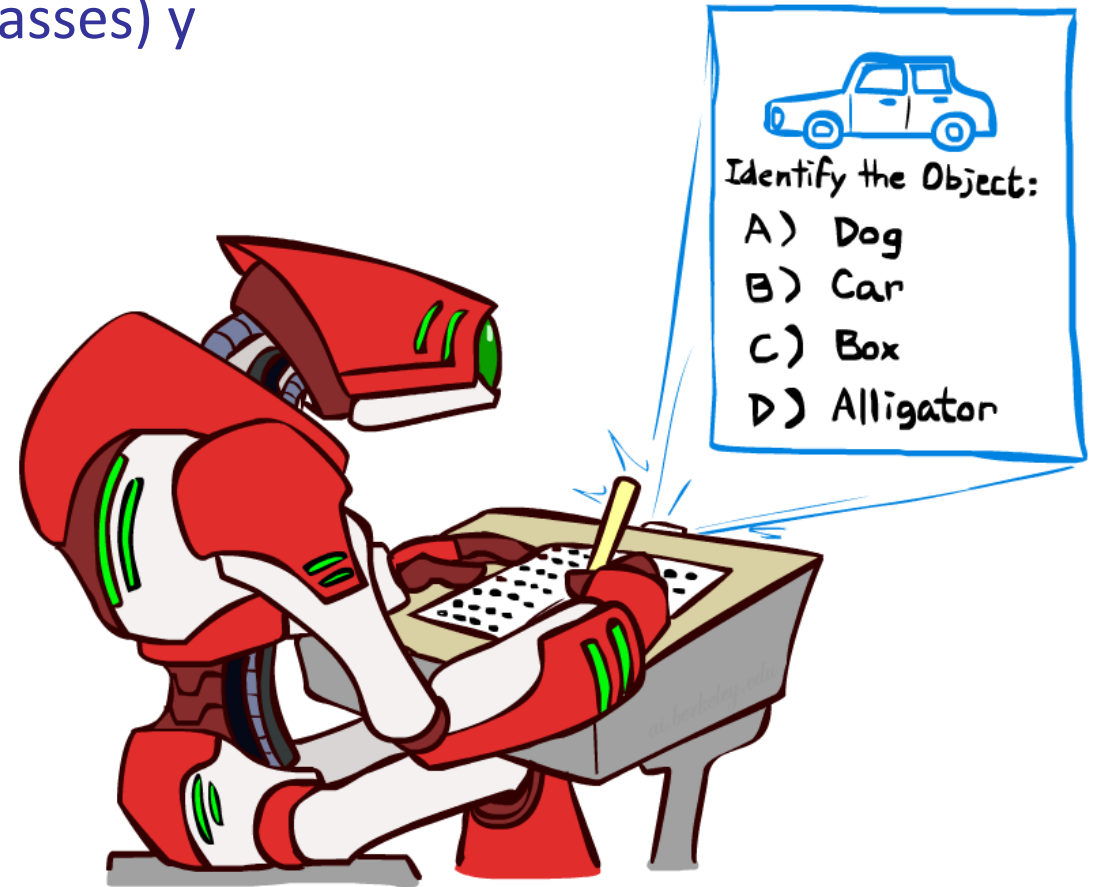


Other Classification Tasks

- Classification: given inputs x , predict labels (classes) y

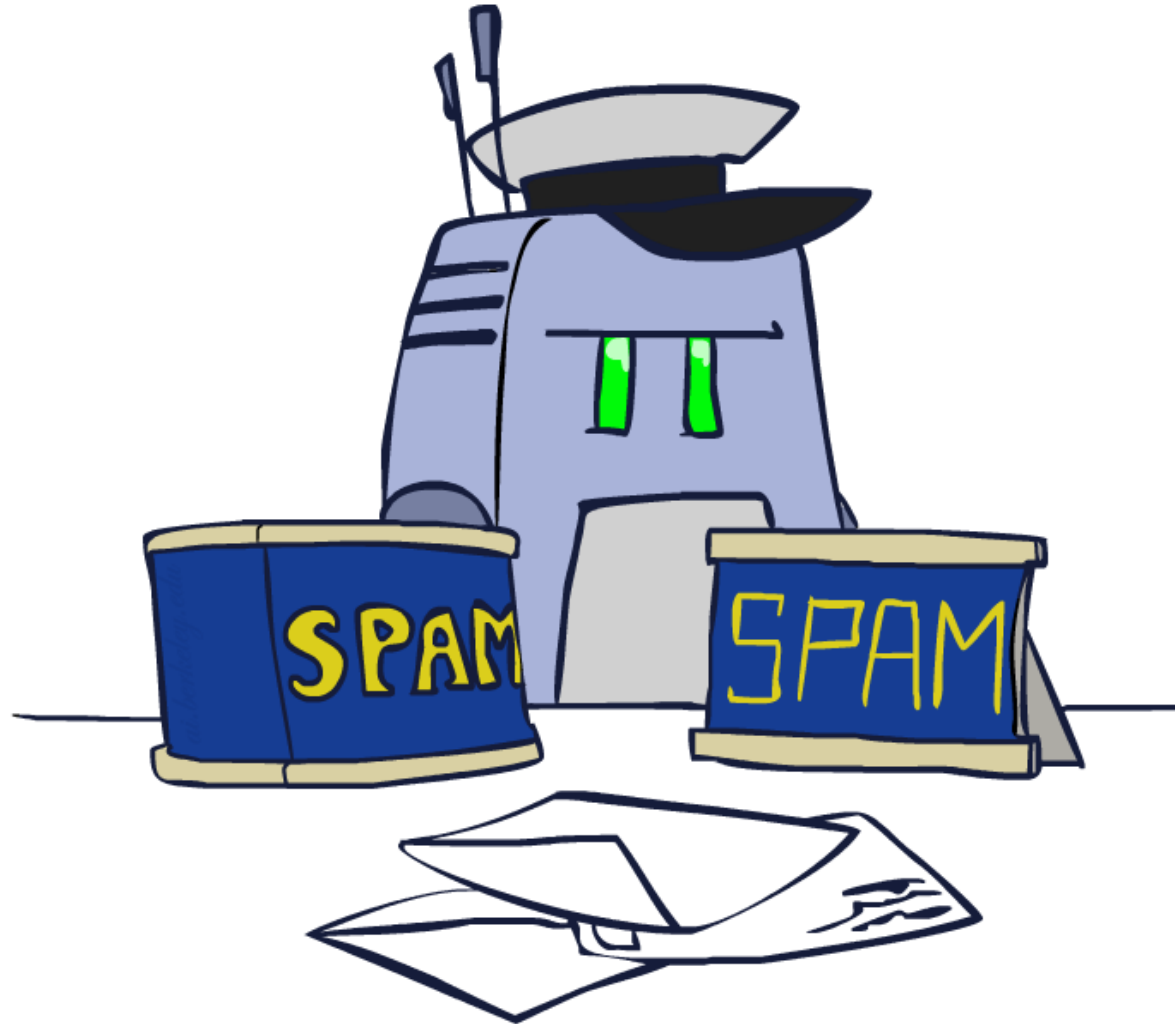
- Examples:

- Medical diagnosis (input: symptoms, classes: diseases)
- Fraud detection (input: account activity, classes: fraud / no fraud)
- Automatic essay grading (input: document, classes: grades)
- Customer service email routing
- Review sentiment
- Language ID
- ... many more



- Classification is an important commercial technology!

Model-Based Classification



Model-Based Classification

- Model-based approach
 - Build a model (e.g. Bayes' net) where both the output label and input features are random variables
 - Instantiate any observed features
 - Query for the distribution of the label conditioned on the features
- Challenges
 - What structure should the BN have?
 - How should we learn its parameters?




Naïve Bayes for Digits

- Naïve Bayes: Assume all features are independent effects of the label

- Simple digit recognition version:

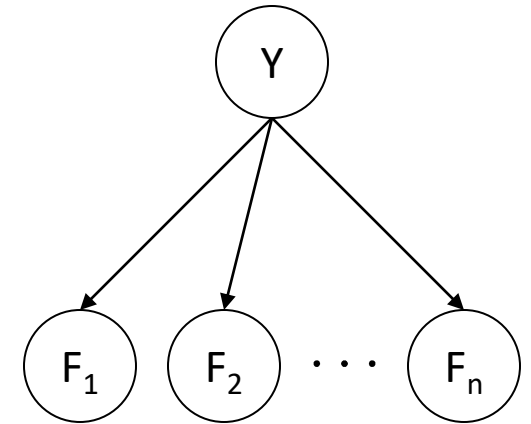
- One feature (variable) F_{ij} for each grid position $\langle i,j \rangle$
- Feature values are on / off, based on whether intensity is more or less than 0.5 in underlying image
- Each input maps to a feature vector, e.g.

 $\rightarrow \langle F_{0,0} = 0 \ F_{0,1} = 0 \ F_{0,2} = 1 \ F_{0,3} = 1 \ F_{0,4} = 0 \ \dots F_{15,15} = 0 \rangle$

- Here: lots of features, each is binary valued

- Naïve Bayes model: $P(Y|F_{0,0} \dots F_{15,15}) \propto P(Y) \prod_{i,j} P(F_{i,j}|Y)$

- What do we need to learn?



