Econ 202N diagnostic midterm

November 1, 2007

Take three hours to do this midterm. It is closed book. The intention is for this "midterm" to give you an idea of how the final will feel. Please turn in the exam by Friday, 11/9. It will be graded on a completed/not completed basis, so you shouldn't feel any compulsion to collaborate, take more than the allotted time, or use the book or notes. Use this as an opportunity to force yourself to study and to see how well you would fare on a real midterm. Solutions will be posted after the exams have been turned in.

1   Revealed preference (25 points)

Consider a two-good economy, and a consumer with complete, transitive, continuous and strictly monotone preferences. Prices in this economy are always positive. Suppose the consumer has wealth 4. We make two observations of consumer choices:

- At prices $(1, 2)$ the choice is $(3, \frac{1}{2})$.

- At prices $(1, 1)$ the choice is $(1, 3)$.

Which of the following observations would be consistent with utility maximization?

(a) Choice $(6, 1)$ at price $(\frac{1}{2}, 1)$

(b) Choice $(1, \frac{1}{2})$ at price $(2, 2)$

(c) Choice $(2, 1)$ at price $(1.6, 0.8)$

Suppose that after making observations 1 and 2, we observe this consumer choosing consumption bundle $(2, 1)$.

(d) What can we conclude about the prices the consumer faced choosing $(2, 1)$?

2 Consumer theory (25 points)

(a) Consider three possible consumption bundles $x_A = (1, 3, 2)$, $x_B = (2, 2, 2)$ and $x_C = (2, 2, 1)$. Suppose Consumer A is rational. At prices $p_A = (2, 1, 1)$, you observe him choose x_A. At prices $p_B = (1, 2, 1)$, you observe him choose x_B. Now he has wealth $w = 7$ and faces prices $p = (1, \frac{3}{2}, 1)$. Can you say for certain if any of the bundles will or will not be chosen? What if As preferences are also locally non-satiated?

(b) Consumer B has the following indirect utility function

$$v(p_1, p_2, w) = \left(\frac{w}{p_1}\right)^{\frac{1}{4}} \left(\frac{w}{p_2}\right)^{\frac{3}{4}} \tag{1}$$

   i. Calculate the expenditure function $e(p_1, p_2, u)$.

   ii. Calculate the Hicksian demand functions $h_1(p_1, p_2, u)$ and $h_2(p_1, p_2, u)$.

   iii. Calculate the equivalent variation if $w = 10$ and prices change from $p = (2, 1)$ to $p_0 = (1, 1)$. Interpret your result.

(c) Suppose that Consumer Cs Marshallian demands for goods 1 satisfies $\frac{\partial x_1}{\partial p_2}(p, w) > 0$ for all (p, w). If good 1 is a normal goods for her, what property can you deduce about her expenditure function $e(p, u)$?

(d) Suppose Consumer D has rational, continuous and locally non-satiated preferences, and that her preferences are also homothetic, i.e. if $x \sim x_0$ then $\alpha x \sim \alpha x_0$. What can you conclude about how Consumer Ds Marshallian demand $x(p, w)$ will vary with w?

3   Rationalizability (25 points)

We observe the profits of a single-output firm as its output price p varies while its input prices stay fixed. These profits are given by the function

$$\pi(p) = \begin{cases} p^\alpha & p \in [0,1] \\ kp^\beta & p > 1 \end{cases} \tag{2}$$

(a) Formulate necessary and sufficient conditions on the parameters α, β, and k for this profit function to be consistent with price-taking, profit-maximizing behavior of the firm.

From now on, suppose that the firm is price-taking and profit-maximizing.

(b) For which price levels can we infer the firm's output choice? Derive this choice.

(c) Use the "inner bound" approach and the findings from part (b) to bound the firm's cost function from above.

(d) Use the "outer bound" approach to bound the firm's cost function from below.

(e) For which parameter values can we infer the firm's cost function exactly? What can we infer about the cost function for other parameter values?

4   Producer theory and taxes (25 points)

Consider a firm that faces a product price p, and chooses inputs k, l to solve:

$$\max_{k,l \geq 0} p \cdot f(k,l) - wl - rk \tag{3}$$

Denote the firm's optimal factor demands by $k(p,w,r)$ and $l(p,w,r)$. Suppose that capital and labor are substitutes in the production process, so that $f_{kl} \leq 0$. Now suppose the government imposes a payroll tax t per unit of labor input. Assume that the prices p, w, r remain unaffected by this tax. Suppose the firm can respond in the short-run only by changing its labor input, but not its capital input (which was chosen optimally before the tax was imposed).

(a) Show that the firm's labor input will decrease in response to this tax.

(b) How will the government's tax revenue depend on the price of capital r?

(c) Can you say how the government's tax revenue will depend on the output price p? Would your answer to this be different if capital and labor were complements?

(d) Argue that the government's tax revenues will be higher in the short-run than in the long-run. Is this a general property when the government taxes intermediate goods? Explain.