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Core Microeconomics

Fall 2016

FINAL EXAM

The exam consists of five questions, divided into parts. The number of points allocated to each part corresponds to the number of minutes allotted for this part, so the total number of points for the exam is 180.

Good Luck!

Part I

Let $\mathcal{N} = \{1, \ldots, n\}$ be a "society" with n members and let X be the set of choices or allocations available to society. In the three problems below, each $i \in \mathcal{N}$ has a *complete*, but not necessarily *transitive*, preference relation $\succcurlyeq_i$. As usual, we define $x \succ_i y$ to mean $y \not\succcurlyeq_i x$. The assumed properties of $\succcurlyeq_i$ may vary among the three parts below.

Question 1. (15 points) Suppose that each individual *i*'s preference relation $\succcurlyeq_i$ is complete and transitive and represented by a utility function $u_i: X \to \mathbb{R}$. Someone has proposed that $W(x) \stackrel{\text{def}}{=} \sum_{i=1}^{n} u_i(x)$ be used as a welfare criterion to rank society's preferences over alternatives $x \in X$. Criticize this proposal.

Question 2. Suppose that each individual *i*'s preferences over consumption bundles in $\mathbb{R}_{++}^k = \{x \in \mathbb{R}^k | x_i > 0 \; for \; i = 1, \ldots, k\}$ satisfies $x \succcurlyeq_i y \Leftrightarrow u_i(x) > u_i(y) - 1$ for some continuous, increasing, quasi-concave function $u_i: \mathbb{R}_{++}^k \to \mathbb{R}$.

(a) (5 points) Show that, with these preferences, $\succcurlyeq_i$ may not be transitive.

(b) (15 points) Someone has proposed that $\sum_{i=1}^{n} u_i(x_i)$ be used as a welfare criterion to rank society's preferences over alternative allocations $x \in \mathbb{R}_{++}^{nk}$, in which $x = (x_i)_{i=1}^{n}$ and x_i is the bundle allocated to individual i. Is this proposal subject to the same criticism as in part (1) above? Explain.

(c) (10 points) Let $k = 2$. Draw a preferred set $P_i(x) = \{y | y \succcurlyeq_i x\}$. What are its properties?

(d) (5 points) Is Walras' Law satisfied for these preferences?

Question 3. Suppose $\succcurlyeq_i$ is complete and that $\succ_i$ is transitive: ($x \succ_i y$ and $y \succ_i z$) $\Rightarrow$ $x \succ_i z$.

(a) (10 points) Show that these two conditions do not imply that $\succcurlyeq_i$ is transitive.

(b) (10 points) Show that $x \succcurlyeq_i y$ and $y \succ_i z$ implies $x \succcurlyeq_i z$.

(c) (10 points) Show that if X is finite, then there exists a "least preferred" element $z \in X$ such that for all $y \in X$, $y \succcurlyeq z$. (Hint: use induction on the size of X.)

(d) (10 points) Using the preferences defined in (2), show that $\succcurlyeq_i$ is complete and $\succ_i$ is transitive.

Part II

Question 4. Consider a decision maker whose preferences over consumption lotteries X (random variables) are described by a utility function of the form

$$u(X) = \mathbb{E}[X] - \frac{1}{2\rho}\mathrm{Var}[X]\,,$$

where $\mathbb{E}[X]$ is the expected value of random variable X, $\mathrm{Var}[X]$ is its variance, and $\rho > 0$ is a parameter describing the agent's "risk tolerance." Answer each question below with either a proof or a counterexample.

(a) (9 points) Do these preferences satisfy the Independence Axiom?

(b) (9 points) Do they always prefer a first-order stochastic improvement?

(c) (9 points) Do they always prefer a mean-preserving second-order stochastic improvement?

Question 5. Consider an economy with uncertainty that may end up in one of *S* different states. The economy consists of *I* agents, each of whom can consume any amount (positive or negative) of a single consumption good in each state, so his state-contingent consumption bundle can be described as $x \in \mathbb{R}^S$. All agents have the same prior beliefs over the states, described by probability vector $p \in \Delta(\{1, \dots, S\})$. Each agent i's preferences over state-contingent consumption bundles (interpreted as random variables) are described by a "mean-variance" utility function of the form described in Question 4 above, with an agent-specific risk-tolerance parameter $\rho^i > 0$.

Before the state is realized, agents can trade *J* + 1 assets. One of them – labeled asset $j = 0$ – is the "safe" asset, yielding return 1 in each state. The other *J* assets are "risky": one unit of risky asset $j = 1, \dots, J$ yields dividend payout d_{js} in state *s*. Let $\mu \in \mathbb{R}^J$ denote the vector of the assets' mean dividend payouts, and $\Omega \in \mathbb{R}^{J\times J}$ denote the variance-covariance matrix of the assets' dividend payouts. Assume that Ω has full rank.

Assume that agents' endowments consist entirely of assets described above: let $e^i \in \mathbb{R}^J$ denote agent i's endowment of the risky assets, and $e_0^i \in \mathbb{R}$ denote his endowment of the safe asset.

Consider an asset-market equilibrium in which each agent chooses his asset portfolio taking asset prices as given. Normalize the price of the safe asset to 1 and let $q \in \mathbb{R}^J$ denote the price vector of the risky assets.

(a) (9 points) Calculate an agent's utility from holding a portfolio consisting of holdings $\alpha \in \mathbb{R}^J$ of the risky assets and amount $\alpha_0 \in \mathbb{R}$ of the safe asset. (Note that the holding of any asset may be either positive or negative, i.e., short sales/borrowing is allowed.) Formulate the agent's problem of choosing his utility-maximizing portfolio subject to his budget constraint.

(b) (9 points) Derive the first-order conditions characterizing the agent's optimal portfolio of risky assets. Compare the optimal risky-asset portfolios of different agents in the economy.

(c) (9 points) Use market-clearing conditions to solve for agents' equilibrium risky-asset portfolios and equilibrium asset prices q.

(d) (9 points) Does this economy have the Gross Substitute property? Give either a proof or a counterexample. [Hint: for simplicity you can focus on the case of two risky assets.]

(e) (9 points) Does this economy exhibit global tatonnement stability? Give either a proof or a counterexample.

(f) (9 points) Define the *net* expected payout on a portfolio of risky assets to be the expected payout on that portfolio net of the portfolio's cost in terms of the safe asset. Let the "market portfolio" denote the aggregate endowment of the risky assets. With the equilibrium prices obtained in part (c), express the net expected payout on the market portfolio through the variance of that portfolio and the agents' risk-tolerance parameters. Then express the equilibrium expected net payout on any risky-asset portfolio α through the portfolio's covariance with the market portfolio and the net expected payout and variance of the market portfolio.

(g) (9 points) Characterize Pareto optimal state-contingent consumption allocations of the economy's aggregate state-contingent endowment [which can be computed as $\sum_{i=1}^{I}(e_0^i + \sum_{j=1}^{J} d_{js} e_j^i)$ in states $s = 1, \ldots, S$]. Compare them to the consumption allocation obtained in the asset market equilibrium characterized in part (c).