Final Exam
Econ 202
Stanford University
Fall 2001

Felix Kubler and Jonathan Levin

December 12, 2001

Instructions:

- You have three hours to complete the exam – there are 4 questions.
- Answer each question in a separate bluebook.
- In the cover of each blue book write the question number and your name.
- The exam is closed book and closed notes.
- Good luck !

Questions:

1. Revealed Preference (25 points)
   Consider a two-good economy, and a consumer with complete, transitive, continuous and locally non-satiated preferences. Prices in this economy are always strictly positive. Suppose the consumer has wealth $w = 4$. You know that at prices $(p_1, p_2) = (1, 1)$, the consumer's unique preferred bundle is $x = (3, 1)$. You then observe this consumer selecting the bundle $y = (1, 2)$, but do not observe the prices at which y was chosen. (You may find it useful to argue your answers graphically, using a clearly labeled and clearly explained picture.)

   (a) What can you conclude about the prices the consumer faced in choosing y?

   (b) Suppose you learn the price of good one was $p_1 = 8/3$. What can you say about the set of bundles to which $x = (3, 1)$ is strictly preferred?

   (c) Suppose the consumer's preferences are also strictly monotone and convex. From the information you have, what can you conclude about the set of bundles that are preferred to y?

   (d) Can you bound the consumer's marginal rate of substitution between goods at y *without* assuming that the indifference curve is smooth at y?

2. Producer Theory & Taxes (25 points)
Consider a firm that faces a product price p, and chooses inputs k, l to solve:

$$\max_{k,l \geq 0} p \cdot f(k,l) - wl - rk.$$

Denote the firm's optimal factor demands by $k(p,w,r)$ and $l(p,w,r)$. Suppose that capital and labor are substitutes in the production process, so that $f_{kl} \leq 0$. Now suppose the government imposes a payroll tax t per unit of labor input. Assume that the prices p, w, r remain unaffected by this tax. Suppose the firm can respond in the short-run only by changing its labor input, but not its capital input (which was chosen optimally before the tax was imposed).

(a) Show that the firm's labor input will decrease in response to this tax.

(b) How will the government's tax revenue depend on the price of capital r?

(c) Can you say how the government's tax revenue will depend on the output price p? Would your answer to this be different if capital and labor were complements?

(d) Argue that the government's tax revenues will be higher in the short-run than in the long-run. Is this a general property when the government taxes intermediate goods? Explain.

3. Choice under Uncertainty (20 points)
Consider a choice problem under uncertainty. The prize space is $\mathcal{Z} = \{0, 10, 100, 1000\}$ Suppose an agent has preferences $\succeq$ over

$$\mathcal{P} = \{p \in I\!R^4_+ : p_1 + p_2 + p_3 + p_4 = 1\}$$

which satisfy the von Neumann-Morgenstern axioms.

(a) Suppose we observe that the agents' preferences satisfy

$$(1,0,0,0) \succ (0,1,0,0) \succ (0,0,1,0) \succ (0,0,0,1).$$

How will the agent choose between $(0.2, 0.2, 0.2, 0.4)$ and $(0.1, 0.3, 0.15, 0.45)$?
Can you say how he will choose between $(0.1, 0.1, 0.1, 0.7)$ and $(0.1, 0.4, 0.4, 0.1)$? Explain.

(b) Risk-aversion.
i) What does it mean to say the agent is 'risk averse' ? Give a general definition and explain how it can be applied to this example, where the prize space is finite.
ii) Suppose $(0.5, 0, 0.5, 0) \succ (0.95, 0, 0, 0.05)$. Is the agent risk averse ? Explain.

(c) Risk aversion allows us to predict an agent's choices even without having observed any of his previous choices. Is this possible without risk aversion (i.e. just assuming that his preferences are von Neumann-Morgenstern) ? Explain.

4. General equilibrium (30 points)
Consider an economy with three commodities and two households. There are two different specifications for preferences

i) Specification 1:

$$u^1(c_1, c_2, c_3) = \min(c_1, c_2, c_3),\ u^2(c_1, c_2, c_3) = \min(c_1, 2c_2, c_3)$$

ii) Specification 2:

$$u^1(c_1, c_2, c_3) = \sqrt{c_1} + c_2,\ u^2(c_1, c_2, c_3) = c_2 + c_3$$

Individual endowments are

$$e^1 = (1, 1, 1) \text{ and } e^2 = (0, 0, 2)$$

(a) Suppose there is no production.
For the two specifications of utility functions tell me if there exists a Walrasian equilibrium. If there exists one compute it. If there is none explain carefully why not and which part of the standard existence proof fails (it is *not* sufficient to say that assumptions (A1)-(A4) from class do not hold).

(b) Now suppose that there is a firm with constant returns to scale technology. The firm can take one unit of commodity 3 as an input and produce 2 units of commodity 1 and 1 unit of commodity 2, its production set is given by

$$Y^1 = \{y \in I\!R^3 : y_3 \leq 0, y_1 \leq -2y_3, y_2 \leq -y_3\}$$

For the two specifications of utility functions tell me if there exists a Walrasian equilibrium. If there exists one compute it.