Fall 2013
Econ 202 Final Exam Solutions

Question 1.

(a) The problem is to choose $N \in \mathbb{N}$ and $q_1, \dots, q_N \in \mathbb{R}_+$ to maximize $\sum_{i=1}^{N}(pq_i - c(q_i)) - g(N)$.

To show that we can get the same optimal value by imposing $q_1 = \dots = q_N$, we denote the value achieved by some list $(N^*, q_1^*, \dots, q_n^*)$ to be Π. Pick $q^* = \arg\max_{q_i}\{pq_i - c(q_i)\}$, we have $N^*(pq^* - c(q^*)) - g(N^*) \geq \Pi$, that is, imposing the restriction does not affect the programmer's ability to achieve any attainable value in the original program.

(b) (i) Yes. Let's denote $v(p) \equiv \sup\{pq - c(q) : q \in \mathbb{R}_+\}$, which is obviously nondecreasing in p. The firm chooses N to maximize $F(N,p) \equiv Nv(p) - g(N)$. From $\frac{\partial F}{\partial p} = Nv'(p)$, we know that F satisfies increasing indifference in (N,p), so the strong set order holds.

(ii) Yes. We know that the chosen q maximizes $G(q,p) \equiv pq - c(q)$. But $\frac{\partial G}{\partial p} = q$, and we have increasing difference again.

(iii) Yes. This is trivial from (i) and (ii).

(c) The problem is to choose $N \in \mathbb{N}$ and $q_1, \dots, q_N \in \mathbb{R}_+$ to maximize $\sum_{i=1}^{N}(P(\sum_{i=1}^{N} q_i)q_i - c(q_i)) - g(N)$.

We suggest the condition that c is convex. To see how this works, we denote the value achieve by some list $(N^*, q_1^*, \dots, q_n^*)$ to be Π. Pick $q^* = \frac{1}{N}\sum_{i=1}^{N} q_i$, we have $N^*(P(Nq^*)q^* - c(q^*)) - g(N^*) \geq \Pi$.

(d) (i) Yes. The problem is to choose N and q to maximize $N(P(Nq)q - C(q)) - kN$. Given N, q is chosen to maximize $P(Nq)q - C(q)$. Let $u(N)$ be the value function of this subproblem, and define $H(N,k) \equiv Nu(N) - kN$. From $\frac{\partial H}{\partial k} = -N$, we know that H satisfies increasing difference in $(N, -k)$, so monotonic comparative statics holds.

(ii) No. Consider the setting with $P(Q) = 1 - Q$ and $C(q) = q^2$. We can solve the subproblem to get $q = \frac{1}{2N+2}$. Substituting this into the original problem, we can solve for optimal N to be $\frac{1}{4\sqrt{k}} - 1$, so $q = 2\sqrt{k}$, increasing in k.

(iii) Yes. The problem is also to choose N and Q to maximize $I(Q,N,k) \equiv P(Q)Q - Nc(\frac{Q}{N}) - kN$. Because $\frac{\partial I}{\partial k} = -N$, I satisfies increasing difference in $(Q,-k)$ and $(N,-k)$. Because $\frac{\partial I}{\partial Q} = P'(Q)Q + P(Q) - c'(\frac{Q}{N})$ and c is convex, I satisfies increasing difference in (Q,N). So I is supermodular in $(Q,N,-k)$, and by Topkis' theorem, monotonic comparative statics works.

Question 2.

(a) The problem is to maximize $q_1 u(x_1) + q_2 u(x_2)$ subject to $p_1 x_1 + p_2 x_2 \leq p_1 e_1 + p_2 e_2$.

The FOC is for $i = 1, 2$,

$$u'(x_i) = \lambda \frac{p_i}{q_i}.$$

We thus know

$$\frac{u'(x_1)}{u'(x_2)} = \frac{p_1 q_2}{p_2 q_1}.$$

(b) The condition is

$$\frac{p_1 q_2}{p_2 q_1} < 1,$$

in which case $u'(x_1) < u'(x_2)$. By the strict monotonicity of u', we can conclude $x_1 > x_2$.

