Question 1.

A single-output firm can choose the number N of plants to operate. When a plant is shut down (does not operate), it does not produce any output and does not incur any cost. When a plant operates, it can produce any output $q \geq 0$ at a cost given by a differentiable function $c(q)$. In addition, the firm incurs a cost $g(N)$ of operating N plants.

(a) Suppose the firm is a price-taker. Formulate its profit-maximization problem. Show that without loss of profit the firm can choose to produce the same output q in each operating plant.

(b) When the output price goes up, will the firm necessarily weakly increase (i) the number of plants it operates, (ii) the output per operating plant, (iii) the total output? Formally justify your answers. (Assume the maximizers are unique, or use the strong set order comparison.)

Now suppose the firm is not a price-taker, instead it faces a differentiable inverse-demand function $P(\cdot)$, with $P'(Q) < 0$.

(c) Formulate the firm's profit-maximization problem. Under what condition on the cost function $c(\cdot)$ can the firm without loss choose to produce the same output q in each operating plant?

(d) Assume that the condition in (c) holds, and that $g(N) = kN$ (with k interpreted as the fixed cost of operating a plant). When the fixed cost k is reduced, will the firm necessarily weakly increase (i) the number of plants it operates, (ii) the output per each operating plant, (iii) the total output? Formally justify your answers. (Assume the maximizers are unique, or use the strong set order comparison.)

Question 2.

Consider a competitive economy with uncertainty with two possible states of the world and one physical good that can be consumed in those states. Before the state is realized, consumers trade state-contingent consumption at the prices $(p_1, p_2) \in \mathbb{R}^2_{++}$. A consumer is a subjective expected-utility maximizer, with subjective probabilities $q_1, q_2 \geq 0$ of the two states ($q_1 + q_2 = 1$), a differentiable, strictly increasing and strictly concave Bernoulli utility function $u(\cdot)$, and a contingent endowment $(e_1, e_2) \in \mathbb{R}^2_{++}$ of the physical good in the two states.

(a) Formulate the consumer's utility maximization problem and write the first-order condition for this problem.

(b) Under what condition will the consumer choose to consume strictly more in state 1 rather than state 2?

(c) Suppose there are *two* consumers of the kind described above, with identical endowments and identical Bernoulli utility functions, but *different subjective probabilities* q_1, q_2 and $\hat{q}_1, \hat{q}_2$ respectively. What can you say about how the two consumers' optimal bundles compare to each other?

(d) Suppose instead there are two consumers of the kind described above, with identical endowments and *identical subjective probabilities* q_1, q_2, but *different Bernoulli utility functions*, denoted by $u(\cdot)$ and $v(\cdot)$ respectively. Formulate the condition on the two utility functions that ensures that the consumption of the consumer with utility function $u(\cdot)$ has a weakly smaller variance between the two states than that of the consumer with utility function $v(\cdot)$, for any $q_1, q_2, p_1, p_2, e_1, e_2$. Relate this condition to the notions of risk aversion studied in class.

Question 3.

Consider an exchange economy with L goods and $I = 2$ consumers with strictly convex and strictly monotone preferences described by differentiable utility functions. The endowment of each consumer $i = 1,2$ is $e^i \in \mathbb{R}^L_{++}$. The economy has a Walrasian equilibrium (p, x), where $p \in \mathbb{R}^2_{++}$ is the equilibrium price vector and $x \in \mathbb{R}^{2L}_{++}$ is the equilibrium allocation (assumed to be interior). When the endowments of the consumers are changed to $\hat{e}^i \in \mathbb{R}^L_{++}$, the economy has a Walrasian equilibrium $(\hat{p}, \hat{x})$. Based on this information, which of the statements (a)-(d) must be true? (Either prove the statement or disprove with a counterexample or an Edgeworth-box picture.)

(a) If $\hat{e}^1 + \hat{e}^2 \gg e^1 + e^2$, then at least one consumer i prefers $\hat{x}^i$ to x^i.

(b) If $\hat{p} \cdot (\hat{e}^1 + \hat{e}^2) > \hat{p} \cdot (e^1 + e^2)$, then at least one consumer i prefers $\hat{x}^i$ to x^i.

(c) If $\hat{e}^1 \gg e^1$ and $\hat{e}^2 = e^2$ then consumer 1 prefers $\hat{x}^1$ to x^1. [Hint: think about what could happen to monopoly profits when it increases its output.]

(d) If $\hat{e}^1 \gg e^1$ and $\hat{e}^1 + \hat{e}^2 = e^1 + e^2$, then consumer 1 prefers $\hat{x}^1$ to x^1.

(e) How do the answers to questions (a)-(d) change, if at all, if both consumers' preferences are known to be *quasilinear in the same good?*