Investigations Into Brownian Motion:

Optical Trapping

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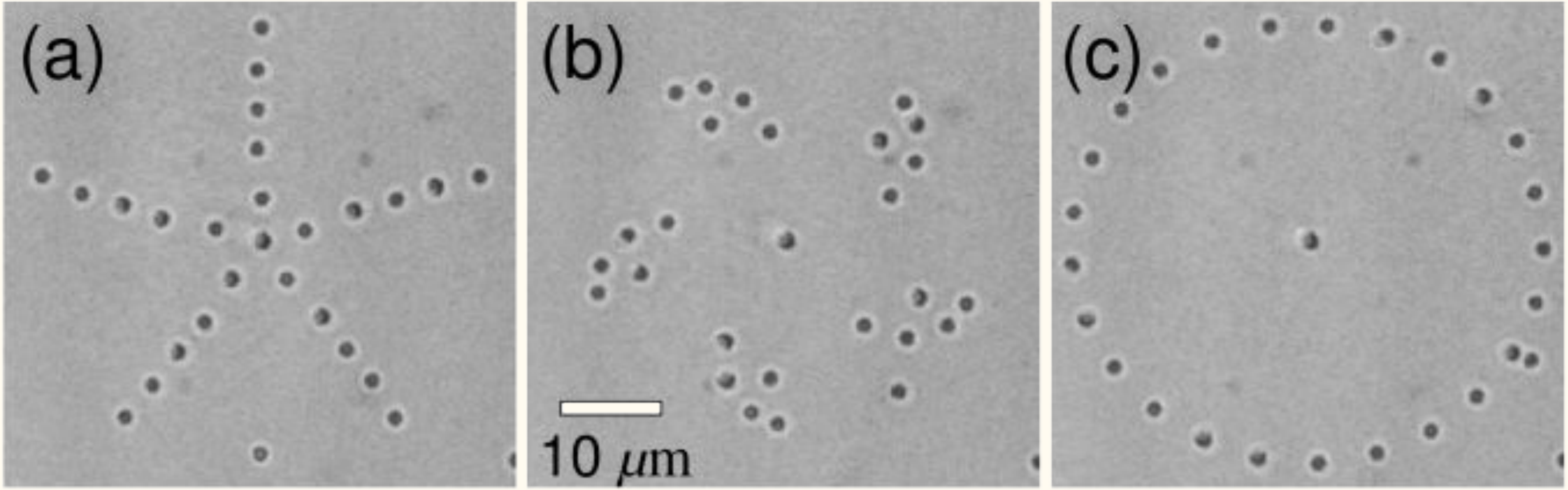
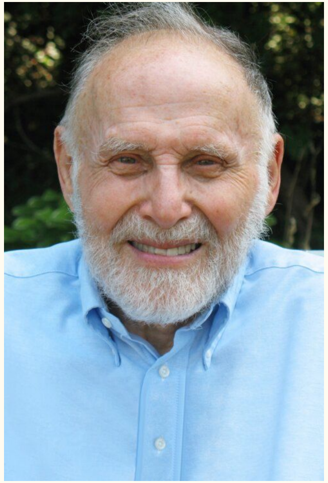


https://physics.nyu.edu/grierlab/nreview2c/

What is Optical Trapping

- Piconewton sized forces on micron sized dielectric objects with infrared laser
- Revolutionary in Biophysics
- Pick up, move around, and interact with small biological systems



Arthur Ashkin

https://www.nobelprize.org/prizes/physics/2018/ashkin/facts/

Goal: Determine the Boltzmann Constant

<u>How has it been done before?</u>

- Dielectric-constant gas thermometry
    - Gas response to electric fields
- Acoustic thermometry
    - Speed of sound in a gas is directly dependent on its temperature

