Politecnico di Torino

# Data Science Lab

## Numpy: Numerical Python

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- Numpy (*Numerical Python*)
  - Store and operate on **dense** data buffers
  - **Efficient** storage and operations
- Features
  - Multidimensional arrays
  - Slicing/indexing
  - Math and logic operations
- Applications
  - Computation with vectors and matrices
  - Provides fundamental Python objects for data science algorithms
    - Internally used by scikit-learn and SciPy

- **Summary**
  - Numpy and computation **efficiency**
  - Numpy **arrays**
  - **Computation** with Numpy arrays
    - Broadcasting
  - **Accessing** Numpy arrays
  - Working with arrays, other functionalities

- **array** is the main object provided by Numpy
- Characteristics
  - Fixed Type
    - All its elements have the **same type**
  - Multidimensional
    - Allows representing vectors, matrices and n-dimensional arrays

- Numpy arrays vs Python lists:
  - Also Python lists allow defining multidimensional arrays
    - E.g. my_2d_list = [[3.2, 4.0], [2.4, 6.2]]
- Numpy advantages:
  - Higher **flexibility** of indexing methods and operations
  - Higher **efficiency** of operations

# Python lists vs NumPy

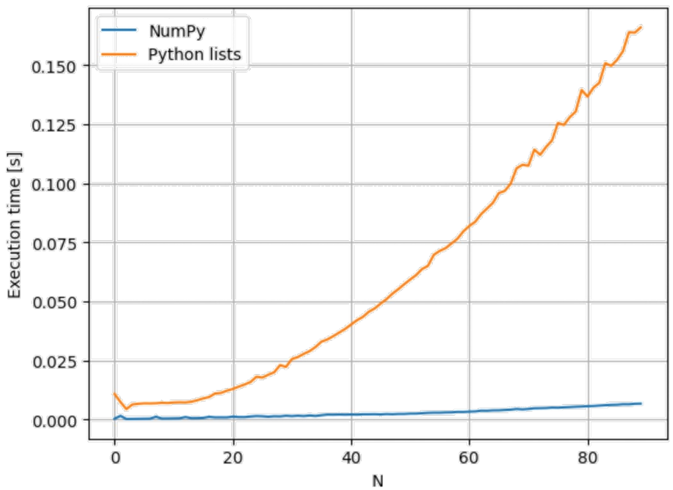
- "Build two randomly initialized NxN matrices A and B, then add them element-wise and place the output in C"

Python lists

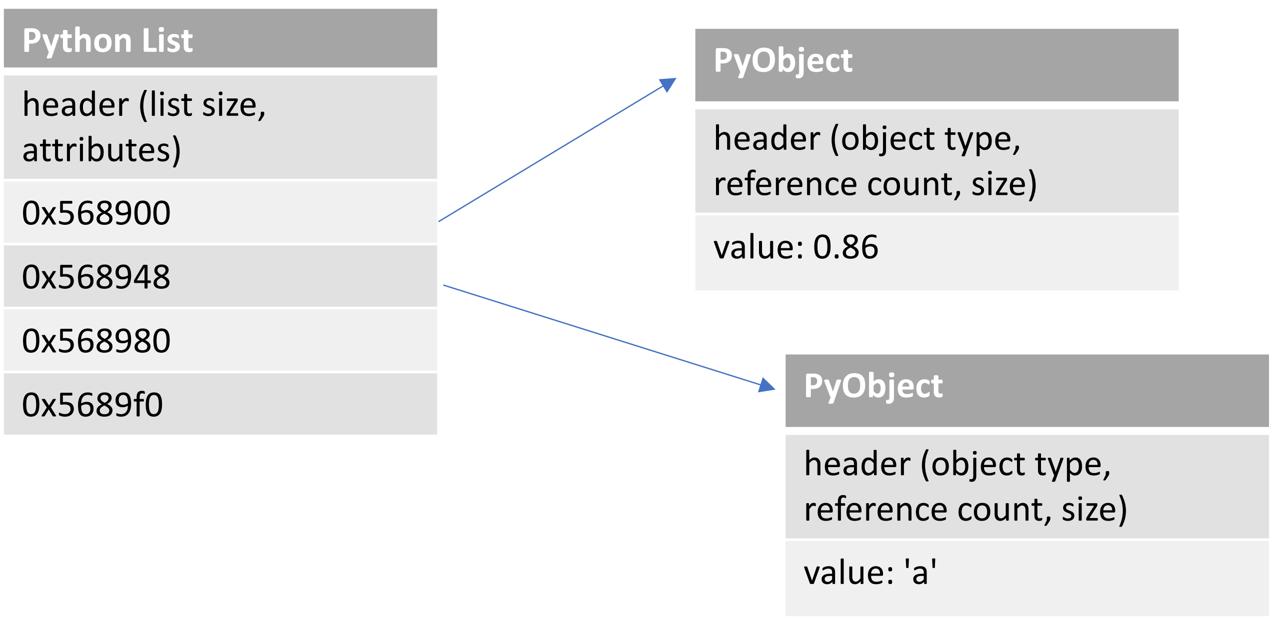
```python
from random import random

def build_random_matrix(n):
    mat = []
    for i in range(n):
        row = []
        for j in range(n):
            row.append(random())
        mat.append(row)
    return mat

n = 100
A = build_random_matrix(n)
B = build_random_matrix(n)

C = []
for i in range(n):
    row = []
    for j in range(n):
        row.append(A[i][j] + B[i][j])
    C.append(row)
```

NumPy

```python
import numpy as np

n = 100
A = np.random.random((n, n))
B = np.random.random((n, n))
C = A + B
```

