

Computer vision for automated experiments with superconducting qubits

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To build a full-scale quantum processor it is necessary to automate as many steps as possible on the physical, hardware level. Circuit quantum electrodynamics (cQED) is a contemporary architecture for dispersive readout and Purcell protection of superconducting qubits of various types, and thus it is necessary to develop software that is able to perform every kind of automatic calibration of such systems from scratch without any human participation. The first step towards this goal is to build a noise-insensitive and accurate computer vision tool to process the three-dimensional spectroscopic data that are ubiquitous in the domain. In this work we present and describe two widely-applicable and scalable algorithms that are capable to robustly and efficiently extract all Hamiltonian parameters from the spectroscopic data.

I. INTRODUCTION

Computation on a quantum computer is realized by analog manipulations of large numbers of physical quantum bits (qubits). One of the most significant challenges is that today's qubit systems are not produced identical and cannot be operated identically^{1,2}: typically, control parameters for a device are determined by following a carefully chosen sequence of calibration experiments, and vary among the devices. Since manual tuning of these parameters is not a scalable solution, operating a few dozens of qubits requires developing an autonomous system able to handle the physical device calibration on its own. Moreover, a lot of research is still being done to find the optimal chip designs and fabrication methods. It includes gathering statistics, i.e. many different samples should be measured and consistently evaluated. An automated tool not only will significantly speed up this process, but as well exclude human error.

In this work, we are proposing and implementing several methods of data processing and computer vision that allow for automatic calibration of circuit QED³ architectures used for the readout of superconducting qubits. Our method is accurate, fast and robust to noise, is applicable to superconducting qubits with a periodic and parametrizable spectrum, and is compatible with the state-of-the-art paradigm of one readout resonator per one qubit^{4,5}. Therefore, the suggested approach is scalable and can be applied to processors with any number of qubits.

The flow of an experiment that characterizes a single qubit on a chip is depicted schematically in Fig. 1(a). It is similar to the classic procedures described in Refs. [2 and 6] for measuring cQED samples with tunable qubits. Additionally, it is close conceptually to the patent application Ref. [7] describing automatic calibration for non-tunable qubits.

First of all, it is necessary to detect the positions of the resonance peaks corresponding to the readout resonators. Resonator search provides the scan range for the probe frequency Δf_p in single-tone spectroscopy (STS). STS records the behaviour of the resonance peak while changing the magnetic field applied to the sample (controlled by some current I). By processing the results of the STS, one can find the parameters of the cQED system. Precisely, coarse estimation $f_{ge}^{(0)}(I)$ of the shape of the dependence of the qubit frequency on current and fine estimation of the resonator frequency $f_r(I)$ and coupling strength g . Using these data, one may set the scan range for the two-tone spectroscopy (TTS) in excitation frequency Δf_{exc} and current ΔI . Next, from TTS, we can obtain various qubit transition frequencies dependence on current $f_{ge}(I), f_{gf/2}(I)$, etc. with accuracy sufficient for pulsed experiments. After that, pulsed calibrations may be started to optimize, for example, single qubit gate fidelities. As one can see, the process is structured so that the results of one measurement generally define the parameters of the next, successively acquiring more accurate information about the physical cQED system.

All the measurement outcomes may be divided in two groups: some results contain 2D data (a single-valued function $f(x)$ on XY plane), and others contain 3D data (heatmap images). The scope of this work is confined to the automatic analysis the latter type of experimental data. In Fig. 1(b),(c) we illustrate examples of the correlated measured data and theoretical model. Fig. 1(b) shows that STS data should be interpreted using two discontinuous curves (upper row) or a single continuous curve (lower row) that are defined by the model. Then, Fig. 1(c) shows how qubit spectral lines should be detected. The main difficulties are that several spectral lines that need detection may be observed even for a single qubit (and thus the model is not single-valued), that spurious non-qubit transitions should be ignored, and the noise.

Since the implementation of our methods relies on the theoretical description of studied physical systems, in Appendices A and B we derive all the necessary equations to form the

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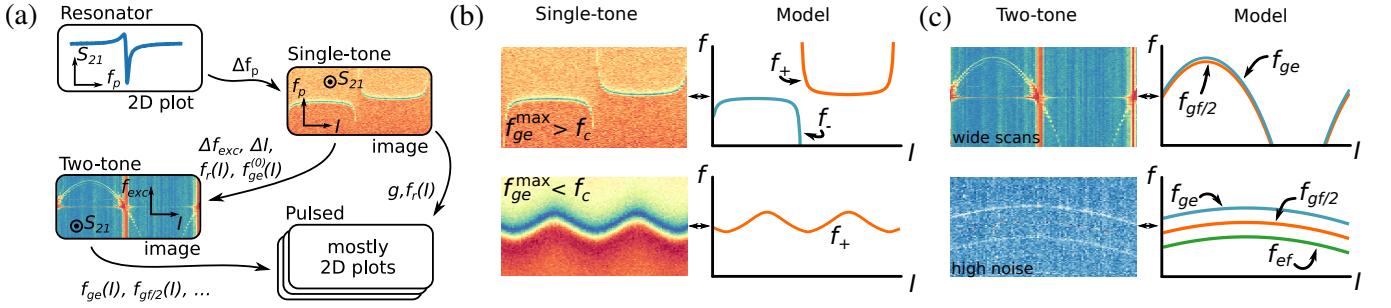


FIG. 1. (a) Common experiment flow for a tunable-frequency qubit: usually, three continuous wave spectroscopic measurements are performed before pulsed experiments, and each next experiment is based on the results of the previous ones. (b) Examples of the desired results after the detection for two qualitatively different STS outcomes. The heatmaps should be brought into correspondence with the 2D model curves. In the first row, two model curves that should be fitted simultaneously since the model is discontinuous. (c) Examples of the desired results after processing TTS data. The heatmap should be brought into correspondence with the model of the qubit spectrum which may not be single-valued since several transitions may be visible.

model curves that should be found in single- and two-tone spectroscopy data. In this work we will take a transmon qubit with asymmetric SQUID as an example, however for different types of qubits the logic will stay the same. For the single-tone spectroscopy, they model curves are (see Fig. 1(b)):

$$f_{\pm}(I) = f_r(I) = \frac{f_c + f_{ge}(I)}{2} \pm \sqrt{g^2 + (f_{ge}(I) - f_c)^2/4}, \quad (1)$$

where \$f_c\$ stands for current-independent bare cavity frequency. Here we note that for the avoided crossings pattern both curves are necessary; alternatively, for the cases when the qubit is entirely below (above) the bare cavity frequency only \$f_+(I)\$ (\$f_-(I)\$) is used.

