Finding The Minimal Energy Configuration Of Charge Particle Within A Circle

Like charges repel each other. But if they are confined to within a circular area, they can only move as far as the edge of the circle. Two particles will obviously want to be at opposite sides of the circle, three will distribute themselves in a triangle. It is obvious that for up to six charges all charges will end up on the boundary. Beyond that, one needs to find the minimum energy configuration through simulation.

As all charges are assumed to be equal, and as the energy may be arbitrarily scaled, you can use the following contribution to the total energy for each pair (i, j) of charges:

$$E_{i,j} = \frac{1}{|_{i,j}|}$$

Here ri,j is the distance between the two particles. If you need the forces that two particles exert on each other, you can use:

$$\vec{F}_{i,j} = \frac{\vec{r}_{i,j}}{|r_{i,j}|^3}$$

This scales incorrectly compared to the energy (a factor two is missing), but that should not matter. Make sure you get the direction of the force vector right.

As regards the elementary moves, just moving a single particle randomly over a limited distance should work (but what will you do when the particle would leave the circle?). You may want to reduce the maximum step size later on in the simulation. Still, there may be more effective strategies, like taking a hint from the total force on the particle.

Experiments that you may conduct:

- First of all, try to find the optimal configuration for smaller problem, up to 11 particles. Then challange your algoritm to find the configuration for 12 particles, *that's when the system starts to behave differently.*
  - When you were confident that your implementation could find the optimal configuration up to 12 particles; then increase the number of particles in the system and try to find higher magic numbers!
- Second, try different *cooling schedules* and observe their effects on the convergence.
- Third, what's the effect of Markov Chain's length on the convergence.
- Finally, try to improve the neighboring procedure. *This is specially very important in this problem because you need to have extra controls over perturbation of the particles at later stages.*