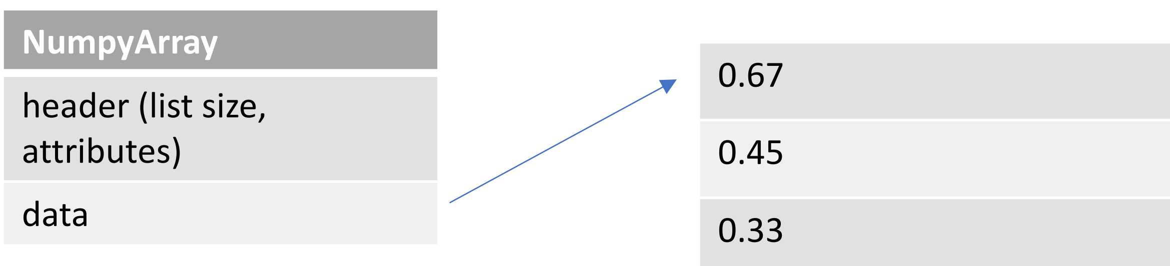


- Since lists can contain heterogeneous data types, they keep **overhead** information
  - E.g. my_heterog_list = [0.86, 'a', 'b', 4]



# Introduction to Numpy

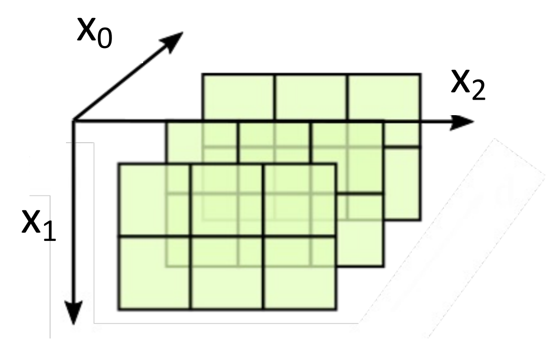
- Characteristics of numpy arrays
  - **Fixed-type** (no overhead)
  - **Contiguous** memory addresses (faster indexing)
  - E.g. my_numpy_array = np.array([0.67, 0.45, 0.33])



- Numpy data types
  - Numpy defines its own data types
  - Numerical types
    - int8, int16, int32, int64
    - uint8, … , uint64
    - float16, float32, float64 (or half, single, double)
  - Boolean values
    - bool

# Multidimensional arrays

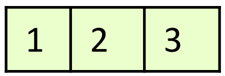
- Collections of elements organized along an arbitrary number of dimensions
- Multidimensional arrays can be represented with
  - Python lists
  - Numpy arrays



# Multidimensional arrays

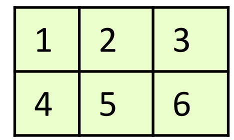
- Multidimensional arrays with **Python lists**
  - Examples:

vector



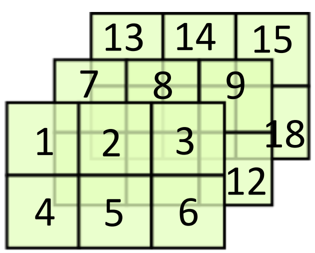

```
list1 = [1, 2, 3]
```

2D matrix




```
list2 = [[1,2,3], [4,5,6]]
```

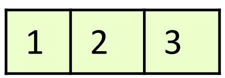
3D array

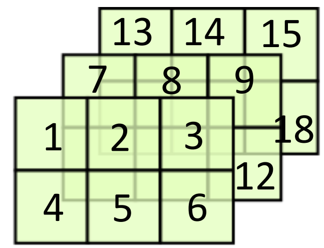



```
list3 = [[[1,2,3], [4,5,6]],
         [[7,8,9], [10,11,12]],
         [[13,14,15], [16,17,18]]]
```

# Multidimensional arrays

- Multidimensional arrays with **Numpy**
  - Can be directly created from Python lists
  - Examples:



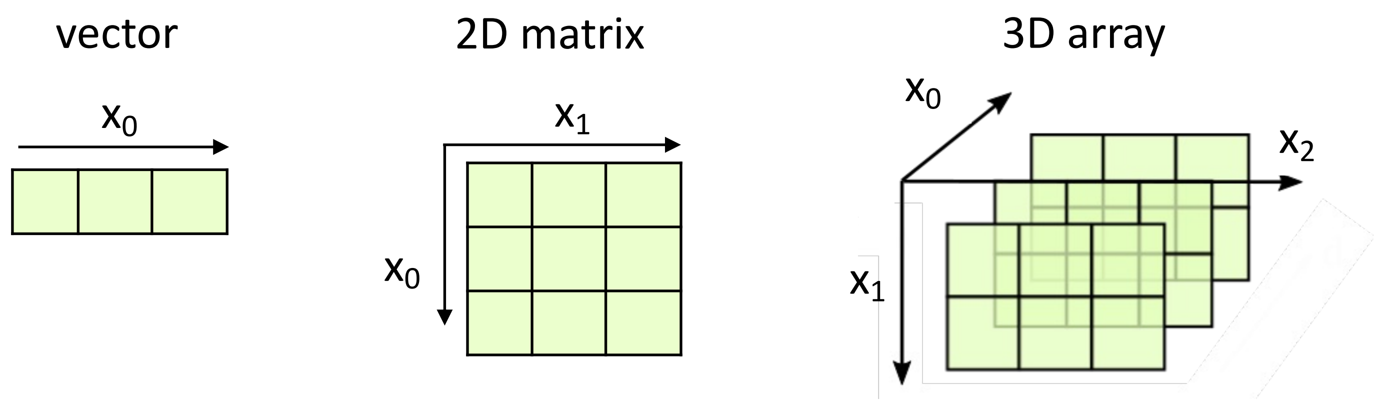




```python
import numpy as np
arr1 = np.array([1, 2, 3])
```