For the two-tone spectroscopy, we take the dependence of the qubit transitions on current \$f_{ge, gf/2, ef}(I)\$ are expressed as

$$f_{ge}(I) = f_{ge}^{\max} \left[\cos^2 \left(\frac{\pi(I - I_{ss})}{\Pi} \right) + d^2 \sin^2 \left(\frac{\pi(I - I_{ss})}{\Pi} \right) \right]^{\frac{1}{4}}, \\ f_{gf/2} = f_{ge} + \alpha/2, \quad f_{ef} = f_{ge} + \alpha \quad (2)$$

where \$\Phi_0\$ is the flux quantum, \$d\$ is the SQUID asymmetry, \$\alpha\$ is the anharmonicity of the transmon equal to its negative charging energy \$-E_C\$, and \$f_{ge}^{\max}\$ is the qubit frequency at \$I = I_{ss}\$; \$I_{ss}\$ is current exactly compensating the non-zero residual flux that is always present in experiment and coming from the local magnetic fields on the chip (sweet-spot current). Finally, \$\Pi\$ is the period of the spectrum in current.

In overall, we have 6 fitting parameters for the single-tone spectroscopy (\$f_c, g, \Pi, I_{ss}, f_{ge}^{\max}, d\$) and 5 parameters for the two-tone (\$\Pi, I_{ss}, f_{ge}^{\max}, d, \alpha\$).

Finally, the structure of the rest of the paper is as follows. First, we will describe our fitting method for the STS data; then, we will cover the algorithm to detect qubit spectral lines on TTS data. Finally, the fitting accuracy, performance and reliability of both procedures will be assessed.

II. METHODS

In this section the actual algorithms that perform fitting and data extraction are discussed. Additionally, we describe important peculiarities of the data itself and some essential experimental details. Without loss of generality, we will consider a tunable transmon⁸ qubit capacitively coupled to a notch-type readout resonator. This architecture is currently widely used⁹ and therefore serves as a good example to describe the algorithm.

A. Analysing resonator spectra

In the real life applications, the spectrum of the resonator is usually recorded with a vector network analyzer which measures the complex scattering parameter \$S_{21}\$ for each probe frequency \$f_p\$ in a certain area around the resonance. Hence, after the VNA scan there are two 2D plots for the amplitude and the phase of the transmission versus frequency. When the magnetic flux sweep is added, the data becomes three-dimensional and has to be represented visually using a heatmap plot. One such heatmap for a notch-port resonator coupled to a transmon qubit is presented in Fig. 2(a).

Fitting the image shown Fig. 2(a) with the model equations (1) is not a trivial task because it belongs to a class of optimization problems where the loss function (i.e. in the least squares algorithm) is ill-defined. This is due to the periodic dependence of the frequencies on current \$I\$ and unknown position of the qubit sweet spot \$I_{ss}\$ due to the local residual flux. Fortunately, it is possible to get a good initial guess for these parameters and then brute force the solution in other variables since we have a well defined bounds for each of the variables left. Finally, we can polish the result in all six parameters using a Nelder-Mead search.

The outline of the method which uses Fig. 2(a) as an example is presented below. The chosen type of the resonator-qubit arrangement which yields the avoided crossing pattern is not universal: there are two other cases when the qubit spectrum lies entirely below or above the resonator frequency, however,

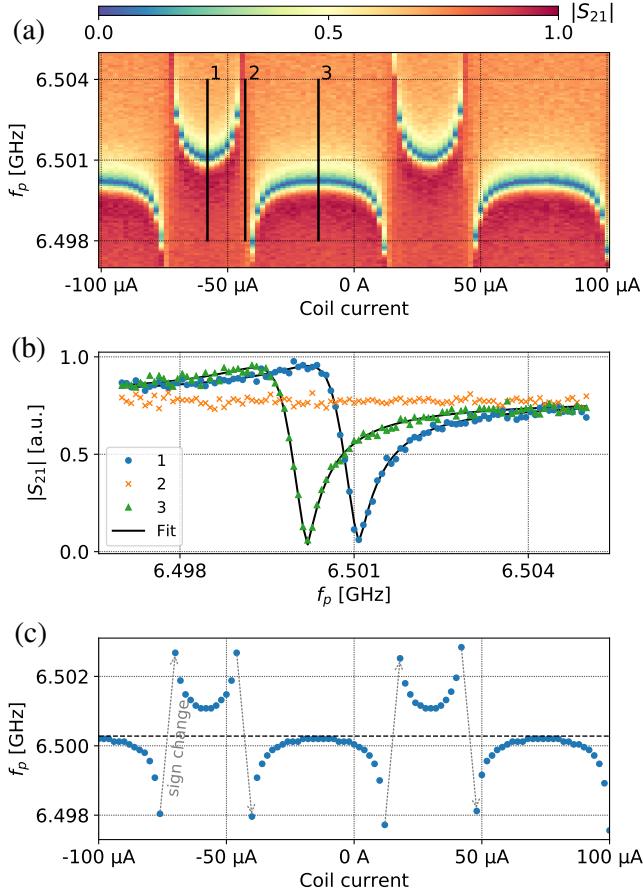


FIG. 2. (a) An experimental spectrum of a resonator strongly coupled to a transmon qubit depending on the coil current I . (b) Slices of the transmission from (a) showing two slices with fits (1,3) and a plateau with no dip (2) present. (c) Extracted data (blue dots) and mean frequency value over all points $\langle f_r \rangle_I$ (black dashed line). Grey arrows show where $f_r - \langle f_r \rangle_I$ changes sign.

they are treated exactly the same way and are simpler in terms of the loss function behaviour at all steps of our algorithm.

1. Extracting $f_r(I)$ from data

The first thing to do is to reduce the dimensionality of the data, i.e. extract the resonance frequency for each I and then be left with the desired $f_r(I)$.

We do this by employing the *circlefit*¹⁰ library which is capable of fitting various types of microwave resonators. For each I we fit the complex transmission $S_{21}(f_p)$ as in Fig. 2(b) (solid black lines) and extract the resonance frequency from the model. The only possible caveat is that for some I values the resonance dip may disappear (see Fig. 2(b)) and the fit will fail. Such slices are thus excluded in advance via a threshold condition.

The resulting plot of extracted frequency is shown in Fig. 2(c) with blue dots. There are some current values located between the branches for which the points are missing,

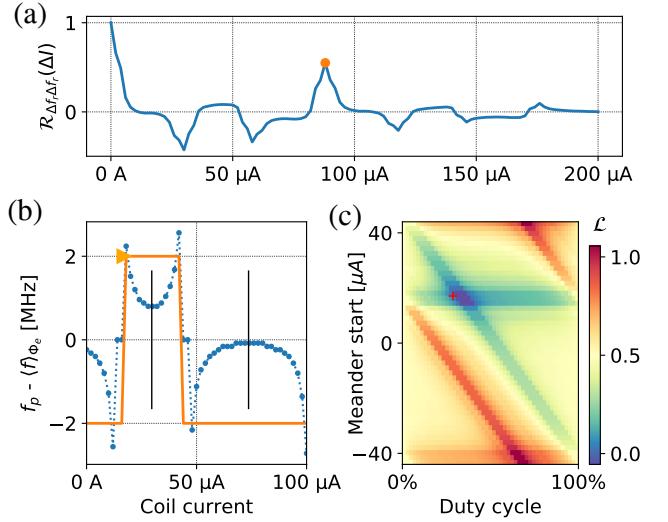


FIG. 3. Period and phase extraction procedure. (a) Autocorrelation function depending on the lag ΔI shows a prominent local maximum at 88 μA (orange dot). (b) Construction for phase estimation; $\Delta f_r = f_r - \langle f_r \rangle_I$ (blue) is fitted with a square wave (orange, start marked with a triangle). Vertical bars mark candidate sweet spots. (c) Loss function (normalized) for the square wave fitting procedure from (b); red cross indicates the parameters of the meander shown there.

as expected. Additionally, we plot here another key parameter which is the mean value of the detected frequencies shown as a dashed black line. Firstly, this parameter will serve as an initial guess for the cavity frequency of the model (1): $f_c \approx \langle f_r \rangle_I$. Secondly, it serves as a baseline used in the following period and phase extraction algorithm which tracks the changes of the sign of the value $\Delta f_r = f_r - \langle f_r \rangle_I$ (marked as grey dashed lines in Fig. 2(c)). These sign changes are crucial in determining the location of the qubit sweet spot.