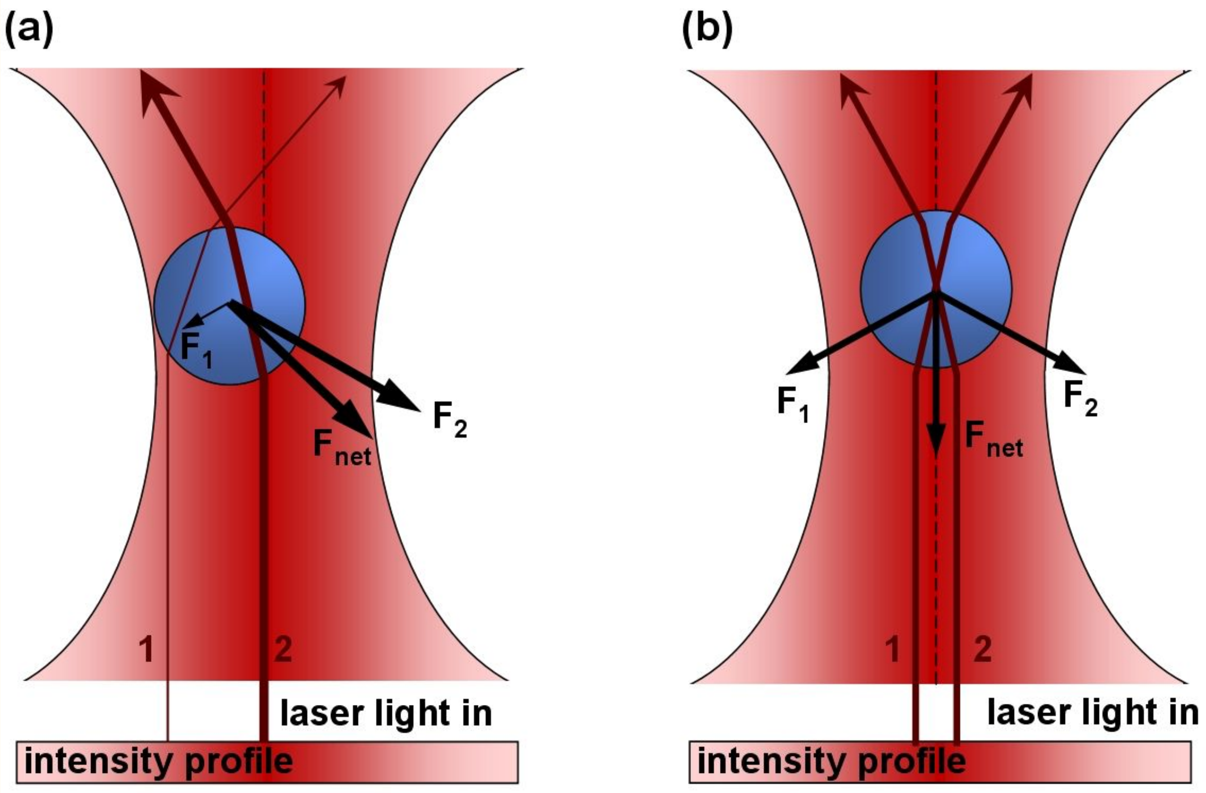
<u>How will we do it?</u>

- Brownian motion and power spectral density using optical trapping

https://www.nist.gov/si-redefinition/kelvin-boltzmann-constant
https://phys.org/news/2013-07-accurate-boltzmann-constant.html

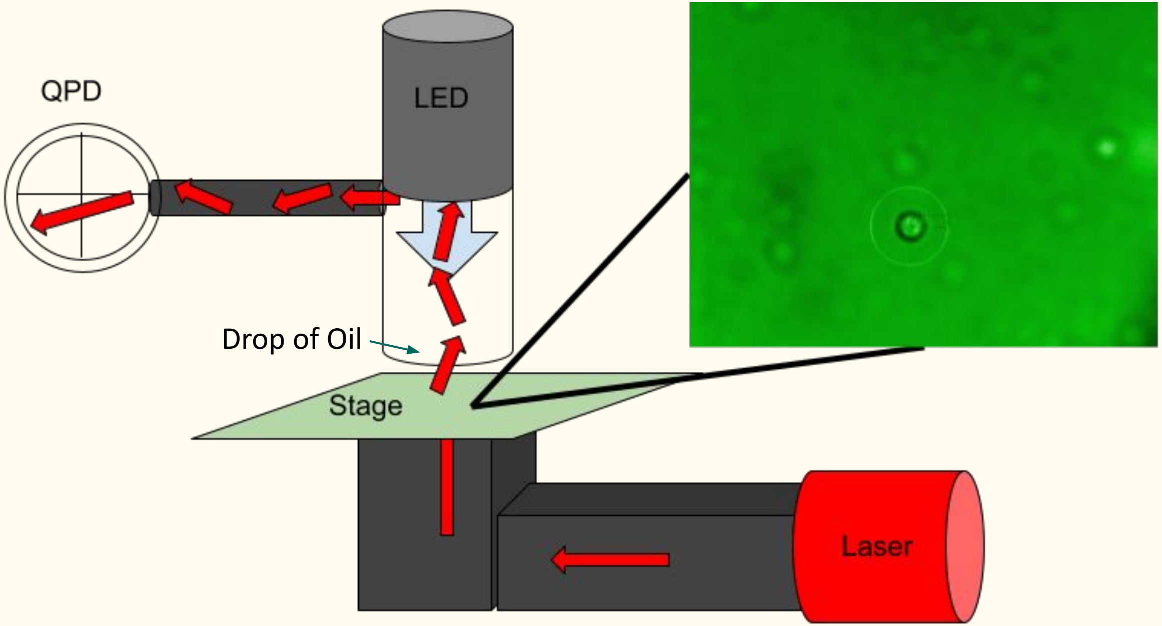
The Physics of Optical Trapping

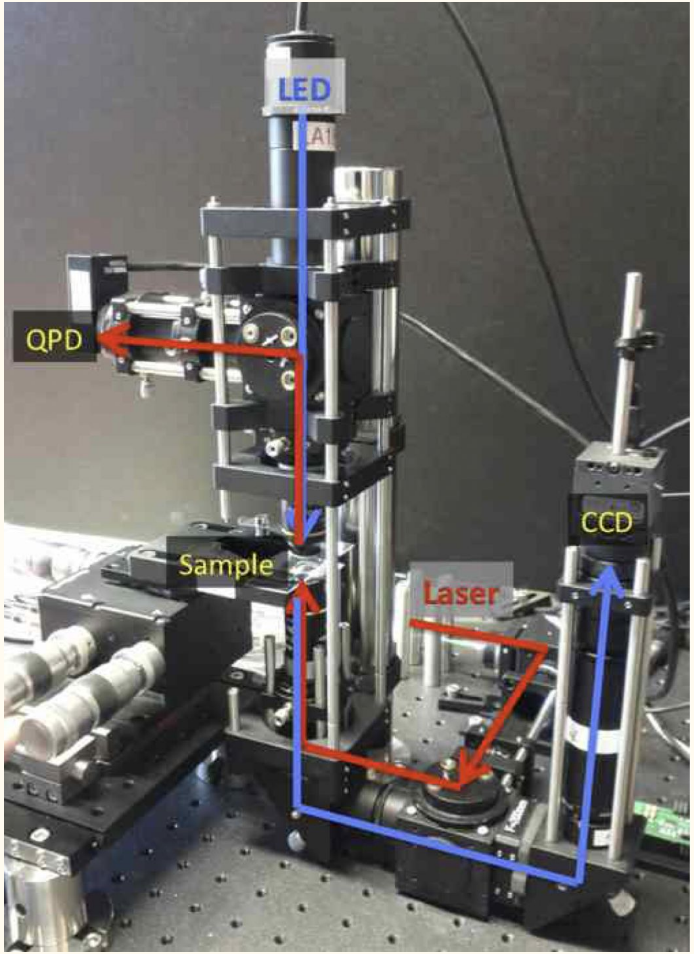
- Momentum
- Scattered light creates a scattering force
- Refracted light creates gradient force



https://commons.wikimedia.org/wiki/File:Optical_Trap_Ray_Optics_Explanation.jpg

Set Up





Trap in X-Y
Focus laser with Z

Math of Optical Trapping

β hydrodynamic drag coefficient

α harmonic optical trap of stiffness

d diameter of bead

η viscosity of the medium

Wiener-Khinchin theorem to define a "power spectral distribution" function

Equipartition theorem...