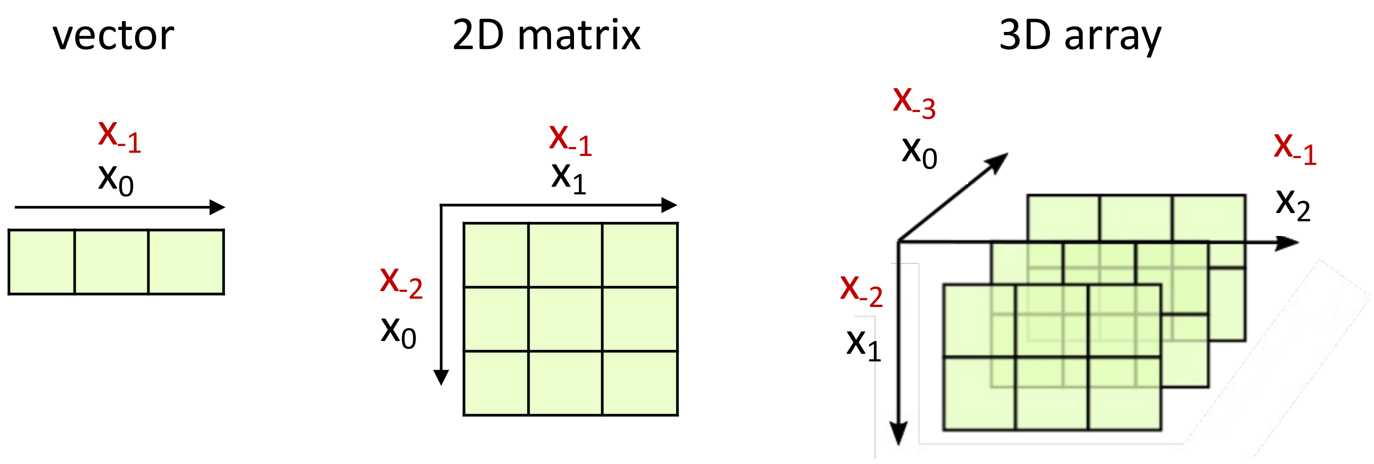
```python
import numpy as np
arr2 = np.array([[[1,2,3], [4,5,6]],
                 [[7,8,9], [10,11,12]],
                 [[13,14,15], [16,17,18]]])
```

# Multidimensional arrays

- Multidimensional arrays with **Numpy**
  - Characterized by a set of **axes** and a **shape**
  - The **axes** of an array define its dimensions
    - a (row) vector has 1 axis (1 dimension)
    - a 2D matrix has 2 axes (2 dimensions)
    - a ND array has N axes

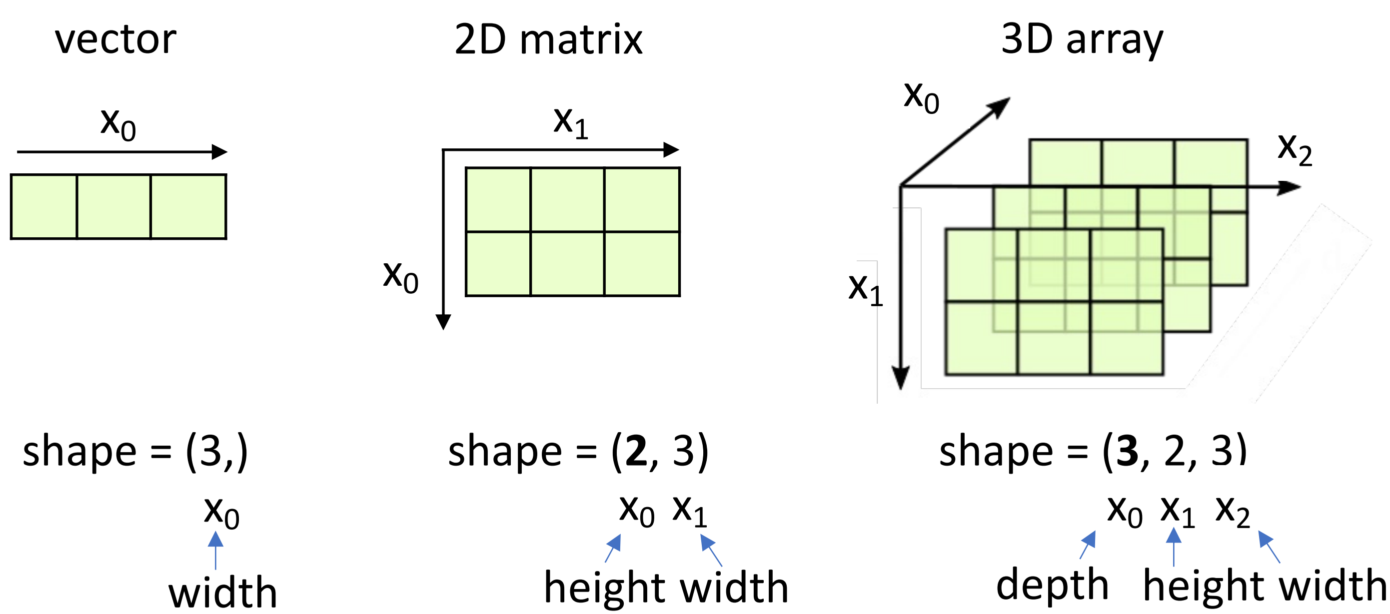


- Multidimensional arrays with **Numpy**
  - Axes can be numbered with negative values
  - Axis −1 is always along the **row** (innermost dimension)



