Core Microeconomics
Economics 202/202N
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Final Exam

Instructions: You have three hours to complete the exam – there are 3 equally weighted questions. Please answer each question in a separate bluebook. Write the question number and your name on the cover of each blue book. The exam is closed book and closed notes. Good luck !

1. (Consumer Theory). Consider a rational price-taking consumer who spends his wealth w on three goods x_1, x_2, x_3 sold at prices p_1, p_2, p_3. We normalize the price of good 3 p_3 to 1 and observe the consumer's demand when prices p_1, p_2 and his wealth w vary locally - i.e. in some open convex set in $\mathbb{R}^3_+$. Suppose that in this set, the consumer's observed demand for goods 1 and 2 is given by

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = a - B \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}.$$

The parameter vector $a = \binom{a_1}{a_2}$ and the parameter matrix $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ are independent of the consumer's wealth w.

(a) State conditions on the parameters in a and B that are needed for the observed demand functions to arise from rational consumer behavior with monotone locally nonsatiated preferences.

(b) For which parameter values are goods 1 and 2 complements and for which are they substitutes? Do the answers depend on whether we use the concepts of gross or compensated complements/substitutes?

(c) Construct an indirect utility function for the consumer that is consistent with the observed demand functions.

(d) Construct a utility function for the consumer that yields the observed local demand functions. [Hint: there are at least two ways to do it: (i) Using the answer to part (c) and the outer-bound method, or (ii) Guessing a function form that yields the linear demands, and then solving for its parameters. Computations are simplified using the matrix notation.]

(e) Suppose now that good 1 is a "long-run" good whose consumption cannot be adjusted in the short run. Calculate the derivative of the short-run demand for good 2 with respect to its price. Compare it to the long-run demand for good 2. How does the comparison depend on the complementarity/substitutability of the two goods? Does the Le Chatelier principle hold?

2. (Profit Maximization under Uncertainty). In this question, you are asked to derive a number of comparative statics results for a firm operating in an uncertain environment. *Make sure you provide proofs for all your answers.*

A competitive firm produces a single output using $M \geq 2$ inputs. Denote the price of output by $p \in \mathbb{R}_+$. The vector of input prices is given by $w \in \mathbb{R}_+^M$. Suppose the price of some input i, w_i, is random, while all other input prices are fixed at some predetermined level $w_{-i} \in \mathbb{R}_+^{M-1}$. The firm's cost function is given by $c(q, w_i)$, where we suppress the dependence of the cost function on w_{-i} to simplify notation.

Suppose the distribution of w_i depends on some parameter $\theta \in \Theta \subseteq \mathbb{R}$. Given θ, w_i is distributed according to the cdf $F(w_i|\theta)$. When choosing the quantity of output it wants to produce, the firm observes θ but cannot observe the realization of w_i. The firm maximizes expected profits, that is, it solves $\max_{q \geq 0}\ pq - C(q, \theta)$, where $C(q, \theta) \equiv \int_{w_i} c(q, w_i)\, dF(w_i|\theta)$ is the expected cost function. Do *not* assume that $C(q, \theta)$ is differentiable in θ.

(a) Suppose that $c_{qw_i} \leq 0$. Further, suppose that if $\theta' > \theta$, then $F(\cdot|\theta')$ first order stochastically dominates $F(\cdot|\theta)$.

   i. Show that $C(q, \theta)$ has decreasing differences in (q, θ) (equivalently, show that $-C(q, \theta)$ has increasing differences in (q, θ)). *Hint: Recall that a function $f(x, t)$ has increasing differences in (x, t) if and only if f_x is weakly increasing in t.*

   ii. How does the firm's optimal output level $q^*(\cdot)$ depend on θ? Explain intuitively why this follows from our assumptions.

   iii. Are expected profits (weakly) increasing or decreasing in θ?

(b) Suppose now that if $\theta' > \theta$, then $F(\cdot|\theta')$ second order stochastically dominates $F(\cdot|\theta)$. Impose an assumption on c_q as a function of w_i that ensures that $q^*(\cdot)$ is weakly decreasing in θ.

(c) Suppose we fix θ at some level $\bar{\theta}$. Consider the following two environments:

   - UNCERTAINTY. w_i is random and distributed according to $F(w_i|\bar{\theta})$.
   - CERTAINTY. w_i is fixed at its expected value given $\bar{\theta}$, i.e., at $\overline{w}_i \equiv \mathbb{E}(w_i|\bar{\theta}) = \int_{w_i} w_i\, dF(w_i|\bar{\theta})$.

   Does the firm prefer to operate under uncertainty or under certainty? Can you explain this intuitively?

3. (General Equilibrium). Consider an economy with two goods: fishing licenses and money. Fishing licenses are indivisible; an agent can consume either no license or a single license. Money is perfectly divisible. There is a unit mass of agents. Each agent has a value v for having a fishing license and an endowment of money w. The pair (v, w) is the agent's type and types are uniformly distributed on the unit square. An agent with value v has utility function $u(x, y) = vx + y$, where $x \in \{0, 1\}$ denotes her license consumption and $y \geq 0$ denotes her money consumption. Note that an agent must consume a non-negative amount of money. We assume that there is a mass $1/2$ of fishing licenses and these are initially held by the government. We will consider three alternative allocation schemes.

Suppose the government sells the licenses for money at a market clearing price and redistributes the revenue uniformly across the population (so if the price is p buyers pay $p/2$ in money and non-buyers receive $p/2$). Assume that the redistribution takes place instantaneously so agents immediately receive their $p/2$ transfer.

(a) Find the market clearing price p^c and the resulting allocation.

(b) Find the average license value of license holders (i.e. the average v).

(c) Suppose that after the initial allocation, a resale market opens. Prove that there is no price p at which further trade could occur.

Now suppose instead that the government sets a price $p^a < p^c$ and randomly assigns the licenses to demanders if demand exceeds supply, with proceeds again distributed uniformly. Assume the randomization is independent of agents' types.

(d) Find the resulting allocation of licenses and money.

(e) Find the average license value of license holders and compare to (b).

Finally, suppose instead that the government sets a price $p^a < p^c$, randomly assigns the licenses if there is excess demand and redistributes the proceeds. Following this, a resale market opens and trade occurs at a market clearing price p^R. Assume that agents cannot hold negative money between the initial and resale markets although they can spend their government transfer $p^a/2$ in the initial market.

(f) Identify the set of agents that initially demand a license.

(g) Find the price p^R that clears the resale market as a function of the initial price p^a (it's enough to write down an equation to which p^R is the solution).

(h) Fixing the initial price p^a, how will the average license value of the eventual license holders compare to the competitive allocation and rationing without resale? (Hint: you may want to start with the case $p^a = 0$ and then consider $0 < p^a < p^c$).