$$\beta x' + \alpha x = F(t)$$

$$\beta = 3\pi\eta d$$

$$S_{xx}(f) = \frac{K_B T}{\pi^2 \beta (f^2 + f_0^2)}$$

$$f_0 = \frac{\alpha}{2\pi\beta}$$

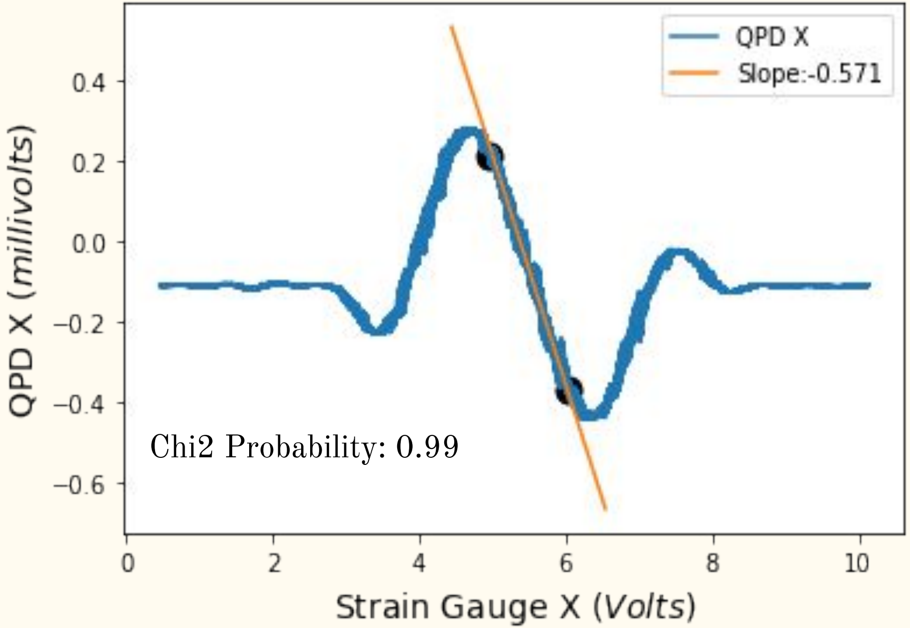
Step One: Obtain Calibration

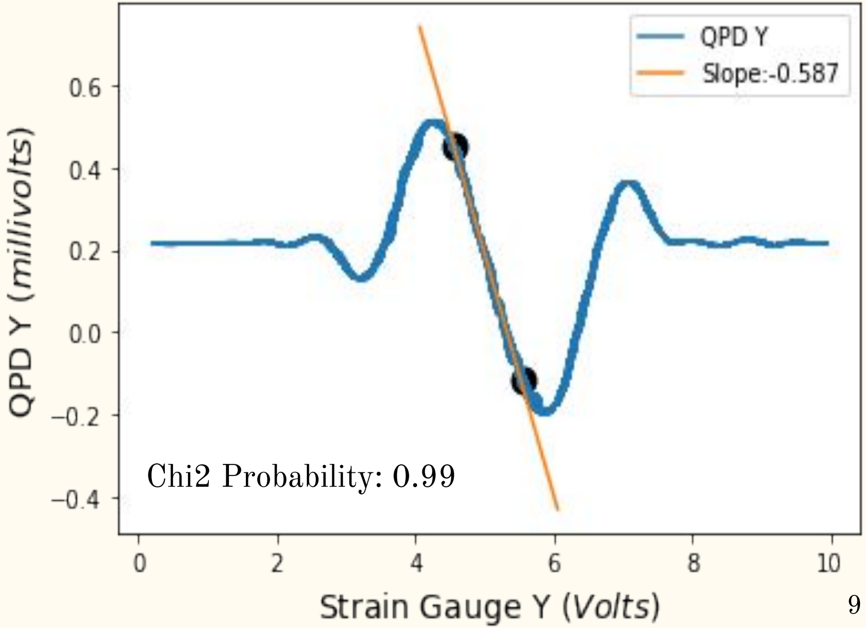
- Problem?
  - QPD - millivolts
    - Photodiode
  - Piezo - volts
    - Strain gauge
- How do we extract the position?
  - Pass through over fixed bead sample!

