Core Microeconomics
Economics 202, Fall 2005

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Final Exam

Instructions: You have three hours to complete the exam – there are 4 equally weighted questions. Please answer each question in a separate bluebook. Write the question number and your name on the cover of each blue book. The exam is closed book and closed notes. Good luck !

1. (Choice Theory) Consider a preference relation $\succeq$ over a finite set X defined by a utility function $u : X \to \mathbb{R}$ in the following way:

$$a \succeq b \Longleftrightarrow u(a) \geq u(b) - 1. \tag{*}$$

(The interpretation is that $a \succeq b$ as long as the utility improvement of b over a is "imperceptible".) Must this preference relation be

(a) reflexive,

(b) complete,

(c) transitive?

The preference relation $\succeq$ defines the corresponding "strictly preferred" relation $\succ$ and "indifference" relation $\sim$ as follows:

$$\begin{aligned} a &\succ b \Longleftrightarrow \text{"}a \succeq b \text{ and not } b \succeq a\text{"} \\ a &\sim b \Longleftrightarrow \text{"both } a \succeq b \text{ and } b \succeq a\text{"} \end{aligned}$$

(d) Must the strictly preferred relation $\succ$ be transitive?

(e) Must the the indifference relation $\sim$ be transitive?

Suppose now that we are instead *given* a complete preference relation $\succeq$ over a finite set X, whose corresponding "strictly preferred" relation is transitive. Can we always find a utility function $u : X \to \mathbb{R}$ such that $\succeq$ is represented by (*)? Prove or disprove with a counterexample, distinguishing between two cases:

(f) X has three elements,

(g) X has more than three elements.

2. (Consumer and Producer Theory) A monopolistic firm sells its output y to N consumers, whose utilities are functions of their consumptions of good y and the numeraire good, and are quasilinear in the numeraire good. Consumers make their purchase decisions taking the price p set by the firm as given (All prices will be in terms of the numeraire good.) Let $Q(p)$ denote the consumers' aggregate demand for good y as a function of its price p.

The firm's production function is $f(k,l)$, where k and l are the inputs of capital and labor, respectively, purchased at prices r and w, respectively. Assume that the production function is increasing and has constant returns to scale.

The firm chooses price p to maximize its profits in terms of the numeraire good.

(a) Derive the total Marshallian surplus of the consumers as a function of the price p set by the firm.

(b) Compare the output resulting from the firm's profit maximization pricing to the Pareto efficient output level.

Suppose now that the government levies a tax τ on each unit of output bought by the consumers, so the total price consumers pay becomes $p+\tau$. You are asked to make monotone comparative statics predictions about the effects of the tax. In some of the questions, you may need to make additional assumptions on the production function or the demand curve to make such a prediction. How does the tax rate τ affect

(c) the firm's output,

(d) its demands for capital and labor,

(e) the price it sets?

Now suppose that we observe the firm's profit-maximizing output $y(\tau)$ at all tax rates τ.

(f) Calculate the firm's cost function.

(g) Is observation of $y(\tau)$ enough to fully infer the firm's production technology?

3. (Choice Under Uncertainty) Consider a manager who is compensated on the basis of performance If profits are x, the manager's compensation is $c(x)$ and her utility is $u(c(x))$, where u is an increasing concave function. The manager must choose between a safe strategy that leads to a known profit or a risky strategy that leads to uncertain profits. The question is how her compensation will affect her choice of strategy. You can assume that profits are always non-negative.
For parts (a) and (b), suppose compensation has the form $c(x) = a + bx$, with $a, b \geq 0$.

(a) What condition on u ensures that a higher choice of a is more likely to induce the risky strategy?

(b) What condition on u ensures that a higher choice of b is less likely to induce the risky strategy?

(c) What compensation schedule guarantees that the manager will choose the strategy that leads to highest expected profit?

4. (General Equilibrium). Consider an economy with two goods: food and club memberships. Each agent has identical preferences $u(x, y) = (1 + \lambda y)x$, where $x \geq 0$ is food consumption, $y \in \{0, 1\}$ is club membership and $\lambda > 0$. There is a unit mass of agents, of which half are endowed with club memberships, so the total mass of memberships is $1/2$. The agents endowed with memberships and those not endowed with memberships both have food endowments uniformly distributed on $[0, 1]$. Suppose the agents can trade both food and club memberships. Normalize the price of food $p_x = 1$ and let $p_y = p$ denote the price of club memberships. (Note that with a continuum of agents, market clearing means that the mass of buyers is just equal to the mass of sellers.)

(a) Derive each agent's Marshallian demand as a function of his endowment.

(b) Explain why in a Walrasian equilibrium there must be trade in club memberships.

(c) Compute the Walrasian equilibrium prices and allocation.

(d) Suppose instead that $u(x, y) = \left(1 + \frac{\lambda}{2Y} y\right) x$, where Y is the aggregate consumption of club memberships, so that club memberships become less valuable as more people consume them. Prove that the the Walrasian equilibrium allocation in this setting is not Pareto efficient, i.e. the first welfare theorem fails.