Final Exam
Econ 202
Stanford University
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Instructions:

- You have three hours to complete the exam – there are 4 questions.
- Answer each question in a separate bluebook.
- On the cover of each blue book write the question number and your name.
- The exam is closed book and closed notes.
- Good luck !

Questions:

1. (Revealed Preference, 25 points). Consider a two-good economy, and a consumer with complete, transitive, continuous and locally non-satiated preferences. Prices in this economy are always strictly positive. Suppose the consumer has wealth $w = 6$. You know that at prices $(p_1, p_2) = (1, 1)$, the consumer's unique preferred bundle is $x = (4, 2)$. You then observe this consumer selecting the bundle $y = (2, 3)$, but do not observe the prices at which y was chosen. (You may find it useful to argue your answers graphically, using a clearly labeled and clearly explained picture.)

    (a) What can you conclude about the prices the consumer faced in choosing y?

    (b) Suppose you learn the price of good one was $p_1 = 2$. What can you say about the set of bundles to which $x = (4, 2)$ is strictly preferred?

    (c) Suppose the consumer's preferences are also strictly monotone and convex. From the information you have, what can you conclude about the set of bundles that are preferred to y?

2. (Producer Theory, 25 points). Consider a firm that chooses two inputs, k and l, to minimize costs under production contracts that specify output level x. If the effectiveness of labor is parameterized by θ, then the cost function is:

$$C(x, r, w, \theta) = \min_{k,l} rk + wl \text{ subject to } f(k, \theta l) = x.$$

In the long run, the firm may also choose to expand output, choosing x. In that case, it solves

$$\pi(p, r, w, \theta) = \max_{x} px - C(x, r, w, \theta) = \max_{k,l} pf(k, \theta l) - rk - wl.$$

Assume that f is differentiable and that the partial derivatives f_l and f_k are positive. You many find it helpful in some of the questions below to use the *isoquant functions* $L(x, k)$ and $K(x, l)$ defined by $x = f(k, L(x, k))$ and $x = f(K(x, l), l)$. Notice that $\partial L/\partial k = -f_k/f_l$ and $\partial K/\partial l = -f_l/f_k$.

(a) Determine sufficient conditions on f, L or K under which an increase in θ (weakly) increases/decreases the short-run demand for capital.

(b) Does your preceding answer question apply if capital is "lumpy"–that is, if k is constrained to be an integer?

(c) Show that l is non-decreasing in θ if $1 + \partial \ln[f_l(k, l)/f_k(k, l)]/\partial \ln l \geq 0$ everywhere and that l is non-increasing if the reverse inequality holds everywhere. Do not assume that the problem is convex.

(d) Suppose that $f_{k,l} \geq 0$. Suppose that labor is fixed in the short-run, perhaps due to training constraints and labor contracts. Given the condition in the preceding part of this problem, compare the long-run and short-run changes in the demand for capital as θ increases.

3. (Choice under Uncertainty, 25 points). Consider a choice problem under uncertainty. The prize space is is some interval $[a, b] \subset I\!R$. Suppose an agent has preferences over lotteries which satisfy the von Neumann-Morgenstern axioms.
For a cumulative distribution function (cdf) F let $U(F) = \int_a^b u(x)dF(x)$ be one utility representation of these preferences and let $u^{(i)}(x)$ denote the i'th derivative of u at x (if u is at least i times differentiable). Define $\mathcal{U}^n$ as the set of all von Neumann Morgenstern utility functions for which u is n times continuously differentiable in the interior of $[a, b]$ and for which

$$u^{(i)} \cdot (-1)^{i+1} \geq 0 \text{ for all } i = 1, ..., n.$$

We say a cdf F n'th order stochastically dominates a cdf G if

$$\int_a^b u(x)dF(x) \geq \int_a^b u(x)dG(x) \text{ for all } u \in \mathcal{U}^n.$$

Note that for $n = 1$ and $n = 2$ this gives the definition of first and second order stochastic dominance from class.

(a) Suppose $a = 0, b = 10$ and consider the following simple probability distributions over $[0, 10]$:

$$F(x) = \begin{cases} 0 & x < 3 \\ 1/3 & x \in [3,5) \\ 1/2 & x \in [5,7) \\ 3/4 & x \in [7,9) \\ 1 & x \geq 9 \end{cases}, \quad G(x) = \begin{cases} 0 & x < 3 \\ 1/6 & x \in [3,6) \\ 1/2 & x \in [6,8) \\ 3/4 & x \in [8,9) \\ 1 & x \geq 9 \end{cases}, \quad H(x) = \begin{cases} 0 & x < 5 \\ 1/3 & x \in [5,6) \\ 1/2 & x \in [6,8) \\ 3/4 & x \in [8,9) \\ 1 & x \geq 9 \end{cases}$$

Can you order these lotteries according to 1st order stochastic dominance ?
Can you order these lotteries according to 2nd order stochastic dominance ?
How would your answer change for 5th order stochastic dominance ?

(b) Prove that for all n, F does not n'th order stochastically dominate G if there exists a $x \in [a, b]$ such that $F(x) > 0$ and $G(x) = 0$.

Hint: Let y be the smallest value at which $G(y) \geq F(y)$ and think about how the Bernoulli function could look like on the interval $[a, y]$

4. (General equilibrium, 25 points). Suppose there are 3 commodities and 3 agents. Suppose that agents' utility functions are separable and invariant across commodities – i.e. they can be written as

$$U^h(c) = v^h(c_1) + v^h(c_2) + v^h(c_3) \text{ for } h = 1, 2, 3.$$

Suppose each v^h is differentiable, strictly increasing and strictly concave.

(a) Suppose that individual endowments $e^h \in I\!R^3_+$ for $h = 1, 2, 3$. Does a Walrasian equilibrium always exist ? Explain in one sentence.

(b) Suppose now that

$$\sum_{h=1}^{3} e_1^h > \sum_{h=1}^{3} e_2^h > \sum_{h=1}^{3} e_3^h$$

Show that for any Walrasian equilibrium we must have $p_1 < p_2 < p_3$.

(c) Suppose now that $v^h(c) = \ln(c)$ for all $h = 1, 2, 3$. Compare two economies with the same aggregate endowments (but possibly different individual endowments). Prove that equilibrium prices have to be the same (up to a normalization) for these two economies.

Is this going to be true whenever preferences are identical or can equilibrium prices change with the distribution of endowments ?