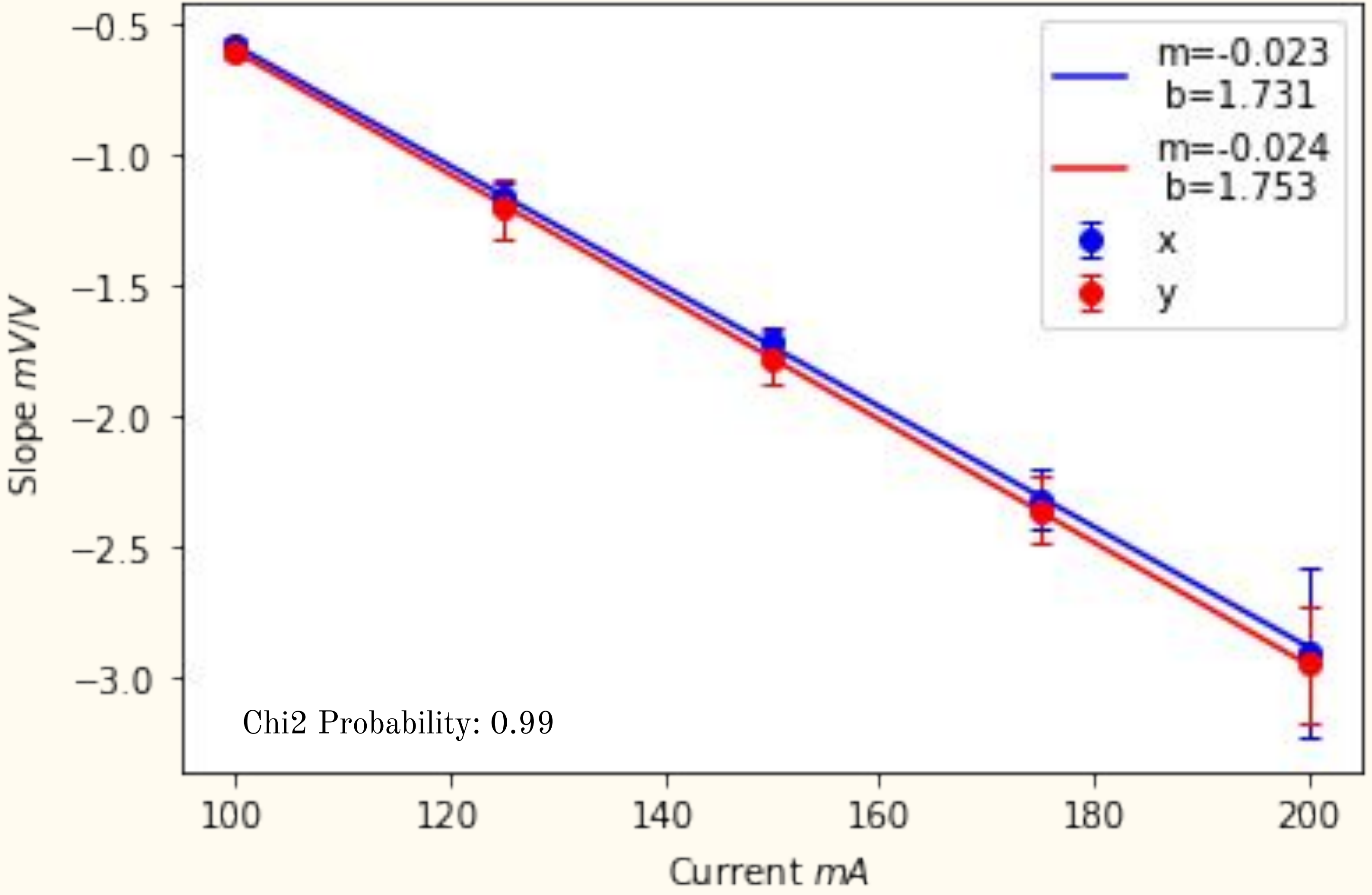
Part 1: Relationship between QPD and Piezos

Reducing systematic errors:

- Three trials for each X and Y at each laser current for the fixed bead sample
- Laser intervals for experiment: 100, 125, 150, 175, 200



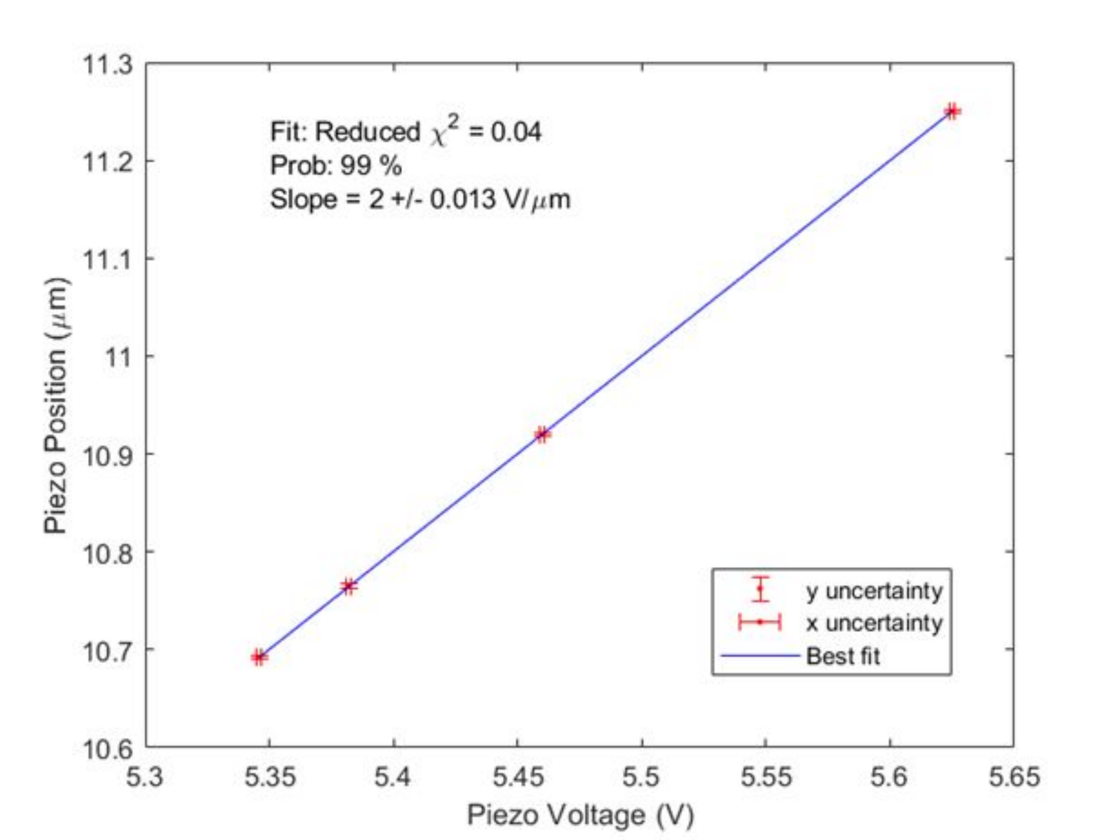




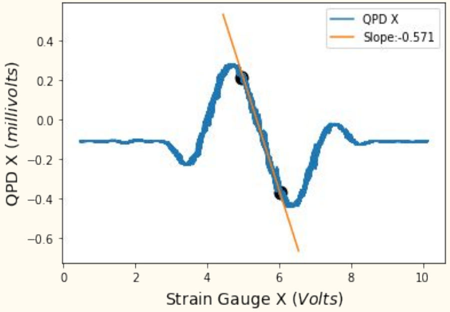
m=-0.023
b=1.731
m=-0.024
b=1.753
x
y
Chi2 Probability: 0.99
Slope mV/V
Current mA
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
100
120
140
160
180
200

