2nd Assignment: Probability and Statistics for Data Analysis

Code ▼

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1. In file "data.txt" (available on the e-class assignments site), you willfind the recorded variables Y, X1, X2, X3, X4 (continuous), and W (categorical with three levels) on 150 cases. Using these data, answer the following questions

```
data <- read.delim("data.txt", header = TRUE, quote = "\"", sep = " ")
View(data)
weightloss <- read.delim("weightloss.txt", header = TRUE, quote = "\"", sep = " ")
View(weightloss)</pre>
```

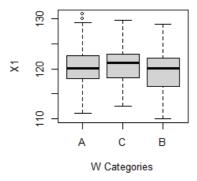
- (a) Run the parametric one-way ANOVA of each of the continuous variables (Y, X1, X2, X3, X4) on the categorical variable (W). Specifically,
- (i) provides a graphical representation of each of the continuous versus the categorical variable

```
Hide
unique_W <- unique(data$W)
cat("Unique W Categories:", unique_W, "\n")

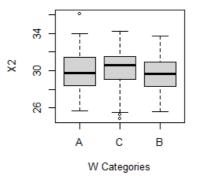
Unique W Categories: A C B</pre>
Hide
```

file:///C:/Users/vassi/OneDrive/Εγγραφα/MSc in Data Science/Fall Semester/Probability and statistics for data analysis/assignment 2/report.nb.h...

X1 vs W Categories



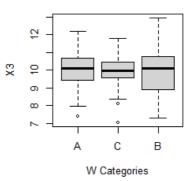
X2 vs W Categories



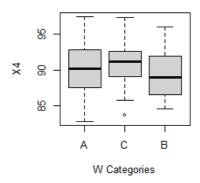
X3 vs W Categories

Hide

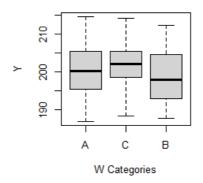
Hide



X4 vs W Categories



Y vs W Categories



(ii) provide the ANOVA output

(iii) check the assumptions.

```
Hide
 fitx1<-aov(X1~factor(W),data=data)</pre>
 summary(fitx1)
                 Df Sum Sq Mean Sq F value Pr(>F)
 factor(W)
                                          2.42 0.0915 .
                       76.3
                                38.13
 Residuals
               197 3104.1
                                15.76
                    0 (***, 0.001 (**, 0.01 (*, 0.05 (., 0.1 ( , 1
 Signif. codes:
                                                                                                                Hide
 layout(matrix(1:4,2,2))
 plot(fitx1)
                                                           Standardized residuals
                    Residuals vs Fitted
                                                                                 Scale-Location
     9
Residuals
     4
                                                                                  120.0
         119.5
                      120.0
                                                                    119.5
                                   120.5
                                                 121.0
                                                                                              120.5
                                                                                                            121.0
                         Fitted values
                                                                                    Fitted values
Standardized residuals
                                                           Standardized residuals
                      Q-Q Residuals
                                                                             Residuals vs Leverage
                                          @ 1460
                                                                N
     N
                                                                0
     0
                                             2
                -2
                                                                    0.000
                                                                               0.005
                                                                                          0.010
                                                                                                     0.015
                     Theoretical Quantiles
                                                                                      Leverage
                                                                                                                Hide
 shapiro.test(fitx1$residuals)
      Shapiro-Wilk normality test
        fitx1$residuals
 W = 0.99123, p-value = 0.268
```

```
Hide
```

```
bartlett.test(X1~factor(W),data=data)
```

```
Bartlett test of homogeneity of variances
```

```
data: X1 by factor(W)
```

```
Bartlett's K-squared = 1.8261, df = 2, p-value = 0.4013
```

```
fligner.test(X1~factor(W),data=data)
```

Fligner-Killeen test of homogeneity of variances

data: X1 by factor(W)

Fligner-Killeen:med chi-squared = 2.3293, df = 2, p-value = 0.312

Hide

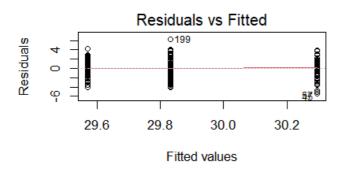
```
fitx2<-aov(X2~factor(W),data=data)
summary(fitx2)</pre>
```

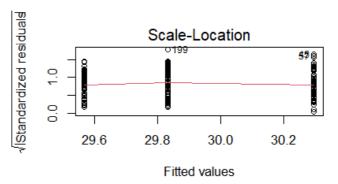
```
Df Sum Sq Mean Sq F value Pr(>F)
```

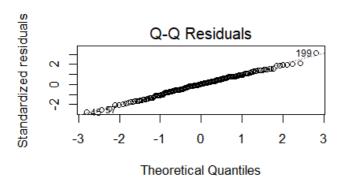
factor(W) 2 17.0 8.489 2.079 0.128

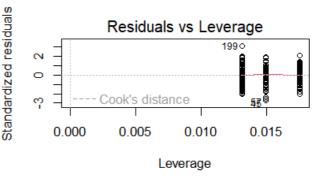
Residuals 197 804.3 4.083

```
layout(matrix(1:4,2,2))
plot(fitx2)
```









shapiro.test(fitx2\$residuals)

Shapiro-Wilk normality test

data: fitx2\$residuals

W = 0.99539, p-value = 0.8049

Hide

bartlett.test(X2~factor(W),data=data)

Bartlett test of homogeneity of variances

data: X2 by factor(W)

Bartlett's K-squared = 2.2362, df = 2, p-value = 0.3269

Hide

fligner.test(X2~factor(W),data=data)

Fligner-Killeen test of homogeneity of variances

data: X2 by factor(W)

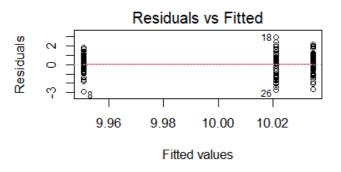
Fligner-Killeen:med chi-squared = 2.1984, df = 2, p-value = 0.3331

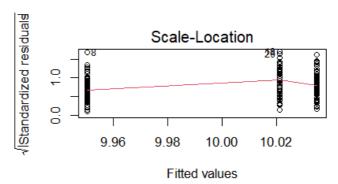
```
fitx3<-aov(X3~factor(W),data=data)
summary(fitx3)</pre>
```

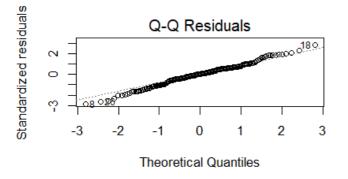
Df Sum Sq Mean Sq F value Pr(>F) factor(W) 2 0.28 0.1397 0.133 0.876 Residuals 197 207.24 1.0520

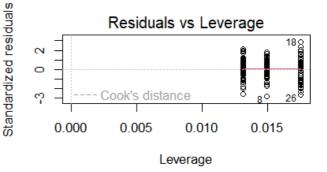
Hide

layout(matrix(1:4,2,2))
plot(fitx3)