(c) WLOG, assume $q_1 > \hat{q}_1$. We can conclude that $x_1 > \hat{x}_1$, so $x_2 < \hat{x}_2$ by Walras' law.

To see this, suppose $x_1 \le \hat{x}_1$. We have $x_2 \ge \hat{x}_2$ by Walras' law. Hence,

$$\frac{p_1 q_2}{p_2 q_1} = \frac{u'(x_1)}{u'(x_2)} \ge \frac{u'(\hat{x}_1)}{u'(\hat{x}_2)} = \frac{p_1 \hat{q}_2}{p_2 \hat{q}_1},$$

contradicting $q_1 > \hat{q}_1$.

(d) The condition is that u is (weakly) more risk averse than v.

When $\frac{p_1 q_2}{p_2 q_1} = 1$, the logic of part (b) gives $x_1 = x_2$ and $\hat{x}_1 = \hat{x}_2$, so both have 0 variance.

When $\frac{p_1 q_2}{p_2 q_1} < 1$, according to part (b), we know that $x_1 > x_2$ and $\hat{x}_1 > \hat{x}_2$, so by Walras' law, u-consumer having a weakly smaller variance is equivalent to $x_1 \le \hat{x}_1$.

Suppose the opposite. By Walras' law, $x_2 < \hat{x}_2 < \hat{x}_1 < x_1$. We can derive from $\frac{u'(x_1)}{u'(x_2)} = \frac{p_1 q_2}{p_2 q_1} = \frac{v'(\hat{x}_1)}{v'(\hat{x}_2)}$ that

$$\frac{u'(x_1)}{v'(x_1)} > \frac{u'(x_1)}{v'(\hat{x}_1)} = \frac{u'(x_2)}{v'(\hat{x}_2)} > \frac{u'(x_2)}{v'(x_2)},$$

contradicting the fact that $\frac{u'(x)}{v'(x)}$ is non-increasing in x (Pratt's theorem).

The case with $\frac{p_1 q_2}{p_2 q_1} > 1$ is symmetric.

★ You can also prove the claim through the Spence-Mirrlees single crossing condition and the "lambda trick."

Question 3.

(a) True. This is implied by (b), but the first welfare theorem also works.

(b) True. Suppose that $u^i(x^i) \ge u^i(\hat{x}^i)$ for $i = 1, 2$. Then $\hat{p} \cdot x^i \ge \hat{p} \cdot \hat{e}^i$. Adding two equations up, we get $\hat{p} \cdot (e^1 + e^2) = \hat{p} \cdot (x^1 + x^2) \ge \hat{p} \cdot (\hat{e}^1 + \hat{e}^2)$, a contradiction.

(c) False. The idea comes from multiple equilibria: while consumer 1 is better endowed in the second economy, the second equilibrium might be a bad one for her. The first graph, though ugly, shows the idea.

(d) False. The first graph still works if we cover up the border.

(e) (a) True. The same proof applies.

(b) True. The same proof applies.

(c) False. The idea is that consumer 1, as a seller of good 2, might flood the market when the increase in e_2^1 is too large. See the second graph, where indifferent curves are drawn to reflect quasi-linearity of preferences.

(d) True. Walrasian equilibria are Pareto efficient by the first welfare theorem. In a quasilinear economy with $u^i(x) = x_1 + v^i(x_{-1})$ for $i = 1, 2$, we know that Pareto efficient $(\tilde{x}^1_{-1}, \tilde{x}^2_{-1})$ maximizes $v^1(A) + v^2(B)$ subject to $A + B \leq e^1_{-1} + e^1_{-1}$. By strict convexity of preferences, this arrangement of $(\tilde{x}^1_{-1}, \tilde{x}^2_{-1})$ is unique. By the differentiability of u^i, we can normalize $p_1 = \hat{p}_2 = 1$ and derive from the FOC of the consumer problem that $p_{-1} = \nabla v(\tilde{x}^1_{-1}) = \hat{p}_{-1}$.

As a result, consumer 1 is facing the same price vector and have a strictly bigger endowment in the second economy, so she must prefer $\hat{x}^1$.