General Naïve Bayes

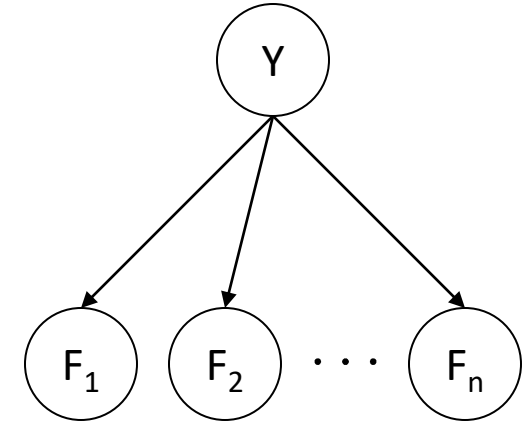
- A general Naive Bayes model:

$|Y|$ parameters

$$P(Y, F_1 \dots F_n) = P(Y) \prod_i P(F_i|Y)$$

$|Y| \times |F|^n$ values

$n \times |F| \times |Y|$
parameters



- We only have to specify how each feature depends on the class
- Total number of parameters is *linear* in n
- Model is very simplistic, but often works anyway

Inference for Naïve Bayes

- Goal: compute posterior distribution over label variable Y
 - Step 1: get joint probability of label and evidence for each label

$$P(Y, f_1 \dots f_n) = \begin{bmatrix} P(y_1, f_1 \dots f_n) \\ P(y_2, f_1 \dots f_n) \\ \vdots \\ P(y_k, f_1 \dots f_n) \end{bmatrix} \Rightarrow \begin{bmatrix} P(y_1) \prod_i P(f_i|y_1) \\ P(y_2) \prod_i P(f_i|y_2) \\ \vdots \\ P(y_k) \prod_i P(f_i|y_k) \end{bmatrix}$$

$$P(f_1 \dots f_n)$$

↪ +

- Step 2: sum to get probability of evidence
- Step 3: normalize by dividing Step 1 by Step 2

$$P(Y|f_1 \dots f_n)$$

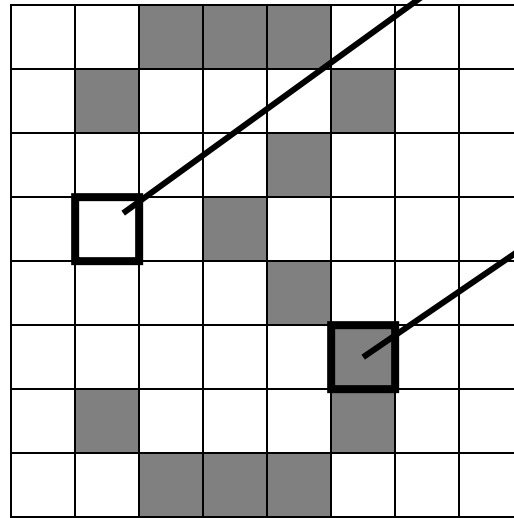
General Naïve Bayes

- What do we need in order to use Naïve Bayes?
 - Inference method (we just saw this part)
 - Start with a bunch of probabilities: $P(Y)$ and the $P(F_i|Y)$ tables
 - Use standard inference to compute $P(Y|F_1...F_n)$
 - Estimates of local conditional probability tables
 - $P(Y)$, the prior over labels
 - $P(F_i|Y)$ for each feature (evidence variable)
 - These probabilities are collectively called the *parameters* of the model and denoted by θ
 - Up until now, we assumed these appeared by magic, but...
 - ...they typically come from training data counts: we'll look at this soon

Example: Conditional Probabilities

$P(Y)$

1	0.1
2	0.1
3	0.1
4	0.1
5	0.1
6	0.1
7	0.1
8	0.1
9	0.1
0	0.1



$P(F_{3,1} = \text{on}|Y)$

1	0.01
2	0.05
3	0.05
4	0.30
5	0.80
6	0.90
7	0.05
8	0.60
9	0.50
0	0.80

$P(F_{5,5} = \text{on}|Y)$

1	0.05
2	0.01
3	0.90
4	0.80
5	0.90
6	0.90
7	0.25
8	0.85
9	0.60
0	0.80

10% of all outputs is 0

both come from data

In 80% of the examples when
output was 0 this pixel was ON

Naïve Bayes for Text

- Bag-of-words Naïve Bayes:

- Features: W_i is the word at position i
- As before: predict label conditioned on feature variables (spam vs. ham)
- As before: assume features are conditionally independent given label
- New: each W_i is identically distributed

- Generative model: $P(Y, W_1 \dots W_n) = P(Y) \prod_i P(W_i|Y)$

*Word at position
 i , not i^{th} word in
the dictionary!*

- “Tied” distributions and bag-of-words

- Usually, each variable gets its own conditional probability distribution $P(F|Y)$
- In a bag-of-words model
 - Each position is identically distributed
 - All positions share the same conditional probs $P(W|Y)$
 - Why make this assumption?
- Called “bag-of-words” because model is insensitive to word order or reordering

Example: Spam Filtering

- Model: $P(Y, W_1 \dots W_n) = P(Y) \prod_i P(W_i|Y)$
- What are the parameters?

$P(Y)$

ham : 0.66
spam: 0.33

$P(W|\text{spam})$

the : 0.0156
to : 0.0153
and : 0.0115
of : 0.0095
you : 0.0093
a : 0.0086
with: 0.0080
from: 0.0075
...

$P(W|\text{ham})$

the : 0.0210
to : 0.0133
of : 0.0119
2002: 0.0110
with: 0.0108
from: 0.0107
and : 0.0105
a : 0.0100
...

- Where do these tables come from?

Spam Example

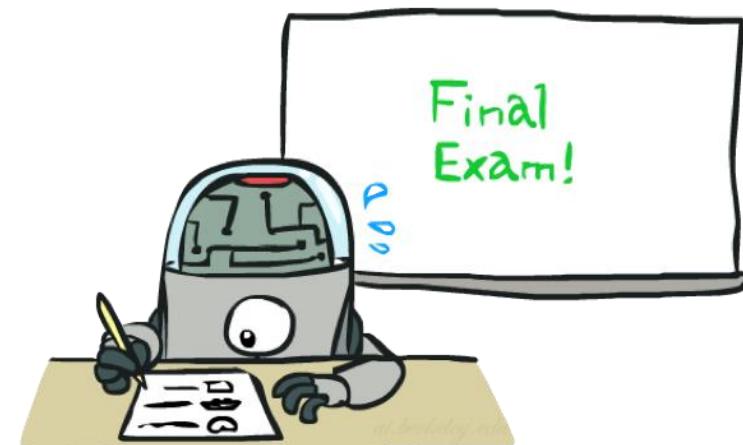
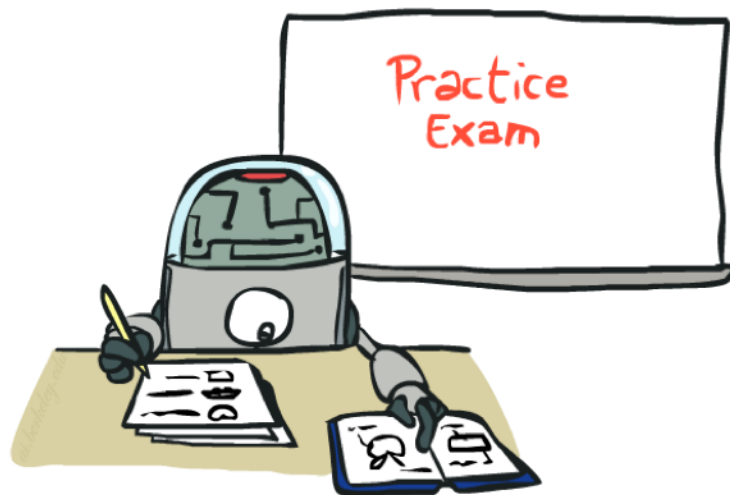
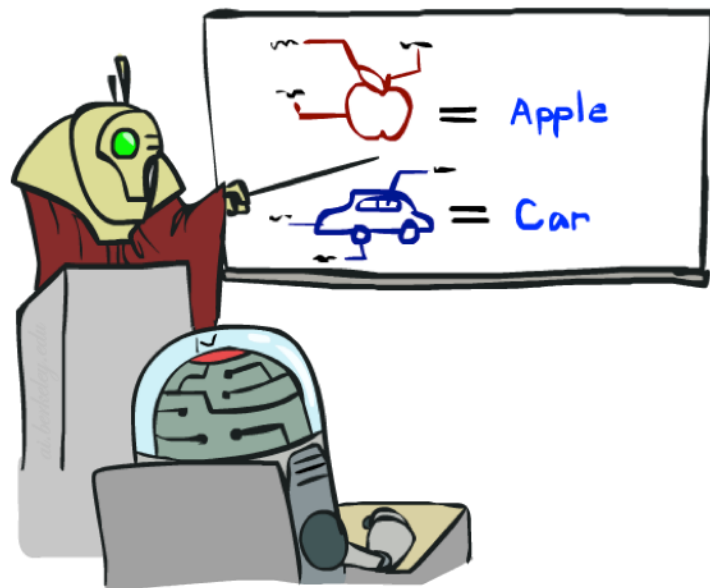
Word	$P(w \text{spam})$	$P(w \text{ham})$	Tot Spam	Tot Ham
(prior)	0.33333	0.66666	-1.1	-0.4
Kire	0.00002	0.00021	-11.8	-8.9
would	0.00069	0.00084	-19.1	-16.0
you	0.00881	0.00304	-23.8	-21.8
like	0.00086	0.00083	-30.9	-28.9
to	0.01517	0.01339	-35.1	-33.2
lose	0.00008	0.00002	-44.5	-44.0
weight	0.00016	0.00002	-53.3	-55.0
while	0.00027	0.00027	-61.5	-63.2
you	0.00881	0.00304	-66.2	-69.0
sleep	0.00006	0.00001	-76.0	-80.5

log of the product of probabilities

$\log(P(\text{spam}, w))$ $\xrightarrow{\text{sum and divide to normalize}}$ $\log(P(\text{ham}, w))$