Hide

shapiro.test(fitx3\$residuals)

Shapiro-Wilk normality test

data: fitx3\$residuals

W = 0.99108, p-value = 0.2555

Hide

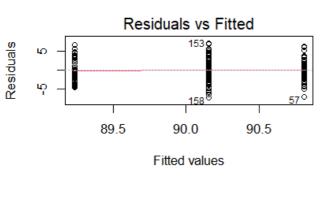
bartlett.test(X3~factor(W),data=data)

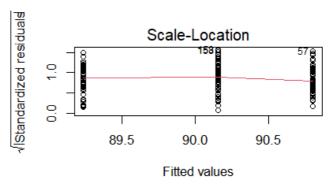
```
Bartlett test of homogeneity of variances
data: X3 by factor(W)
Bartlett's K-squared = 14.682, df = 2, p-value = 0.0006485
                                                                                          Hide
fligner.test(X3~factor(W),data=data)
    Fligner-Killeen test of homogeneity of variances
data: X3 by factor(W)
Fligner-Killeen:med chi-squared = 12.686, df = 2, p-value = 0.001759
                                                                                          Hide
fitx4<-aov(X4~factor(W),data=data)</pre>
summary(fitx4)
            Df Sum Sq Mean Sq F value Pr(>F)
            2 75.8 37.89 4.171 0.0168 *
factor(W)
Residuals 197 1789.6 9.08
```

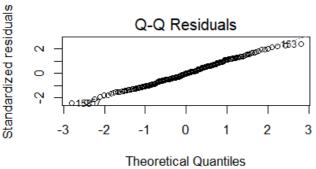
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1

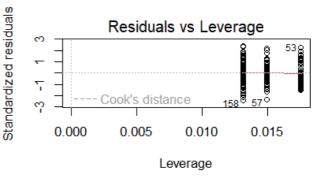
Hide

layout(matrix(1:4,2,2)) plot(fitx4)









shapiro.test(fitx4\$residuals)

Shapiro-Wilk normality test

data: fitx4\$residuals

W = 0.99272, p-value = 0.4243

Hide

bartlett.test(X4~factor(W),data=data)

Bartlett test of homogeneity of variances

data: X4 by factor(W)

Bartlett's K-squared = 4.0286, df = 2, p-value = 0.1334

Hide

fligner.test(X4~factor(W),data=data)

Fligner-Killeen test of homogeneity of variances

data: X4 by factor(W)

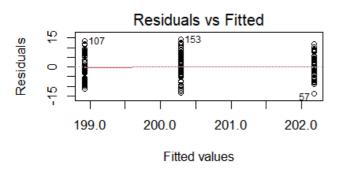
Fligner-Killeen:med chi-squared = 4.3185, df = 2, p-value = 0.1154

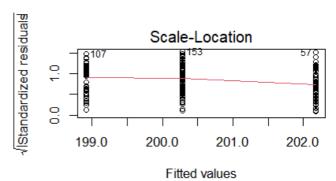
```
fity<-aov(Y~factor(W),data=data)
summary(fity)</pre>
```

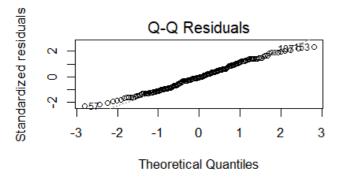
```
Df Sum Sq Mean Sq F value Pr(>F)
factor(W) 2 333 166.71 4.352 0.0141 *
Residuals 197 7546 38.31
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

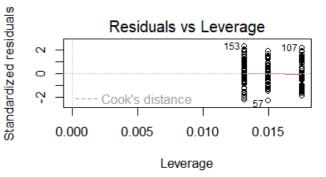
Hide

layout(matrix(1:4,2,2))
plot(fity)









Hide

shapiro.test(fity\$residuals)

Shapiro-Wilk normality test

data: fity\$residuals

W = 0.98923, p-value = 0.1374

Hide

bartlett.test(Y~factor(W),data=data)

```
Bartlett test of homogeneity of variances

data: Y by factor(W)

Bartlett's K-squared = 4.6956, df = 2, p-value = 0.09558
```

```
fligner.test(Y~factor(W),data=data)
```

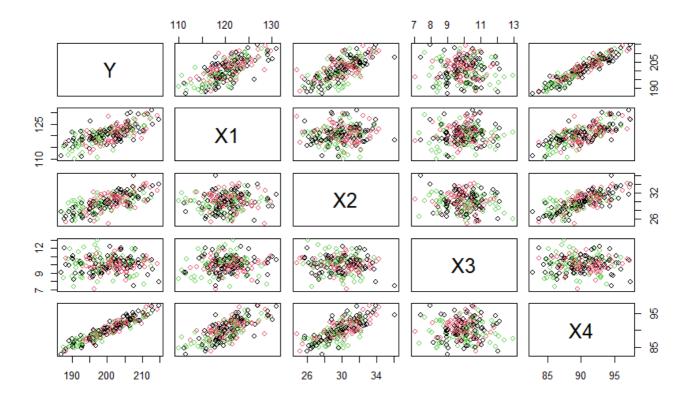
```
Fligner-Killeen test of homogeneity of variances

data: Y by factor(W)
Fligner-Killeen:med chi-squared = 7.0448, df = 2, p-value = 0.02953
```

As we can see above, foe all 5 models the residuals are normally distributed, with constant variance and mean value close to 0. So the assumptions exist in our models.

(b) Provide a scatter-plot matrix of Y, X1, X2, X3, and X4, annotating the different levels of W in each plot using a different color.

```
data$W <- as.factor(data$W)
pairs(data[, c("Y", "X1", "X2", "X3", "X4")], col = as.numeric(data$W))
legend("topright", legend = levels(data$W), col = 1:length(levels(data$W)), pch = 1, title =
"W")</pre>
```



(c) Run the regression model of Y on X4

```
Hide
model \leftarrow lm(Y \sim X4, data = data)
summary(model)
lm(formula = Y ~ X4, data = data)
Residuals:
   Min
            1Q Median
                             3Q
                                   Max
-5.5133 -1.3818 0.1039 1.4803 5.9044
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 26.1973 4.4449
                                 5.894 1.6e-08 ***
Х4
                        0.0493 39.243 < 2e-16 ***
             1.9347
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.129 on 198 degrees of freedom
Multiple R-squared: 0.8861,
                              Adjusted R-squared: 0.8855
F-statistic: 1540 on 1 and 198 DF, p-value: < 2.2e-16
```

(d) Run the regression model of Y on all the remaining variables (X1,X2, X3, X4, W), including the non-additive terms (i.e., interactions of the continuous predictors with the categorical).