# Multidimensional arrays

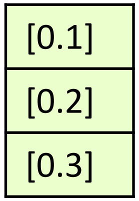
- Multidimensional arrays with **Numpy**
  - The **shape** of a Numpy array is a tuple that specifies the number of elements along each axis
    - Examples:



# Multidimensional arrays

- Column vector vs row vector

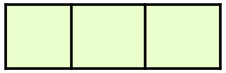
e.g. np.array([[0.1], [0.2], [0.3]])




shape = (3, **1**)

**Column vector is a 2D matrix!**

e.g. np.array([0.1, 0.2, 0.3])



shape = (3,)

# Numpy arrays

- Creation from list:
  - np.array(my_list, dtype=np.float16)
    - Data type inferred if not specified
- Creation from scratch:
  - np.zeros(shape)
    - Array with all 0 of the given shape
  - np.ones(shape)
    - Array with all 1 of the given shape
  - np.full(shape, value)
    - Array with all elements to the specified value, with the specified shape

- Creation from scratch: examples

In [1]:
```
np.ones((2,3))
```

Out[1]:
```
[[1, 1, 1],
 [1, 1, 1]]
```

In [2]:
```
np.full((2,1)), 1.1)
```

Out[2]:
```
[[1.1],
 [1.1]]
```

# Numpy arrays

- Creation from scratch:
  - np.linspace(start, stop, num)
    - Generates *num* samples from *start* to *stop* (included)
    - np.linspace(0,1,11) → [0.0, 0.1, … , 1.0]
  - np.arange(start, stop, step)
    - Generates numbers from *start* to *stop* (excluded), with step *step*
    - np.arange(1, 7, 2) → [1, 3, 5]
  - np.random.normal(mean, std, shape)
    - Generates random data with normal distribution
  - np.random.random(shape)
    - Random data uniformly distributed in [0, 1]

# Numpy arrays

- Main attributes of a Numpy array
  - Consider the array
    - x = np.array([[2, 3, 4],[5,6,7]])
  - **x.ndim:** number of dimensions of the array
    - Out: 2
  - **x.shape**: tuple with the array shape
    - Out: (2,3)
  - **x.size**: array size (product of the shape values)
    - Out: 2*3=6

**Summary:**

- **Universal functions** (Ufuncs):
  - **Binary** operations (+,−,*,…)
  - **Unary** operations (exp(),abs(),…)
- **Aggregate** functions
- **Sorting**
- **Algebraic** operations (dot product, inner product)

- **Universal functions** (Ufuncs): element-wise operations
  - **Binary** operations with arrays of the **same shape**
    - `+`, `-`, `*`, `/`, `%` (modulus), `//` (floor division), `**` (exponentiation)

# Computation on Numpy

- Example:

In [1]:
```
x=np.array([[1,1],[2,2]])
y=np.array([[3, 4],[6, 5]])
x*y
```

Out[1]:
```
[[3, 4], [12, 10]]
```

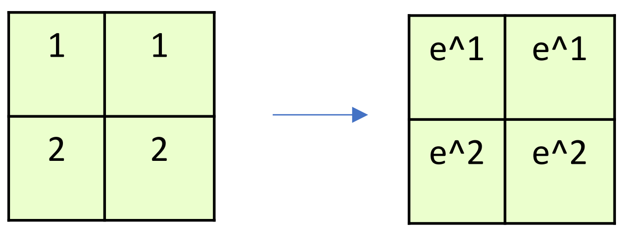
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 1*3 & 1*4 \\ 2*6 & 2*5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 12 & 10 \end{bmatrix}$$

- **Universal functions** (Ufuncs):
  - **Unary** operations
    - np.abs(x)
    - np.exp(x), np.log(x), np.log2(x), np.log10(x)
    - np.sin(x), cos(x), tan(x), arctan(x), …
  - They apply the operation separately to each element of the array

- Example:

In [1]:
```python
x=np.array([[1,1],[2,2]])
np.exp(x)
```

Out[1]:
```
[[2.718, 2.718],[7.389, 7.389]]
```




- **Note: original array (x) is not modified**

- **Aggregate functions**
  - **Return** a single value from an array
    - np.min(x), np.max(x), np.mean(x), np.std(x), np.sum(x)
    - np.argmin(x), np.argmax(x)
  - Or equivalently:
    - x.min(), x.max() x.mean(), x.std(), x.sum()
    - x.argmin(), x.argmax()
  - Example

In [1]:
```
x=np.array([[1,1],[2,2]])
x.sum()
```

Out[1]:
```
6
```

- For 1-dimensional array x → position of the smallest/largest element of x

```
x = np.array([5, 3, 9, 0, 7])
                0  1  2  3  4
x.argmin()
```

```
3
```

- For N-dimensional array x → position of the smallest/largest element of the *flattened* version of x
  - *Flattened* = collapsed into one dimension, `x.flatten()`

```
x = np.array([[8, 3, 9],
              [4, 2, 9]])
x.argmin()
```

x.flatten()

[ 8, 3, 9, 4, 2, 9 ]
0 1 2 3 4 5

```
4
```

## Aggregate functions along axis

- Allow specifying the **axis** along with performing the operation
- Examples

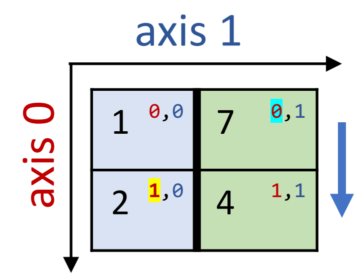
In [1]:

```
x=np.array([[1,7],[2,4]])
x.argmax(axis=0)
```

