Economics 202
Core Microeconomics

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## Final Exam Solutions

**1**. (Consumer Theory)

(a) Observe that $p^D \cdot x^A = 7.5 > 7$ is unaffordable, so it won't be chosen. On the other hand, $x^B$ is affordable and nothing is revealed preferred to it, so it could be chosen. Finally, $x^C$ in the interior of the budget set at prices $p^B$ and isn't chosen so $x^B \succeq x^C$ and under local non-satiation $x^B \succ x^C$. So $x^C$ could be chosen if $A$'s preferences are rational but can't be chosen if $A$'s preferences are also locally non-satiated.

(b) The expediture function is:

$$e(p,u) = up_1^{1/4}p_2^{3/4}.$$

The Hicksian demands are:

$$\begin{aligned} h_1(p,u) &= \frac{\partial e}{\partial p_1} = \frac{1}{4}up_1^{-3/4}p_2^{3/4} \\ h_2(p,u) &= \frac{\partial e}{\partial p_2} = \frac{3}{4}up_1^{1/4}p_2^{-1/4} \end{aligned}$$

The equivalent variation is $e(p',u') - e(p,u')$, where $e(p',u') = w$ and $u' = v(p',w) = 10$. Computing this out

$$EV = 10 - 10 \cdot 2^{1/4} = 10 \cdot (1 - 2^{1/4}) < 0.$$

which says that to achieve the same utility before the price chance as one does after one would have needed $10(1 - 2^{1/4})$ fewer dollars.

(c) If $\partial x_1(\mathbf{p};w)/\partial p_2 > 0$ and good 1 is normal, the slutsky equation tells us that $\partial h_1(\mathbf{p};w)/\partial p_2 > 0$. Therefore by Shephards Lemma,

$$\frac{\partial^2 e(p,u)}{\partial p_1 \partial p_2} > 0.$$

(d) Homotheticity implies that $x(p,\alpha w) = \alpha x(p,w)$. So D's Marshallian demands scale up proportionately with $w$, as in the Cobb-Douglas case.

**2**. (Producer Theory)

(a) The optimal assignment is assortative: $\tau^*(j) = j$ for all $j = 1, ..., n$. To see this, consider some other $\tau$ with the property that for some $k < l$, $\tau(l) > \tau(k)$ (any $\tau$ but $\tau^*$ has this property). Now consider swapping managers $\tau(l)$ and $\tau(k)$. This increases output because

$$f(x_{\tau(l)}, y_k) + f(x_{\tau(k)}, y_l) > f(x_{\tau(k)}, y_k) + f(x_{\tau(l)}, y_l).$$

The inequality follows directly from $f_{xy} > 0$, which implies that for any $x' > x$ and $y' > y$:

$$f(x', y') + f(x, y) > f(x', y) + f(x, y').$$

It follows that any $\tau \neq \tau^*$ is strictly suboptimal.

(b) The optimal assignment is still assortative — the above proof relies only on $f_{xy} > 0$.

(c) Any assignment that maximizes total revenue also maximizes

$$\log R(\tau) = \sum_{j=1}^{n} \log f(x_{\tau(j)}, y_j).$$

A sufficient condition for the optimal assignment to be assortative therefore is that $(\log f)_{xy} > 0$, or that $\log f$ is supermodular.

(d) Precisely the same assumptions that allow one to conclude the optimal assignment given a fixed set of managers is assortative, namely $f_{xy} > 0$ in the sum of outputs case and $(\log f)_{xy} > 0$ in the product of outputs case. To see this, note that one chose $x_i > x_{i+1}$, we could assign manager $i$ to team $i+1$ and vice-versa, pay the same total salary and increase revenue!

(e) Let $\Pi(\mathbf{x}, \mathbf{y}) = \mathbf{R}(\mathbf{x}, \mathbf{y}) - \sum_i w(x_i)$. In the sum of outputs case, $\Pi$ has increasing differences in $(x_i, y_i)$ but is separable across teams. Therefore an increase in $y_j$ will increase $x_j$ but will not affect $x_i$ for any $i \neq j$. In the product of outputs case,

$$\frac{\partial \Pi}{\partial x_i} = f_x(x_i, y_i) \prod_{j \neq i} f(x_j, y_j) - w'(x_i)$$

will be increasing in $x_j, y_j$ and $y_i$ if $f_x, f_y, f_{xy} > 0$. So $\Pi$ is supermodular in $\mathbf{x}$ and has increasing differences in every $(x_i, y_j)$ pair. So an increase in $y_1$ will increase the whole vector $x_1, ..., x_n$.