$P(\text{spam} | w) = 0.989$

Training and Testing

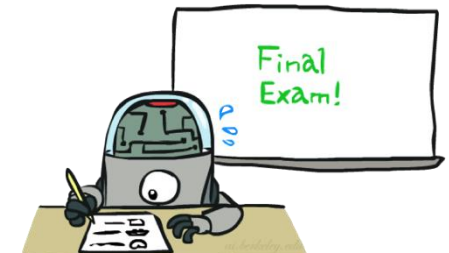
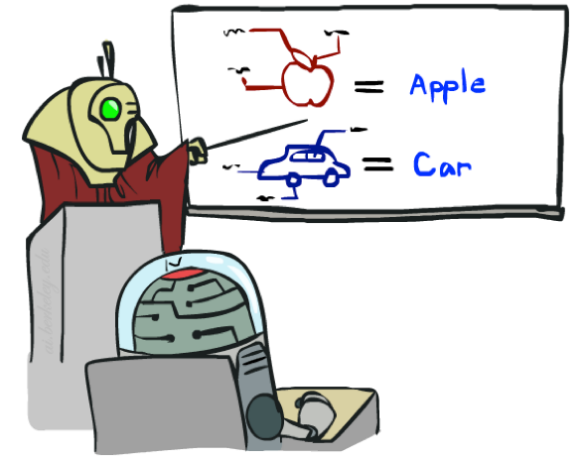
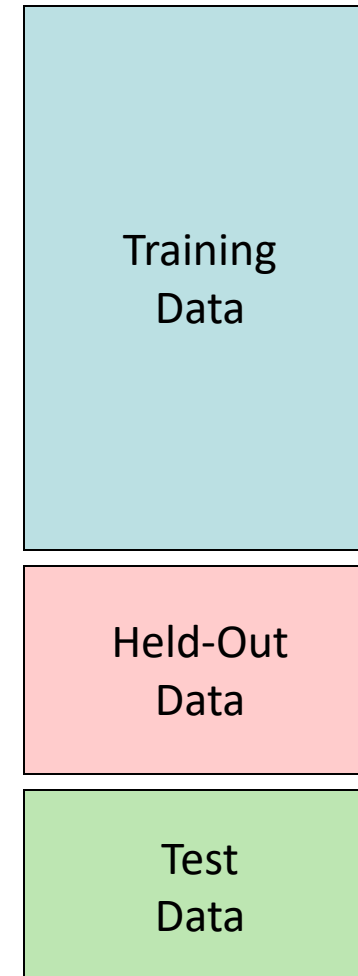


Empirical Risk Minimization

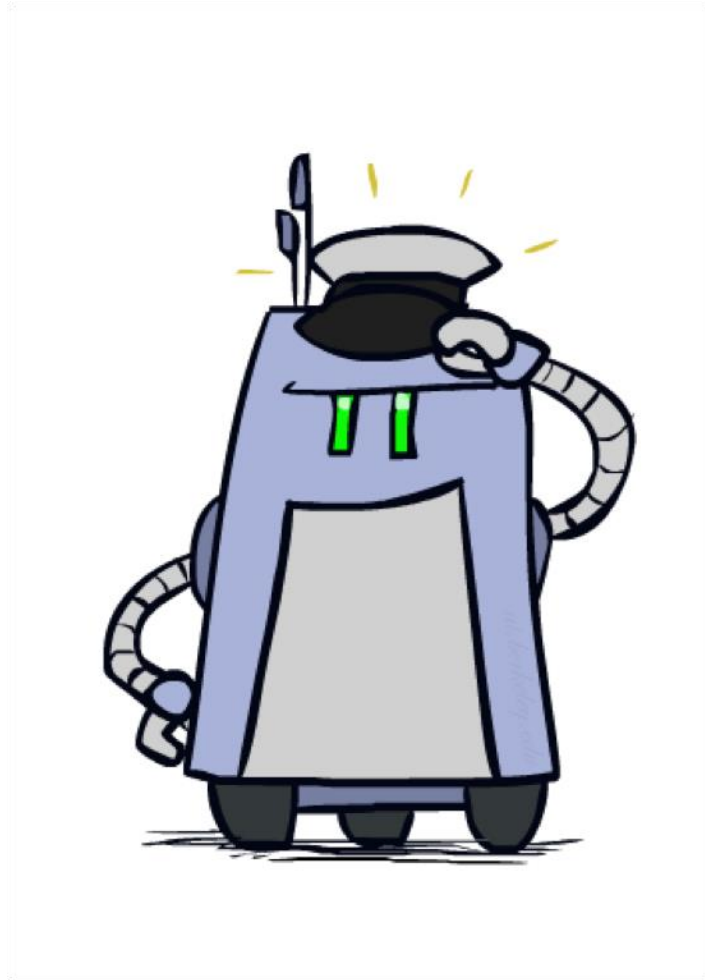
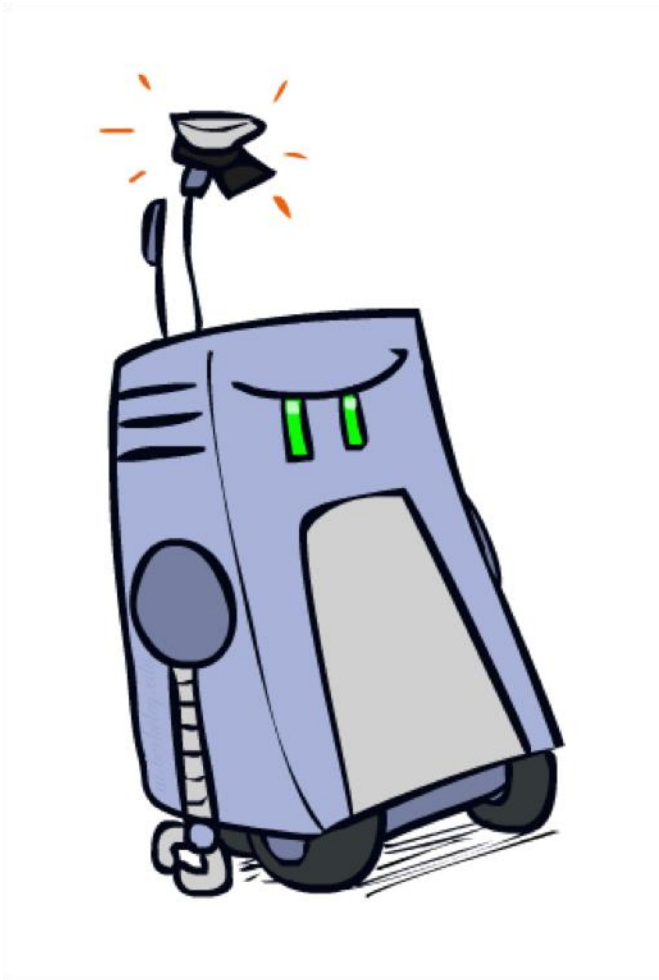
- Empirical risk minimization
 - Basic principle of machine learning
 - We want the model (classifier, etc) that does best on the true test distribution
 - Don't know the true distribution so pick the best model on our actual training set
 - Finding “the best” model on the training set is phrased as an optimization problem
- Main worry: overfitting to the training set
 - Better with more training data (less sampling variance, training more like test)
 - Better if we limit the complexity of our hypotheses (regularization and/or small hypothesis spaces)