Out[1]:

```
[1, 0]
```

(index of maximum element within each column)



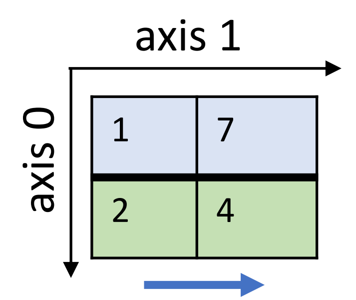

In [2]:

```
x.sum(axis=1)    # or axis=-1
```

Out[2]:

```
[8, 6]
```

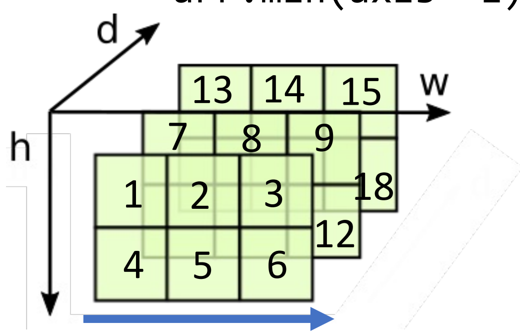
(sum the elements of each row)



# Computation on Numpy

- **Aggregate functions along axis**
  - The aggregation dimension is **removed** from the output

```
arr.min(axis=-1)
```

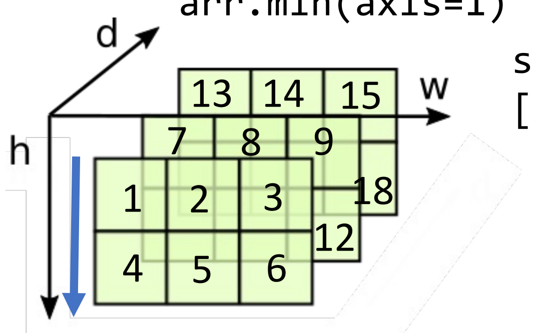



```
shape = (3, 2, 1)
[[[ 1], [ 4] ],
 [[ 7], [10]],
 [[13], [16]]]
```

**Final output**

```
shape = (3, 2)
[[ 1,  4],
 [ 7, 10],
 [13, 16]]
```

```
arr.min(axis=1)
```




```
shape = (3, 1, 3)
[[[ 1,  2,  3]],
 [[ 7,  8,  9]],
 [[13, 14, 15]]]
```

```
shape = (3, 3)
[[ 1,  2,  3],
 [ 7,  8,  9],
 [13, 14, 15]]
```

- **Sorting**
  - **np.sort(x):** creates a sorted copy of x
    - x is not modified
  - **x.sort():** sorts x inplace (x is modified)

## Sorting

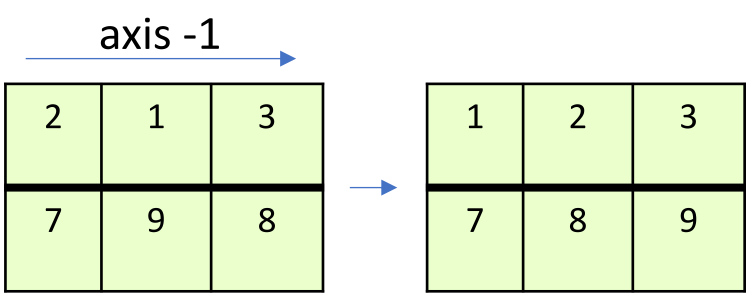
- Array is sorted along the last axis (-1) by default

In [1]:

```
x = np.array([[2,1,3],[7,9,8]])
np.sort(x)          # Sort along rows (axis -1)
```

Out[1]:

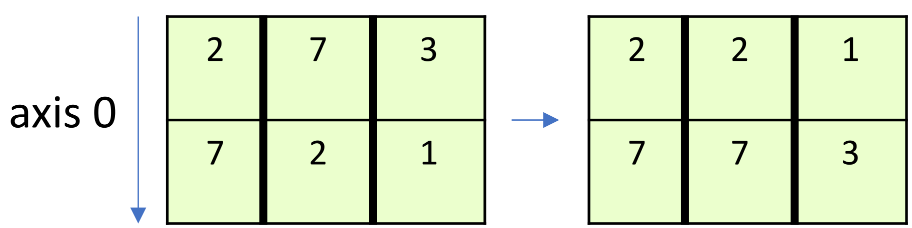
```
[[1,2,3],[7,8,9]]
```



- **Sorting**
  - Allows specifying the axis being sorted

In [1]:
```
x = np.array([[2,7,3],[7,2,1]])
np.sort(x, axis=0)    # Sort along columns
```

