Economics 202 Paul Milgrom
Core Microeconomics Jonathan Levin

Final Exam

Instructions: You have three hours to complete the exam – there are 4 questions. Please answer each question in a separate bluebook. Write the question number and your name on the cover of each blue book. The exam is closed book and closed notes. Good luck !

1. (Consumer Theory) Consider a two-good economy, and a consumer with complete, transitive, continuous and strictly monotone preferences. Prices in this economy are always positive. Suppose the consumer has wealth 4. We make two observations of consumer choices:

   - At prices (1, 2) the choice is $(3, \frac{1}{2})$.
   - At prices (1, 1) the choice is (1, 3).

   Which of the following observations would be consistent with utility maximization?

   (a) Choice (6, 1) at price $(\frac{1}{2}, 1)$
   (b) Choice $(1, \frac{1}{2})$ at price (2, 2)
   (c) Choice (2, 1) at price (1.6, 0.8)

   Suppose that after making observations 1 and 2, we observe this consumer choosing consumption bundle (2, 1).

   (d) What can we conclude about the prices the consumer faced choosing (2, 1)?

2. (Producer Theory) A certain wholesaler acquires products from a variety of sources, incurring a total cost of $C(x)$ to acquire input vector $x \in \mathbb{R}^3$. It sells its output at price vector p, and its maximum net profit is $\pi(p) = \max_x p \cdot x - C(x)$. Let $x^*(p)$ denote the wholesaler's profit-maximizing choice.

   (a) In what way does the profit function defined above differ from the standard profit function of producer theory? Must such a profit function be homogeneous of degree one?

   (b) Is it possible that, in some open neighborhood of some price vector p^*, $\pi(p) = \sum_{i=1}^{3} p_i - \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} p_i p_j$? Explain.

   (c) Suppose that for some vector $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in \mathbb{R}^3_+$, the maximum profit function satisfies the equation: $\pi(p) = \sum_{i=1}^{3} \alpha_i \max(0, p_i - 2p_i^{0.5})$. Suppose that $p = (1, 9, 3)$. How many units of good 2 does the wholesaler sell, that is, what is $x_2^*(p)$?

   For the next two parts, suppose that C is twice continuously differentiable, that all relevant optima involve positive quantities, and that $C_{ij}(x) < 0$ for all $i \neq j$. Do not assume that π takes the form given in (a) or (b).

   (d) If the price of good 1 increases from p_1' to $p_1'' > p_1'$ with other prices unchanged (that is, $p_2'' = p_2'$ and $p_3'' = p_3'$), what can you conclude about how the price increase alters the wholesaler's demand for each good? Explain.

   (e) Suppose the quantity demanded of good 3 is limited, so that the wholesaler's profit maximization problem is changed to $\max_x p \cdot x - C(x)$ subject to $x_3 \leq x_3^*(p')$. Denote the solution of this constrained problem by $\overline{x}(p; p')$ (which we abbreviate as $\overline{x}(p)$). Compare the four quantities: $x^*(p')$, $x^*(p'')$, $\overline{x}(p')$, and $\overline{x}(p'')$.

3. (Choice Under Uncertainty) Consider an agent with vN-M utility $U(L) = \sum u(z)\Pr(z)$, where $u : \mathbb{R} \to \mathbb{R}$ is increasing. Suppose the agent has a single dollar and three investment options. For each dollar invested, the returns on these investments are:

$$\begin{aligned} L_1 &= 2 \text{ for sure} \\ L_2 &= x \text{ w/ Prob. } p \text{ and } 1 \text{ w/ Prob. } 1-p, \text{ where } x > 1 \\ L_3 &= y \text{ w/ Prob. } q \text{ and } 0 \text{ w/ Prob. } 1-q, \text{ where } y > 0. \end{aligned}$$

Let $U(L_i)$ denote the expected utility from investing the entire dollar in L_i.

(a) Find all values of q, y that guarantee $U(L_2) \geq U(L_3)$. (NB: hold p and x fixed).

(b) Assuming u is also concave, find all values of p, x that guarantee $U(L_1) \geq U(L_2)$.

Now suppose the agent can spread his wealth across two lotteries, by investing α in L_i and $1-\alpha$ in L_j, where α can be chosen optimally from $[0,1]$. Let $p = q = 1/2$, $x = 4$ and $y = 7$.

(c) Assume u is concave as well as increasing. Show that the agent might prefer investing everything in L_2 to investing everything in L_3, but he necessarily prefers an optimal mix of L_1 and L_3 to an optimal mix of L_1 and L_2. Explain this result.

Finally consider a second agent with vN-M utility $V(L) = \sum v(z)\Pr(z)$, where $v(z) = u(z) + h(z)$, where $h : \mathbb{R} \to \mathbb{R}$ is increasing and convex. Again let $p = q = 1/2$, $x = 4$ and $y = 7$, and now assume $U(L_3) \geq U(L_2) \geq U(L_1)$.

(d) Compare $V(L_1)$, $V(L_2)$ and $V(L_3)$. Justify your comparison.

4. (General Equilibrium) Consider a pure exchange economy with two agents and two commodities. Assume that utility functions u^1 and u^2 are strictly increasing, strictly concave and continuously differentiable. In the first setting, endowments are e^1 and e^2, and there is a Walrasian equilibrium (p, x^1, x^2). In the second setting, endowments are $\overline{e}^1$ and $\overline{e}^2$, and there is a Walrasian equilibrium $(\overline{p}, \overline{x}^1, \overline{x}^2)$. Assume that all endowments are strictly positive, that total endowment is the same in both settings (i.e. $e^1 + e^2 = \overline{e}^1 + \overline{e}^2$), and that agent 1's endowment is strictly greater in the second setting (i.e. $\overline{e}^1 \gg e^1$). (You may find it useful to argue your answers graphically, using a clearly labeled and clearly explained picture).

(a) Is it possible that in the second setting, agent 1 is worse off and agent 2 is better off than in the first setting? In other words, is it possible that $u^1(\overline{x}^1) < u^1(x^1)$ and $u^2(\overline{x}^2) > u^2(x^2)$?

(b) Is it possible that both agents are worse off in the second setting? In other words, is it possible that $u^1(\overline{x}^1) < u^1(x^1)$ and $u^2(\overline{x}^2) < u^2(x^2)$?

(c) Suppose also that the Walrasian equilibrium is unique in the second setting. Is it possible that in the second setting, agent 1 is worse off and agent 2 is better off than in the first setting?

(d) Suppose also that $u^1(c^1, c^2) = u^2(c^1, c^2) = \log c_1 + \log c_2$. Prove that the Walrasian equilibrium is unique for all endowment levels.