Final Exam
Econ 202
Stanford University
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Instructions:

- You have three hours to complete the exam – there are 4 questions.
- Answer each question in a separate bluebook.
- In the cover of each blue book write the question number and your name.
- The exam is closed book and closed notes.

1. (20 points)

   Consider an agent who needs to choose how much of an addictive substance to consume. The substance can only be consumed at two levels: $c_D = 0, 1$, where c_D denotes the consumption of the drug. The price of the drug is p and the agent has income y. Let x denote the amount spend in all other goods and services.

   An empiricist has been observing the agent for a while and has made the following observations:

   - The agent consumes the drug a fraction $\alpha < 1/2$ of the time,
   - The agent consumes the drug only when the weather is grey, which happens a fraction α of the time.

(a) (8 points) Is the behavior of this agent compatible with utility maximization? Be precise. (Think of the data as repeated observations from the same static choice problem)

(b) (8 points) Write the SIMPLEST model that generates this behavior.

(c) (4 points) Consider an economy in which all of the consumers have preferences as in (b) and firms produce the drug at a constant marginal cost c. Does the market generate Pareto optimal allocation? Be precise.

2. (30 points)

Consider an agent with preferences $u(x,l) = \sum_i \lambda_i . x_i - l$, where x_i denotes consumption of good i, l denote the amount of labor supplied, and $\lambda_i > 0$ for all i. Let w denote the wage rate and p_i the price of good i. The agent is endowed with $\bar{l}$ units of labor and has to choose how much to work and how much to consume of each good. However, if he earns $e = wl$ in the labor market, he needs to pay labor income taxes equal to $T(e)$.

(a) (3 points) Write down the optimization problem for the agent. Be precise.

(b) (9 points) Show that if $T(.)$ is a convex function, then the budget constraint is a convex set.

(c) (9 points) Show that the indirect utility function for this problem is a convex function on the weights λ.

(d) (9 points) State a condition on $T(.)$ that makes the indirect utility function differentiable and provide a precise justification for your answer.

3. (15 points)

Suppose an economic agent has von-Neumann Morgenstern preferences over simple money-lotteries, which can be represented by the following utility function

$$U(p) = \sum_z p(z)u(z)$$

with the Bernoulli function being

$$u(z) = -exp(-az) \text{ for some } a > 0$$

(a) (5 points) Define the coefficient of absolute risk aversion. How does it change with wealth for our economic agent?

(b) (10 points) Suppose the certainty equivalent of a gamble where the agent wins 1000 dollars with probability 1/2 and zero with probability 1/2 is 488 dollars. What can you say about the certainty equivalent of a gamble where the agent wins 1500 dollars with probability 1/2 and 500 dollars with probability 1/2?

4. (35 points)

Consider a Walrasian exchange economy with 2 agents and 4 commodities. Suppose the two agents have the following utility functions

$$u^1(x_1, x_2, x_3, x_4) = \log(x_1) + \log(x_2) + \log(x_3) + \log(x_4)$$

$$u^2(x_1, x_2, x_3, x_4) = -\frac{1}{x_1} - \frac{1}{x_2} - \frac{1}{x_3} - \frac{1}{x_4}$$

Throughout this exercise, suppose individual endowments are such that

$$e^1 + e^2 = \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \end{pmatrix}.$$

(a) (5 points) Consider the allocation

$$x^1 = (x_1^1, x_2^1, x_3^1, x_4^1) = (2, 3, 6, 7)$$

$$x^2 = (x_1^2, x_2^2, x_3^2, x_4^2) = (8, 7, 4, 3)$$

Is this allocation Pareto-efficient? (Justify your answer!)

(b) (7 points) Consider the allocation

$$x^1 = (x_1^1, x_2^1, x_3^1, x_4^1) = (2, 2, 2, 2),$$

$$x^2 = (x_1^2, x_2^2, x_3^2, x_4^2) = (8, 8, 8, 8).$$

Construct individual endowments $(\tilde{e}^h)_{h=1,2}$ and a price p^* such that $((x^h)_{h=1,2}, p^*)$ is a Walrasian equilibrium for the economy $(u^h, \tilde{e}^h)_{h=1,2}$.

(c) (7 points) Use the first welfare theorem to prove that whenever

$$e^1 + e^2 = \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \end{pmatrix}$$

all equilibrium allocations must satisfy $x_l^h = x_{l'}^h$ for $h = 1, 2$ and all $l, l' = 1, 2, 3, 4$.

(d) (8 points) Use your result in c) to prove that Walrasian equilibrium must be globally unique if

$$e^1 + e^2 = \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \end{pmatrix}.$$

(e) (8 points) Now somebody in this economy invents a linear technology which takes 1 unit of commodity 1 and turns it into A units of commodity 2, B units of commodity 3 and C units of commodity 4 i.e. there is now a linear activity

$$a = (-1, A, B, C) \text{ with } A, B, C \geq 0.$$

Give conditions on A, B and C which ensure that the allocation in the Walrasian equilibrium with production still satisfies $x_l^h = x_{l'}^h$ for $h = 1, 2$ and all $l, l' = 1, 2, 3, 4$. Show that these conditions are necessary and sufficient.