Econ202 Final Exam 2009
Solutions

December 13, 2009

1.

(a) Budget constraints are binding each period so they are 200 for period 1 and $120 + 8t_2$ for period 2.

(b) $x_1 \succ_p x_2$ if $200 > 120 + 10t_2$, so $t_2 < 8$.

(c) $x_2 \succ_p x_1$ if $120 + 8t_2 > 10 \times 10 + 10 \times 8$, so $t_2 > 7.5$.

(d) It is not rationalizable iff $x_1 \succ_p x_2$ and $x_2 \succ_p x_1$, so for $t_2 \in (7.5, 8)$. Otherwise it's rationalizable.

(e) We will think of cost minimization problem. I decompose the changes in consumption into the substitution and utility effects (wealth effect should have the same sign as utility effect by local nonsatiation).

He is better off in Year 1 if $t_2 < 8$. The substitution effect from Year 2 to Year 1 works in favor on Saxophones since their relative price goes down. Therefore the substitution effect makes you buy more Saxophones. However he actually buys less Saxophones in Year 1, so the good must be inferior (negative utility effects). The answer is $t_2 \leq 7.5$ (they need to be rationalizable).

(f) Suppose he is better off in Year 2 ($t_2 > 8$) and he buys less Telephones if $t_2 \leq 10$. Telephones are strictly relatively cheaper in Year 2 so the substitution effect makes him buy more Telephones. However he buys less, so it must be that they are inferior. The answer is $t_2 \in (8, 10]$.

2.

(a) $U(X) = \mathbb{E}[X^2] = \mathbb{E}[u(X)]$, where $u(x) = x^2$. So U can be represented with von Neumann-Morgerstern utility function.

$U(X) = (\mathbb{E}[X])^2$, respresents the same preferences over lotteries as $U'(X) = \mathbb{E}[X]$. Therefore we can conclude that the preferences are consistant with von Neumann-Morgerstern utility theory.

$U(X) = \mathbb{E}[X] - \mathrm{Var}(X)$ is inconsistent. There are several ways to see it. For example one may show that the indifference curves are non-linear. Lets suppose that X has just two outcomes that come with probability p_1 and p_2. Then

$$U(X) = p_1x_1 + p_2x_2 - p_1x_1^2 - p_2x_2^2 + (p_1x_1 + p_2x_2)^2 =$$

$$= p_1x_1 + p_2x_2 - p_1x_1^2 - p_2x_2^2 + p_1^2x_1^2 + 2p_1p_2x_1x_2 + p_2^2x_2^2 = \bar{U}$$

One can already see that indifference curve $p_1(p_2)$ is non-linear. I will confirm it using implicit function theorem.

$$\frac{\partial U(X)}{\partial p_1} = x_1 - x_1^2 + x_1^2 + 2p_2x_1x_2$$

$$\frac{\partial U(X)}{\partial p_2} = x_2 - x_2^2 + x_2^2 + 2p_1x_1x_2$$

$$\frac{dp_1}{dp_2} = -\frac{x_2 - x_2^2 + x_2^2 + 2p_1x_1x_2}{x_1 - x_1^2 + x_1^2 + 2p_2x_1x_2}$$

Which is not constant in p_2. Because the indifference curve is non-linear the preferences are not consistant with von Neumann-Morgerstern utility theory.

(b) First is risk lowing. Since $u(x) = x^2$ is convex we know by Jensens inequality that $U(X) = \mathbb{E}[X^2] \geq (\mathbb{E}X)^2 = U(\mathbb{E}[X])$
Second is risk neutral since $U(X) = (\mathbb{E}X)^2 = U(\mathbb{E}[X])$
Last is risk averse, because $U(X) = \mathbb{E}[X] - \mathrm{Var}(X) \leq \mathbb{E}[X] = U(\mathbb{E}[X])$

(c) For the first two one-can just compute the absolute risk aversion parameters

$$\lambda_1 = -\frac{u_1''}{u_1'} = -2x$$

$$\lambda_2 = -\frac{u_2''}{u_2'} = 0$$

For the last one we need to use the definition, and check the wealth effects.

$$U(X+w) = \mathbb{E}[X+w] - \mathrm{Var}(X+w) = \mathbb{E}[X] + w - \mathrm{Var}(X) = U(X) + w$$

so consumers risk premium does not change with wealth (constant absolute risk aversion).

3.