Out[1]:
```
[[2,2,1],
[7,7,3]]
```

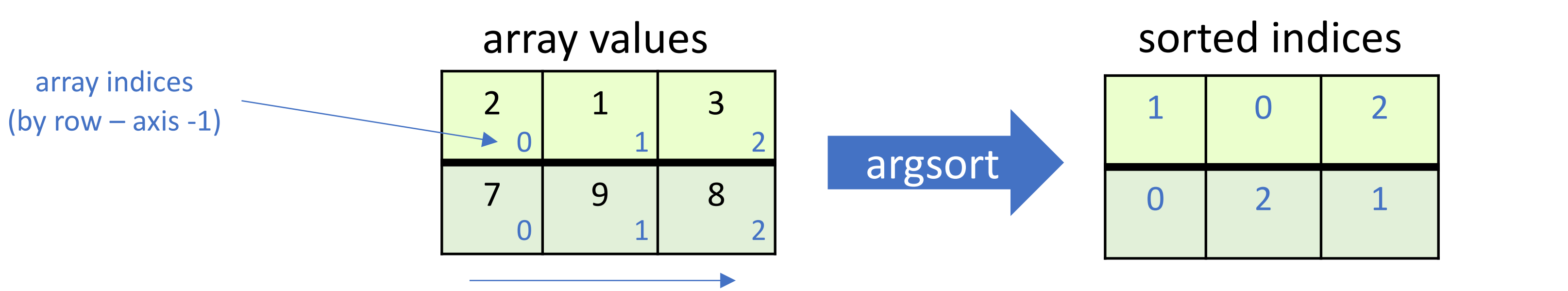


## Sorting

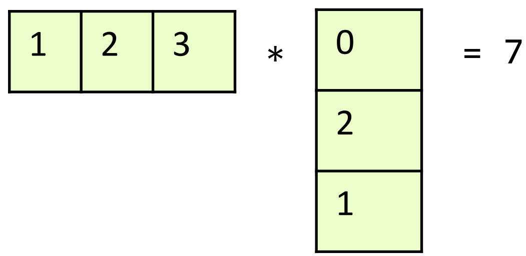
- **np.argsort(x):** return the position of the indices of the sorted array (sorts by default on axis −1)

In [1]:
```python
x = np.array([[2,1,3],[7,9,8]])
np.argsort(x)      # Sort along rows (axis -1)
```

Out[1]:
```
[[1,0,2],[0,2,1]]
```



- **Algebraic operations**
  - x @ y
    - inner product if x and y are two 1-D arrays




In [1]:

```
x=np.array([1, 2, 3])
y=np.array([0, 2, 1])  # works even if y is a row vector
x @ y
```

Out[1]:

```
7
```

- **Algebraic operations**
  - X @ y
    - matrix multiplied by vector

$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix} * \begin{bmatrix}2\\ 3\end{bmatrix} = \begin{bmatrix}5\\ 10\end{bmatrix}$$

In [1]:

```
X=np.array([[1,1],[2,2]])
y=np.array([2, 3]) # works even if y is a row vector
X @ y
```

Out[1]:

```
[5, 10]    # result is a row vector
```

- **Algebraic operations**
  - X @ Y
    - matrix multiplied by matrix

$$\begin{bmatrix}1 & 1\\2 & 2\end{bmatrix} * \begin{bmatrix}2 & 2\\1 & 1\end{bmatrix} = \begin{bmatrix}3 & 3\\6 & 6\end{bmatrix}$$

