Econ 202 Final Exam
Draft Nov 27, 2015

1. A long-standing finding in psychology is that individuals cannot distinguish small quantity differences, and that the probability that a difference is distinguishable depends on the ratio of the two quantities. Suppose that there is some $\delta > 1$ such that, given two quantities $x > y > 0$ of a good, the two can be distinguished if and only if $x > \delta y$.

   With that motivation, let us define a preference relation for a single good as follows:
   (i) $x \succ y$ if $x > \delta y$,
   (ii) $x \succcurlyeq y$ if it is not the case that $y \succ x$, and
   (iii) $x \sim y$ if $x \succcurlyeq y$ and $y \succcurlyeq x$.

   Prove or disprove each of the following.

   a) The relation $\succcurlyeq$ is complete.
   b) The relation $\succ$ is transitive.
   c) The relation $\sim$ is transitive.
   d) ($x \succ y$ and $y \sim z$) $\Rightarrow x \succcurlyeq z$.

We will say that the function *U* *cardinally represents* $\succcurlyeq$ if for all $x, y > 0$,

$$x \succcurlyeq y \text{ if and only if } U(x) \geq U(y) - 1.$$

   e) Prove or disprove that there is some function $U(\cdot)$ that cardinally represents $\succcurlyeq$.

2. Consider a firm with two inputs, labor and capital, and one output. The firm produces output according to the Cobb-Douglass production function $f(k, l) = Al^{\alpha} k^{\beta}$, for some $\alpha, \beta \in (0,1)$ and $A > 0$. Let the price of output be p and denote the input prices by w (wage) for labor and r (rental rate) for capital.

   a) Derive the profit function $\pi(p, w, r)$. Under what conditions is profit finite?
   b) Derive the supply correspondence.
   c) What fraction of the firm's total cost is the labor cost?

3. Suppose that all consumers have quasi-linear preferences expressible as $U(x) = x_1 + v(x_2, \ldots, x_n)$. Fix good #1 to be numeraire (that is, fix its price always to be 1). Assume that Marshallian demand is a singleton, that is, $x(p, w)$ is the unique solution to $max_x U(x)$ subject to $p \cdot x \leq w$.

   a) Show that Marshallian demand must satisfy the following identity for all p, w, w':

$$x_j(p, w) = \begin{cases} x_j(p, w') + w - w' & \text{for } j = 1 \\ x_j(p, w') & \text{for } j = 2, \ldots, n \end{cases}$$

b) Show that for goods 2,…,n, the law of demand applies, that is, for any two price vectors $p, p' \in \mathbb{R}_+^n$ with $p_1 = p_1' = 1$ and any wealth level w, any Marshallian demand function must satisfy $(p - p') \cdot \big(x(p,w) - x(p',w)\big) \leq 0$.

c) Write the Slutzky decomposition and use it to show that for goods $j = 2, \dots, n$, Hicksian demand coincides with Marshallian demand: $h_j(p,u) \equiv x_j(p,w)$.

d) Show that for any wealth levels $(w^1, \dots, w^I)$, any utility levels $(u^1, \dots, u^I)$, and any goods supplies $\bar{x} = (\bar{x}_1, \dots, \bar{x}_L)$, the price vector p^* with $p_1^* = 1$ clears all markets if $p^* \in argmax_p \sum_{i=1}^{I} e^i(p, u^i) - p \cdot \bar{x}$.

e) Suppose that $n = 3$ and $\frac{\partial^2 v(x_2,x_3)}{\partial x_2 \partial x_3} > 0$. Show that goods 2 and 3 are gross complements.

4. Consider lotteries for which the prize space is $\mathcal{X} = \{1,2,3,4,5\}$. These are all money amounts, so higher prizes are preferred to smaller ones. Suppose a risk-averse expected-utility maximizer has to compare the following three gambles:

$$p = (.2, .2, .2, .1, .3) \quad q = (.4, 0, .2, 0, .4) \quad r = (.2, .2, .2, .2, .2)$$

What can be said unambiguously about how she would rank these three gambles?

5. Consider an economy with I consumers. Let the finite set S denote the possible states of the world. There is a single consumption good in each state and consumers maximize expected utility and have constant absolute risk aversion. That is,

$$U^i(x) = -\sum_{s \in S} p_s \exp(-r^i x_s^i).$$

Consumer *i* has a state-contingent endowment e^i that pays $e_s^i > 0$ of the consumption good in state s.

Let R_{js} denote the payment by security *j* in state s, for $j = 1, \dots, J$. Let $y^i = (y_1^i, \dots, y_J^i)$ denote consumer *i*'s portfolio of securities, with corresponding expected utility

$$V^i(y) = -\sum_{s \in S} p_s \exp\left(-r^i (e_s^i + \sum_{j=1}^{J} y_j^i R_{js})\right)$$

Let $q = (q_1, \dots, q_J)$ denote the vector of security prices. The securities are all in zero net supply.

Suppose that the full set of securities $R_1, \dots, R_J$ is linearly independent, so that no two different portfolios yield the same state-contingent payoff profile.

a) Write down the first-order conditions for the consumer i's portfolio problem.
b) Argue that the problem has a unique solution.
c) Let security #1 be a safe security that pays $R_1(s) = 1$ in every state s. Show that preferences over portfolios of securities are then quasi-linear, with the first security serving as numeraire.
d) Write down the "Pareto problem" of choosing an allocation of securities in zero net supply to maximize the utility of consumer #1, subject to minimum utilities of $\left(\bar{u}^i\right)_{i=2}^{I}$ for the other consumers, and consider what may change when the endowment of consumer 1 is increased by one in every state: $\hat{e}_s^1 \equiv e_s^1 + 1$. Show that at the optimum, y^i remains unchanged for all i.