2. Extracting period, phase and sweet spot locations

As we have said above, one of the serious obstacles for the fitting is that the periodicity of the data on one of the fitting parameters. Along with the equally unknown phase of the signal, this leads to the presence of many local minima in the loss function which severely impede the progress of iterative optimization algorithms. In other words, the unknown parameters Π and I_{ss} are preventing us to find reliably the global minimum we need. Fortunately, it is possible to find the period and the sweep spot location (or, the phase of the data) without fitting the full model (1) which alleviates the stated problem. In the absence of noise, this would be a trivial task. However, in the real-life conditions the noise to some extent is always present, so below we engineer a solution that is insensitive to local sporadic perturbations of the data.

A powerful tool for finding the period in a given dataset y especially when it contains just a few periods is the autocorrelation function $\mathcal{R}_{yy}(l) = \sum_n y_n y_{n-l}$. The location of the largest of its local maxima (except for the $l = 0$) equals ex-

actly the sought period. However, this is only true when the mean value of the function is zero; otherwise, $\mathcal{R}_{yy}(l)$ would be linear with a very steep slope smearing out all the extrema. Therefore, instead of directly calculating the autocorrelation of $f_r(I)$ we consider the function $\Delta f_r(I)$ introduced above which has zero mean and the same period. For this function, which looks exactly like one from Fig. 2(c) but centered vertically around zero, the autocorrelation function $\mathcal{R}_{\Delta f_r \Delta f_r}(\Delta I)$ is shown in Fig. 3(a). As one can see, it depends on the current offset ΔI which spans $200 \mu\text{A}$ just as the data itself. This means that the data is being zero-padded at all ΔI except for $\Delta I = 0$, and this is why we get a diminishing value of the correlation peaks when the lag reaches Π or 2Π . The orange dot in the plot shows the highest local extremum of $\mathcal{R}_{\Delta f_r \Delta f_r}(\Delta I)$ at $88 \mu\text{A}$. It is a very prominent peak and can be easily distinguished among all others. There is as well a small peak at $176 \mu\text{A}$ which corresponds to $\Delta I = 2\Pi$. Note that the the autocorrelation function is not smooth. This is because there were some missing points in the data (corresponding to the plateaus of Fig. 2(b)) that were replaced by zeros to ensure correct mapping between current indices and current values. This can be noticed as well in Fig. 3(b).

Now, having found the period it is possible to precisely determine the phase of the signal. This is done via finding a global maximum of zero-lag correlation function $\mathcal{R}_{\Delta f_r S}(0)$ between Δf_r and a square wave $S(I, \Pi, \phi, D)$ signal having the same period but unknown phase ϕ and duty cycle D . An illustration of a meander function satisfying the optimal condition is presented in Fig. 3(b) in orange. The phase ϕ (in μA) denotes the x-coordinate of the first point after the rising edge, and is marked with a triangle. The high and low levels of the square must be opposite in value, i.e. 1 and -1, and the absolute value does not matter. Generally, the idea behind this is to robustly detect sign changes that were shown back in Fig. 2(c). We can not rely on a more simple algorithm that walks through the points and marks where the function changes its sign because such algorithm will fail in the presence of a strong noise. Conversely, the optimization algorithm described above is not sensitive to local short-term sign changes.

The global optimal ϕ and D are found using a brute force algorithm. It calculates the loss function $\mathcal{L} = -\mathcal{R}_{\Delta f_r S}(0)$ on a 50×50 grid of (ϕ, D) and takes the minimal value of all. It is feasible due to the evident boundaries on $\phi \in [-\Pi/2, \Pi/2]$ and $D \in [0, 1]$. The loss function topography for the avoided crossing patterns is nicely structured and for our example is shown in Fig. 3(c). One can see that instead of a single minimum it has an area of the same minimal value. Again, the roots of this effect lie in the missing f_r points. However, any value from this valley suits well enough for our purposes, and the algorithm finds no difficulty in locating it.

With values for Π , ϕ and D we now can calculate the currents of the transmon sweet spots in the case of the avoided crossings pattern and the smooth patterns:

$$I_{ss} = \begin{cases} \phi + \Pi(1+D)/2, & \text{for avoided crossings} \\ \phi + \Pi D/2, & \text{otherwise} \end{cases} \quad (3)$$

To distinguish between these two cases when the noise is not

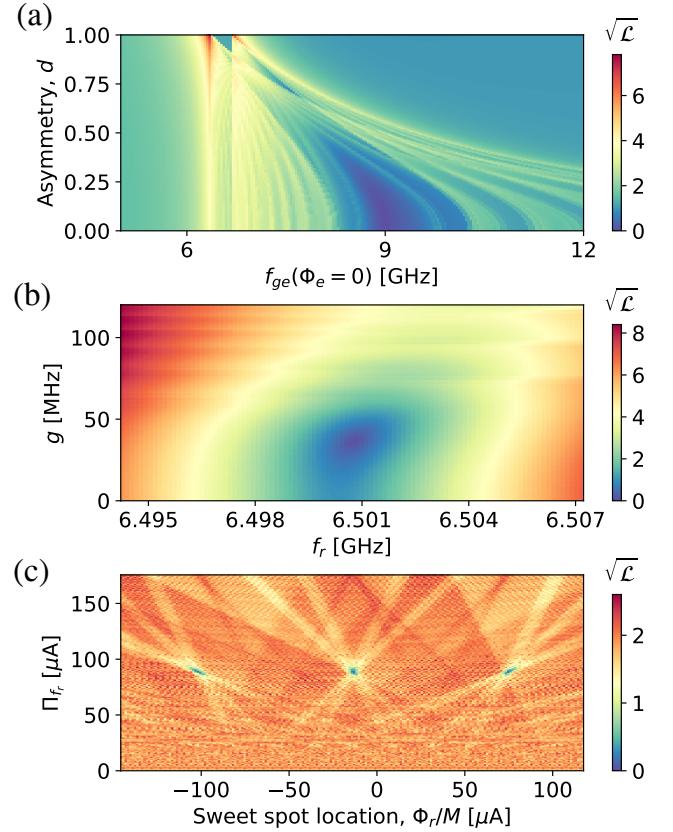


FIG. 4. Slices of the loss function (4) for the full model and experimental data from the example made around the optimal point. Root-mean-square per-point loss $\sqrt{\mathcal{L}}$ is in MHz. (a) f_{ge}^{\max} and d are varied, other parameters optimal. A large valley is located near 9 GHz, and some smaller locally minimal ones are present all around. (b) g and f_r are varied, others optimal. The loss function for these two parameters is well-conditioned near the optimum. (c) Period $\Pi_{f_r} = \Phi_0/M$ and Φ_r/M varied, others optimal. This subplot illustrates a complex structure of local minima around the true one which we find analytically.