In [1]:

```
X=np.array([[1,1],[2,2]])
Y=np.array([[2,2],[1,1]])
X @ Y
```

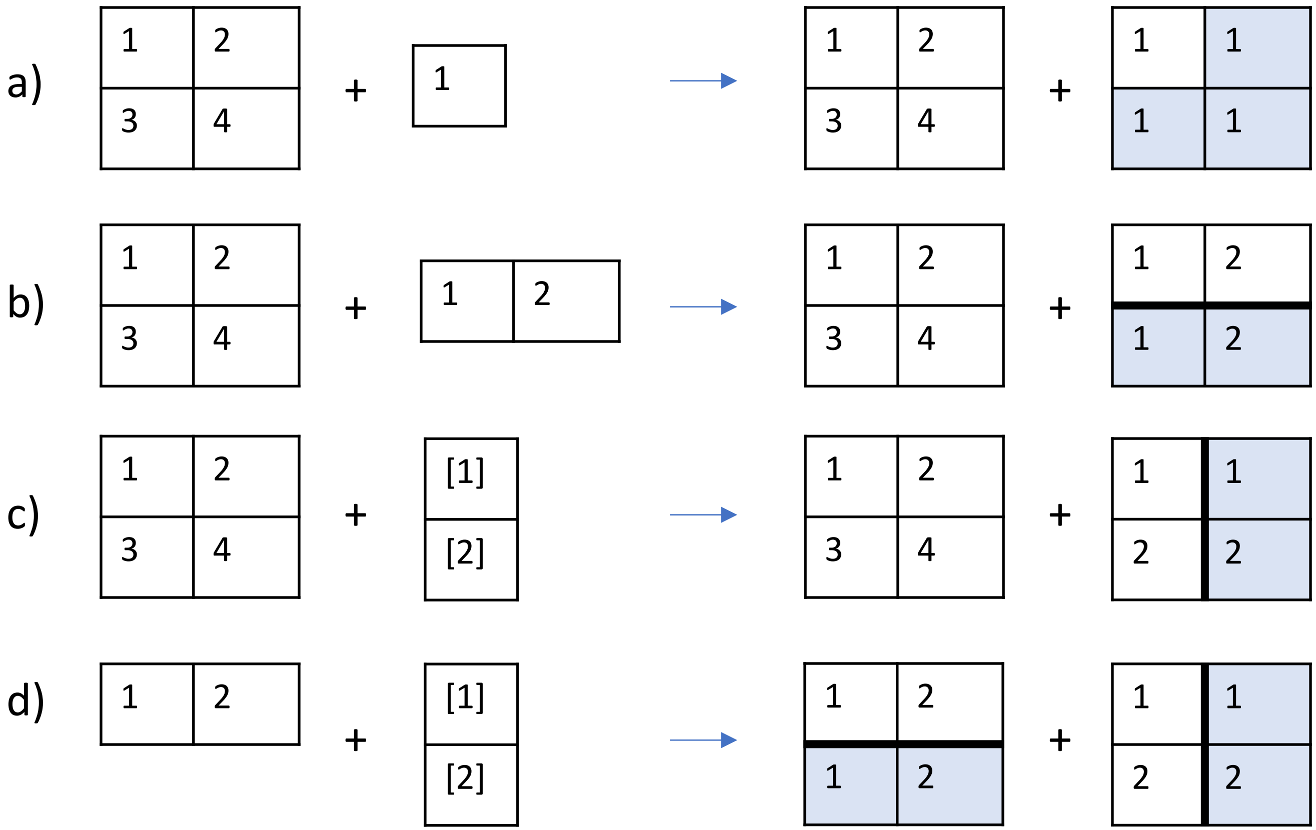
Out[1]:

```
[[3,3],[6,6]]
```

# Notebook Examples

- **2-Numpy Examples.ipynb**
  - **1) Computation with arrays**

- Pattern designed to perform operations between arrays with **different shape**



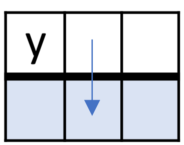
# Broadcasting

- Rules of broadcasting

1. The shape of the array with **fewer dimensions** is **padded** with leading ones

   x.shape = (2, 3), y.shape = (3) → y.shape = (**1**, 3)

2. If the shape along a dimension is 1 for one of the arrays and >1 for the other, the array with shape = 1 in that dimension is **stretched to match the other array**

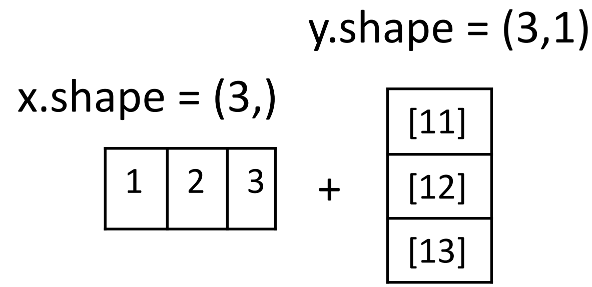



   x.shape = (2, 3), y.shape = (**1**, 3) → stretch: y.shape = (**2**, 3)

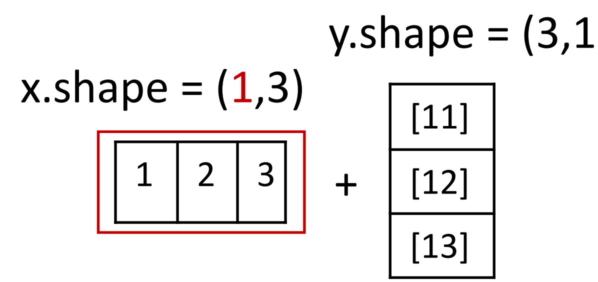
3. If there is a dimension where both arrays have shape >1 and those shapes differ, then broadcasting **cannot be performed**

# Broadcasting

- Example: compute x + y
  - x = np.array([1, 2, 3])
  - y = np.array([[11], [12], [13]])
  - z = x + y




  - Apply Rule 1
    - x.shape becomes (**1**, 3): x=[[1,2,3]]
  - Apply Rule 2:
    - extend x on the vertical axis, y on the horizontal one




$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 1 & 2 & 3 \\ \hline 1 & 2 & 3 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline 11 & 11 & 11 \\ \hline 12 & 12 & 12 \\ \hline 13 & 13 & 13 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 12 & 13 & 14 \\ \hline 13 & 14 & 15 \\ \hline 14 & 15 & 16 \\ \hline \end{array}$$

# Broadcasting

- Example: compute x + y
  - x = np.array([[1, 2],[3,4],[5,6]])   x.shape = (3, 2)
  - y = np.array([11, 12, 13])   y.shape = (3,)
  - z = x + y

  - Apply Rule 1
    - y.shape becomes (**1**, 3): y=[[11,12,13]]
  - Apply Rule 3
    - shapes (3, **2**) and (1, **3**) are incompatibles
    - Numpy will raise an **exception**

| | | |
|---|---|---|
| 11 | 12 | 13 |

| | |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |

# Notebook Examples

- **2-Numpy Examples.ipynb**
  - **2) Broadcasting: dataset normalization**

# Accessing Numpy Arrays

- Numpy arrays can be accessed in many ways
  - Simple indexing
  - Slicing
  - Masking
  - Fancy indexing
  - Combined indexing
- Slicing provides **views** on the considered array
  - Views allow **reading** and **writing** data on the **original** array
- Masking and fancy indexing provide **copies** of the array

# Accessing Numpy Arrays

- **Simple indexing**: read/write access to element
  - x[i, j, k, … ]

In [1]:
```
x = np.array([[2, 3, 4],[5,6,7]])
el = x[1, 2]          # read value (indexing)
print("el =", el)
```

Out[1]:
```
el = 7
```

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 3 | 4 |
| 1 | 5 | 6 | 7 |

In [2]:
```
x[1, 2] = 1           # assign value
print(x)
```

Out[2]:
```
[[2, 3, 4], [5, 6, 1]]
```

# Accessing Numpy Arrays

- **Simple indexing:** returning elements **from the end**
- Consider the array
  - x = np.array([[2, 3, 4],[5,6,7]])
- x[0, **-1**]
  - Get last element of the first row: 4
- x[0, **-2**]
  - Get second element from the end of the first row: 3

