

# Fall 2014

## Economics 202 Final Exam

*All the three questions will be given the same number of points (60). Within each question, the points will be divided equally among its parts .*

*Good Luck!*

### Question 1.

A firm can produce  $K$  nonnegative outputs,  $q_1, \dots, q_K \geq 0$ . In order to produce these outputs, the firm needs to install some capacity level  $q_0 \geq 0$  at a cost  $C_0(q_0)$ , enabling it to produce any output  $q_k \leq q_0$  of each product  $k \geq 1$  at the additional cost of  $C_k(q_k)$ . Thus, the firm's total production cost is  $C_0(q_0) + \sum_{k=1}^K C_k(q_k)$ . The firm makes its production decisions to maximize profits taking the output prices  $p_1, \dots, p_K$  as given.

- (a) (15 points) Formulate the firm's maximization problem .
- (b) (15 points) Show that the optimal output of any product  $k$  is nondecreasing in the price of any product  $j$ . (Assume for simplicity that the optimal output is unique.)
- (c) (15 points) Assuming all the cost functions are differentiable, write down the Kuhn-Tucker first-order conditions for the problem formulated in part (a), including the applicable complementary slackness conditions. [For simplicity, you may ignore the nonnegativity constraints for the rest of this question.]
- (d) (15 points) Now assume that the cost functions are continuously differentiable and strictly concave. For which product pairs  $j, k$ , with  $j \neq k$ , will the firm *strictly* increase its optimal output of product  $j$  in response to a small increase in the price of product  $k$ ?

## Question 2.

This question asks you to consider the same setting as in question 1, but in which the firm has to choose its capacity  $q_0$  before some uncertainty about the future price  $p_K$  of product  $K$  is resolved. After the firm chooses its capacity, the price  $p_K$  is drawn from a distribution  $F$ , and at that point the firm chooses its outputs of all the products. Assume that firm knows the prices  $p_1, \dots, p_{K-1}$  of the other  $K - 1$  products from the outset.

- (a) (15 points) Let  $q_k^*(p_K, q_0)$  denote the firm's optimal choice of output  $k \geq 1$  after it has chosen capacity  $q_0$  and observed price realization  $p_K$ , and let  $V(p_K, q_0)$  be the firm's resulting ex-post profit. Use the envelope theorem to write the partial derivative of  $V$  with respect to  $p_K$  when it exists.

The following questions ask you about comparative statics on the distribution  $F$ . Answer them with rigorous arguments.

- (b) (15 points) Would the firm's profits increase or decrease if  $F$  improves in the First-Order Stochastic Dominance sense? Under what conditions is the effect strict?
- (c) (15 points) Would the firm's profits increase or decrease if  $F$  improves in the Second-Order Stochastic Dominance sense? Under what conditions is the effect strict?
- (d) (15 points) How is the firm's optimal choice of  $q_0$  affected if  $F$  improves in the First-Order Stochastic Dominance sense?

### Question 3.

Consider an exchange economy with two goods (labeled  $x$  and  $y$ ) and two consumers (labeled A and B), with the consumers' described by utility functions

$$u_A(x_A, y_A) = x_A + y_A,$$

$$u_B(x_B, y_B) = y_B \cdot e^{x_B}.$$

Each consumer can only consume a nonnegative amount of each good. The economy's aggregate endowment of each good is 2 units.

In answering the questions below, note that there are different regions to consider. The points will be allocated equally among the regions, so that a correct answer for each region will be worth the same number of points.

- (a) (30 points) Characterize the set of Pareto Optimal allocations in this economy, and depict them in an Edgeworth box.
- (b) (30 points) Calculate the Walrasian Equilibrium of this economy as it depends on the allocation of initial endowments between the consumers.