```
Hide
model2 \leftarrow lm(Y \sim X1 + X2 + X3 + X4 + W + X1:W + X2:W + X3:W + X4:W, data = data)
summary(model2)
Call:
lm(formula = Y \sim X1 + X2 + X3 + X4 + W + X1:W + X2:W + X3:W +
    X4:W, data = data)
Residuals:
    Min
          10 Median
                                  30
-3.8807 -1.3656 -0.0337 1.0723 5.4653
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 28.3612 7.1589 3.962 0.000106 ***
                X1
X2
               0.3221 0.2313 1.393 0.165391
-0.5859 0.5015 -1.168 0.244184
X3
X4
               -8.2392 11.6561 -0.707 0.480544
WB
              -24.4132 10.7774 -2.265 0.024658 *
WC
               -0.2119 0.3432 -0.617 0.537741
X1:WB
X1:WC
               -0.4392 0.3618 -1.214 0.226304
               -0.9233 0.7186 -1.285 0.200463
-1.3562 0.7368 -1.841 0.067257 .
X2:WB
X2:WC

      0.2838
      0.3743
      0.758
      0.449266

      -0.3090
      0.3076
      -1.005
      0.316355

      0.6572
      0.6797
      0.967
      0.334848

      1.3478
      0.7030
      1.917
      0.056730

X3:WB
X3:WC
X4:WB
X4:WC
                              0.7030 1.917 0.056730 .
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

(e) Examine the regression assumptions and provide alternatives if any of them fails.

Adjusted R-squared: 0.9108

Residual standard error: 1.879 on 185 degrees of freedom

F-statistic: 146.2 on 14 and 185 DF, p-value: < 2.2e-16

Hide

shapiro.test(model2\$residuals)

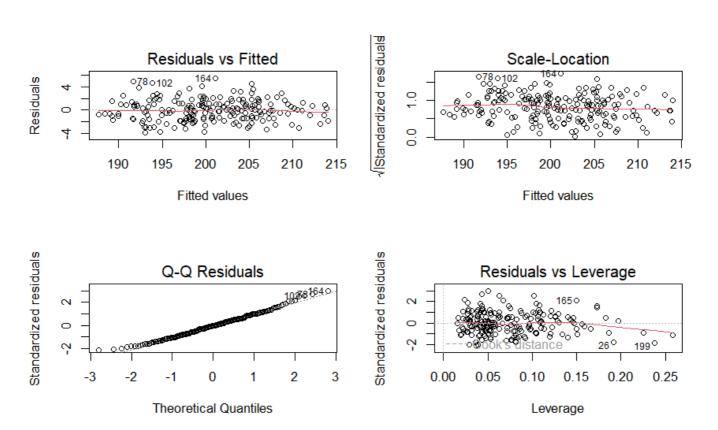
Multiple R-squared: 0.9171,

```
Shapiro-Wilk normality test

data: model2$residuals

W = 0.9907, p-value = 0.2253
```

```
layout(matrix(1:4,2,2))
plot(model2)
```



The residuals of our model seem to be normal, with constant variance and mean equal to 0, so the assumptions of our model exist.

(f) Use the "stepwise regression" approach to examine whether you can reduce the dimension of the model.

```
Start: AIC=736.75
Y ~ 1
      Df Sum of Sq
                   RSS AIC
+ X4
         6982.1 897.7 304.30
+ X2
      1
         3966.9 3912.8 598.74
+ X1
    1 3892.7 3987.0 602.50
+ W
     2 333.4 7546.3 732.10
                 7879.7 736.75
<none>
+ X3 1
            6.2 7873.5 738.59
Step: AIC=304.3
Y ~ X4
      Df Sum of Sq
                  RSS AIC
+ X1
         11.6 886.1 303.70
                  897.7 304.30
<none>
+ W
      2
            14.8 882.9 304.97
+ X2 1
             1.0 896.7 306.09
+ X3 1
            0.8 896.9 306.13
- X4
           6982.1 7879.7 736.75
Step: AIC=303.7
Y \sim X4 + X1
      Df Sum of Sq RSS
                          AIC
       1 145.70 740.4 269.78
+ X2
<none>
                  886.1 303.70
+ W
          16.10 870.0 304.04
- X1 1
           11.58 897.7 304.30
+ X3 1
            0.31 885.8 305.63
- X4
         3100.94 3987.0 602.50
    1
Step: AIC=269.78
Y \sim X4 + X1 + X2
      Df Sum of Sq
                   RSS
                          AIC
- X4
           1.293 741.70 268.12
           10.964 729.44 268.79
+ X3
       1
<none>
                 740.40 269.78
+ W
     2 14.393 726.01 269.85
- X2
       1 145.696 886.10 303.70
- X1
    1 156.321 896.73 306.09
Step: AIC=268.12
Y \sim X1 + X2
      Df Sum of Sq
                  RSS
                          AIC
+ X3
            12.1 729.6 266.84
<none>
                  741.7 268.12
+ W
      2
            14.4 727.3 268.19
+ X4
      1
             1.3 740.4 269.78
- X1 1 3171.1 3912.8 598.74
      1
- X2
           3245.3 3987.0 602.50
```

```
Step: AIC=266.84
Y \sim X1 + X2 + X3
      Df Sum of Sq RSS
                          AIC
       2 15.2 714.5 266.64
+ W
                  729.6 266.84
<none>
             12.1 741.7 268.12
- X3
+ X4
            0.2 729.4 268.79
         3111.9 3841.6 597.06
- X1
- X2 1
         3251.7 3981.3 604.21
Step: AIC=266.64
Y \sim X1 + X2 + X3 + W
      Df Sum of Sq
                  RSS
                          ATC
+ X1:W 2 27.66 686.8 262.74
                  714.5 266.64
<none>
- W
     2
          15.18 729.6 266.84
+ X2:W 2
           11.42 703.0 267.42
          12.81 727.3 268.19
- X3
+ X3:W 2
           8.57 705.9 268.23
+ X4 1
            0.15 714.3 268.60
- X1 1 3037.25 3751.7 596.33
- X2
      1 3160.70 3875.2 602.80
Step: AIC=262.74
Y \sim X1 + X2 + X3 + W + X1:W
      Df Sum of Sq RSS
                          AIC
<none>
                  686.8 262.74
+ X2:W 2
           12.73 674.1 263.00
+ X3:W 2
            9.57 677.2 263.94
- X3 1
           12.53 699.3 264.36
+ X4
      1
            0.14 686.7 264.70
- X1:W 2 27.66 714.5 266.64
- X2
       1 3149.67 3836.5 604.80
```

The dimensionallity of the model was indeed reduced, from 9 independent variables to 5

