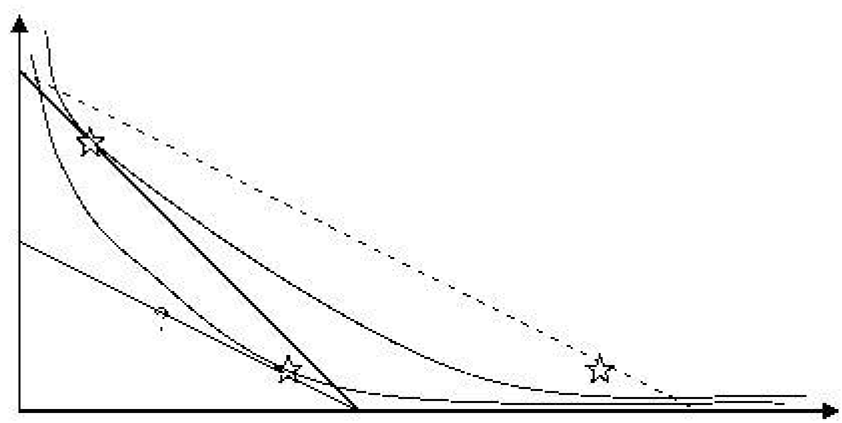
Econ 202N diagnostic midterm - Solutions

November 1, 2007

1 Revealed Preference (25 points)

Consider a two-good economy, and a consumer with complete, transitive, continuous and strictly monotone preferences. Prices in this economy are always positive. Suppose the consumer has wealth 4. We make two observations of consumer choices:

- At prices $(1, 2)$ the choice is $(3, \frac{1}{2})$.
- At prices $(1, 1)$ the choice is $(1, 3)$.



Which of the following observations would be consistent with utility maximization?

(a) Choice $(6, 1)$ at price $(\frac{1}{2}, 1)$
Consistent.

(b) Choice $(1, \frac{1}{2})$ at price $(2, 2)$
Inconsistent: violates Walras law

(c) Choice $(2, 1)$ at price $(1.6, 0.8)$
Inconsistent: by revealed preferences, $(1, 3) > (2, 1) \geq (1, 3)$.

Suppose that after making observations 1 and 2, we observe this consumer choosing consumption bundle $(2, 1)$.

(d) What can we conclude about the prices the consumer faced choosing $(2, 1)$?
Answer: We must have
$2p_1 + p_2 < p_1 + 3p_2 \Rightarrow p_1 < 2p_2$ and
$2p_1 + p2 < 3p_1 + \frac{1}{2}p_2 \Rightarrow p_1 > \frac{1}{2}p_2$.
We conclude that prices must satisfy
$p_1 \in [\frac{1}{2}p_2, 2p_2]$, with $2p_1 + p_2 = 4$.

2 Consumer theory (25 points)

(a) Observe that $p^D \cdot x^A = 7.5 > 7$ is unaffordable, so it won't be chosen. On the other hand, x^B is affordable and nothing is revealed preferred to it, so it could be chosen. Finally, x^C in the interior of the budget set at prices p^B and isn't chosen so $x^B \succsim x^C$ and under local non-satiation $x^B \succ x^C$. So x^C could be chosen if A's preferences are rational but can't be chosen if A's preferences are also locally non-satiated.

(b) The expediture function is:

$$e(p, u) = up_1^{1/4}p_2^{3/4} \tag{1}$$

The Hicksian demands are:

$$\begin{aligned} h_1(p, u) &= \tfrac{\partial e}{\partial p_1} = \tfrac{1}{4}up_1^{3/4}p_2^{3/4} \\ h_2(p, u) &= \tfrac{\partial e}{\partial p_2} = \tfrac{3}{4}up_1^{3/4}p_2^{3/4} \end{aligned} \tag{2}$$

The equivalent variation is $e(p', u') - e(p, u')$, where $e(p', u') = w$ and $u' = v(p', w) = 10$. Computing this out

$$EV = 10 - 10 \cdot 2^{1/4} = 10 \cdot (1 - 2^{1/4}) < 0 \tag{3}$$

which says that to achieve the same utility before the price chance as one does after one would have needed $10(1 - 2^{1/4})$ fewer dollars.