too large, it is enough to calculate the maximal absolute differential of the frequency data $\max_{i>0} |f_{r,i} - f_{r,i-1}|$ and compare it to the peak-to-peak amplitude $\max_{i,j} |f_{r,i} - f_{r,j}|$. For the avoided crossings these values are close and for the smooth curves they are not. However, in the presence of noise this indicator may fail, and one will have to check both current values to be I_{ss} and choose based on the best fit among the two possibilities.

3. Full model fitting

Having performed the aforementioned preliminary steps, it is now possible to fit the full model to the extracted points. To do this, we employ brute force optimization combined with Nelder-Mead¹¹ search. For both methods, we use a common loss function defined as follows. For the known probe frequency span of the data Δf_p (Fig. 2(a), whole y-axis) and the

set of N extracted points $\{p_i\} = \{(I_i, f_{r,i})\}$, we calculate the loss function as

$$\mathcal{L} = \frac{1}{N} \sum_{i=0}^N [f_{r,i} - \mathcal{M}(I_i, \Pi, I_{ss}, f_c, g, f_{ge}^{max}, d)]^2, \quad (4)$$

$$\mathcal{M} = \begin{cases} f_+, |f_+ - f_c| < \Delta f_p/2 \\ f_-, \text{ otherwise,} \end{cases} \quad (5)$$

where, as in the previous section,

$$f_{\pm}(I_i) = f_{\pm}(I_i, \Pi, I_{ss}, f_c, g, f_{ge}^{max}, d)$$

The condition of Eq. (5) means that we choose only the frequencies that lie within a window Δf_p around the model f_c parameter. This ensures, firstly, that in the optimum we will not take any excess points outside the frequency scan, and, secondly, that we have a single model value for each current.

To substantiate the choice of the optimization algorithms, we present in Fig. 4 three visualizations of the the defined loss function. The plots show how the function behaves if a certain pair of 6 model parameters is varied. From the plots it is obvious that it is ill-defined and has a lot of local minima which would impede robustness in real life applications. The f_{ge}^{max} and d parameters present the most significant difficulty in terms of false minima as can be seen from Fig. 4(a). In contrast, f_c presents the least difficulty, as can be seen from Fig. 4(b). The last plot Fig. 4(c) shows a very complex structure of the loss function and a very small-sized optimal valley and serves as an illustration of why the period and phase extraction is important.

B. Analysing qubit spectra

Unfortunately, the resonator fitting procedure accuracy in terms of the qubit frequency (>100 MHz) is not enough in practical applications where it is necessary to obtain exact value of the qubit frequency down to 1-10 MHz. For the cQED systems, it may be done, in general, by trying to excite the qubit at different frequencies and then read it out expecting the e -state to be measured. The task is to find the exact location and shape of all the spectral lines depending on the magnetic flux that will be detected using such a method.

Particularly, in many groups, the qubit spectrum is searched for using the so-called two-tone spectroscopy¹². It consists of sending two microwave signals at the cQED system from which one, the probe tone, has a fixed frequency near f_c , and the frequency f_{exc} of the other, excitation tone, is varied. When the second signal becomes resonant with some transition (i.e., qubit ge transition), a shift of the cavity frequency occurs, and thus the transmission at the frequency of the first tone changes. This way, a heatmap showing the dependence of the frequency of various system transitions on the magnetic flux can be obtained.

An example from our database for a tunable transmon coupled to a notch-port resonator is displayed in Fig. 5. To obtain the two-tone spectrum in Fig. 5(b), for each current value the

frequency of the first tone f_p is chosen to be equal to the frequency of the minimal transmission from the single-tone spectroscopy (STS), see Fig. 5(a). At the avoided crossing, this is not possible, and thus the frequency f_p may be chosen to be arbitrary within the scan area of the STS. This does not pose a problem since the two-tone spectroscopy is still only effective when the qubit and the resonator are detuned far from each other, i.e. in the dispersive regime¹³, and would not work well near the resonant regime.

The resulting plot Fig. 5(b) shows the dependence of S_{21} on the magnetic flux and the excitation frequency. Most of the scan is in blue meaning that the resonator is not shifted and the probe tone gets almost entirely reflected (see colorbar). However, there are some different colour areas. Firstly, a bright and narrow horizontal line is observed near 6.5 GHz where the resonator is located. Likewise, there are two sharp horizontal lines just above 6.75 GHz which are from the crosstalk coupling to another on-chip resonator. Secondly, bright vertical stripes are seen at the current values corresponding to the avoided crossing areas. Finally, two transmon transitions of varying frequency are visible. The upper one is the ge transition, and the lower one is the two-photon $gf/2$ one; at each coil current, they are separated by $E_C/2h$, or half the anharmonicity of the transmon (see Table III).

The presence of several bright lines along with noise makes it in general impossible to accurately fit the data with the model (A3) using the standard curve fitting procedures. Additionally, when several multi-photon transitions are visible, it is hard for an algorithm to distinguish between them to find only the ge transition. This means that for robust qubit detection we need to fit all possible single- and multi-photon spectral lines

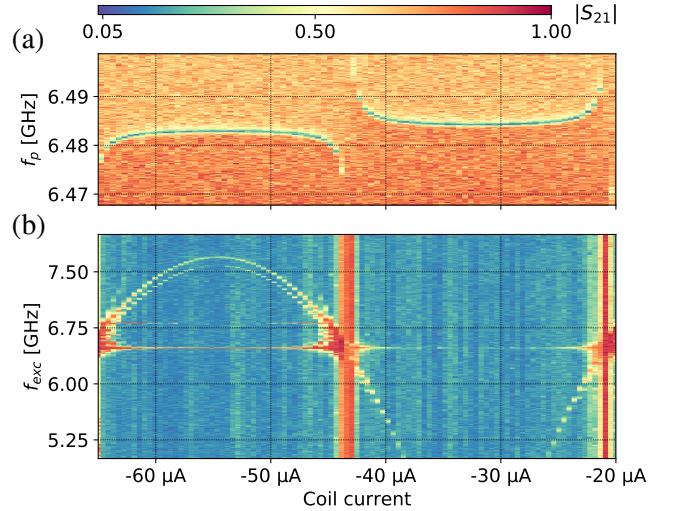


FIG. 5. Two types of spectroscopy results side by side; the colorbar is common, coil current scan range is the same. (a) Single-tone spectroscopy (one period on I). (b) Two-tone spectroscopy result. Clear flux-dependent transmon transitions are visible (ge and $gf/2$), and some fixed-frequency resonator excitations are present as well. Red vertical stripes are observed when the qubit and the cavity are close to resonance.

simultaneously. Below, we present our approach to solve this problem.