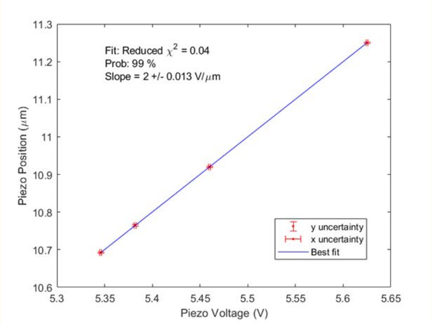
Part 2: Relationship between Piezo and Position

- Four different points
- Preset at two



The Complete Conversion from QPD to Position





$$QPD(mV) \times \frac{Piezo\ (V)}{QPD(mV)} \times \frac{Position\ (\mu m)}{Piezo\ (V)} = \mu m$$

$[Part\ 1{:}\ Slope]^{-1}$ $\quad$ $Part\ 2{:}\ Slope$

Step Two: Power Spectral Distribution

- Fourier transform QPD data using signal processing trick
- Fit
  - Extract boltzmann constant
  - Extract stiffness
- Function to fit:

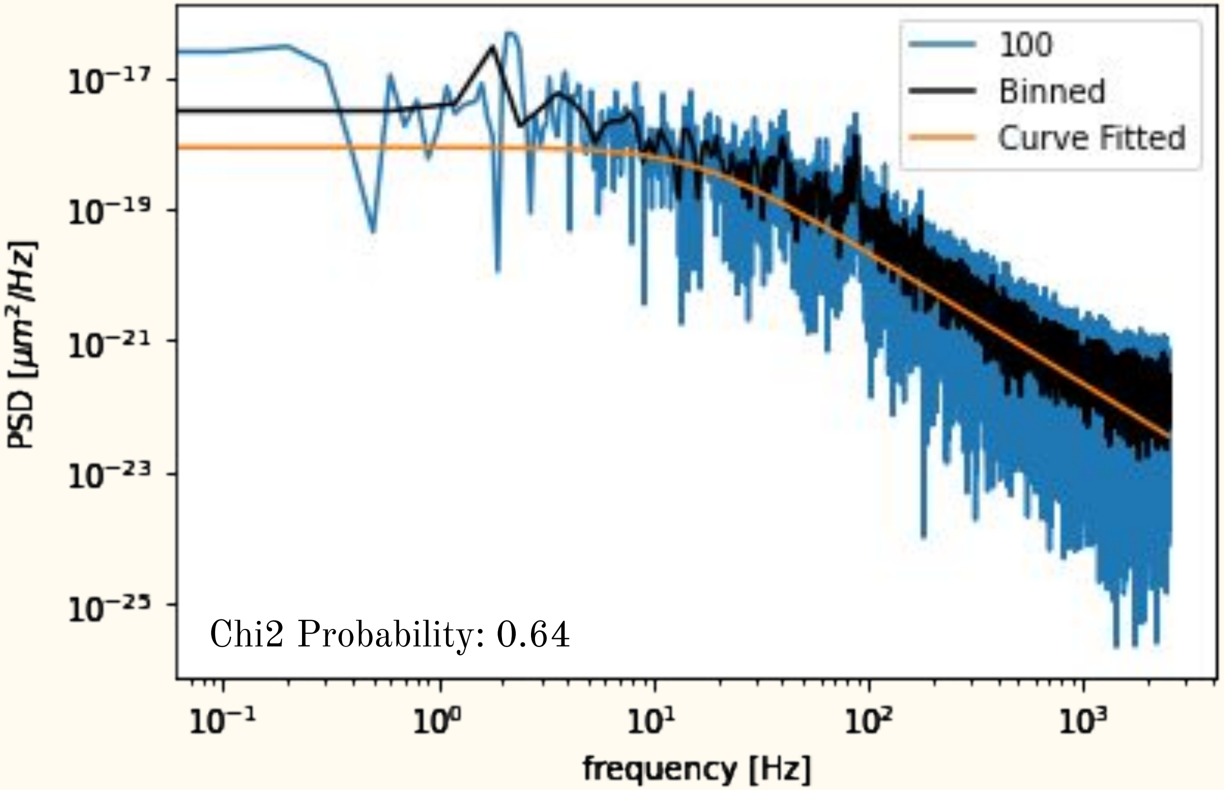
$$S_{xx}(f) = \frac{A}{f^2 + f_0^2}$$

$$\beta = 3\pi\eta d$$

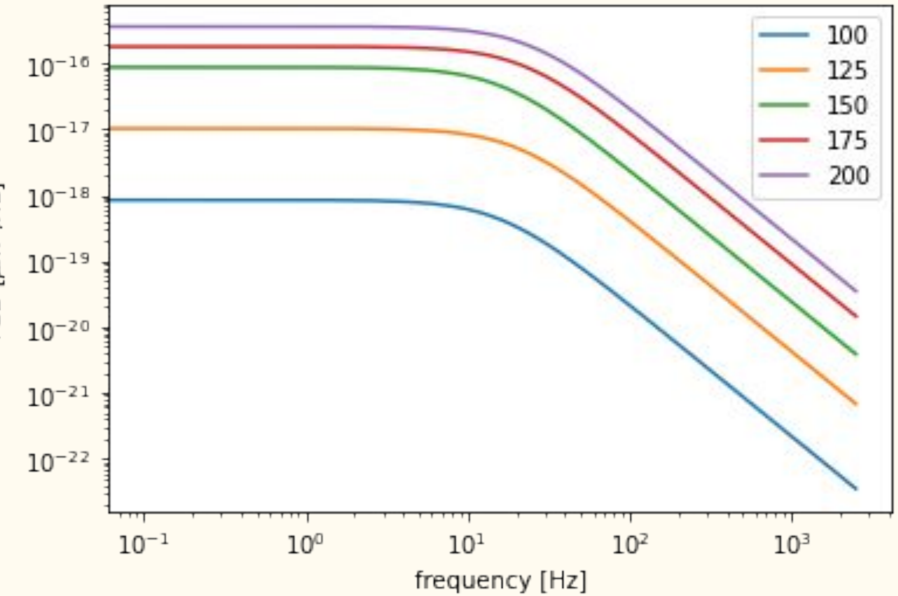
$$S_{xx}(f) = \frac{K_B T}{\pi^2 \beta (f^2 + f_0^2)}$$