(c) If $\frac{\partial x_1}{\partial p_2}(p,w) > 0$ and good 1 is normal, the Slutsky equation tells us that $\frac{\partial h_1}{\partial p_2}(p,w) > 0$. Therefore by Shephard's Lemma,

$$\frac{\partial e}{\partial p_1 \partial p_2}(p,u) > 0 \tag{4}$$

(d) Homotheticity implies that $x(p, \alpha w) = \alpha x(p,w)$. So D's Marshallian demands scale up proportionately with w, as in the Cobb-Douglas case. For a more complete solution, see the Consumer Theory I notes.

3 Rationalizability (25 points)

(a) So, the necessary and sufficient conditions for a profit function to be rationalizable are

- $\pi(p,w)$ is concave in (p,w)
- $\pi(p,w)$ is homogeneous of degree one in (p,w)

We don't observe the input prices, so we can't make a statement about homogeneity. We can say that π must be convex. This, along with the fact that profit must be increasing in output price p tells us that $\alpha, \beta \geq 1$. Now, what about $p = 1$. If $k \neq 1$ there is a jump discontinuity, which will lead to non-convexity. So, we require that $k = 1$. Finally, we must have our two curves meet such that they don't make a dimple. Hence $\alpha \geq \beta$. Thus, the conditions are $\boxed{k = 1, \alpha \geq \beta \geq 1}$.

(b) Wherever profit is differentiable, the envelope theorem dictates that $\pi'(p) = y^*(p)$. Hence, we know the output choice everywhere except at $p = 1$, unless $\alpha = \beta$, in which case $y^*(p) = \alpha p^{\alpha - 1}, \forall p$. Otherwise, we have

$$y^*(p) = \begin{cases} \alpha p^{\alpha-1} & p \in [0,1) \\ \beta p^{\beta - 1} & p > 1 \\ ? & p = 1 \end{cases} \tag{5}$$

(c) So, the inner bound approach tells us that all observed production levels must be feasible. We observe π and y^*. The definition of the arg max then tells us that the cost of producing y^* is $\pi(p) - py^*$. This cost can't be less than the minimum cost, so we arrive at $c(y) \geq py - \pi p$ for all y observed.

As p goes from 0 to 1, we see y^* go from 0 to α. In this regime, $p = \left(\frac{y}{\alpha}\right)^{\frac{1}{\alpha-1}}$. As p goes from 1 to ∞, we see y^* go from β to ∞. In this regime, $p = \left(\frac{y}{\beta}\right)^{\frac{1}{\beta-1}}$. Put together, this yields

$$c(y) \leq \begin{cases} \left(\frac{y}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} [\alpha - 1] & y < \alpha \\ \left(\frac{y}{\beta}\right)^{\frac{\beta}{\beta-1}} [\beta - 1] & y > \beta \\ \text{unconstrained} & y \in [\alpha, \beta] \end{cases} \tag{6}$$

(d) By the optimality of profit, we have $\pi(p) \geq py - c(y)$. Hence, we derive a lower bound for the cost function – $c(y) \geq py - \pi(p), \forall y$. Hence, we must solve

$$c(y) \geq \max_p [py - \pi(p)] \tag{7}$$

for all $y \notin [\alpha, \beta]$. To start, we solve the optimization, assuming that $p \in [0, 1)$ case. The first order conditions quickly yield

$$p = \left(\frac{y}{\alpha}\right)^{\frac{1}{\alpha-1}} \tag{8}$$

So, as long as $y < \alpha$, this is a self-consistent solution. What if $p > 1$? Then, the first order conditions yield

$$p = \left(\frac{y}{\beta}\right)^{\frac{1}{\beta-1}} \tag{9}$$

which is self-consistent so long as $y > \beta$. Finally, we must consider what happens if $y \in [\alpha, \beta]$. In these cases, we hit corner solutions. Since we have exhausted all other possibilities, it must be that $p = 1$ optimizes in these cases. Hence we have that $c(y) \leq y - 1$ on this interval.