Important Concepts

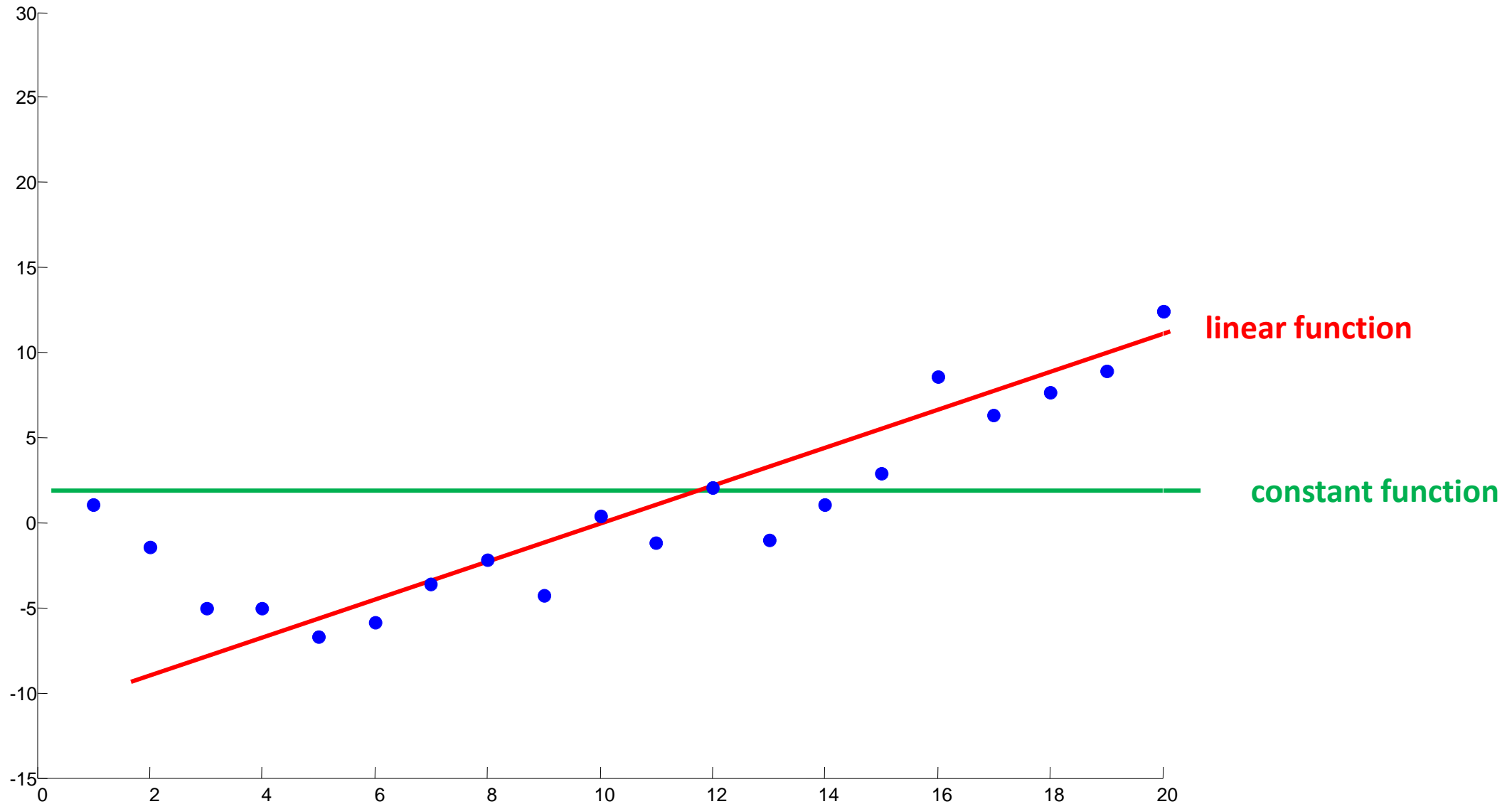
- Data: labeled instances (e.g. emails marked spam/ham)
 - Training set
 - Held out set
 - Test set
- Features: attribute-value pairs which characterize each x
- Experimentation cycle
 - Learn parameters (e.g. model probabilities) on training set
 - (Tune hyperparameters on held-out set)
 - Compute accuracy of test set
 - Very important: never “peek” at the test set!
- Evaluation (many metrics possible, e.g. accuracy)
 - Accuracy: fraction of instances predicted correctly
- Overfitting and generalization
 - Want a classifier which does well on *test* data
 - Overfitting: fitting the training data very closely, but not generalizing well
 - We’ll investigate overfitting and generalization formally in a few lectures



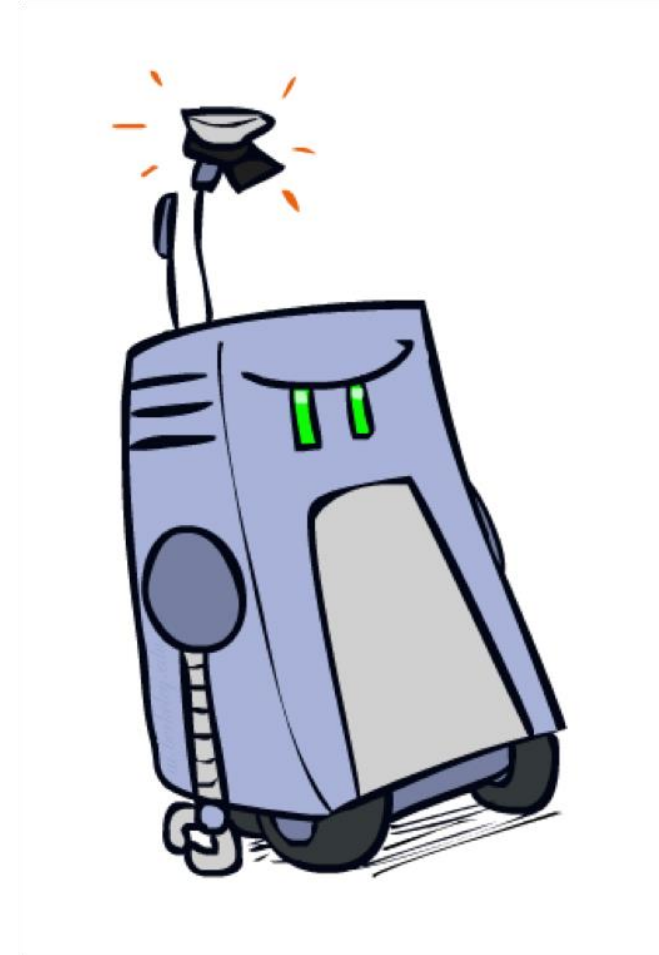
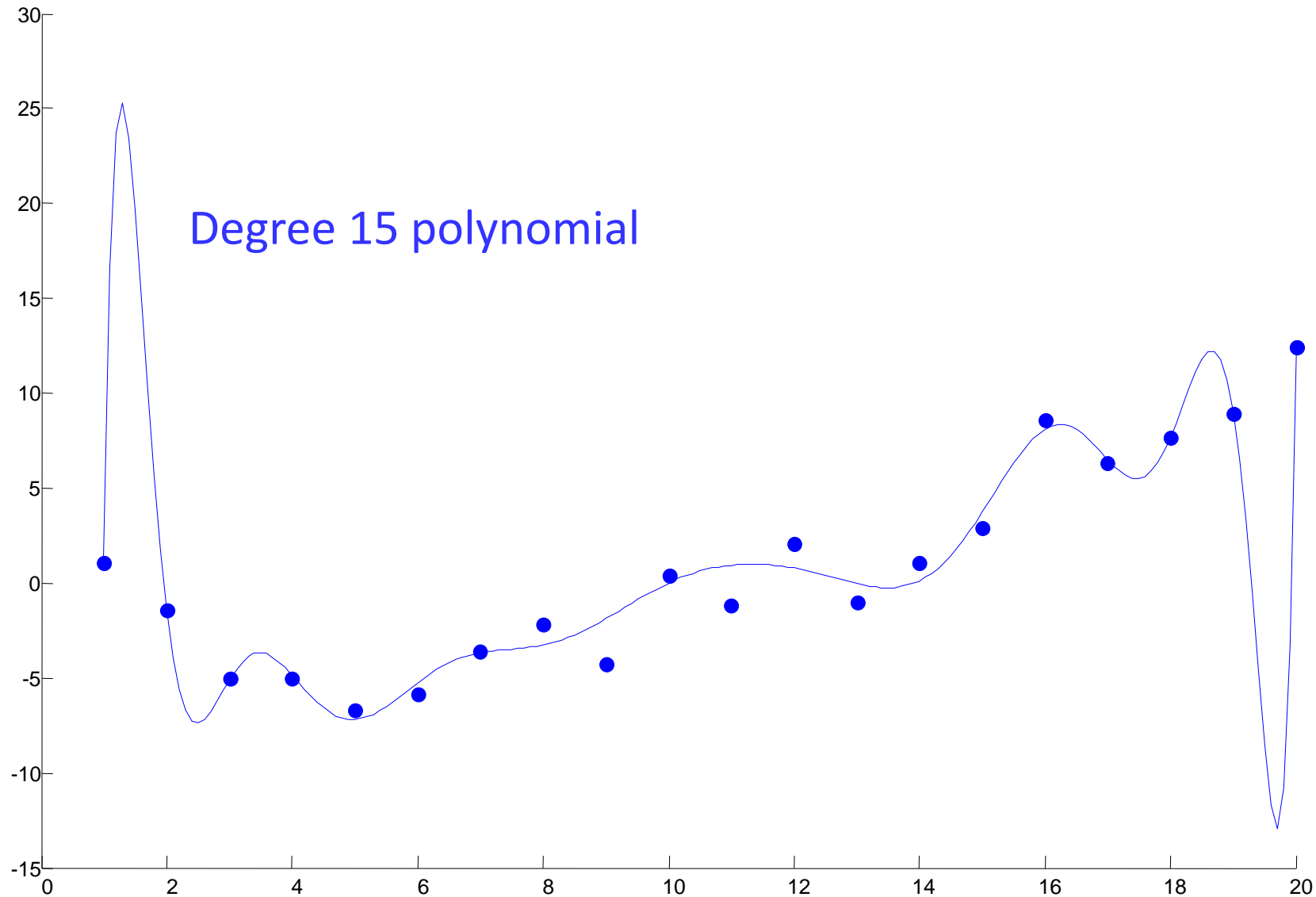
Generalization and Overfitting