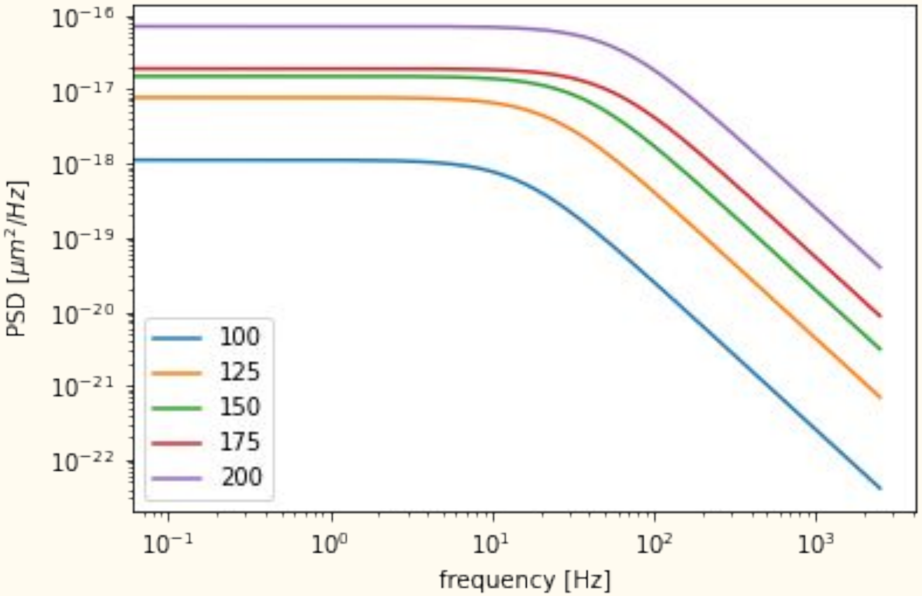
$$f_0 = \frac{\alpha}{2\pi\beta}$$

Power Spectral Distribution Plot



Fitted Power Spectrum Density: QPD X and QPD Y





Putting It All Together

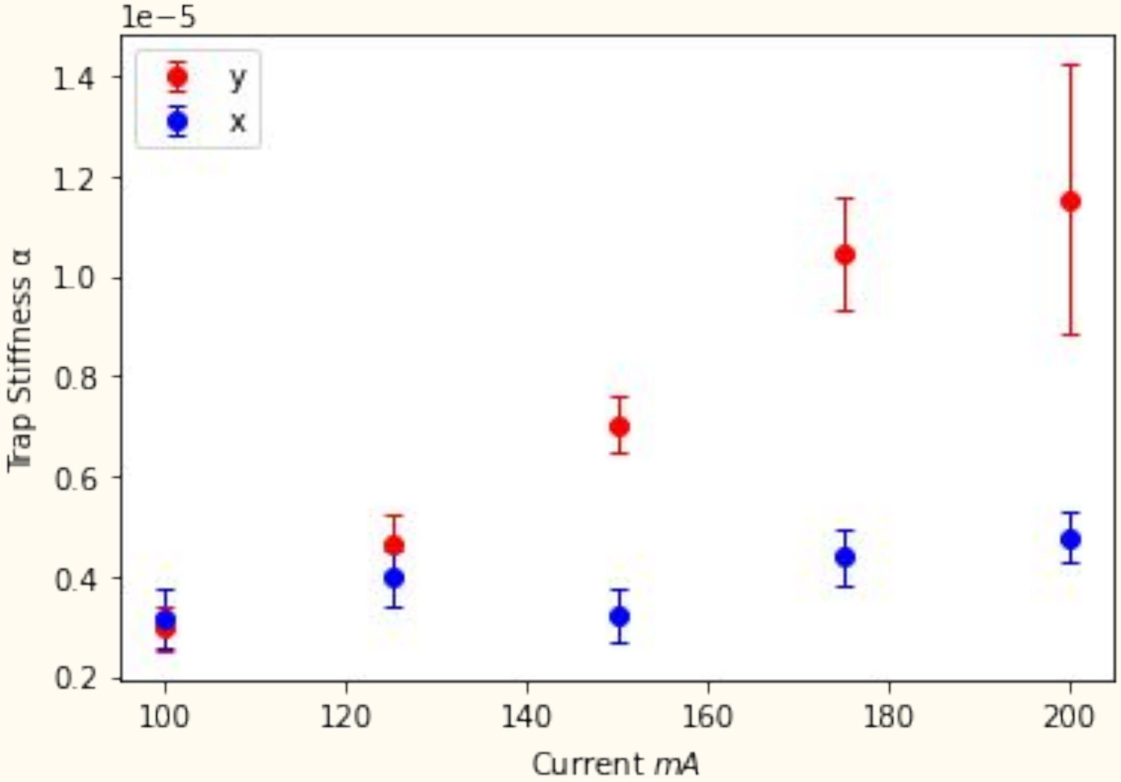
$$S_{xx}(f) = \frac{A}{f^2 + f_0^2}$$

$$\beta = 3\pi\eta d$$

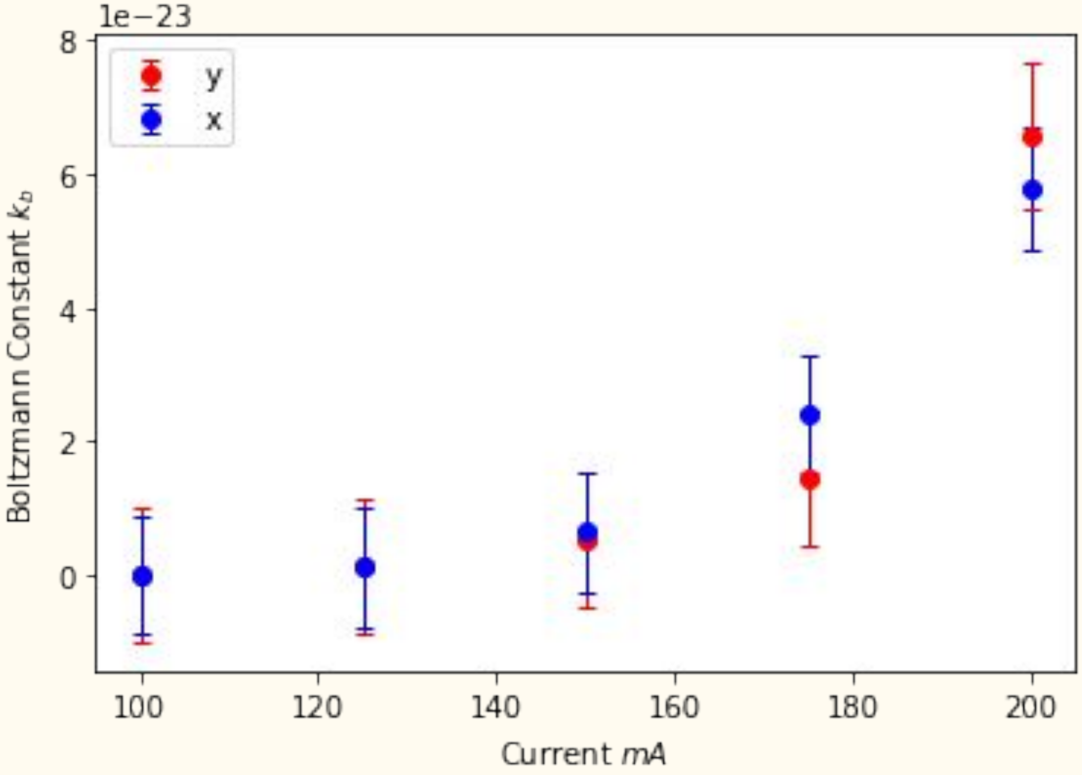
$$S_{xx}(f) = \frac{K_B T}{\pi^2 \beta (f^2 + f_0^2)}$$

$$f_0 = \frac{\alpha}{2\pi\beta}$$

Trap Stiffness for X and Y



Determination of Boltzmann Constant for X and Y



Grand Finale: Boltzmann Constant

$$k_B = QPD\ X\colon 1.79 \times 10^{-23} \pm 1.10 \cdot 10^{-23} \quad \frac{J}{K}$$

$$k_B = QPD\ Y\colon 1.72 \times 10^{-23} \pm 3.94 \cdot 10^{-24} \ \frac{J}{K}$$

Error Discussion

- Heating due to laser $T = 293 \pm 5\,K$ in reality...
- Piezo electronics (strain gauge)
- Lens and coverslip
- Other
- Propagation (statistical)

- 10%
- 5%
- 3%
- 1%

Conclusion

$$k_B = QPD\ X{:}\ 1.79 \times 10^{-23} \pm 1.10 \cdot 10^{-23}\ (stat) \pm 0.34 \times 10^{-23} (sys)\ \frac{J}{K}$$

$$k_B = QPD\ Y{:}\ 1.72 \times 10^{-23} \pm 0.39 \cdot 10^{-23}\ (stat) \pm 0.32 \times 10^{-23} (sys)\ \frac{J}{K}$$

- Calibration but inherent dependence on electronics
- Measure active temperature of water
- No way to quantify interactions between stage, slip, oil, and lens

They were within error of each other different from the true value of $1.38 \times 10^{-23} \frac{J}{K}$