(g) Using the model found in (f), provide a point estimate and a 95% confidence interval for the prediction of Y when: (X1, X2, X3, X4, W) = (120, 30, 10, 90, B)

```
step_model = lm(Y ~ X1 +X2 + X3 + W + X1:W,data=data)
summary(step_model)
```

```
Call:
lm(formula = Y \sim X1 + X2 + X3 + W + X1:W, data = data)
Residuals:
   Min
         10 Median
                      3Q
-4.2069 -1.2826 0.0146 1.1169 5.6274
Coefficients:
         Estimate Std. Error t value Pr(>|t|)
(Intercept) 30.78245 6.87991 4.474 1.31e-05 ***
          Х1
X2
          Х3
          WB
         -13.51907 10.31593 -1.311 0.19159
WC
         -26.58912 9.69468 -2.743 0.00667 **
X1:WB
          0.11739 0.08529 1.376 0.17032
X1:WC
          Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.891 on 192 degrees of freedom
Multiple R-squared: 0.9128,
                        Adjusted R-squared: 0.9097
F-statistic: 287.3 on 7 and 192 DF, p-value: < 2.2e-16
```

```
new_data <- data.frame(X1 = 120, X2 = 30, X3 = 10, W = "B")
prediction_with_interval <- predict(step_model, newdata = new_data, interval = "confidence")
predicted_Y <- prediction_with_interval[1]
lower_bound <- prediction_with_interval[2]
upper_bound <- prediction_with_interval[3]
cat("Predicted Y:", predicted_Y, "\n")</pre>
```

```
Predicted Y: 200.6028
```

```
cat("95% Confidence Interval:", lower_bound, "to", upper_bound, "\n")
```

```
95% Confidence Interval: 200.1282 to 201.0774
```

(h) Using the cut() function, create a categorical variable (named Z) with three levels based on the quantiles of X4. Provide the contingency table of X4 and W.

Hide

Hide

 $data$Z \leftarrow cut(data$X4, breaks = quantile(data$X4, probs = c(0, 1/3, 2/3, 1)), labels = c("Lev = continuous co$ el1", "Level2", "Level3")) contingency_table <- table(data\$X4, data\$W)</pre> print(contingency_table)

| | Α | В | С |
|----------------|--------|--------|--------|
| 82.86 | 1 | 0 | 0 |
| 83.74 | 0 | 1 | 0 |
| 83.8 | 1 | 0 | 0 |
| 84.42 | 1 | 0 | 0 |
| 84.63 | 0 | 0 | 1 |
| 84.74 | 1 | 0 | 0 |
| 84.83 | 1 | 0 | 0 |
| 84.95 | 0 | 0 | 1 |
| 84.98 | 0 | 0 | 1 |
| 85.16 | 0 | 0 | 1 |
| 85.18 | | | 1 |
| 85.36 | 0 | | 1 |
| 85.39 | 1 | 0 | 0 |
| 85.45 | | | |
| 85.47 | | 0 | |
| 85.61 | 1 | 0 | 0 |
| 85.68 | 1 | 0 | |
| 85.73 | | 0 | |
| 85.81 | 0 | 0 | 1 |
| 85.84 | | 1 | 0 |
| 86.06 | | 0 | |
| 86.13 | 0 | 0 | 1 |
| 86.31 | 0 | 0 | 1 |
| 86.33 | 1 | 0 | |
| 86.36 | | 0 | 1 |
| 86.4 | 1 | 0 | |
| 86.44 86.45 | | | |
| 86.51 | 0 | 1 0 | 0 1 |
| 86.58 | 1 | | 0 |
| 86.73 | | 1 | |
| 86.75 | 0 1 | 0 | 0 0 |
| 86.79 | 1 | 0 | 0 |
| 86.93 | 1 | 0 | 0 |
| 87.01 | 0 | 1 | 0 |
| 87.09 | 1 | 0 | 0 |
| 87.12 | 0 | 0 | 1 |
| 87.12 | 0 | 1 | 0 |
| 87.23 | 0 | 0 | 1 |
| 87.43 | 1 | 0 | 0 |
| 87.44 | 1 | 0 | 0 |
| 87.46 | | 0 | 1 |
| 87.5 | 0 | 1 | 0 |
| 87.52 | 1 | 0 | 0 |
| 87.59 | 0 | 0 | 1 |
| 87.6 | 1 | 0 | 1 |
| 87.63 | 0 | 1 | 0 |
| 87.68 | 1 | 0 | 0 |
| 87.71 | 0 | 1 | 0 |
| 87.74 | 0 | 0 | 1 |
| 87.8 | 0 | 0 | 1 |
| 87.81 | 0 | 1 | 0 |
| 87.87 | 1 | 0 | 0 |
| | | | |

```
31/12/23, 8:23 µ.µ.
```

```
93.52 1 0 0
93.65 0 1 0
93.85 0 0 1
94.02 1 0 0
94.13 0 0 1
94.15 1 0 0
94.16 0 1 0
94.23 0 1 0
94.29 1 0 0
94.33 1 0 0
94.42 0 1 0
94.54 1 0 0
94.56 1 0 0
94.62 0 1 0
94.78 1 0 0
94.98 0 0 1
95.17 1 0 0
95.18 1 0 0
95.6 1 0 0
95.93 1 0 1
96.11 0 1 0
96.91 0 1 0
97.06 1 0 0
97.26 0 1 0
97.36 1 0 0
```

(i) Run the parametric two-way ANOVA of Y on the categorical variables W and Z (including the interaction term). Provide the fit, examine the assumptions, and comment on the significance of the terms.

```
model2wayAnova <- aov(Y ~ W * Z, data = data)
summary(model2wayAnova)</pre>
```

```
Df Sum Sq Mean Sq F value Pr(>F)

W 2 328 164.2 19.102 2.76e-08 ***

Z 2 5704 2852.0 331.883 < 2e-16 ***

W:Z 4 28 6.9 0.808 0.521

Residuals 190 1633 8.6

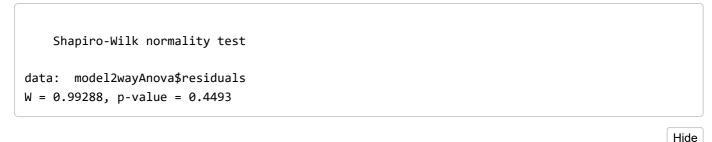
---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

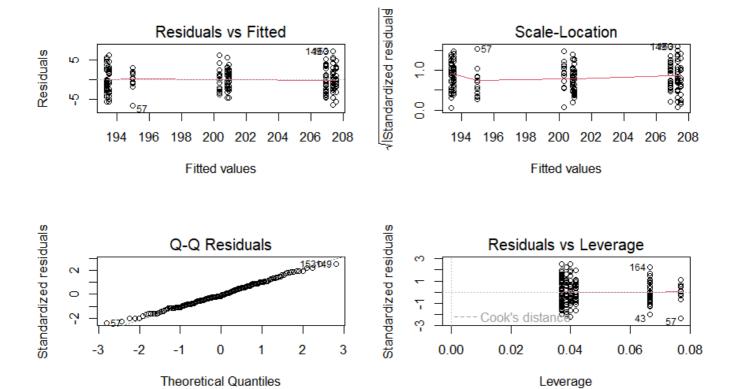
1 observation deleted due to missingness
```

```
Hide
```

```
shapiro.test(model2wayAnova$residuals)
```



layout(matrix(1:4,2,2))
plot(model2wayAnova)