(a) Maximize the weighted sum of utilities (necessary and sufficient for PO because U is strictly increasing)

$$\max_{x_s^A, x_s^B} \lambda \left[p_1u(x_1^A) + p_2u(x_2^A)\right] + (1-\lambda)\left[pu(x_1^B) + p_2u(x_2^B)\right]$$

$$\text{s.t.} x_s^A + x_s^B = e_s$$

FOCs are given by

$$\lambda p_s u'(x_s^A) = \sigma_s$$
$$(1-\lambda)p_s u'(x_s^B) = \sigma_s$$

(b) Dividing FOCs we get that

$$\frac{\lambda}{1-\lambda}\left(\frac{x_s^A}{x_s^B}\right)^{-\rho} = 1$$

That gives $x_s^A = Kx_s^B$, and K is the same for all s

(c) We know FWE holds so WE is PO We have that $x_s^A = Kx_s^B$, and by adding up we get $K = \frac{e^A}{e^B}$. Denote prices by q_s. From FOC are

$$\lambda^i p_s = q_s (x_s^i)^\rho$$

Dividing we get that

$$\frac{x_2^i}{x_1^i} = \sqrt[\rho]{\frac{q_2}{q_1}\frac{p_1}{p_2}}$$

This ratio does not depend on consumer so it must be that

$$\frac{x_2^i}{x_1^i} = \frac{e_2}{e_1}$$

It follows from the adding up condition. Normalize $q_2 = 1$ to get

$$q_1 = \left(\frac{e_2}{e_1}\right)^\rho \frac{p_1}{p_2}$$

Now write the budget constraint for consumer i

$$q_1 x_1^i + x_2^i = q_1 e_1^i + e_2^i$$

$$x_1^i = \frac{q_1 e_1^i + e_2^i}{q_1 + \frac{e_1}{e_2}} = \frac{\left(\frac{e_2}{e_1}\right)^\rho \frac{p_1}{p_2} e_1^i + e_2^i}{\left(\frac{e_2}{e_1}\right)^\rho \frac{p_1}{p_2} + \frac{e_1}{e_2}} = \frac{e_2^\rho p_1 e_1^i + e_1^\rho p_2 e_2^i}{e_2^\rho p_1 + e_1^\rho p_2}$$

We can think of this solution as if probabilities are adjusted by good scarcity and risk aversion. Then each person is getting expected value of their endowment according to the adjusted probabilities $\frac{p_1 e_2^\rho}{p_2 e_1^\rho + p_1 e_2^\rho}, \frac{p_2 e_1^\rho}{p_2 e_1^\rho + p_1 e_2^\rho}$

(d) In the first period agents trade as if in (c), and in the second period they retrade according to new probabilities $P(s|\sigma)$ treating allocation from (c) as initial endowment. Let x_s^i be trades in the first stage and $z_s^i(\sigma)$ be the trades in the second stage, conditional on realization of σ It must be that - from the results in (c)

$$\frac{z_1^i(\sigma)}{z_2^i(\sigma)} = \frac{e_1}{e_2} = \frac{x_1^i}{x_2^i}$$

So $z_s^i(\sigma) = x_s^i$ from the budget constraint. Therefore the is no trade and the allocation ex-ante PO.

(e) I will guess the equilibrium. Suppose we take the allocation from (c) and there is no trade in the second period. Let's use backwards induction. Given allocation in (c) there is no trade in the second period by the same arguments as in (d). Now, we know that given no trade in the second period the signal doesn't matter so the proposed allocation maximizes consumer utility in the first period. Moreover this allocation is ex-ante PO. Equilibrium is unique by FWT.

(f) First compute

$$P(s|\sigma = 1) = \frac{P(\sigma = 1|s)P(s)}{P(\sigma = 1|s = 1)P(s = 1) + P(\sigma = 1|s = 2)P(s = 2)} = \frac{q_s p_s}{q_1 p_1 + q_2 p_2}$$

For each σ, solution similar to (c) applies (just substitute different probabilities). The trades conditional on each signal will give different allocations if $P(s|\sigma = 1) \neq p_s$. Ex-ante Pareto optimality however requires the same ratios of consumptions for each σ i.e.

$$\frac{x_1^i(\sigma)}{x_2^i(\sigma)} = \frac{e_1}{e_2}$$

so this allocation would not be PO. So there is a loss of efficiency if the markets are not complete.

(g) Already done; there is loss of efficiency in (f) because the markets are not complete.