Underfitting



Overfitting



Example: Overfitting

$P(\text{features}, C = 2)$

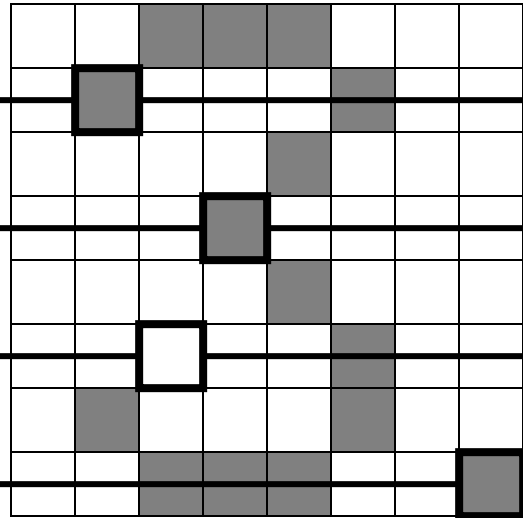
$P(C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.8$

$P(\text{on}|C = 2) = 0.1$

$P(\text{off}|C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.01$



$P(\text{features}, C = 3)$

$P(C = 3) = 0.1$

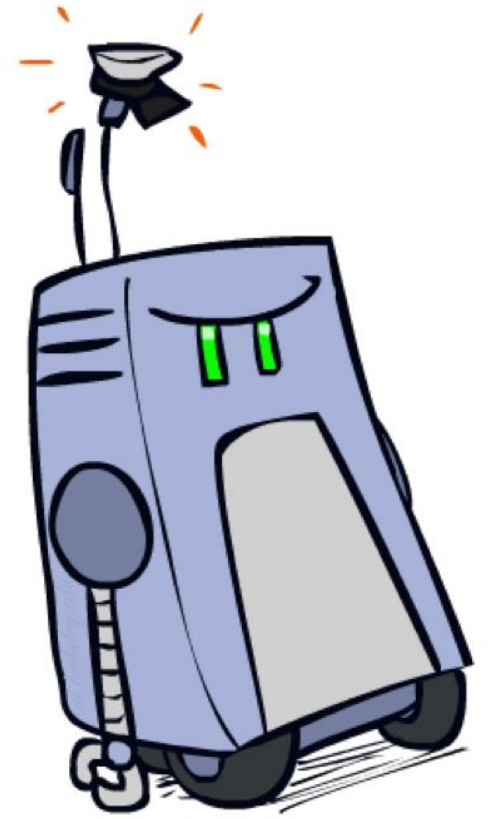
$P(\text{on}|C = 3) = 0.8$

$P(\text{on}|C = 3) = 0.9$

$P(\text{off}|C = 3) = 0.7$

$P(\text{on}|C = 3) = 0.0$

2 wins!!



Example: Overfitting

- Posterior determined by *relative* probabilities (odds ratios):

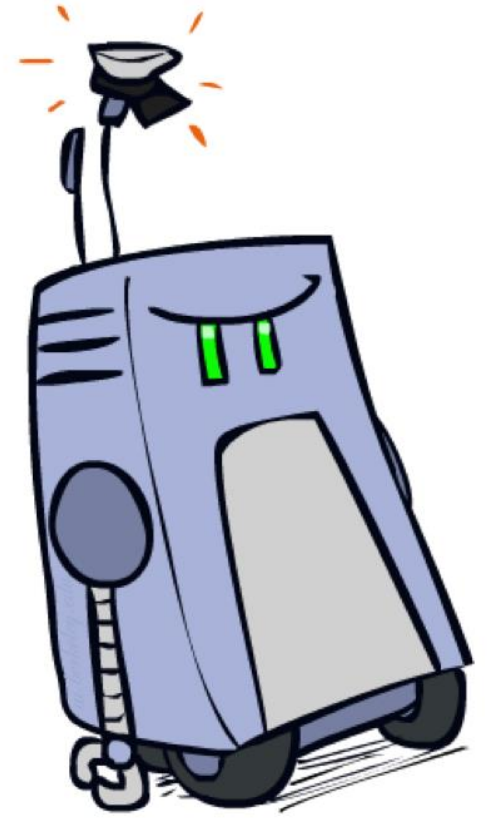
$$\frac{P(W|\text{ham})}{P(W|\text{spam})}$$

south-west	:	inf
nation	:	inf
morally	:	inf
nicely	:	inf
extent	:	inf
seriously	:	inf
...		

$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

screens	:	inf
minute	:	inf
guaranteed	:	inf
\$205.00	:	inf
delivery	:	inf
signature	:	inf
...		

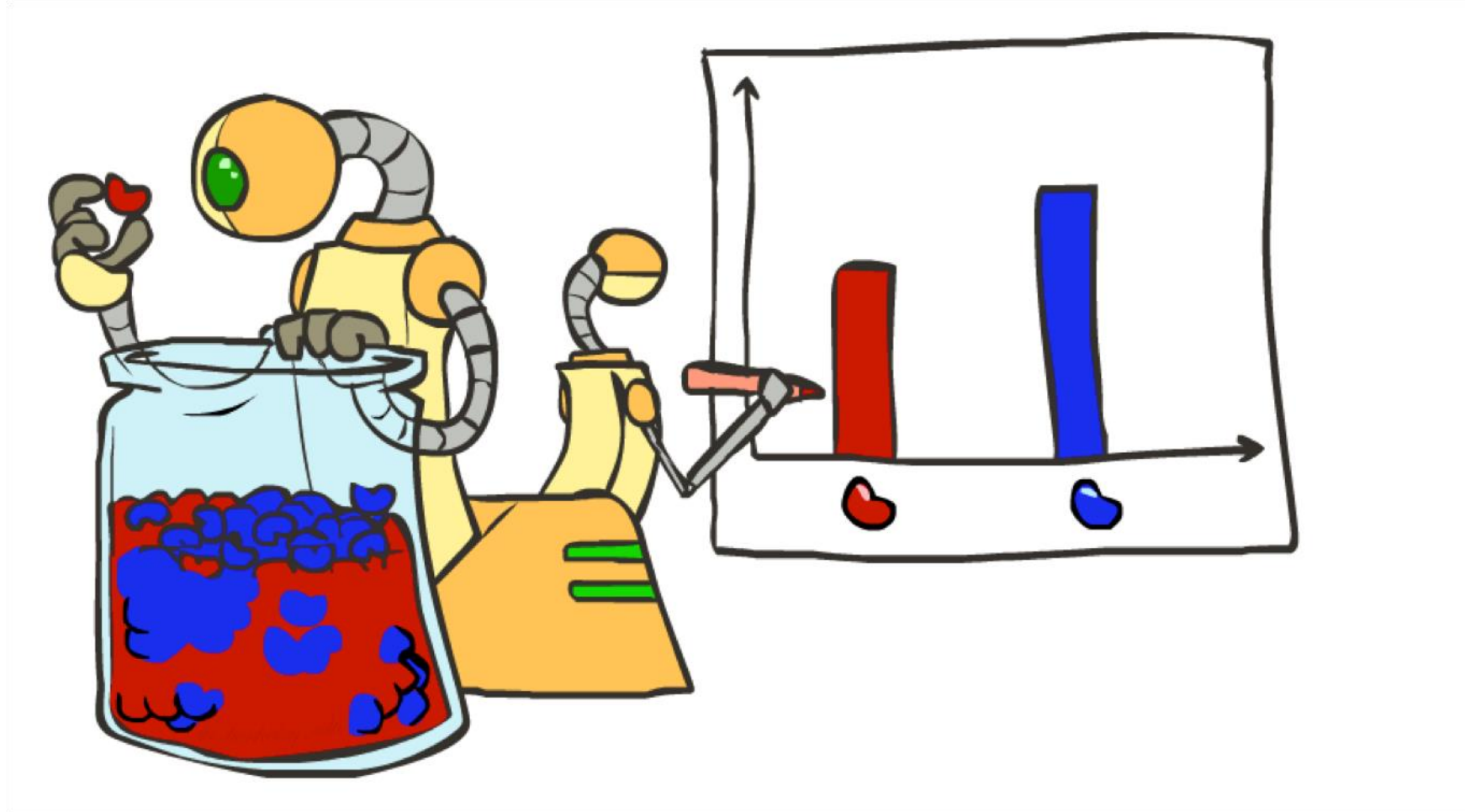
What went wrong here?