1. Thresholding the data

Similar to the reasoning in section II A 1, the first thing to do is to reduce the three-dimensional picture Fig. 5 to two dimensions. This is done in several steps.

First, we calculate the median absolute transmission median $[S_{21}]_{f_{exc}}(I)$ for each current value (or frequency column). It gives us the background dependent only on I and not on f_{exc} which can be then subtracted from the data; then, the absolute value of the difference $Z = |S_{21} - \text{median}[S_{21}]_{f_{exc}}|$ may be plotted. The subtraction both removes the background and in each data column highlights the points that are distinct in either amplitude or phase. For our example, the resulting image can be seen in Fig. 6. As one can notice, the background subtraction removes most of the vertical stripes that were present in Fig. 5(b).

Next, the data points may be extracted to form a 2D point array where the qubit lines may be searched for. This procedure is done in several steps. First of all, it is necessary to do thresholding of the data to obtain a binary image. We have tested various thresholding methods, i.e. Otsu's method¹⁴; however, the best performance in distinguishing local peaks from background was obtained using another method that is based on estimating the noise in the data. We estimate noise level σ by calculating the numerical difference between adjacent points in a row for each flux value, and then calculate the median absolute value:

$$\sigma = \text{median}[|Z_{i,j+1} - Z_{i,j}|]_{i,j},$$

where i numerates current values, and j numerates excitation frequency values. Using median instead of mean reduces the impact of singular bright extrema that may be present in the row.

Next, for each row we use the *find_peaks* routine of the Scipy library to detect all local extrema there, and then exclude those whose height is less than σ as probably originating from the noise.

After finishing the procedures above, we get an array of points that correspond to the most prominent peaks in the data. However, already at this point it is possible to remove all points that come from the fixed-frequency resonator spectral lines. This is done by excluding horizontal rows of points that have exactly the same frequency value within the resolution of the data.

The final extracted points are shown in Fig. 6 in black crosses. As one can see, the horizontal lines are correctly not included, and two qubit lines are almost fully represented. Background noise is mostly ignored, too.

2. Fitting binary data

a. General outline of the method. In most cases, even the filtered binary data contains noise and, importantly, is not

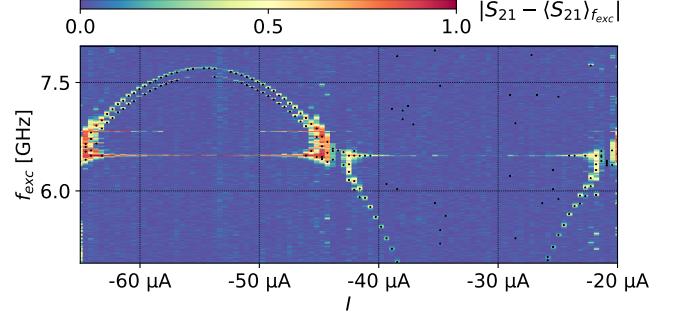


FIG. 6. Processed data and extracted and filtered datapoints (black crosses) that will be subject to fitting.

single-valued for each flux value. This means that standard curve fitting methods are not applicable, and thus we resort to a hybrid algorithm that combines features of the Hough transform¹⁵ for global optimization and Nelder-Mead search local optimization. The algorithm bases on three main parts: the point selection procedure, solution ranking and parameter sampling. The selection procedure is universal, and the latter two parts come in two variations depending on which type of optimization is used.

The general idea for the first part is illustrated in Fig. 7. Here, three narrow frequency bands of Δf are chosen around the model spectral lines calculated for tested parameters by (2). Points outside the bands are considered far away and are ignored. All the points inside a band are attributed to its line and included in its loss function. If at a certain flux there are several points inside a band, only the closest one is chosen and the rest (contentious) are ignored. Let $P_{1,2,3}$ be the sets of points selected for corresponding transition lines $ge, gf/2, gd/3$ (in blue, orange and green in Fig. 7); the number of points in the sets are $N_{1,2,3}$, $N_\Sigma = \sum_i N_i$, and M is the total number of measured magnetic flux values. Finally, let $D_{1,2,3}$ be the corresponding sets of vertical distances between the selected points and the corresponding spectral lines.

The second part of the algorithm now has to evaluate the quality of the fit, i.e. be able to rank a supplied sets of optimization parameters. The ranking differs for the brute search and for the local optimization.

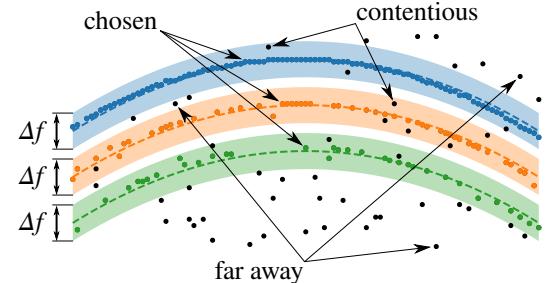


FIG. 7. An illustration for the loss function definition. In different colours are the frequency bands and the corresponding selected points, black points are ignored.

In case of the brute search, the evaluation is based both on the total number of selected points N_Σ and the normalized squared distances $\langle D_\Sigma^2 \rangle = \sum_i \sum_{P_i} D_i^2 / N_1$ (note that we normalize based just on N_1 to respect the majority voting in case of low N_2, N_3). Maximizing the total number of points included ought to have a higher priority than minimizing the average distances, since otherwise a single selected point lying exactly on the spectral line would win. However, in the presence of spurious spectral lines this prioritization may lead to wrong results since an incorrect solution with a slightly larger N_Σ but high normalized distance measure would overshadow the correct one with a low distance measure. To battle this effect and increase the influence of the distance loss, we quantize N_Σ into bins of size N_b equal to 25% of the total number of fluxes M , and then choose from the best bin the solution having the lowest distance measure.

Alternatively, for the Nelder-Mead algorithm one should build a loss function that returns just a single value instead of a double-valued rank as before. Since we still want to both maximize the number of points and minimize the mean distances, we take again same binned value $[N_\Sigma/N_b]N_b$ and $\langle D_\Sigma^2 \rangle$ but now calculate the function

$$\mathcal{L}_{N\text{-}M} = \frac{1}{[N_\Sigma/N_b]N_b} + \langle D_\Sigma^2 \rangle. \quad (6)$$

This loss function has a convex shape near the sought extremum; this makes it possible to run the Nelder-Mead algorithm without risk of escaping the optimal valley.

Finally, the third part of the algorithm is either the brute force search to find the global extremum or the Nelder-Mead descent to polish the found parameters.

Note the similarity between this construction and the Hough transform. The transform is implemented by quantizing the Hough parameter space into finite intervals or accumulator cells that each point votes for. Likewise, we use the frequency band (frequency quantization) to find the points voting for a certain parameter set; other parameters are as well quantized on a brute search grid. However, our approach is extended by a subsequent local optimization necessary since we want to improve the accuracy beyond brute grid resolution.

