Fall 2011
Economics 202 Final Exam

Each question has a number of points associated with it, which roughly corresponds to the number of minutes you can expect to spend on it.
Good Luck!

Question 1. [*15 points*]

A friend tells you his method of choosing wine from restaurants' wine lists. Is this method consistent with making a uniquely optimal choice according to complete and transitive preferences? Is it consistent with the preferences satisfying the von Neumann-Morgenstern's Independence Axiom (in the case of (f))? Justify your answers.

(a) [*3 points*] Choose the cheapest wine on the list.

(b) [*3 points*] Choose the second-cheapest wine on the list.

(d) [*3 points*] If French wine is available, choose the cheapest French wine, otherwise choose the cheapest wine regardless of origin.

(e) [*3 points*] If both French and New Zealand wine is available, choose the cheapest French wine, otherwise choose the cheapest wine regardless of origin.

(f) [*3 points*] Toss a coin. If it falls heads, choose the cheapest French wine, otherwise choose the cheapest California wine.

Question 2. [35 *points*]

A von-Neumann-Morgenstern (vNM) decision maker chooses between two lotteries, p and q, which put positive probabilities on three monetary payoffs: $\$1, \$2$, and $\$3$. Let $p_i \geq 0$ and $q_i \geq 0$ denote the probabilities of payoff $\$i$ $(i = 1, 2, 3)$ in the two respective lotteries, with $\sum_{i=1}^{3} p_i = \sum_{i=1}^{3} q_i = 1$.

(a) [*10 points*] State a necessary and sufficient condition for any vNM decision maker who prefers higher certain monetary payoffs to weakly prefer lottery p to lottery q.

(b) [*5 points*] State the condition that lotteries p and q have the same expected value.

(c) [*10 points*] Assuming that the condition in part (b) holds, state *the simplest possible* necessary and sufficient condition for any risk-averse vNM decision maker to weakly prefer lottery p to lottery q.

(d) [*10 points*] Assuming that the condition in part (b) holds, state *the simplest possible* necessary and sufficient condition for any vNM decision maker with constant and nonnegative absolute risk aversion to weakly prefer lottery p to lottery q.

Question 3. [*60 points*]

Consider an economy with two goods, labeled x and y. The economy has N consumers, with the utility of each consumer $i = 1, ..., N$ from consumption bundle (x, y) taking the form $u^i(x, y) = v^i(x) + y$, where $v^i(\cdot)$ is an increasing function. In addition, there is a firm that can produce any output $q \geq 0$ of good x, using up $c(q)$ units of good y. Each consumer i has a zero endowment of good x, a large endowment of good y so that he never runs out of it, and an ownership share $\alpha^i \geq 0$ in the firm (with $\sum_i \alpha^i = 1$).

(a) [*15 points*] Write the conditions characterizing a Walrasian equilibrium of the economy. Characterize the equilibrium output of the firm.

Suppose now that the firm does not take prices as given, but recognizes its effect on prices. Namely, after the firm chooses it output q, the prices emerge to clear the market for good x with price-taking consumers. (The market for good y will then also clear by Walras' Law.) Let $P(q)$ denote the resulting price of good x in terms of good y (i.e., with the price of good y normalized to 1).

(b) [*15 points*] Suppose that the firm's objective is to maximize its profit expressed in terms of good y. Characterize the firm's optimal output choice and compare it to the Walrasian equilibrium output characterized in part (a).

(c) [*15 points*] Now suppose instead that the firm chooses its output to maximize the utility of its controlling shareholder, who is consumer k holding share α_k of the firm's profits. Characterize the firm's optimal output choice and compare it to that in part (b).

(d) [*15 points*] Suppose that all the consumers have the same utility function. Compare the firm's output choice in part (c) to that in part (a).

Question 4. [*70 points*]

Consider an exchange economy with N consumers and L goods. Each consumer i has continuous, strictly monotone, and strictly convex preferences $\succeq_i$ and initial endowment $e^i \gg 0$. Given an allocation $x = (x^1, ..., x^N) \in \mathbb{R}_+^{LN}$, say that i *envies* j in x if $x^j \succ_i x^i$. An allocation x is *envy-free* if there exists no pair of consumers i and j such that i envies j in x.

(a) [*10 points*] Is every Walrasian equilibrium allocation envy-free? Prove or disprove with a counter example.

An allocation contains an *envy cycle* if there exists a set of consumers $\{i_1, ..., i_k\}$ such that $i_1 = i_k$ and $x^{i_{n+1}} \succ_{i_n} x^{i_{n+1}}$ for each $n = 1, ..., k$.

(b) [*10 points*] Can an equilibrium allocation contain an envy cycle? Prove or disprove.

Consider a social planner who has to make sure that all allocations are envy-free. Suppose that the social planner can change consumers' initial endowments by reallocating the aggregate endowment across consumers.

(c) [*10 points*] Can the social planner change the initial endowments such that after the change, every Walrasian equilibrium allocation is envy-free?

Now suppose that the social planner cannot change initial endowments, but can instead impose a progressive tax that is proportional to the difference between a consumer's wealth and the average wealth in the economy. More specifically, the tax takes the following form

$$T^i(p) = \alpha(p \cdot e^i - \frac{p \cdot \sum_i e^i}{N}),$$

where $\alpha \in [0, 1]$.

(d) [*10 points*] Define a Walrasian equilibrium with progressive tax.

(e) [*10 points*] Is every Walrasian equilibrium with progressive tax Pareto Optimal? Prove or disprove.

(f) [*10 points*] Can every interior Pareto Optimal allocation be supported as an equilibrium with some progressive tax $\alpha \in [0, 1]$?

(g) [*10 points*] Can the social planner choose some $\alpha \in [0, 1]$ such that every equilibrium with progresstive tax is envy-free?