Generalization and Overfitting

- Relative frequency parameters will **overfit** the training data!
 - Just because we never saw a 3 with pixel (15,15) on during training doesn't mean we won't see it at test time
 - Unlikely that every occurrence of "minute" is 100% spam
 - Unlikely that every occurrence of "seriously" is 100% ham
 - What about all the words that don't occur in the training set at all?
 - In general, we can't go around giving unseen events zero probability
- As an extreme case, imagine using the entire email as the only feature (e.g. document ID)
 - Would get the training data perfect (if deterministic labeling)
 - Wouldn't *generalize* at all
 - Just making the bag-of-words assumption gives us some generalization, but isn't enough
- To generalize better: we need to **smooth** or **regularize** the estimates

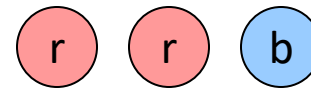
Parameter Estimation



Parameter Estimation

- Estimating the distribution of a random variable
- *Elicitation*: ask a human (why is this hard?)
- *Empirically*: use training data (learning!)
 - E.g.: for each outcome x , look at the *empirical rate* of that value:

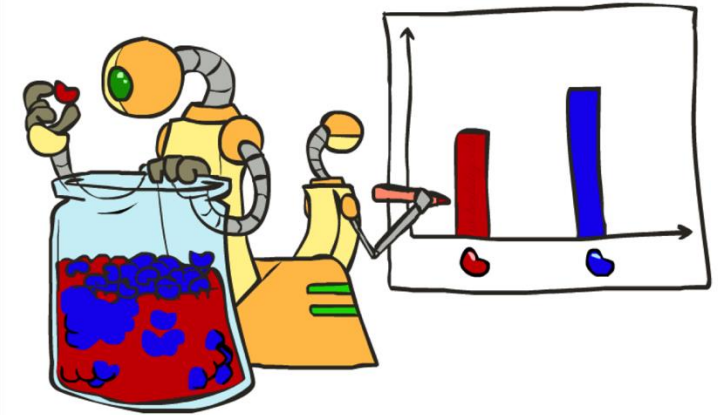
$$P_{\text{ML}}(x) = \frac{\text{count}(x)}{\text{total samples}}$$



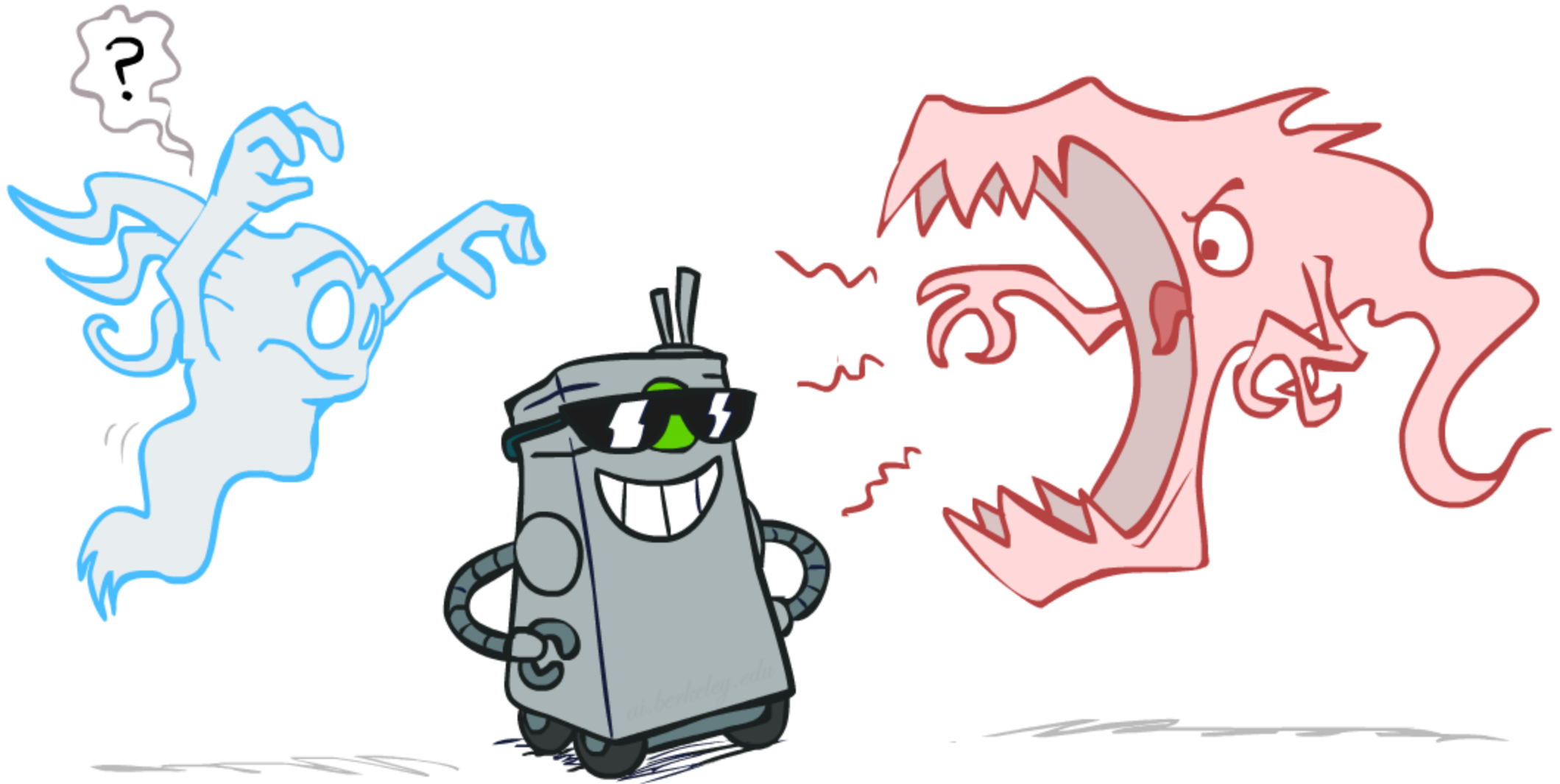
$$P_{\text{ML}}(r) = 2/3$$

- This is the estimate that maximizes the *likelihood of the data*

$$L(x, \theta) = \prod_i P_{\theta}(x_i)$$



Smoothing



Maximum Likelihood?

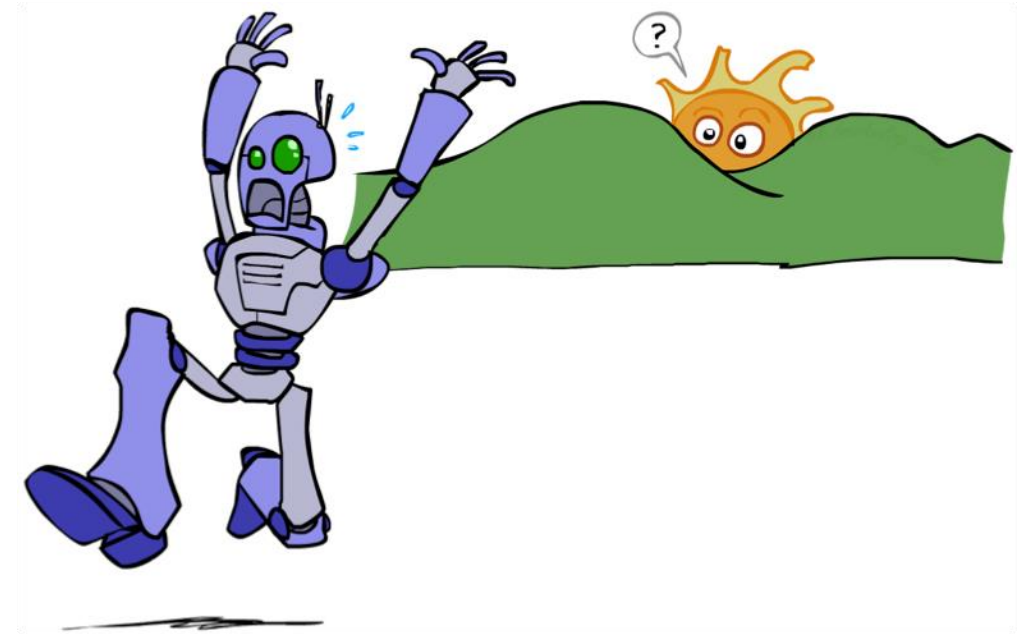
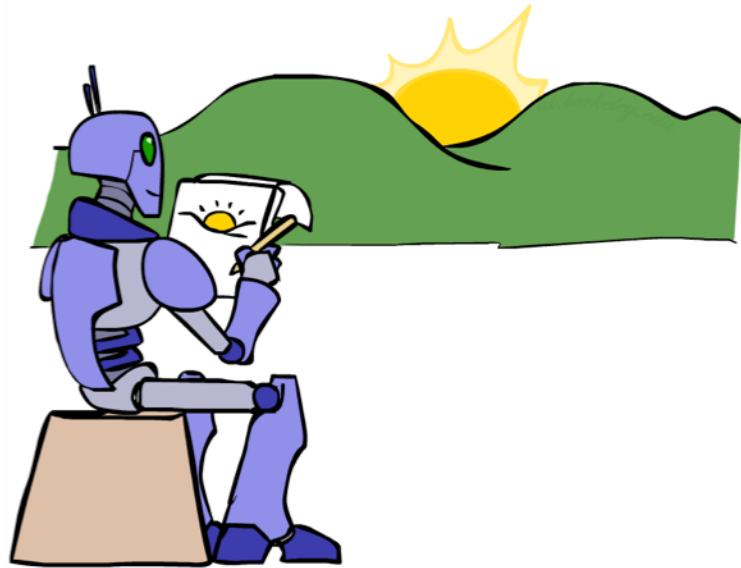
- Relative frequencies are the maximum likelihood estimates

$$\begin{aligned}\theta_{ML} &= \arg \max_{\theta} P(\mathbf{X}|\theta) \\ &= \arg \max_{\theta} \prod_i P_{\theta}(X_i)\end{aligned} \quad \Rightarrow \quad P_{ML}(x) = \frac{\text{count}(x)}{\text{total samples}}$$