To sum up, the local optimization is done on points belonging to a narrow frequency band around the spectral line when this band contains as many points as possible. This logic is applicable both to single-line and multi-line fitting. In the following, we will use single-line fitting to find at least one spectral line in the point cloud, and then full multi-line fitting to detect all lines that may be present.

b. Single-line optimization First of all, we refine the given coarse initial values for the qubit parameters. We assume that the supplied period Π is accurate which is true if the initial guess stems from the single-tone spectroscopy fitting, and that the other parameters may be inaccurate due to noise/magnetic flux shifts. So, only the sweet spot location $I_{ss}^{(0)}, f_{ge}^{max,(0)}$ and $d^{(0)}$ are optimized by brute force. The refinement is performed using a single spectral line by two brute force searches and then a Nelder-Mead optimization on the chosen points.

Optimization		I_{ss}	f_{ge}^{max}	d
$\Delta f=100$ MHz	range	$I_{ss}^{(0)} \pm .05 \Pi$	$f_{ge}^{max,(0)} \pm 30\%$	0.1 - 0.9
	steps	10	50	8
$\Delta f=50$ MHz	range	$I_{ss}^{(1)} \pm .02 \Pi$	$f_{ge}^{max,(1)} \pm 100$ MHz	$d^{(1)} \pm 0.1$
	steps	10	20	10

TABLE I. Grid specifications for the brute force algorithm in the single-line fitting. First iteration is a coarse search and the second is more accurate.

First brute force is done with a wide frequency band $\Delta f=100$ MHz around it and a grid specified in Table I. We do a rather wide search of 10% of the period Π around the initial sweet spot position $I_{ss}(0)$ since in our data there are sometimes inconsistencies between the single-tone and the two-tone spectroscopy that have been recorded on different days. In the real experiment, though, it is not usually necessary since flux drifts are rare if the two-tone scan is taken right after the single-tone. Due to rather large Δf value, the first search is used only to generally locate the extracted points in the parameter space. The found coarse optimum is denoted as $I_{ss}^{(1)}, f_{ge}^{max,(1)}$ and $d^{(1)}$.

Then, the second brute force is launched with contracted parameter ranges around the coarse optimum (see Table I) and reduced Δf . This search already is aimed to locate the near-optimal point in the parameter space. Importantly, this step may not find exactly the highest line $f_{ge}(I)$, but it certainly will find one of the lines that has the most points in it, i.e. $f_{gf/2}(I)$, and thus get an incorrect value for $f_{ge}^{max,(2)}$. This behaviour can't be overcome except than by fitting multiple lines at the same time. For the other two parameters $I_{ss}^{(2)}$ and $d^{(2)}$ there is no ambiguity, and they are estimated accurately.

Finally, the chosen points from the second brute force are used to fit the model using the Nelder-Mead algorithm. This procedure is allowed to polish the 3 parameters from the second brute search and

III. RESULTS

A. Resonator spectra

The brute force algorithm acts on the grid specified in Table II. The ranges in the grid are based on the usual design parameters of the modern qubit samples and the number of steps is chosen to be able to catch the optimal valley in the loss function.

After the coarse brute force optimization is done and the optimal valley is located, we apply the Nelder-Mead simplex downhill algorithm to polish the brute force result. This algorithm works very well in our case, as one can see from the Table II comparing the brute estimation and the final result after Nelder-Mead is performed. The significant improvement in the loss value of the polished result compared to the brute estimation is due to the more accurate determination of the cavity frequency f_c which leads as well to major shifts in op-

Parameter	Value range	Steps #	Brute est.	Final
f_c	$\langle f_r \rangle_I \pm 1$ MHz	3	6.5003 GHz	6.5007 GHz
g	20 - 40 MHz	5	24 MHz	36.1 MHz
f_{ge}^{max}	4 - 12 GHz	80	8 GHz	8.9 GHz
d	0 - 0.9	9	0.5	0.08

TABLE II. Grid specifications for the brute force algorithm and the resulting optimal points. The number of steps is taken with some margin to ensure correct location of the optimal valley even for difficult cases. The optimal parameters estimated by brute force and the final ones have the RMS loss values of 0.258 MHz and 0.024 MHz, respectively.

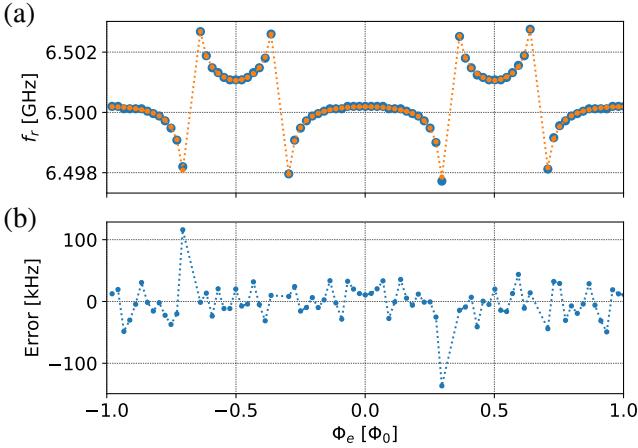


FIG. 8. The result of the algorithm execution. (a) The data (blue dots) and the model (orange connected dots). (b) Residual error which is mostly random and is of order of 25 kHz per point.

timal g , f_{ge}^{max} and d .

Finally, the resulting fit is presented in Fig. 8 along with the plot of the residuals. The match is almost perfect, and the noise in the residuals is random and thus shows no systematic errors. In Fig. 9 we also present the fitting results for different powers of added noise. The signal-to-noise ratio (SNR) is defined for our data in a similar manner as the SNR in the resonator fitting tests¹⁰; we take distance $2r$ between two maximally remote points of $S_{21}(f_p, I)$ among any pair on the complex plane (i.e., the diameter of the resonance circle) as the signal amplitude, and the noise of the form $\frac{\xi_1 + i\xi_2}{\sqrt{2}}$, where ξ_1, ξ_2 distributed normally with zero mean and variance (amplitude) σ . The SNR then is defined as r/σ . As one can see from the graph, the algorithm stays robust even at very low SNRs that are even lower than the low limit SNRs for the *circlefit* stability. This is possible because a fallback strategy of taking the smallest amplitude point as the resonance is applied when *circlefit* fails. Ultimately, the key point of this stress test is to show that the algorithm can be applied in real-life scenarios when the data may be of low quality.