**3.** (Choice Under Uncertainty)

(a) Ann's problem is:

$$\max_{x_1,\ldots,x_n} U(x) = \sum p_i u(w + x_i - r \cdot x)$$

(b) Let $w_i = w + x_i - r \cdot x$ be final wealth if horse $i$ wins. If $x_i > 0$, the first order condition is:

$$\frac{p_i}{r_i} u'(w_i) = \sum_{k=1}^{n} p_k u'(w_k)$$

Suppose that $p_i/r_i > p_j/r_j$, that optimally $x_j > x_i \geq 0$. Then:

$$\begin{aligned} \frac{dU}{dx_i} &= p_i u'(w_i) - r_i \sum_{k=1}^{n} p_k u'(w_k) \\ &= p_i u'(w_i) - r_i \frac{p_j}{r_j} u'(w_j) > 0 \end{aligned}$$

contradicting the assertion of optimality.

(c) Rank the horses to that $p_1/r_1 \geq \ldots \geq p_n/r_n$. From above, we know that optimally, $x_1 \geq \ldots \geq x_n$. Let $i$ be the least number such that $p_i/r_i > 1$ for some $i$ but $x_i = 0$. If there is no such $i$, we're done. If there is, then for all $k < i$, $x_k > 0$ and hence $w_k > w_i$, while for all $k > i$, $x_k = 0$ so $w_k = w_i$.

$$\frac{dU}{dx_i} \overset{sgn}{=} \frac{p_i}{r_i} - \sum_{k=1}^{n} p_k \frac{u'(w_k)}{u'(w_i)} > 0$$

To see this, observe that the first term is strictly greater than 1 while the second term, because each $u'(w_k)/u'(w_i)$ is weakly less than 1 is also weakly less than 1 (because $\sum p_k = 1$.

(d) If $p_1/r_1 = p_2/r_2$, there is no investment regardless of $w$, so assume that $p_1/r_1 > p_2/r_2$. It is easy to see from (b) and (c) that for all $w$, $x_1(w) > 0 = x_2(w)$. The claim is that $x_1(w)$ will be increasing in $w$ under DARA preferences. To see this, observe that the first order condition for $x_1$ can be written as:

$$\frac{p_1}{r_1} = p_1 + p_2 \frac{u'(w - r_1 x_1)}{u'(w + x_1 - r_1 x_1)}.$$

The last term is always increasing in $x_1$. Under DARA it is decreasing in $w$. So $x_1(w)$ will be increasing in $w$.

**4.** (General Equilibrium).

(a) Marshallian demands are:

$$\begin{aligned} x^1 &= \frac{\alpha p_y}{p_x} \qquad y^1 = 1-\alpha \\ x^2 &= \beta \qquad y^2 = \frac{(1-\beta)p_x}{p_y} \end{aligned}$$

(b) Market clearing for good $x$ requires:

$$x^1 + x^2 = \alpha p_y + \beta = 1 = e^1 + e^2.$$

So $p_y = \frac{1-\beta}{\alpha}$. Mr 1 consumers $(1-\beta, 1-\alpha)$ and Mr 2 consumes $(\beta, \alpha)$.

For parts (c) and (d), note that because the firm has a linear production function, it will not operate if $p_x > p_y$, will operate an infinite amount if $p_x < p_y$, and will also not operate if $p_x = p_y$. The last, which contrasts the the in-class activity analysis model, is because of the first cost. With $p_x = p_y$ the firm's profits are $-F$ if it operates at any positive scale. All of this means that if there is a Walrasian equilibrium, it must involve no production and hence requires $p_x \geq p_y$. The question then is whether there is an exchange economy equilibrium with $p_x \geq p_y$.

(c) If $\alpha = \frac{2}{5}$ and $\beta = \frac{4}{5}$, the exchange economy equilibrium prices are $p_x = 1$ and $p_y = 1/2$. So there is a production equilibrium at these prices where the firm does not produce.

(d) If $\alpha = \frac{1}{5}$ and $\beta = \frac{3}{5}$, the only way to clear markets without the exchange economy is with $p_x = 1$ and $p_y = 2$. But this isn't a WE because the firm would want to produce an infinite amount. So there is no WE.