- Another option is to consider the most likely parameter value given the data

$$\begin{aligned}\theta_{MAP} &= \arg \max_{\theta} P(\theta|\mathbf{X}) \\ &= \arg \max_{\theta} P(\mathbf{X}|\theta)P(\theta)/P(\mathbf{X}) \quad \Rightarrow \quad \text{????} \\ &= \arg \max_{\theta} P(\mathbf{X}|\theta)P(\theta)\end{aligned}$$

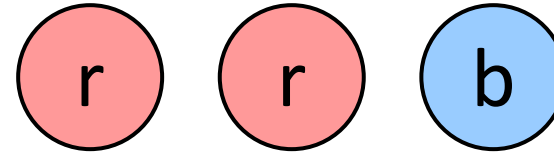
Unseen Events



Laplace Smoothing

- Laplace's estimate:
 - Pretend you saw every outcome once more than you actually did

$$\begin{aligned}P_{LAP}(x) &= \frac{c(x) + 1}{\sum_x [c(x) + 1]} \\ &= \frac{c(x) + 1}{N + |X|}\end{aligned}$$



$$P_{ML}(X) = \left\langle \frac{2}{3}, \frac{1}{3} \right\rangle$$

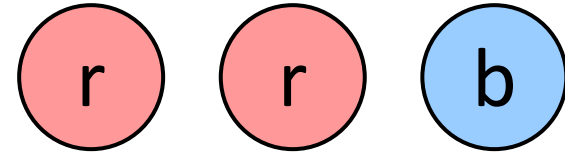
$$P_{LAP}(X) = \left\langle \frac{3}{5}, \frac{2}{5} \right\rangle$$

Laplace Smoothing

- Laplace's estimate (extended):
 - Pretend you saw every outcome k extra times

$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

- What's Laplace with $k = 0$?
- k is the **strength** of the prior



$$P_{LAP,0}(X) = \left\langle \frac{2}{3}, \frac{1}{3} \right\rangle$$

$$P_{LAP,1}(X) = \left\langle \frac{3}{5}, \frac{2}{5} \right\rangle$$

$$P_{LAP,100}(X) = \left\langle \frac{102}{203}, \frac{101}{203} \right\rangle$$

Real NB: Smoothing

- For real classification problems, smoothing is critical
- New odds ratios:

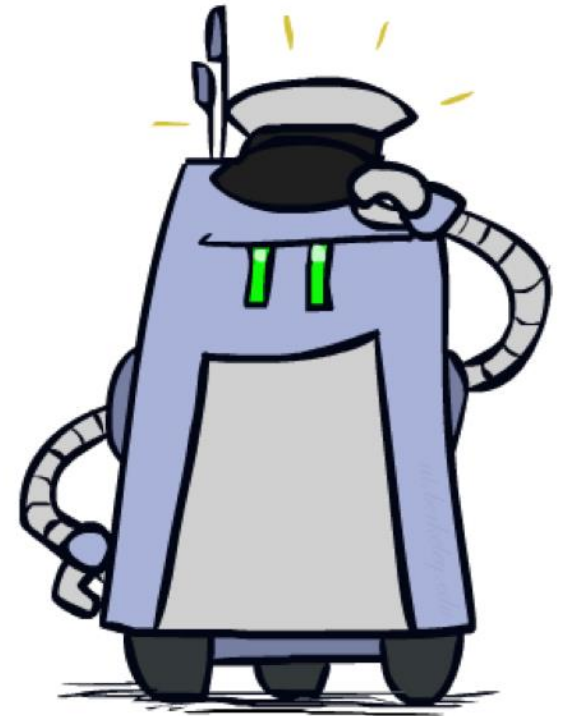
$$\frac{P(W|\text{ham})}{P(W|\text{spam})}$$

helvetica	:	11.4
seems	:	10.8
group	:	10.2
ago	:	8.4
areas	:	8.3
...		

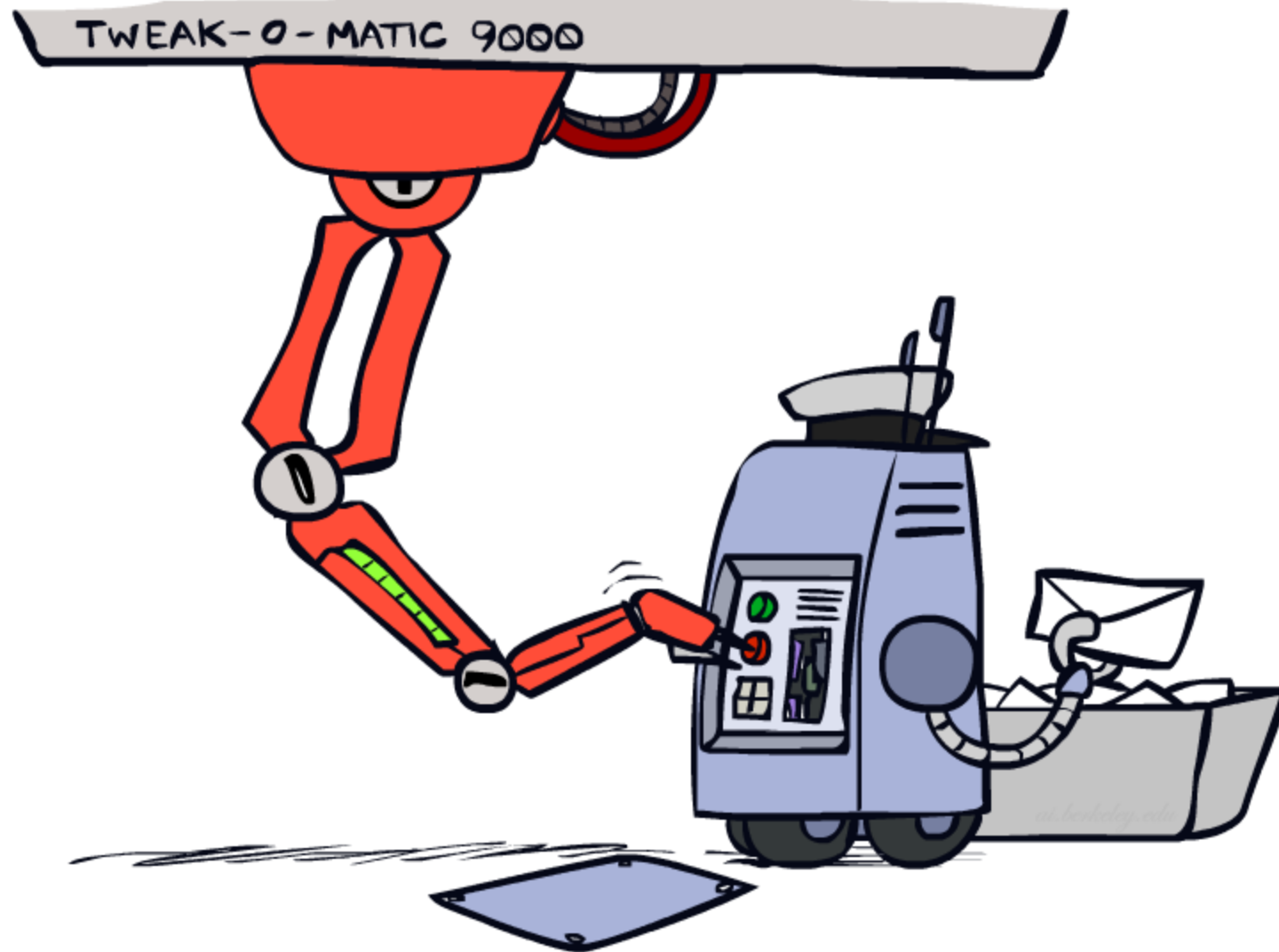
$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

verdana	:	28.8
Credit	:	28.4
ORDER	:	27.2
	:	26.9
money	:	26.5
...		

Do these make more sense?