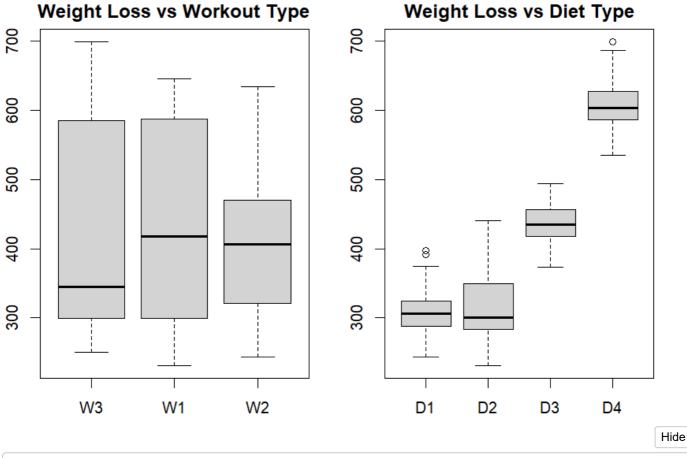
The residuals of our model are normal with constant variance and 0 mean. Both W and Z variables were statistically significant for our ANOVA model, but the interaction between them wasn't. That could mean that our dependent variable Y has a greater variance due to W and Z variables, but we can't be certain. For a different model the significance values could be different. Still, the p-values for these categorical variables were almost 0, meaning that they were very significant for our model.

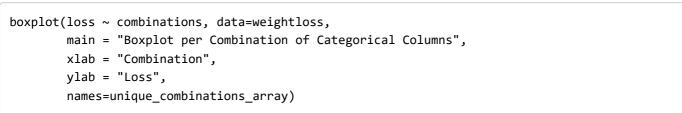
2. In the file "weightloss.txt" (available on the e-class assignments site) you will find the recorded variables work (categorical with three levels), diet (categorical with four levels), and loss (continue, in calories). More specifically, the data provide the weight loss per day in a 3×4 factorial experiment. The

two factors include 3 types of workout and 4 types of diet. Each combination of the two factors is used to be completely randomized.

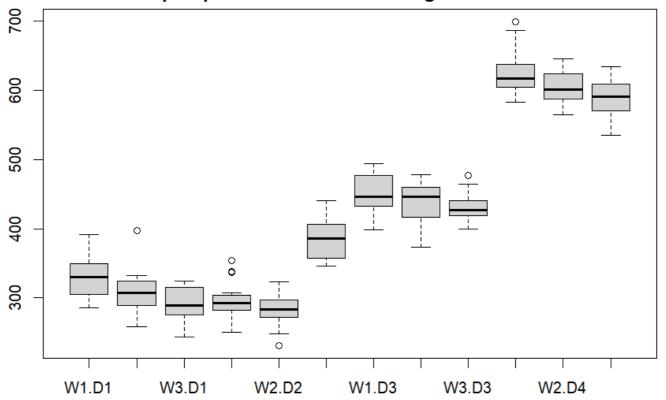
(a) Provide boxplots of the weight loss per workout, per diet, and for the combinations of the two categorical factors.

```
Hide
unique_work <- unique(weightloss$workout)</pre>
cat("Unique Workout Categories:", unique_work, "\n")
Unique Workout Categories: W3 W1 W2
                                                                                                Hide
unique_d <- unique(weightloss$diet)</pre>
cat("Unique Diet Categories:", unique_d, "\n")
Unique Diet Categories: D1 D2 D3 D4
                                                                                                Hide
par(mfrow = c(1, 2))
par(mar = c(2, 2, 2, 2))
boxplot(loss~factor(workout),data=weightloss,
        main=paste("Weight Loss", "vs Workout Type"),
        xlab="Workout Types", ylab="Loss",
        names=c("W3","W1","W2"))
boxplot(loss~factor(diet),data=weightloss,
        main=paste("Weight Loss", "vs Diet Type"),
        xlab="Workout Types", ylab="Loss",
        names=c("D1","D2","D3","D4"))
                                                                                                Hide
weightloss$combinations <- interaction(weightloss$workout, weightloss$diet)</pre>
unique combinations <- levels(weightloss$combinations)</pre>
unique_combinations_array <- as.array(unique_combinations)</pre>
par(mfrow = c(1, 1))
```





Boxplot per Combination of Categorical Columns



(b) Fit a One-Way ANOVA model with the weight loss as a response and the workout (as a factor). Interpret the model parameters.

```
model_work <- aov(loss ~ workout, data = weightloss)
summary(model_work)

Df Sum Sq Mean Sq F value Pr(>F)
workout 2 13831 6916 0.4 0.671
Residuals 237 4101316 17305
```

The most important parameters we have to interpret in this model are the mean sum of squares for the workout variable and the residuals, to see which of these have a great effect. As we can see the mean SoS for our variable workout is very low comparing the residuals one (very high p-value of the F-Statistic) meaning that our dependent variable Y doesn't vary depending to the workout but more on other effects.

(c) In the ANOVA model of (b), is the expected difference between W2 and W3 significant? [TIP: change the reference level appropriately and refit the ANOVA model of question (b)]

```
Hide
weightloss$workout <- factor(weightloss$workout)</pre>
weightloss$workout <- relevel(weightloss$workout, ref = "W3")</pre>
model_ref_W3 <- aov(loss ~ workout, data = weightloss)</pre>
summary(model ref W3)
             Df Sum Sq Mean Sq F value Pr(>F)
workout
              2 13831
                            6916
                                     0.4 0.671
            237 4101316
Residuals
                           17305
                                                                                               Hide
weightloss$workout <- factor(weightloss$workout)</pre>
weightloss$workout <- relevel(weightloss$workout, ref = "W2")</pre>
model ref W2 <- aov(loss ~ workout, data = weightloss)</pre>
summary(model_ref_W2)
             Df Sum Sq Mean Sq F value Pr(>F)
                                     0.4 0.671
                  13831 6916
workout
              2
Residuals
            237 4101316
                           17305
```

We can see that both models with reference to W2 or W3 are the same.