Hence, we have

$$c(y) \leq \begin{cases} \left(\frac{y}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} [\alpha - 1] & y < \alpha \\ \left(\frac{y}{\beta}\right)^{\frac{\beta}{\beta-1}} [\beta - 1] & y > \beta \\ y - 1 & y \in [\alpha, \beta] \end{cases} \tag{10}$$

(e) So, we only know the cost function exactly when there is no unconstrained interval in (c), i.e. when $\alpha = \beta$. At other parameter values, we know the cost function exactly when $y \notin [\alpha, \beta]$, but when $y \in [\alpha, \beta]$, all we can say is that $c(y) \geq y - 1$.

4 Producer theory (25 points)

To begin, let φ denote the firm's objective function.

(a) We are maximizing over l (k is fixed) and we are concerned about changing the parameter w (a tax is essentially an increase in w). We are told that the production function f has a mixed partial, so we don't need to worry about the non-differentiable definition of supermodularity. We can just go right to taking derivatives.

$$\begin{aligned} \frac{\partial \varphi}{\partial w} &= -l \\ \frac{\partial^2 \varphi}{\partial w \partial l} &= -1 \end{aligned} \tag{11}$$

This mixed partial tells us that we have increasing differences in $(l, -w)$

So, we are able to conclude that l is weakly decreasing in w, by Topkis' theorem.

(b) The government's tax revenue is proportional to the firm's labor choice, hence we are concerned with how l changes with r. The mixed partial is clearly given by $\frac{\partial^2 \varphi}{\partial l \partial r} = 0$. Hence the objective is both super and sub modular in (l, r). So Topkis tells us that l must be both weakly increasing and weakly decreasing in r. The only way for this to happen, though, is for l to remain constant. **So l does not vary as r is changed.** This solution makes a lot of sense. When k is held constant, the $-rk$ term is just a constant in the objective, and as we know from the first-order conditions, additive constants are ignored in optimization.

(c) We start by noting that $\frac{\partial^2 \varphi}{\partial p \partial l} = f_l$. So, as long as we assume that labor is a "good" and not a "bad", this will be positive. Hence, Topkis' theorem tells us that **labor is weakly increasing in output price**. This conclusion is independent of whether capital and labor are substitutes or complements.

(d) The short run problem maximizes with a fixed k. This problem is

$$l(p,w,r,k) = \arg\max_{l} \underbrace{pf(k,l) - wl - rk}_{\varphi} \tag{12}$$

The long run problem can be expressed in terms of the arg max of the short run problem as follows

$$k(p,w,r) = \arg\max_{k} \underbrace{pf(k,l(p,w,r,k)) - wl(p,w,r,k) - rk}_{\psi} \tag{13}$$

Let's consider how long run k changes with the tax (i.e. an increase in w). Taking derivatives, we find

$$\frac{\partial \psi}{\partial w} = pf_l(k,l(p,w,r,k))\frac{\partial l}{\partial w}(p,w,r,k) - l(p,w,r,k) - w\frac{\partial l}{\partial w}(p,w,r,k) \tag{14}$$

This is a mess! How might we simplify? The first order condition of the short run problem states

$$pf_l(k,l(p,w,r,k)) = w \tag{15}$$

Using this to simplify, we find

$$\frac{\partial \psi}{\partial w} = -l(p,w,r,k) \tag{16}$$

So, whether the long run objective ψ is supermodular or submodular in (k,w) depends entirely on how the short run arg max varies in the fixed level of capital k. To check this, we examine the relevant cross-partial

$$\frac{\partial^2 \varphi}{\partial l \partial k} = pf_{kl} \leq 0 \tag{17}$$

So, Topkis tells us that $l(p,w,r,k)$ is decreasing in k. Hence, we find that ψ is supermodular. Topkis' theorem then tells us that $k(p,w,r)$ is increasing in w. So, we then see that

$$\begin{aligned} l_{SR} &= l(p,w+t,r,k(p,w,r)) \\ l_{LR} &= l(p,w+t,r,k(p,w+t,r)) \end{aligned} \tag{18}$$

So, since $k(p, w, r)$ is increasing in w, and $l(w, p, r, k)$ is decreasing in k, we must conclude that

$$l_{SR} \geq l_{LR} \tag{19}$$

Hence tax revenue will decrease from short run to long run, as rational maximizers will be more able to dodge a tax as they have more instruments freed with which to do so. This is quite general – it is the LeChâtelier principle.