The fitting algorithm was implemented in Python using brute and Nelder-Mead routines from Scipy. The fitting example above runs for 3.2 seconds on a 5-year old Intel Core i5-3337U CPU, from which 2.7 seconds are for the brute and

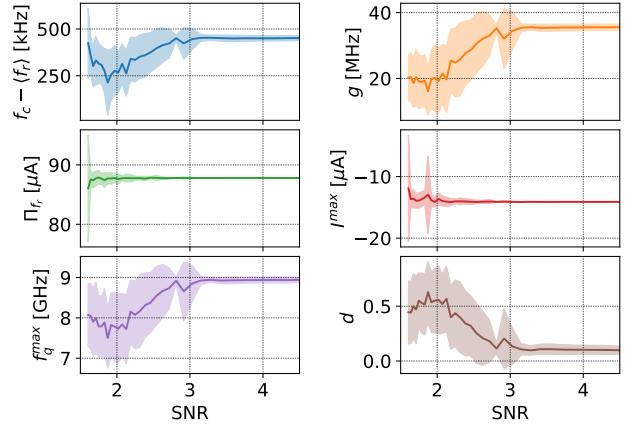


FIG. 9. Behaviour of the algorithm on the data with the gaussian noise of varying noise power added. The clouds show standard deviations of optimal parameters after 50 tries, solid lines show mean values. The algorithm is very stable above SNR=8 but is still robust at detecting avoided crossings down to SNR=3-4 but with reduced accuracy in f_{ge}^{max} , g and d . Other three parameters are still very accurate.

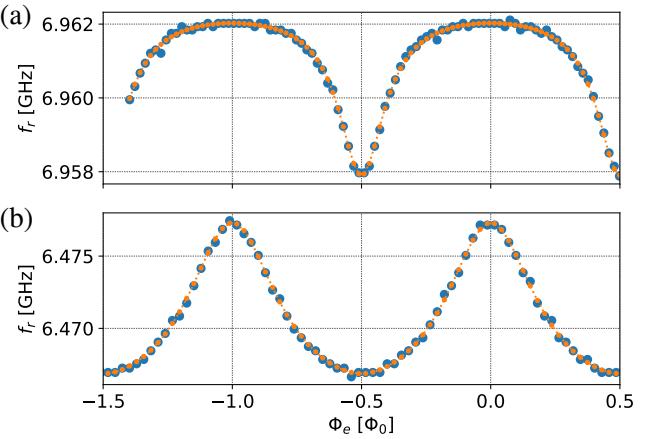


FIG. 10. Fitting two other samples of our dataset presenting other two possible cases (blue dots are data, orange connected dots are the model). (a) The qubit is entirely above the resonator. Per-point RMS error is 60 kHz. (b) The qubit is below the resonator. Error is 150 kHz (larger due to the proportionally larger amplitude of the frequency variation).

0.4 of a second is for the Nelder-Mead minimization. On a contemporary Intel Core i7-7700 CPU, it takes about one second to run the whole procedure. The time costs mostly come from the square root calculation necessary for Eqs. (A3) and (B3) and because that the brute routine is not parallelized. However, this is still fast, firstly, because recording a resonator spectrum such as one in the example takes around 20 seconds, and, secondly, it is much faster than a human would do.

We have tested the algorithm on more than 100 real spectra that we have in our experimental database, and it finds an optimum for any kind of the dependence of the resonance on the

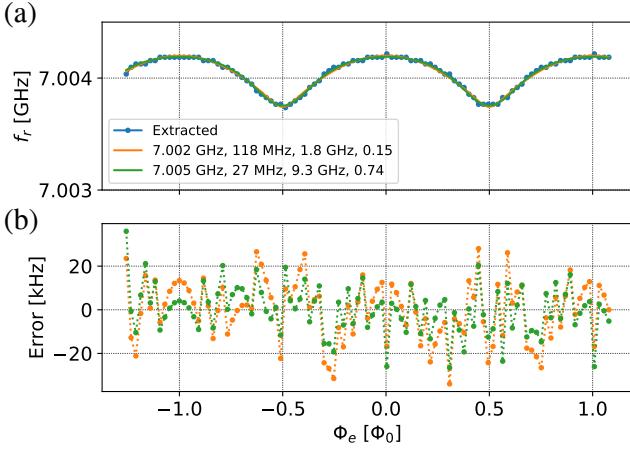


FIG. 11. (a) Two alternative fits (orange and green lines) for the same data (blue dots). The corresponding parameters f_c , g , f_{ge}^{max} , d for the two alternative fits are in shown the legend. (b) Residuals for the two fits, in orange (qubit below) and green (qubit above). RMS errors are 14 kHz and 11 kHz, respectively.

flux caused by various dispositions of the qubit and the resonator, as in Fig. 13. Example fitting results for the variants in Fig. 13(b,c) are provided in Fig. 10. There is a caveat, however, in some of such cases. It turns out, that for large qubit-cavity detunings two different sets of parameters minimizing the loss function equally well are possible. An illustration for this statement can be seen in Fig. 11(a),(b). The algorithm was launched using different grid ranges for f_q^{max} . In one case (orange) the search was done below, and in the other (green) above the mean resonance frequency $\langle f_c \rangle_I$. The resulting fits are equally accurate, and without other information about the system it would not be possible to choose between the two. Therefore, some hints should be passed to the algorithm to avoid ambiguity; for instance, whether to look for the qubit above or below the resonator. Otherwise, it would be necessary to check both possibilities with other methods, i.e. via two-tone spectroscopy.

IV. DISCUSSION

Appendix A: Transmon Hamiltonian

The simplest version of this qubit consists of a Josephson junction shunted with a large capacitor. Flux tunability of the frequency is attained by replacing a single Josephson junction with a SQUID as in Fig. 12(a) and applying external magnetic flux Φ_e to its loop. This configuration can be equivalently represented with a shunted junction of tunable energy, Fig. 12(b). The Hamiltonian for such equivalent circuit is as follows:

$$\hat{H}_{tr} = 4E_C\hat{n}^2 - E_J(\Phi_e)\cos\hat{\phi}, \quad (\text{A1})$$

where $E_{J1,2}$ are the Josephson energies, $E_C = e^2/2C_\Sigma$, $C_\Sigma = C_s + C_1 + C_2$, is the charging energy, \hat{n} and $\hat{\phi}$ are the operators for the Cooper pair number and the phase of the qubit island.

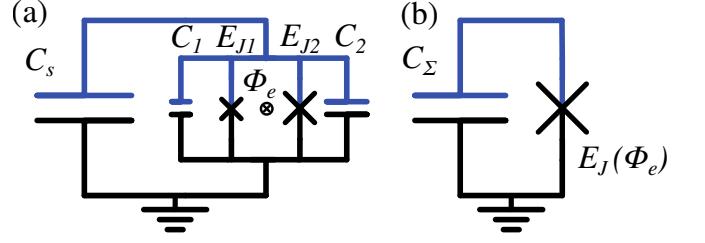


FIG. 12. (a) A tunable transmon circuit with an asymmetric SQUID, $E_{J1} \neq E_{J2}$. (b) Equivalent transmon with tunable energy $E_J(\Phi_e)$ and unified capacitance C_Σ . The qubit island containing its single degree of freedom is in blue.

Level	Energy	Transition	Frequency
$g(E_0)$	0	ge	ω_{ge}
$e(E_1)$	$\sqrt{8E_J E_C} - E_C$	$gf/2$	$\omega_{ge} - 0.5E_C$
$f(E_2)$	$2\sqrt{8E_J E_C} - 3E_C$	$ef, gd/3$	$\omega_{ge} - E_C$
$d(E_3)$	$3\sqrt{8E_J E_C} - 6E_C$	$ed/2$	$\omega_{ge} - 1.5E_C$
E_4	$4\sqrt{8E_J E_C} - 10E_C$	$fd, eE_4/3$	$\omega_{ge} - 2E_C$

TABLE III. Energies and some transition (single and multi-photon) frequencies for the first 5 levels of the transmon calculated with (A3).