(d) Fit a One-Way ANOVA model for the weight loss as response and diet. Interpret the model parameters. Are all treatments significant?

```
Hide
model_diet <- aov(loss ~ diet, data = weightloss)</pre>
summary(model_diet)
             Df Sum Sq Mean Sq F value Pr(>F)
                                959.3 <2e-16 ***
diet
              3 3803266 1267755
Residuals 236 311882
                           1322
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                                                                                            Hide
TukeyHSD(model_diet)
 Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = loss ~ diet, data = weightloss)
$diet
            diff
                        lwr
                                  upr
                                          p adj
D2-D1 9.411629 -7.419278 26.24254 0.4714878
D3-D1 128.105386 110.036557 146.17421 0.0000000
D4-D1 297.752757 281.371398 314.13412 0.0000000
D3-D2 118.693757 100.437764 136.94975 0.0000000
D4-D2 288.341128 271.753553 304.92870 0.0000000
D4-D3 169.647372 151.804985 187.48976 0.0000000
                                                                                            Hide
weightloss_excluded <- subset(weightloss, diet != "D1")</pre>
model_excluded <- aov(loss ~ diet, data = weightloss_excluded)</pre>
summary(model_excluded)
             Df Sum Sq Mean Sq F value Pr(>F)
diet
              2 2711589 1355795
                                    942 <2e-16 ***
           173 248982
Residuals
                           1439
Signif. codes: 0 '***, 0.001 '**, 0.01 '*, 0.05 '.', 0.1 ', 1
```

We can see that in the Tuckey test, the pair D2-D1 isn't significant, meaning that for these diets loss varies the same and aren't both significant for our ANOVA model (same information)

(f) Fit a Two-Way ANOVA model of main effects. Provide the interpretation for the parameters.

```
model2wayAnova_loss <- aov(loss ~ workout + diet, data = weightloss)
summary(model2wayAnova_loss)</pre>
```

```
Df Sum Sq Mean Sq F value Pr(>F)
workout 2 13831 6916 5.349 0.00535 **
diet 3 3798759 1266253 979.329 < 2e-16 ***
Residuals 234 302557 1293
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Hide

```
TukeyHSD(model2wayAnova_loss)
```

```
Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = loss ~ workout + diet, data = weightloss)
$workout
           diff
                      lwr
                                        p adj
                                upr
W3-W2 -14.41912 -27.63088 -1.207355 0.0286318
W1-W2 -17.09412 -30.53083 -3.657402 0.0083608
W1-W3 -2.67500 -16.30712 10.957122 0.8886991
$diet
            diff
                        lwr
                                          p adj
        9.108733 -7.540375 25.75784 0.4908204
D3-D1 125.560756 107.687098 143.43441 0.0000000
D4-D1 295.777422 279.573006 311.98184 0.0000000
D3-D2 116.452023 98.393222 134.51082 0.0000000
D4-D2 286.668689 270.260285 303.07709 0.0000000
D4-D3 170.216667 152.567004 187.86633 0.0000000
```

We can see that both diet and workout have significant F-Statistic values, meaning that the mean Sum of Squares for each variable wasn't significantly (or at all) smaller than the one for the Residuals and these categorical values affect the variability of the weight loss.

(g) Exclude the non-significant levels of the factors and refit the model. Provide the interpretation for the parameters of the new, simplified, model.

```
weightloss_excluded_2 <- subset(weightloss, !(diet == "D2"))
weightloss_excluded_2 <- subset(weightloss_excluded_2, !(workout == "W3"))
model2wayAnova_loss_ex <- aov(loss ~ workout + diet, data = weightloss_excluded_2)
summary(model2wayAnova_loss)</pre>
```

```
Df Sum Sq Mean Sq F value Pr(>F)
workout 2 13831 6916 5.349 0.00535 **
diet 3 3798759 1266253 979.329 < 2e-16 ***
Residuals 234 302557 1293
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

We can see that the results of the new model are the same as the old, without diet D2, meaning that D1 could by itself explain the variability of the weightloss.

(h) Fit a Two-Way ANOVA model with interactions. Are all the parameters significant

```
Hide
```

```
model2wayAnova_loss_IN <- aov(loss ~ workout * diet, data = weightloss)
summary(model2wayAnova_loss_IN)</pre>
```

```
Df Sum Sq Mean Sq F value Pr(>F)

workout 2 13831 6916 9.836 7.99e-05 ***

diet 3 3798759 1266253 1800.976 < 2e-16 ***

workout:diet 6 142252 23709 33.721 < 2e-16 ***

Residuals 228 160305 703

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Hide

TukeyHSD(model2wayAnova_loss_IN)

```
Tukey multiple comparisons of means
    95% family-wise confidence level
Fit: aov(formula = loss ~ workout * diet, data = weightloss)
$workout
          diff
                     lwr
                               upr
                                       p adj
W3-W2 -14.41912 -24.16328 -4.674960 0.0016683
W1-W2 -17.09412 -27.00419 -7.184048 0.0001915
W1-W3 -2.67500 -12.72919 7.379188 0.8050786
$diet
           diff
                       lwr
                                 upr
                                         p adj
D2-D1
       9.108733 -3.170853 21.38832 0.2226182
D3-D1 125.560756 112.378000 138.74351 0.0000000
D4-D1 295.777422 283.825819 307.72903 0.0000000
D3-D2 116.452023 103.132714 129.77133 0.0000000
D4-D2 286.668689 274.566634 298.77074 0.0000000
D4-D3 170.216667 157.199119 183.23421 0.0000000
$`workout:diet`
                 diff
                              lwr
                                         upr
                                                 p adj
W3:D1-W2:D1 -16.416667
                       -43.720202 10.886869 0.7027219
W1:D1-W2:D1 20.924242 -6.906098 48.754583 0.3561075
W2:D2-W2:D1 -24.035088 -52.837173
                                   4.766997 0.2065725
W3:D2-W2:D1 78.611111 49.422404 107.799819 0.0000000
W1:D2-W2:D1 -13.375000 -40.678536 13.928536 0.9010037
W2:D3-W2:D1 129.533333 101.083700 157.982966 0.0000000
W3:D3-W2:D1 122.859649 94.057564 151.661734 0.0000000
W1:D3-W2:D1 143.083333 105.874886 180.291781 0.0000000
W2:D4-W2:D1 297.261905 270.807392 323.716418 0.0000000
W3:D4-W2:D1 280.701754 251.899669 309.503839 0.0000000
W1:D4-W2:D1 317.190476 289.063534 345.317419 0.0000000
W1:D1-W3:D1 37.340909 11.494627 63.187191 0.0002031
W2:D2-W3:D1 -7.618421 -34.508243 19.271401 0.9986765
W3:D2-W3:D1 95.027778 67.724242 122.331314 0.0000000
             3.041667 -22.236496 28.319829 0.9999998
W1:D2-W3:D1
W2:D3-W3:D1 145.950000 119.438040 172.461960 0.0000000
W3:D3-W3:D1 139.276316 112.386494 166.166138 0.0000000
W1:D3-W3:D1 159.500000 123.751280 195.248720 0.0000000
W2:D4-W3:D1 313.678571 289.319925 338.037218 0.0000000
W3:D4-W3:D1 297.118421 270.228599 324.008243 0.0000000
W1:D4-W3:D1 333.607143 307.441759 359.772527 0.0000000
W2:D2-W1:D1 -44.959330
                       -72.383905 -17.534755 0.0000100
W3:D2-W1:D1 57.686869
                       29.856528 85.517209 0.0000000
W1:D2-W1:D1 -34.299242 -60.145524 -8.452961 0.0010669
W2:D3-W1:D1 108.609091 81.554908 135.663274 0.0000000
W3:D3-W1:D1 101.935407
                       74.510832 129.359982 0.0000000
W1:D3-W1:D1 122.159091
                       86.006418 158.311764 0.0000000
W2:D4-W1:D1 276.337662 251.389948 301.285377 0.0000000
W3:D4-W1:D1 259.777512 232.352937 287.202087 0.0000000
W1:D4-W1:D1 296.266234 269.551592 322.980876 0.0000000
W3:D2-W2:D2 102.646199
                       73.844114 131.448284 0.0000000
W1:D2-W2:D2 10.660088
                       -16.229734 37.549910 0.9772520
```

W2:D3-W2:D2 153.568421 125.515595 181.621248 0.0000000