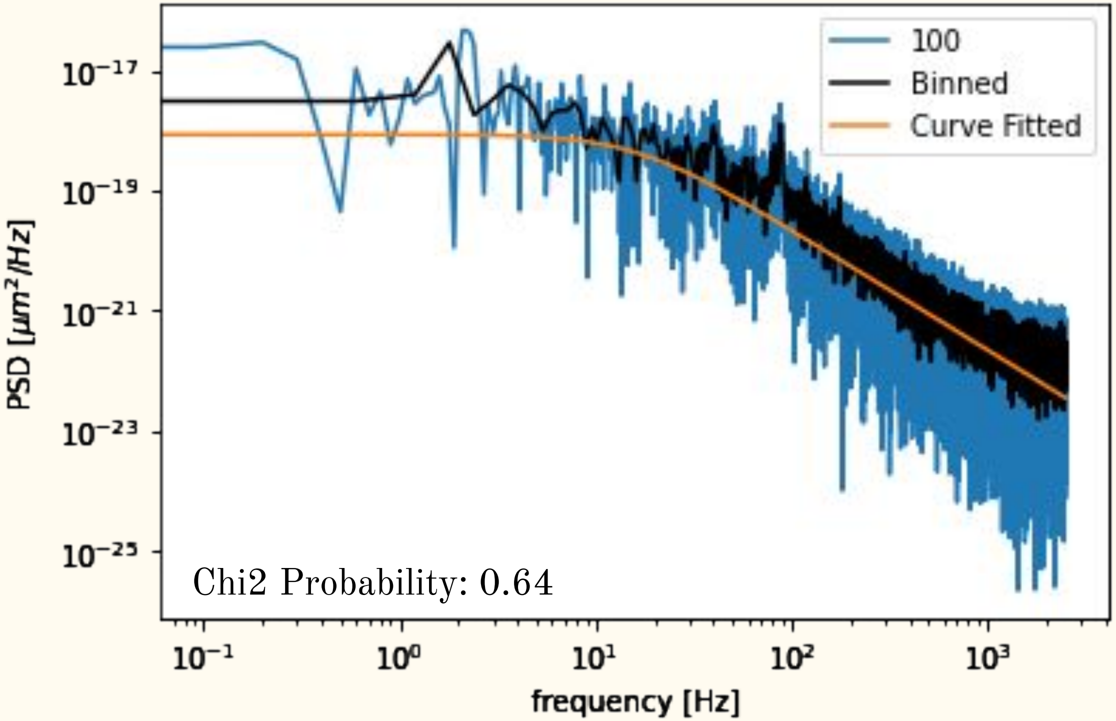
Resources

[1] J. L. Staff, Photoelectric effect lab guide (2018), jLab E-Library, URL http://web.mit.edu/8.13/www/JLExperiments/JLExp51.pdf

[2] UC. B. Staff, Advanced Experimentation Laboratory (2018), University of California, Berkeley, URL http://experimentationlab.berkeley.edu/sites/default/files/writeups/OTZ.pdf

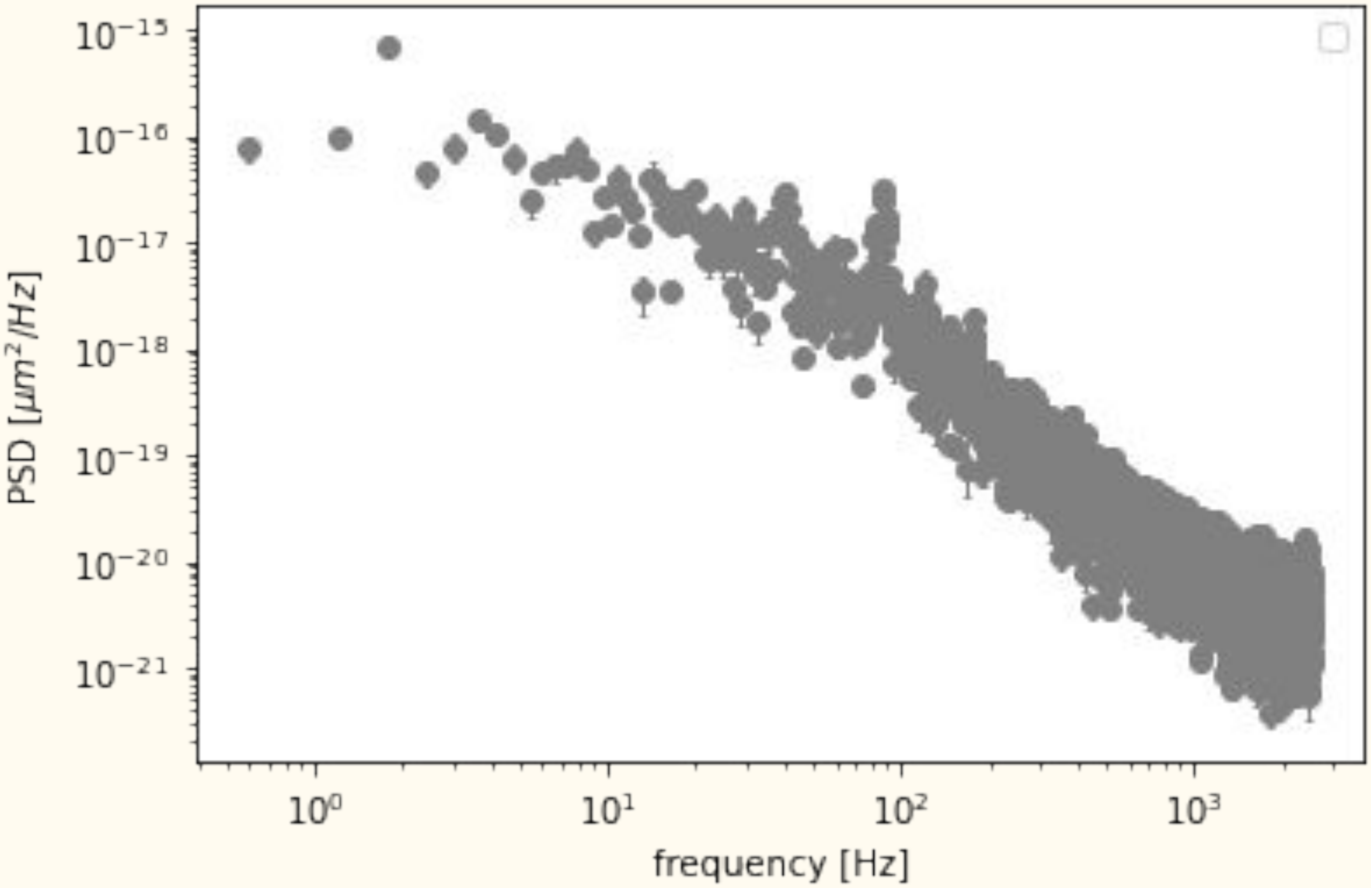
[3] P. Bevington and D. Robinson, Data Reduction and Error Analysis for the Physical Sciences (McGraw-Hill, 2003).

Thank You



Average Chi Square Probability:
0.702 , 0.771

Standard Deviation: 0.261, 0.229



10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
PSD [$\mu m^2/Hz$]
10^0
10^1
10^2
10^3
frequency [Hz]