Economics 202
Final Examination

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Read and follow the following instructions carefully:

1. Please fill out the acknowledgment and acceptance of the honor code on the cover of each bluebook, *writing your Stanford Student ID number in place of your name*
2. You have three hours to complete this exam
3. Please answer each of question in a separate bluebook, writing the question number on the cover of the bluebook
4. You may not use any aids (e.g., notes, books, calculators, etc.)
5. Answering these questions may require you to make reasonable assumptions that are not explicitly stated in the problem; please try to be explicit whenever you do so

Good luck!

1 Production in the short and long run (20 points)

Consider the following production set

$$Y = \{(y_1, y_2, y_3) \in \mathbb{R} \times (-\infty, 0] \times \{0, -1\} : \; -y_1 \geq y_3\sqrt{-y_2}\}.$$

Assume throughout that $p > \mathbf{0}$.

(a) Sketch a picture (or pictures) of the production set. 5 points

(b) Provide an economic interpretation of y_3 and p_3. What do $y_3 = 0$ and $y_3 = -1$ correspond to? 5

(c) Find the firm's optimal supply correspondence when y_3 is fixed (i.e., the "short run"). 5

(d) Find the firm's optimal supply correspondence when y_3 is variable (i.e., the "long run"). 5

2 Thanksgiving complements (30 points)

A consumer has preferences defined over turkey (x_1) and gravy (x_2). Assume her preferences are Leontief, and are represented by utility function $u(x_1, x_2) = \min\{x_1, x_2\}$.

Let prices be $p \equiv (p_1, p_2)$ and the consumer's wealth be y.

(a) Show that preferences are homothetic. 5

(b) Find the

- (i) indirect utility function, 2
- (ii) expenditure function, 2
- (iii) Hicksian demand, and 2
- (iv) Marshallian demand. 2

(c) Verify that the Slutsky equation holds. Comment briefly on the relative size of the wealth and substitution effects. 8

(d) Suppose initially that $p_1^0 = p_2^0 = 1$ and $y = 100$. Assume that a climate shock causes a large increase in the price of turkey, to $p_1^1 = 3$. Find the

- (i) Equivalent variation, 3
- (ii) Compensating variation, and 3
- (iii) (Change in Marshallian) Consumer surplus. 3

3 Limited liability (25 points)

Consider a von Neumann-Morganstern decision maker who would be risk neutral, except that she has "limited liability": whenever the realization of a risky gamble leaves her with a negative payout, her debt is forgiven and consumption is zero.

Thus, her Bernoulli utility function is

$$v(c) = \max\{x, 0\}.$$

(a) Is this decision maker Risk Averse, Risk Neutral, or Risk Loving? 5

(b) Formulate a definition of Decreasing/Constant/Increasing Absolute Risk Aversion that is appropriate for this environment. In particular, it should not involve derivatives of the Bernoulli utility function. 5

(c) Does this decision maker have Decreasing, Constant, or Increasing Absolute Risk Aversion? 5

(d) Repeat parts (b)–(c), replacing "Absolute Risk Aversion" with "Relative Risk Aversion. 5

(e) Suppose the decision maker has initial wealth w, and of which she invests γ in a project with a random net return R. She can invest up to $\bar{\gamma}$ in the project, so $\gamma \in [0, \bar{\gamma}]$. The wealth not invested in the project is invested in a safe asset at zero interest rate, so the agent's payout is

$$x = w + \gamma R.$$

(i) Describe how the optimal choice of γ depends on initial wealth w. 3

(ii) Show that either $\gamma^* = 0$ or $\gamma^* = \bar{\gamma}$ is always an optimal choice. 2

4 The Sneetches (25 points)

Now the Star-bellied Sneetches had bellies with stars.
The Plain-bellied Sneetches had none upon thars.
The stars weren't so big; they were really quite small.
You would think such a thing wouldn't matter at all.
But because they had stars, all the Star-bellied Sneetches
would brag, "We're the best kind of Sneetches on the beaches."

The Sneetches live on the coast of an island. There are two goods in the Sneetchean economy, stars and food. A star is a green tattoo a Sneetch wears on its belly. Every Sneetch is endowed with one unit of food and all Sneetches start without a star. Each individual can buy at most one star.

All the Sneetches on the beaches are concerned about "status." Status is acquired by having a star or not. Having a star does not intrinsically give more status, it depends on what the other Sneetches think about it. Sneetches believe they acquire status by making the same choice as those who worry more about acquiring status. This may be circular reasoning, but the Sneetches have managed to convince themselves that having status actually means something.

Each individual has a parameter θ that represents how obsessed it is with acquiring status. There is a unit mass of Sneetches whose θ's are independently drawn from a uniform distribution on $[0, 1]$. The utility of each individual is given by:

$$u(x, y; \theta) = x(2\pi_1\theta + y) + (1 - x)(2\pi_0\theta + y),$$

where y is food and $x \in \{0, 1\}$, with $x = 1$ meaning the individual has a star on its belly. π_1 is the average θ of the Sneetches who have stars, and π_0 is the average θ of those who do not.

Stars must be bought from a single firm that tattoos them using a strange machine. This machine can tattoo stars at a marginal cost of $0 < c < 1/2$ units of food. The firm has no other cost for the stars.

Let p be the price of getting a star, and normalize the price of food to 1. This question asks you to look for the Walrasian Equilibria of this economy. Always assume that a positive fraction of the Sneetches get a star and a positive fraction do not.

(a) Explain why $\pi_0 \geq \pi_1$ cannot happen in equilibrium. 2

(b) Show that individual demand for stars is non-decreasing in type θ. 5

(c) Assume $p < 1$. Solve for the unique type that is indifferent between buying a star or not. 2

(d) Solve for the Walrasian equilibrium of this economy. 7

Now suppose the government imposes a tax $t \in [0, 1 - c]$ on those who buy stars and redistributes the total revenue as a lump-sum transfer evenly distributed among all Sneetches.

(e) Solve for the Walrasian equilibrium with a tax t. 2

(f) Find the optimal tax to maximize the average utility in a Walrasian equilibrium with tax. 5

(g) Prove or disprove that the First Welfare Theorem obtains in this economy when $t = 0$? If it doesn't hold, explain what assumption fails. 2