Economics 202 | Jonathan Levin
Core Microeconomics | Fall 2004

Final Exam

Instructions: You have three hours to complete the exam – there are 4 questions. Please answer each question in a separate bluebook. Write the question number and your name on the cover of each blue book. The exam is closed book and closed notes. Good luck !

1. (Consumer Theory, 25 points)

(a) Consider three possible consumption bundles $x^A = (1, 3, 2)$, $x^B = (2, 2, 2)$ and $x^C = (2, 2, 1)$. Suppose Consumer A is rational. At prices $p^A = (2, 1, 1)$, you observe him choose x^A. At prices $p^B = (1, 2, 1)$, you observe him choose x^B. Now he has wealth $w = 7$ and faces prices $p = (1, \frac{3}{2}, 1)$. Can you say for certain if any of the bundles will or will not be chosen? What if A's preferences are also locally non-satiated?

(b) Consumer B has the following indirect utility function

$$v(p_1, p_2, w) = \left(\frac{w}{p_1}\right)^{\frac{1}{4}} \left(\frac{w}{p_2}\right)^{\frac{3}{4}}$$

i. Calculate the expenditure function $e(p_1, p_2, u)$.

ii. Calculate the Hicksian demand functions $h_1(p_1, p_2, u)$ and $h_2(p_1, p_2, u)$.

iii. Calculate the equivalent variation if $w = 10$ and prices change from $p = (2, 1)$ to $p' = (1, 1)$. Interpret your result.

(c) Suppose that Consumer C's Marshallian demands for goods 1 satisfies $\partial x_1(p;w)/\partial p_2 > 0$ for all (p,w). If good 1 is a normal goods for her, what property can you deduce about her expenditure function $e(p, u)$?

(d) Suppose Consumer D has rational, continuous and locally non-satiated preferences, and that her preferences are also *homothetic*, i.e. if $x \sim x'$ then $\alpha x \sim \alpha x'$. What can you conclude about how Consumer D's Marshallian demand $x(p, w)$ will vary with w?

2. (Producer Theory) Consider a firm that has n teams that need to be managed. Suppose team j has productivity y_j with $y_1 < ... < y_n$. If a manager of ability x assigned to team j its output will be $f(x, y_j)$, where f is continuously differentiable.

For parts (a)-(c), assume that the firm has already hired n managers, denoted $i = 1, ..., n$. Each manager i has ability x_i with $x_1 < ... < x_n$. Let $\tau : \{1, ..., n\} \rightarrow \{1, ..., n\}$ denote an assignment of managers to teams where $\tau(i) \neq \tau(j)$.

(a) Suppose firm revenue is given by $\sum_{j=1}^{n} f(x_{\tau(j)}, y_j)$. Assume $f_x, f_y, f_{xy} > 0$. What assignment maximizes firm revenue? Prove your claim.

(b) Suppose we maintain $f_{xy} > 0$, but relax the assumptions that $f_x, f_y > 0$. Is the optimal assignment necessarily the same?

(c) Suppose firm revenue is given by $\prod_{j=1}^{n} f(x_{\tau(j)}, y_j)$. Identify sufficient conditions on f for the revenue-maximizing assignment to be assortative, i.e. $\tau(j) = j$.

For parts (d)-(e), assume that rather than being endowed with managers, the firm must hire them on the outside labor market. Hiring a manager of ability x requires paying a salary $w(x)$, where $w(\cdot)$ is continuous and increasing. A hiring policy is a vector $(x_1, ..., x_n)$, where x_j is the ability of the manager hired to manage team j. An optimal hiring policy maximizes firm profit, i.e. its revenue minus its salary expenditures.

(d) What assumptions would allow one to conclude that the optimal hiring policy satisfies $x_1 \leq ... \leq x_n$ if firm revenue the sum of team outputs? If firm revenue is the product of team outputs?

(e) Suppose that $f_x, f_y, f_{xy} > 0$. Suppose the productivity of team 1 increases. How will this affect the optimal hiring rule if firm revenue is the sum of team outputs? If firm revenue is the product of team output?

3. (Choice Under Uncertainty) Ann is an expected utility maximizing individual with a strictly increasing, strictly concave and twice-differentiable Bernoulli utility function $u(\cdot)$ defined over wealth. She has initial wealth w. One day she goes to the race-track and has to opportunity to bet on a race with $n \geq 2$ horses, $i = 1, ..., n$. Ann assesses the probabilities of horses $1, ..., n$ winning as $p_1, ..., p_n$, with $\sum_i p_i = 1$. A ticket that pays 1 in the event that horse i wins costs r_i, with $\sum_i r_i = 1$. Ann can buy as many tickets as she likes (assume she can buy ticket fractions and also that she never wants to spend more than her whole initial wealth) taking the prices as fixed. Let $x_1, ..., x_n$ denote her purchases.

(a) Write Ann's optimization problem.

(b) Show that if $p_i/r_i > p_j/r_j$ then Ann will bet more on horse i than on horse j.

(c) Show that if $p_i/r_i > 1$, then Ann will gamble a strictly positive amount on horse i.

(d) Assume that $n = 2$ and that Ann has DARA preferences. Show how her optimal ticket purchases will change with her initial wealth w.

4. (General Equilibrium, 25 points). Consider an economy with 2 agents and 2 commodities. The agents have the following Cobb-Douglas utility functions:

$$u^1(x, y) = \alpha \ln(x) + (1 - \alpha) \ln(y)$$
$$u^2(x, y) = \beta \ln(x) + (1 - \beta) \ln(y)$$

with $\alpha \in (0, 1)$ and $\beta \in (0, 1)$.

Individual endowments are given by $e^1 = (0, 1)$ and $e^2 = (1, 0)$.

(a) Derive each agent's Marshallian demand a function of p_x and p_y.

(b) Normalize $p_x = 1$. Compute the Walrasian equilibrium allocation and prices.

Now suppose a firm enters this market. The firm is a profit maximizer and produces good y using x as an input. The firm's production function is given by $f(x) = x$, but (unlike what we have studied in class) has a fixed cost of production given by $F > 0$. The firm only pays the fixed cost if it produces. If the firm produces nothing, it does not have to pay the fixed cost.

(c) Suppose $\alpha = \frac{2}{5}$ and $\beta = \frac{4}{5}$. Does a Walrasian equilibrium exist? If so give the Walrasian equilibrium allocation and prices. If not, explain why an equilibrium does not exist.

(d) Suppose $\alpha = \frac{1}{5}$ and $\beta = \frac{3}{5}$. Does a Walrasian equilibrium exist? If so give the Walrasian equilibrium allocation and prices. If not, explain why an equilibrium does not exist.