Утверждение. Пусть $X, Y \in W^{\Phi}$, тогда $X \not\sim_i^{\Phi} Y \Rightarrow \vdash \underline{X} \to K_i\neg\underline{Y}$

1	$X \not\sim_i^{\Phi} Y$	$\triangleright \vdash \underline{X} \to K_i\neg\underline{Y}$
2	$\exists\theta \in \Phi : K_i\theta \in X, \theta \notin Y$ или $K_i\theta \in Y, \theta \notin X$	
3	$K_i\theta \in X, \theta \notin Y$	$\triangleright \vdash \underline{X} \to K_i\neg\underline{Y}$
4	$\neg\theta \in Y$	экономное отрицание?
5	$Y \vdash \neg\theta$	
6	$\vdash \underline{Y} \to \neg\theta$	
7	$\vdash \theta \to \neg\underline{Y}$	
8	$\vdash K_i\theta \to K_i\neg\underline{Y}$	
9	$\vdash \underline{X} \to K_i\theta$	
10	$\vdash \underline{X} \to K_i\neg\underline{Y}$	
11	$K_i\theta \in Y, \theta \notin X$	$\triangleright \vdash \underline{X} \to K_i\neg\underline{Y}$
12	$\neg\theta \in X$	
13	$\vdash \underline{X} \to \neg\theta$	
14	$\vdash \theta \to \neg\underline{X}$	
15	$\vdash K_i\theta \to K_i\neg\underline{X}$	
16	$\vdash \underline{Y} \to K_i\theta$	
17	$\vdash \underline{Y} \to K_i\neg\underline{X}$	
18	$\vdash \hat{K}_i\underline{X} \to \neg\underline{Y}$	
19	$\vdash K_i\hat{K}_i\underline{X} \to K_i\neg\underline{Y}$	
20	$\vdash \underline{X} \to K_i\hat{K}_i\underline{X}$	
21	$\vdash \underline{X} \to K_i\neg\underline{Y}$	
22	$\vdash \underline{X} \to K_i\neg\underline{Y}$	

Следствие. Пусть $X, Y \in W^{\Phi}$, тогда $\underline{X}, \hat{K}_i \underline{Y} \nvdash \bot \Rightarrow X \sim_i^{\Phi} Y$

$\varphi' = K_i\varphi \ (\Rightarrow)$

1	$K_i\varphi \in X$	$\triangleright\ M^\Phi, X \models K_i\varphi \Leftrightarrow \triangleright\ \forall Y(X \sim_i^\Phi Y \Rightarrow M^\Phi, Y \models \varphi)$
2	$\boxed{Y}\ X \sim_i^\Phi Y$	$\triangleright\ M^\Phi, Y \models \varphi$
3	$K_i\varphi \in Y$	из 1, 2
4	$\varphi \in Y$	из 3 т.к. $\varphi \in \Phi$ и $\vdash K_i\varphi \to \varphi$
5	$M^\Phi, Y \models \varphi$	из 4 по ПИ
6	$\forall Y(X \sim_i^\Phi Y \Rightarrow M^\Phi, Y \models \varphi)$	
7	$M^\Phi, X \models K_i\varphi$	

$\varphi' = K_i\varphi$ ($\Leftarrow$)

Обозначения: $\boxed{K_iX := \{K_i\psi \mid K_i\psi \in X\}}$, $\boxed{\neg K_iX := \{\neg K_i\psi \mid \neg K_i\psi \in X\}}$

1	$K_i\varphi \notin X$	$\triangleright\ M^\Phi, X \not\models K_i\varphi \Leftrightarrow \triangleright\ \exists Y(X \sim_i^\Phi Y \wedge M^\Phi, Y \not\models \varphi)$
2	$\neg K_i\varphi \in X$	
3	$\vdash \underline{X} \to \neg K_i\varphi$	
4	$\quad y_0 = K_iX \cup \neg K_iX \cup \{\neg\varphi\} \vdash \bot$	$\triangleright$ «$\bot$»
5	$\quad K_iX, \neg K_iX \vdash \varphi$	
6	$\quad \vdash ((K_i\psi_1 \wedge \dots \wedge K_i\psi_n) \wedge (\neg K_i\chi_1 \wedge \dots \wedge \neg K_i\chi_m)) \to \varphi$	
7	$\quad \vdash K_i((K_i\psi_1 \wedge \dots \wedge K_i\psi_n) \wedge (\neg K_i\chi_1 \wedge \dots \wedge \neg K_i\chi_m)) \to K_i\varphi$	
8	$\quad \vdash ((K_iK_i\psi_1 \wedge \dots \wedge K_iK_i\psi_n) \wedge (K_i\neg K_i\chi_1 \wedge \dots \wedge K_i\neg K_i\chi_m)) \to K_i\varphi$	
9	$\quad \vdash ((K_i\psi_1 \wedge \dots \wedge K_i\psi_n) \wedge (\neg K_i\chi_1 \wedge \dots \wedge \neg K_i\chi_m)) \to K_i\varphi$	
10	$\quad \vdash \underline{X} \to ((K_i\psi_1 \wedge \dots \wedge K_i\psi_n) \wedge (\neg K_i\chi_1 \wedge \dots \wedge \neg K_i\chi_m))$	
11	$\quad \vdash \underline{X} \to K_i\varphi$	
12	$\quad$ «$\bot$»	
13	$y_0 = K_iX \cup \neg K_iX \cup \{\neg\varphi\} \not\vdash \bot$	