For the equivalent Josephson energy E_J one obtains

$$E_J(\Phi_e) = E_{J\Sigma} \cos(\pi\Phi_e/\Phi_0) \sqrt{1 + d^2 \tan^2(\pi\Phi_e/\Phi_0)}, \quad (\text{A2})$$

where $E_{J\Sigma} = E_{J1} + E_{J2}$, $d = \frac{E_{J1} - E_{J2}}{E_{J1} + E_{J2}}$ is the asymmetry of the SQUID. As one can notice, the dependence is periodic in Φ_e .

It is also possible to derive analytical expressions for the energy levels and transition frequencies for this type of qubits. The energy of the m^{th} level is⁸

$$E_m = m\sqrt{8E_J(\Phi_e)E_C} - \frac{E_C}{12}(6m^2 + 6m), \quad (\text{A3})$$

and some of the transition frequencies are presented in Table III. The qubit frequency may be approximated as

$$\begin{aligned} f_{ge}(\Phi_e) &\approx \sqrt{8E_J(\Phi_e)E_C} \\ &= f_{ge}^{max} \sqrt{\cos(\pi\Phi_e/\Phi_0)} \sqrt{1 + d^2 \tan^2(\pi\Phi_e/\Phi_0)}, \end{aligned} \quad (\text{A4})$$

where $f_{ge}^{max} = \sqrt{8E_J(0)E_C}$. This simplifies the expression for the frequency since now it depends on two parameters instead of three.

One final note is that in real-life applications is not possible to know directly the flux Φ_e that is threaded through the SQUID. The experimenter usually knows only the current I (or voltage) which he applies to some coil that is connected inductively to the SQUID. Then $\Phi_e = MI + \Phi_r$, where M stands for the mutual inductance of the coil and the SQUID, and Φ_r is some residual flux inherent to the sample.

Appendix B: Circuit QED

The readout of the superconducting qubits is now predominantly done using an ancilla system which is usually im-

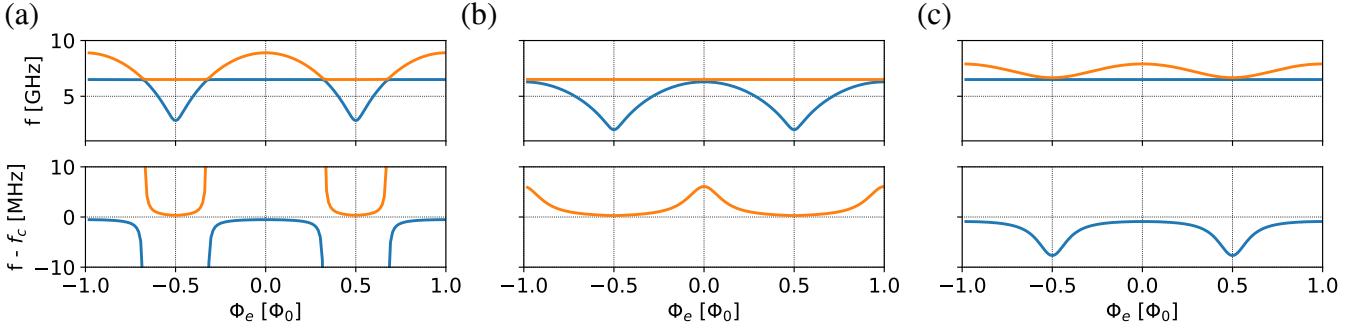


FIG. 13. Frequency spectrum of the transmon-resonator system. Parameters used: $f_{ge}(0) \approx \sqrt{8E_C E_J(0)}/2\pi = 8.5$ GHz, $d = 0.3$, $f_r = 6.4$ GHz, $g = 30$ MHz. For each subplot two transition branches $f_{\pm} = (E_{\pm,0} - E_{g,0})/2\pi$ are shown (orange and blue, respectively) both forming the resonator and qubit lines. As one can notice, there are three qualitatively different cases of the resonator-qubit disposition. Lower row shows a zoomed area around f_c that looks differently in each case.

plemented as a superconducting microwave resonator which acts as an electromagnetic cavity in the standard cavity QED. Truncating the qubit to two levels, one may obtain the following Hamiltonian for the compound cavity-qubit system (in RWA):

$$\hat{H}/\hbar = \frac{f_q}{2} \hat{\sigma}_z + f_c \hat{a}^\dagger \hat{a} + g(\hat{\sigma}^- \hat{a}^\dagger + \hat{\sigma}^+ \hat{a}), \quad (\text{B1})$$

where f_q is the qubit frequency, f_c is the cavity frequency and g is the coupling strength. As long as the RWA is done, this Hamiltonian may be diagonalized analytically¹³:

$$E_{g,0}/\hbar = \frac{f_c - f_q}{2}, \quad (\text{B2})$$

$$E_{\pm,n}/\hbar = (n+1)f_c \pm \frac{1}{2} \sqrt{4g^2(n+1) + (f_q - f_c)^2}. \quad (\text{B3})$$

This is very convenient for our purposes. By substituting the dependence of the qubit frequency $f_q \equiv f_{ge}(\Phi_e)$ into these equations, we can get straightforwardly the full system spectrum in dependence on the magnetic flux. In Fig. 13 we have used the equations (A3), (B2) and (B3) to model a tunable transmon interacting with a cavity for various Φ_e and various f_{ge}^{max} , d . In the lower row of the figure, one can see that it is possible to extract the dependence of the cavity frequency f_c on Φ_e ; for example, the well-known avoided crossing pattern can be directly observed in Fig. 13(a), and the other two possible behaviours for the qubit entirely above or below the resonator in Fig. 13(b),(c). To shorten the notation, in the following we will define the corresponding branch frequencies of (B3) as $f_{\pm} = (E_{\pm,0} - E_{g,0})/2\pi$.

In Fig. 13(a) it is also possible to see the entire spectrum of a transmon *ge* transition predicted by equations (A2), (A3). It has a cosine-like periodic shape with a period of one flux quantum Φ_0 . Consequently, it has two extrema, the upper and the lower which are called “sweet spots” due to the first-order insensitivity to Φ_e , and thus to possible flux noise. In the following, by saying sweet spot we will assume the upper one whose frequency is $f_{ge}(\Phi_e = 0) \equiv f_{ge}^{max}$.

We will use the model (B2), (B3) to fit the resonator frequency that we can find in an experiment. The only conceptual problem for the fitting that is left now is that the function

we want to use as a model is not single-valued. Indeed, in Fig. 13(a, top) for each value of magnetic flux we always find two frequency points corresponding to the qubit and to the resonator, respectively. However, in practice only a narrow scan around the resonator frequency such as in Fig. 13(a, bottom) is required, and thus no ambiguity occurs.

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