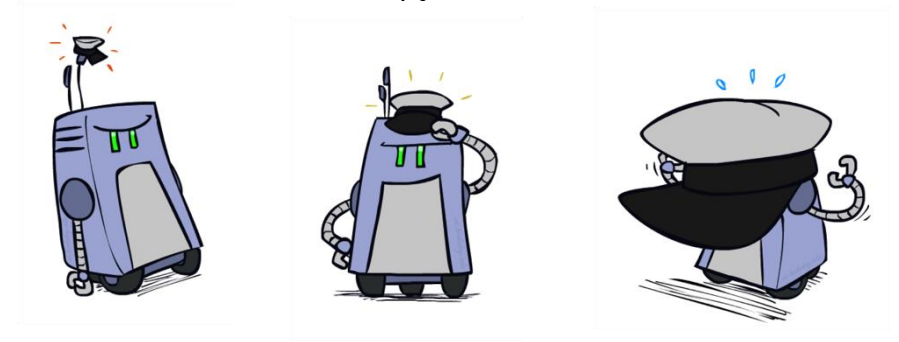
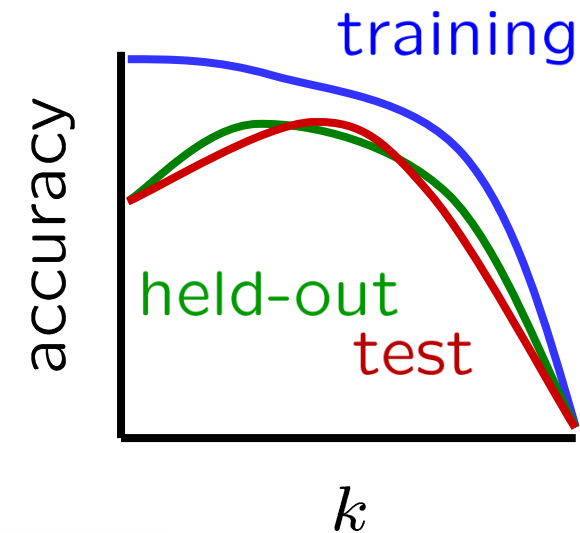


Tuning

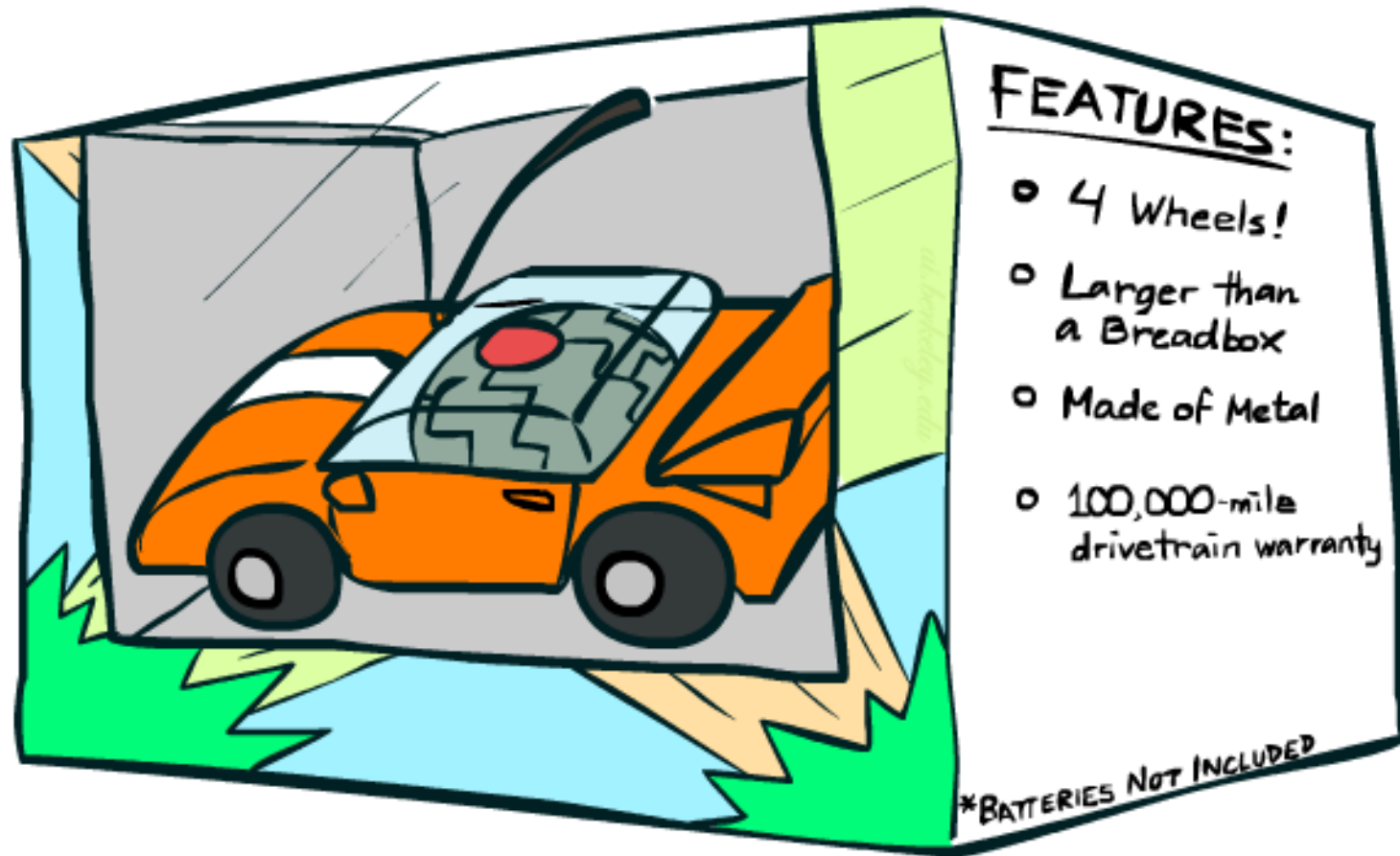


Tuning on Held-Out Data

- Now we've got two kinds of unknowns
 - Parameters: the probabilities $P(X|Y)$, $P(Y)$
 - Hyperparameters: e.g. the amount / type of smoothing to do, k , α
- What should we learn where?
 - Learn parameters from training data
 - Tune hyperparameters on different data
 - Why?
 - For each value of the hyperparameters, train and test on the held-out data
 - Choose the best value and do a final test on the test data



Features



Errors, and What to Do

- Examples of errors

Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the latest version of OmniPage Pro, for just \$99.99* - the regular list price is \$499! The most common question we've received about this offer is - Is this genuine? We would like to assure you that this offer is authorized by ScanSoft, is genuine and valid. You can get the . . .

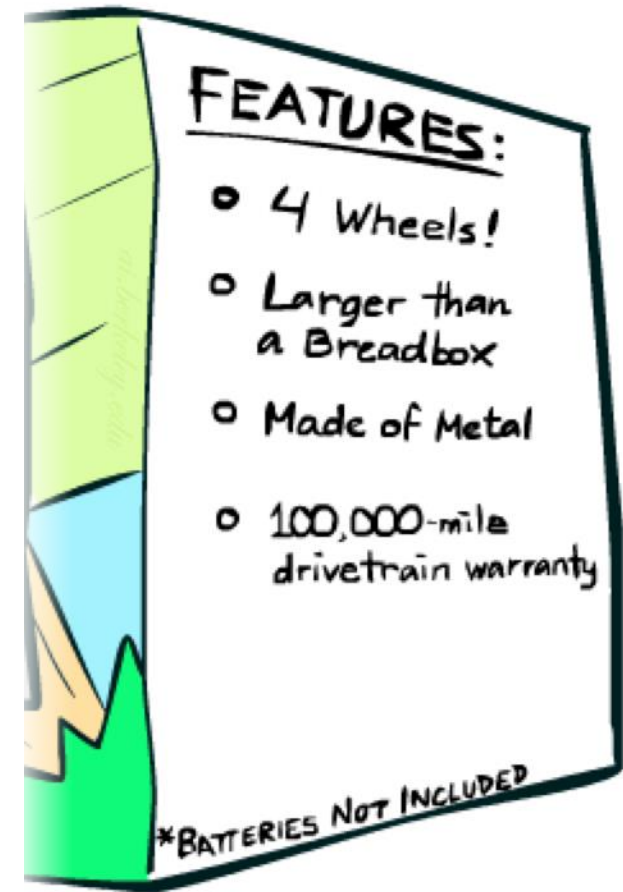
. . . To receive your \$30 Amazon.com promotional certificate, click through to

<http://www.amazon.com/apparel>

and see the prominent link for the \$30 offer. All details are there. We hope you enjoyed receiving this message. However, if you'd rather not receive future e-mails announcing new store launches, please click . . .

What to Do About Errors?

- Need more features— words aren't enough!
 - Have you emailed the sender before?
 - Have 1K other people just gotten the same email?
 - Is the sending information consistent?
 - Is the email in ALL CAPS?
 - Do inline URLs point where they say they point?
 - Does the email address you by (your) name?
- Can add these information sources as new variables in the NB model



Summary

- Bayes rule lets us do diagnostic queries with causal probabilities
- The naïve Bayes assumption takes all features to be independent given the class label
- We can build classifiers out of a naïve Bayes model using training data
- Smoothing estimates is important in real systems
- We use held-out data to tune hyperparameters
- Errors are common in classification, one way to handle it is using more and/or better features