```
W3:D3-W2:D2 146.894737 118.484535 175.304939 0.0000000
W1:D3-W2:D2 167.118421 130.212486 204.024356 0.0000000
W2:D4-W2:D2 321.296992 295.269686 347.324299 0.0000000
W3:D4-W2:D2 304.736842 276.326640 333.147044 0.0000000
W1:D4-W2:D2 341.225564 313.500046 368.951082 0.0000000
W1:D2-W3:D2 -91.986111 -119.289647 -64.682575 0.0000000
W2:D3-W3:D2 50.922222 22.472589 79.371855 0.0000008
W3:D3-W3:D2 44.248538 15.446453 73.050623 0.0000513
W1:D3-W3:D2 64.472222 27.263775 101.680670 0.00000021
W2:D4-W3:D2 218.650794 192.196281 245.105306 0.0000000
W3:D4-W3:D2 202.090643 173.288558 230.892728 0.0000000
W1:D4-W3:D2 238.579365 210.452423 266.706307 0.0000000
W2:D3-W1:D2 142.908333 116.396373 169.420294 0.0000000
W3:D3-W1:D2 136.234649 109.344827 163.124471 0.0000000
W1:D3-W1:D2 156.458333 120.709613 192.207053 0.0000000
W2:D4-W1:D2 310.636905 286.278258 334.995551 0.0000000
W3:D4-W1:D2 294.076754 267.186932 320.966576 0.0000000
W1:D4-W1:D2 330.565476 304.400092 356.730860 0.0000000
W3:D3-W2:D3 -6.673684 -34.726511 21.379142 0.9997455
W1:D3-W2:D3 13.550000 -23.081537 50.181537 0.9867743
W2:D4-W2:D3 167.728571 142.091836 193.365307 0.0000000
W3:D4-W2:D3 151.168421 123.115595 179.221248 0.0000000
W1:D4-W2:D3 187.657143 160.297942 215.016343 0.0000000
W1:D3-W3:D3 20.223684 -16.682250 57.129619 0.8112155
W2:D4-W3:D3 174.402256 148.374949 200.429563 0.0000000
W3:D4-W3:D3 157.842105 129.431904 186.252307 0.0000000
W1:D4-W3:D3 194.330827 166.605309 222.056345 0.0000000
W2:D4-W1:D3 154.178571 119.074025 189.283117 0.00000000
W3:D4-W1:D3 137.618421 100.712486 174.524356 0.0000000
W1:D4-W1:D3 174.107143 137.725653 210.488633 0.0000000
W3:D4-W2:D4 -16.560150 -42.587457 9.467157 0.6224704
W1:D4-W2:D4 19.928571 -5.349591 45.206734 0.2840098
W1:D4-W3:D4 36.488722 8.763204 64.214240 0.0012371
```

From the Tuckey Test we can see that not all parameters of the model are significant and we can see many pairs of combinations that provide a non significant p-value (for example W3:D1-W2:D1).

(i) Using the stepwise method, choose a model based on the AIC criterion starting from the full model (including the main effects and the interaction term). Are all coefficients significant?

```
weightloss$workout = as.factor(weightloss$workout)
weightloss$diet = as.factor(weightloss$diet)
dummy_variables <- model.matrix(~ workout - 1, data = weightloss)</pre>
weightloss <- cbind(weightloss, dummy_variables)</pre>
dummy variables <- model.matrix(~ diet - 1, data = weightloss)</pre>
weightloss <- cbind(weightloss, dummy_variables)</pre>
fitnull<-lm(loss ~ workoutW1 +workoutW2 + workoutW3 + dietD1 + dietD2 + dietD3 + dietD4 + wor
koutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 + workoutW1:dietD4 + workoutW2:dietD1 + w
orkoutW2:dietD2 + workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1 + workoutW3:dietD2 +
workoutW3:dietD3 + workoutW3:dietD4,data=weightloss)
stepSR<-step(fitnull, scope=list(lower = ~ 1,</pre>
                                  upper = ~ workoutW1 +workoutW2 + workoutW3 + dietD1 + dietD2
+ dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 + workoutW1:dietD4
+ workoutW2:dietD1 + workoutW2:dietD2 + workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD
1 + workoutW3:dietD2 + workoutW3:dietD3 + workoutW3:dietD4),
             direction="both", data=weightloss)
```

```
Start: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
    workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1 +
    workoutW3:dietD2 + workoutW3:dietD3 + workoutW3:dietD4
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
   dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
    workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1 +
    workoutW3:dietD2 + workoutW3:dietD3
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
    workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1 +
    workoutW3:dietD2
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
    workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
    workoutW2:dietD3 + workoutW2:dietD4
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
   workoutW1:dietD4 + workoutW2:dietD1 + workoutW2:dietD2 +
   workoutW2:dietD3
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + dietD4 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW2:dietD1 + workoutW2:dietD2 + workoutW2:dietD3
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + workoutW3 + dietD1 + dietD2 +
    dietD3 + workoutW1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 +
    workoutW2:dietD1 + workoutW2:dietD2 + workoutW2:dietD3
```

