

Q.2:→

New Instance 1:→

$$\begin{aligned} P(y=\text{Low} | x=\text{High School, Service, } < 3) &= P(x=\text{H.S} | y=\text{Low}) * P(x=\text{Service} | y=\text{Low}) * \\ &\quad * P(x=<3 | y=\text{Low}) * P(y=\text{Low}) \\ &= \frac{4}{6} * \frac{4}{6} * \frac{2}{6} * \frac{6}{10} \end{aligned}$$

After Applying Laplace Smoothing: -

$$\begin{aligned} &= \frac{4+1}{6+2} * \frac{4+1}{6+2} * \frac{2+1}{6+3} * \frac{6}{10} \\ &= \frac{5}{8} * \frac{5}{8} * \frac{3}{9} * \frac{6}{10} \\ &= \frac{450}{5760} = \boxed{7.80\%} \end{aligned}$$

$$\begin{aligned} P(y=\text{High} | x=\text{HS, Service, } < 3) &= P(x=\text{HS} | y=\text{High}) * P(x=\text{Service} | y=\text{High}) * P(x=<3 | y=\text{High}) \\ &\quad * P(y=\text{High}) \end{aligned}$$

$$= \frac{1}{4} * \frac{1}{4} * \frac{1}{4} * \frac{4}{10}$$

After Applying Laplace Smoothing:→

$$\begin{aligned} &= \frac{1+1}{4+2} * \frac{1+1}{4+2} * \frac{1+1}{4+3} * \frac{4}{10} \\ &= \frac{2}{6} * \frac{2}{6} * \frac{2}{7} * \frac{4}{10} = \frac{32}{2520} \\ &= \boxed{1.27\%} \end{aligned}$$

Since the $y=\text{Low}$ Probability is high

Predicted Class = Low

For Instance 2:-

$$P(y=Low | X=College, Retail, <3) = P(X=College/y=Low) \times P(X=Retail/y=Low) \times P(X=<3/y=Low) \times P(y=Low)$$

$$= \frac{3}{6} \times \frac{0}{6} \times \frac{3}{6} \times \frac{6}{10}$$

After applying Laplace Smoothing :-

$$= \frac{3+1}{6+2} \times \frac{0+1}{6+3} \times \frac{3+1}{6+3} \times \frac{6}{10}$$

$$= \frac{3}{8} \times \frac{1}{9} \times \frac{3}{9} \times \frac{6}{10}$$

$$= \frac{54}{6480}$$

$$= \boxed{0.83\%}$$

$$P(y=high | X=College, Retail, <3) = P(X=College/y=high) \times P(X=Retail/y=high) \times P(X=<3/y=high) \times P(y=Low)$$

$$= \frac{3}{4} \times \frac{0}{4} \times \frac{1}{4} \times \frac{4}{10}$$

After applying Laplace Smoothing :-

$$= \frac{3+1}{4+2} \times \frac{0+1}{4+3} \times \frac{1+1}{4+3} \times \frac{4}{10}$$

$$= \frac{4}{6} \times \frac{1}{7} \times \frac{2}{7} \times \frac{4}{10}$$

$$= \frac{32}{2040}$$

$$= \boxed{1.1\%}$$

Final Predicted Class = High

For Instance 3:-

$$\begin{aligned} P(y=\text{Low} | X=\text{Graduate, Service, 3-10}) &= P(X=\text{Graduate} | y=\text{Low}) * P(X=\text{Service} | y=\text{Low}) \\ &\quad * P(X=3-10 | y=\text{Low}) * P(y=\text{Low}) \\ &= \frac{0}{6} * \frac{4}{6} * \frac{3}{6} * \frac{6}{10} \end{aligned}$$

After Laplace Smoothing :-

$$\begin{aligned} &= \frac{0+1}{6+3} * \frac{4+1}{6+2} * \frac{3+1}{6+3} * \frac{6}{10} \\ &= \frac{1}{9} * \frac{5}{8} * \frac{4}{9} * \frac{6}{10} \\ &= \frac{90}{6480} \\ &= \boxed{1.4\%} \end{aligned}$$

$$\begin{aligned} P(y=\text{high} | X=\text{Graduate, Service, 3-10}) &= P(X=\text{Graduate} | y=\text{high}) * P(X=\text{Service} | y=\text{high}) \\ &\quad * P(X=3-10 | y=\text{high}) * P(y=\text{high}) \\ &= \frac{0}{4} * \frac{1}{4} * \frac{1}{4} * \frac{4}{10} \end{aligned}$$

After Laplace Smoothing :-

$$\begin{aligned} &= \frac{0+1}{4+3} * \frac{1+1}{4+2} * \frac{1+1}{4+3} * \frac{4}{10} \\ &= \frac{1}{7} * \frac{2}{6} * \frac{2}{7} * \frac{4}{10} \\ &= \frac{16}{2940} \\ &= \boxed{0.54\%} \end{aligned}$$

Final Predicted class = Low