Q. 2:-

a)

$$m_1 = \frac{1}{3}\left[\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\right] = \frac{1}{3}\left[\begin{pmatrix}6\\6\end{pmatrix}\right]$$

$$\boxed{m_1 = \begin{pmatrix}2\\2\end{pmatrix}}$$

$$m_2 = \frac{1}{5}\left[\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}6\\2\end{pmatrix}+\begin{pmatrix}7\\2\end{pmatrix}+\begin{pmatrix}8\\2\end{pmatrix}+\begin{pmatrix}9\\2\end{pmatrix}\right]$$

$$= \frac{1}{5}\left[\begin{pmatrix}35\\10\end{pmatrix}\right]$$

$$\boxed{m_2 = \begin{pmatrix}7\\2\end{pmatrix}}$$

b)

$$\text{Total mean } (m) = \frac{1}{8}\left[\begin{pmatrix}6\\6\end{pmatrix}+\begin{pmatrix}35\\10\end{pmatrix}\right]$$

$$m = \frac{1}{8}\begin{bmatrix}41\\16\end{bmatrix}$$

$$\boxed{m = \begin{pmatrix}5.125\\2\end{pmatrix}}$$

c)

$$S_1 = \left[\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]\left[\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]^T + \left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]\left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]^T + \left[\begin{pmatrix}3\\3\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]\left[\begin{pmatrix}3\\3\end{pmatrix}-\begin{pmatrix}2\\2\end{pmatrix}\right]^T$$

$$= \begin{bmatrix}-1\\-1\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}^T + \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^T + \begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}^T$$

$$= \begin{bmatrix}1&1\\1&1\end{bmatrix} + \begin{bmatrix}0&0\\0&0\end{bmatrix} + \begin{bmatrix}1&1\\1&1\end{bmatrix}$$

$$\boxed{S_1 = \begin{bmatrix}2&2\\2&2\end{bmatrix}}$$

$$S_2 = \left[\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]\left[\begin{pmatrix}5\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]^T + \left[\begin{pmatrix}6\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]\left[\begin{pmatrix}6\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]^T + \left[\begin{pmatrix}7\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]\left[\begin{pmatrix}7\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]^T$$

$$+ \left[\begin{pmatrix}8\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]\left[\begin{pmatrix}8\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]^T + \left[\begin{pmatrix}9\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]\left[\begin{pmatrix}9\\2\end{pmatrix}-\begin{pmatrix}7\\2\end{pmatrix}\right]^T$$

$$S_2 = \begin{bmatrix}-2\\0\end{bmatrix}\begin{bmatrix}-2\\0\end{bmatrix}^T + \begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}-1\\0\end{bmatrix}^T + \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}^T + \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}^T + \begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}^T$$

$$S_2 = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}$$

$$\boxed{S_2 = \begin{bmatrix}10 & 0\\0 & 0\end{bmatrix}}$$

d)

$$S_W = S_1 + S_2$$

$$S_W = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} + \begin{bmatrix}10 & 0\\0 & 0\end{bmatrix}$$

$$\boxed{S_W = \begin{bmatrix}12 & 2\\2 & 2\end{bmatrix}}$$

e)

$$S_B = 3\left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}5.125\\2\end{pmatrix}\right]\left[\begin{pmatrix}2\\2\end{pmatrix}-\begin{pmatrix}5.125\\2\end{pmatrix}\right]^T + 5\left[\begin{pmatrix}7\\2\end{pmatrix}-\begin{pmatrix}5.125\\2\end{pmatrix}\right]\left[\begin{pmatrix}7\\2\end{pmatrix}-\begin{pmatrix}5.125\\2\end{pmatrix}\right]^T$$

$$S_B = 3\begin{bmatrix}-3.125\\0\end{bmatrix}\begin{bmatrix}-3.125\\0\end{bmatrix}^T + 5\begin{bmatrix}1.875\\0\end{bmatrix}\begin{bmatrix}1.875\\0\end{bmatrix}^T$$

$$S_B = \begin{bmatrix}29.3 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}17.6 & 0\\0 & 0\end{bmatrix}$$

$$\boxed{S_B = \begin{bmatrix}46.9 & 0\\0 & 0\end{bmatrix}}$$

f)

$$\frac{tr(S_B)}{tr(S_W)} = \frac{46.9+0}{12+2} = \frac{46.9}{14}$$

$$\boxed{\frac{tr(S_B)}{tr(S_W)} = 3.35}$$