```
Step: AIC=1585.01
loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD1 +
   workoutW1:dietD2 + workoutW1:dietD3 + workoutW2:dietD1 +
   workoutW2:dietD2 + workoutW2:dietD3
                 Df Sum of Sq RSS
                                       AIC
                       0 160305 1583.0
- workoutW2:dietD1 1
- workoutW1:dietD1 1
                           4 160309 1583.0
                        512 160817 1583.8
workoutW2:dietD3 1
workoutW1:dietD3 1
                        952 161257 1584.4
<none>
                              160305 1585.0
- workoutW2:dietD2 1 72304 232609 1672.3
- workoutW1:dietD2 1 83585 243890 1683.7
Step: AIC=1583.01
loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD1 +
   workoutW1:dietD2 + workoutW1:dietD3 + workoutW2:dietD2 +
   workoutW2:dietD3
                 Df Sum of Sq RSS AIC
- workoutW1:dietD1 1 6 160311 1581.0
workoutW2:dietD3 1
                        647 160953 1582.0
- workoutW1:dietD3 1
                        1001 161306 1582.5
<none>
                              160305 1583.0
+ workoutW2:dietD1 1 0 160305 1585.0
- workoutW1:dietD2 1 90363 250668 1688.3
workoutW2:dietD2 1
                       91886 252191 1689.8
Step: AIC=1581.02
loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD2 +
   workoutW1:dietD3 + workoutW2:dietD2 + workoutW2:dietD3
                  Df Sum of Sq
                                  RSS
                         656 160967 1580.0
- workoutW2:dietD3 1
workoutW1:dietD3 1
                        1246 161557 1580.9
<none>
                               160311 1581.0
+ workoutW1:dietD1 1
                           6 160305 1583.0
+ workoutW2:dietD1 1
                           2 160309 1583.0
- workoutW2:dietD2 1 92221 252533 1688.1
- workoutW1:dietD2 1 115582 275894 1709.3
                 1 2847391 3007703 2282.7
- dietD1
Step: AIC=1580
loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD2 +
   workoutW1:dietD3 + workoutW2:dietD2
                                 RSS
                 Df Sum of Sq
                                        ATC
- workoutW1:dietD3 1 733 161701 1579.1
<none>
                               160967 1580.0
+ workoutW2:dietD3 1
                        656 160311 1581.0
+ workoutW2:dietD1 1
                          76 160891 1581.9
+ workoutW1:dietD1 1
                          15 160953 1582.0
- workoutW2:dietD2 1
                       96430 257397 1690.7
- workoutW1:dietD2 1
                       115793 276760 1708.1
```

```
- dietD1
                1 2870636 3031603 2282.6
Step: AIC=1579.09
loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD2 +
   workoutW2:dietD2
               Df Sum of Sq
                              RSS AIC
                           161701 1579.1
<none>
+ workoutW1:dietD3 1
                       733 160967 1580.0
+ workoutW1:dietD1 1
                      204 161497 1580.8
+ workoutW2:dietD3 1
                      144 161557 1580.9
                     17 161684 1581.1
+ workoutW2:dietD1 1
- workoutW2:dietD2 1 96431 258132 1689.3
- workoutW1:dietD2 1 117266 278967 1708.0
- dietD1
               1 2869903 3031603 2280.6
```

```
final_model = lm(loss~workoutW1 +workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD2 + wo
rkoutW2:dietD2, data=weightloss )
summary(final_model)
```

```
Call:
lm(formula = loss ~ workoutW1 + workoutW2 + dietD1 + dietD2 +
      dietD3 + workoutW1:dietD2 + workoutW2:dietD2, data = weightloss)
Residuals:
                 1Q Median
      Min
                                          3Q
                                                    Max
-65.910 -17.769 -1.066 18.106 90.009
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
                        590.645 4.393 134.462 < 2e-16 ***
(Intercept)
                          33.201 5.038 6.590 2.93e-10 ***
13.496 4.700 2.871 0.00446 **
workoutW1
workoutW2

      dietD1
      -297.151
      4.631
      -64.169
      < 2e-16</td>
      ***

      dietD2
      -204.368
      7.617
      -26.831
      < 2e-16</td>
      ***

      dietD3
      -165.231
      5.053
      -32.703
      < 2e-16</td>
      ***

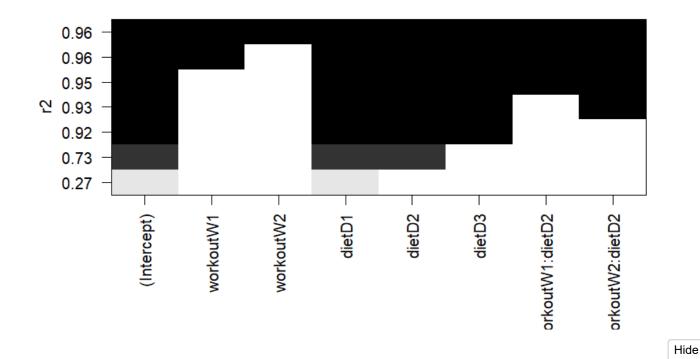
      workoutW1:dietD2
      -125.187
      9.651
      -12.971
      < 2e-16</td>
      ***

                                          9.874 -11.762 < 2e-16 ***
workoutW2:dietD2 -116.142
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
Residual standard error: 26.4 on 232 degrees of freedom
Multiple R-squared: 0.9607,
                                            Adjusted R-squared: 0.9595
F-statistic: 810.3 on 7 and 232 DF, p-value: < 2.2e-16
```

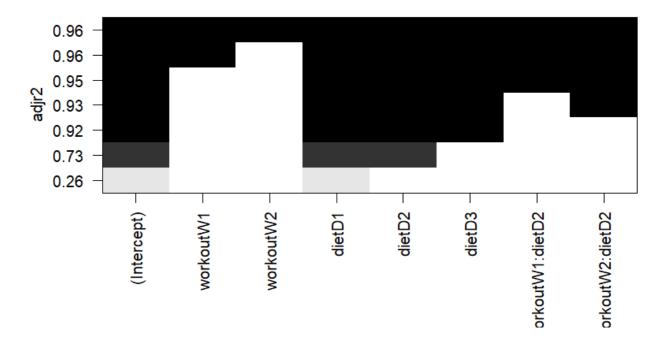
Yes all the coefficients of the final model are statistically significant.

(j) Provide a graphical representation for the final model.

leaps<-regsubsets(loss~workoutW1 +workoutW2 + dietD1 + dietD2 + dietD3 + workoutW1:dietD2 + w
orkoutW2:dietD2, data=weightloss)
plot(leaps,scale="r2")</pre>



plot(leaps,scale="adjr2")



(k) Compare the constant model against the (full) main effects model and the (full) interaction model. Are the models different?

```
fitM0<-lm(loss~workoutW1 +workoutW2 + workoutW3 + dietD1 + dietD2 + dietD3 + dietD4 + workout
W1:dietD1 + workoutW1:dietD2 + workoutW1:dietD3 + workoutW1:dietD4 + workoutW2:dietD1 + worko
utW2:dietD2 + workoutW2:dietD3 + workoutW2:dietD4 + workoutW3:dietD1 + workoutW3:dietD2 + workoutW3:dietD3 + workoutW3:dietD4,data=weightloss)
fitM1<-lm(loss~workoutW1 +workoutW2 + workoutW3 + dietD1 + dietD2 + dietD3 + dietD4,data=weightloss)
fitM2<-lm(loss~1,data=weightloss)
anova(fitM0,fitM2)</pre>
```

```
anova(fitM1,fitM2)
```

Hide

```
aic_model0 <- AIC(fitM0)
aic_model1 <- AIC(fitM1)
aic_model2 <- AIC(fitM2)
print(c(AIC = c(aic_model0,aic_model1, aic_model2)))</pre>
```

```
AIC1 AIC2 AIC3
2268.097 2408.543 3024.982
```

Yes, the models are completely different. It can be seen from the AIC values of the larger models that are much lower from the simple model AIC, even though they have more params, meaning that they have a significantly larger likelihood value.