Econ 202
Final Exam

Professors T. Bresnahan and I. Segal

December 10, 2007

You have three hours to complete this exam. You are not allowed to use notes, books, calculators, etc. Please solve each problem in a separate blue book. Good luck!

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Professor L. Stein
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1 Subsistence (30 points)

This question concerns a consumer who has wealth W and who consumes in two time periods. Consumption in each time period t is given by c_t and the consumer's budget constraint is $W \geq p_1 c_1 + p_2 c_2$. You observe the following demand system for the consumer depending on her wealth (your econometricians have estimated $\gamma > 0$ and α for you.) First, if $W < \gamma(p_1 + p_2)$, the consumer sets $c_2 = 0$. Second, if $W \geq \gamma(p_1 + p_2)$ the consumer sets $c_2 = \gamma + \frac{\alpha}{p_2}[W - \gamma(p_1 + p_2)]$

(a) Under the assumption that this consumer is locally non-satiated, what is her demand for c_1? It will be helpful to distinguish between two ranges of wealth. (HINT: Making c_1 look similar to c_2 should improve your intuition for this problem.)

(b) The boundary case in our problem is $W = \gamma(p_1 + p_2)$. What is the most straightforward interpretation of this wealth level?

(c) If this consumer always had $W < \gamma(p_1 + p_2)$, is her behavior in setting consumption in both time periods consistent with utility maximization? Prove either that it is or is not. If it is, give a utility function that rationalizes it when it is always the case that $W < \gamma(p_1 + p_2)$.

(d) If this consumer always had $W \geq \gamma(p_1 + p_2)$, is her behavior in setting consumption in both time periods consistent with utility maximization? Prove either that it is or is not. If it is, give a utility function that rationalizes it when it is always the case that $W \geq \gamma(p_1 + p_2)$. It may be helpful to restrict the domain of your utility function.

(e) Let W be unrestricted. Can the consumer's behavior be rationalized by a utility function $u(c_1, c_2)$ with the property that $\forall \delta > 0$, $\exists x(\delta) > 0$ such that there is an indifference curve that runs through both $(\gamma + \delta, \gamma + \delta)$ and $(x(\delta), 0)$?

(f) Write a utility function, not necessarily continuous, which rationalizes the consumer's behavior, or explain why this cannot be done.

2 Risky assets (30 points)

A risk-averse von Neumann-Morgenstern decision maker with a three-times differentiable Bernoulli utility function u and a positive initial wealth w chooses the **fraction** α of his wealth to invest in an asset with a non-negative, but risky, return x, keeping the rest in a riskless asset with return r. The risky return x is distributed according to the cdf $G(x)$.

(a) Write the decision-maker's problem.

(b) Let $F(x)$ be another possible cumulative distribution of the return. Suppose we are interested in whether α would be larger in the world where the asset's return is distributed according to F or in the world where the asset's return is distributed according to G. Give a condition on how the objective functions in the two worlds relate to each other that is both global in α and sufficient to ensure that the optimum α is non-decreasing as the distribution jumps from G to F.

(c) Say that $F \geq_{FOSD} G$. Give a sufficient condition on the decision-maker's coefficient of relative risk aversion that ensures that the condition in part (b) holds, thereby providing a sufficient condition for α to be weakly greater when the distribution of returns jumps to a new distribution that first-order stochastically dominates the old one.

(d) Say that $F \geq_{SOSD} G$. Give a sufficient condition on the decision-maker's coefficient of relative prudence $\left(P(x) = -x\frac{u'''(x)}{u''(x)}\right)$ that ensures that the condition in part (b) holds, thereby providing a sufficient condition for α to be weakly greater when the distribution of returns jumps to a new distribution that second-order stochastically dominates the old one.

3 Cars, externality, and general equilibrium (40 points)

A consumer is endowed with w units of money, which he can trade for two types of goods: vehicles and gasoline. The price of money is normalized to 1. The price of gasoline is γ per gallon. Suppose she can choose between two types of vehicles: a scooter (s) and a car (c). For $i \in \{s, c\}$,

- The price to purchase a type i vehicle is p_i; suppose $p_s \leq p_c < w$.
- A type i vehicle can be driven θ_i miles per gallon of gasoline used; suppose $\theta_s \geq \theta_c > 0$.
- The probability of an accident for a driver of a type i vehicle is π_i; suppose $\pi_s \geq \pi_c$.
- Driving a type i vehicle for x miles gives (Bernoulli) utility $u_i(x)$ if there is no accident, and utility 0 if there is an accident. Note there is zero utility from consuming money, and no reason to purchase zero or more than one vehicle.
  - u_i is strictly increasing, and concave, with $u_i(0) = 0$.
  - $u_s'(x) \leq u_c'(x)$ for all x.

(a) Write the consumer's optimization problem.

(b) Show that s and c are gross substitutes.

(c) Show that if $\theta_s = \theta_c$ (i.e., the vehicles are equally fuel efficient), c is a normal good.

From now on, suppose the following functional forms and calibrations:

- $\pi_s = \frac{1}{2}$.
- $u_s(x) = x^{1/3}$, and $u_c(x) = x^{1/2}$.
- $\theta_s = 40$, $\theta_c = 5$; $w = 1000$.
- Gasoline is produced by firms with a linear (CRS) technology that can turn 1 unit of money into a gallon of gasoline.
- Scooters are produced by firms with a linear (CRS) technology that can turn 800 units of money into a scooter.
- Cars are produced by firms with a linear (CRS) technology that can turn 955 units of money into a car.

(d) Suppose all firms price at marginal cost. For what values of π_c will the consumer prefer a scooter? For what values will she prefer a car?

Now suppose that there are N identical consumers, where N is extremely large. Each consumer chooses between s and c; let N_s be the number who choose s, and N_c the number who choose c with $N = N_s + N_c$.

(e) Suppose that at Walrasian equilibrium $N_s > 0$ and $N_c > 0$. What must the prices γ, p_s, and p_c be at Walrasian Equilibrium? Explain why. (This question should not require any math.)

Finally, suppose that the more consumers who drive cars, the more dangerous cars become: in particular, $\pi_c = \frac{1}{2}(N_c/N)$. As above, $\pi_s = \frac{1}{2}$.

(f) At equilibrium, what fraction of consumers choose s? What fraction choose c?

(g) Consider a social planner who aims to maximize the utility of the average consumer (i.e., who maximizes a Samuelson-Bergson social welfare function with equal weights for all consumers). Assume the planner can only tell each consumer what kind of car she must buy, but cannot reallocate wealth (or gasoline) across consumers. At this planner's social optimum, what fraction of consumers choose s? What fraction choose c?

(h) Show that the outcome you describe in (g) Pareto dominates the outcome in (f). Very briefly, why do the answers to the last two questions differ? That is, why does the First Welfare Theorem appear to fail?

(i) Suppose the government wanted to encourage fewer people to drive big cars *in equilibrium* (i.e., without forcing them). In particular, suppose it wants there to be the same fraction of car drivers as you found in part (g). What level of taxes on gasoline prices achieve this goal? You may assume that the government throws away tax revenue.

(j) Do you think gasoline taxes are a good instrument for realizing this policy goal? Why or why not? Can you suggest anything better?