# Accessing Numpy Arrays

- **Slicing**: access contiguous elements
  - x[start:stop:step, …]
    - Creates a *view* of the elements from *start* (included) to *stop* (excluded), taken with fixed step
    - **Updates on the view yield updates on the original array**
    - Useful shortcuts**:**
      - **omit start** if you want to start from the beginning of the array
      - **omit stop** if you want to slice until the end
      - **omit step** if you don't want to skip elements

# Accessing Numpy Arrays

- **Slicing**: access contiguous elements
  - Select **all rows** and the **last 2 columns**:

In [1]:
```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
x[:, 1:]        # or x[0:3, 1:3]
```

Out[1]:
```
[[2,3], [5,6], [8,9]]
```

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 4 | 5 | 6 |
| 2 | 7 | 8 | 9 |

  - Select the **first two rows** and the **first** and **third columns**

In [2]:
```
x[:2, ::2]      # or x[0:2, 0:3:2]
```

Out[2]:
```
[[1, 3], [4, 6]]
```

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 1 | 4 | 5 | 6 |
| 2 | 7 | 8 | 9 |

# Accessing Numpy Arrays

## Update a sliced array

In [1]:

```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
x[:, 1:] = 0
print(x)
```

Out[1]:

```
[[1,0,0], [4,0,0], [7,0,0]]
```

# Accessing Numpy Arrays

- **Update a view**

In [1]:

```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
view = x[:,1:]
view[:,:] = 0
print(x)
```

Out[1]:

```
[[1,0,0], [4,0,0], [7,0,0]]
```

- To avoid updating the original array use .**copy**()
    - x1=x[:,1:].copy()

# Accessing Numpy Arrays

- **Masking**: use boolean masks to select elements
  - x[mask]
    - mask
      - **boolean** numpy array that specifies which elements should be selected (select if True)
      - **same shape** as the original array
  - The result is a **one-dimensional vector** that is a **copy** of the original array elements selected by the mask

# Accessing Numpy Arrays

- **Mask creation**
  - x *op* value (e.g *x==4*)
  - where *op* can be >, >=, <, <=, ==, !=
- Examples

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x > 4
```

Out[1]:
```
[False, True, False, True]
```

In [2]:
```
x2 = np.array([[1.2, 4.1], [1.5, 4.5]])
x2 >= 4
```

Out[2]:
```
[[False, True], [False, True]]
```

- **Operations with masks (boolean arrays)**
  - Numpy allows boolean operations between masks with the same shape (bitwise operators)
    - & (and), | (or), ^ (xor), ~ (negation)
  - Example
    - mask = ~((x < 1) | (x > 5)) ⇔ ((x >= 1) & (x <= 5))
    - elements that are between 1 and 5 (included)

# Accessing Numpy Arrays

PoliTo DBMG

- **Masking** examples

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[x > 4]
```

Out[1]:
```
[4.1, 4.5]
```

In [2]:
```
x2 = np.array([[1.2, 4.1], [1.5, 4.5]])
x2[x2 >= 4]
```

Out[2]:
```
[4.1, 4.5]
```

- Even if the shape of x2 is (2, 2), the result is **a one-dimensional** array containing the elements that satisfy the condition

## Update a masked array

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[x > 4] = 0        # Assignment is allowed
x
```

Out[1]:
```
[1.2, 0, 1.5, 0]
```

- **Masking does not create views, but copies**

In [2]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
masked = x[x > 4]  # Masked is a copy of x
masked[:] = 0      # Assignment does not affect x
x
```

Out[2]:
```
[1.2, 4.1, 1.5, 4.5]
```

- **Fancy indexing**: specify the **index** of elements to be selected
  - Example: select elements from 1-dimensional array

x[1] x[3]

In [1]:
```
x = np.array([7.0, 9.0, 6.0, 5.0])
x[[1, 3]]
```

Out[1]:
```
[9.0, 5.0]
```

# Accessing Numpy Arrays

- **Fancy indexing:** selection of **rows** from a 2-dimensional array

| | | | |
|---|---|---|---|
| | 0.0 | 1.0 | 2.0 |
| x[1,:] | 3.0 | 4.0 | 5.0 |
| x[2,:] | 6.0 | 7.0 | 8.0 |

In [1]:
```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0],
              [6.0, 7.0, 8.0]])
x[[1, 2]]
```

Out[1]:
```
[[3.0, 4.0, 5.0], [6.0, 7.0, 8.0]]
```

# Accessing Numpy Arrays

- **Fancy indexing**: selection of elements with coordinates
  - Result contains a 1-dimensional array with selected elements

| | | |
|---|---|---|
| 0,0 | 0,1 | 0,2 |
| 1,0 | 1,1 | 1,2 |
| 2,0 | 2,1 | 2,2 |

In [1]:

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0],
              [6.0, 7.0, 8.0]])
x[[1, 2], [0, 2]]  →  1,0 | 2,2   (indices being selected)
```

Out[1]:

```
[3.0, 8.0]
```

# Accessing Numpy Arrays

- Similarly to masking, fancy indexing provides **copies** (not views) of the original array

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[[1, 3]] = 0      # Assignment is allowed
x
```

Out[1]:
```
[1.2, 0, 1.5, 0]
```

In [2]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
sel = x[[1, 3]]    # sel is a copy of x
sel[:] = 0         # Assignment does not affect x
x
```

Out[2]:
```
[1.2, 4.1, 1.5, 4.5]
```

- **Combined indexing:**
  - Allows mixing the indexing types described so far
  - Important rule:
    - The number of dimensions of selected data is:
      - **The same as the input** if you mix:
        - masking+slicing, fancy+slicing
      - **Reduced by one** for each axis where simple indexing is used
        - Because simple indexing takes only 1 **single** element from an axis

# Accessing Numpy Arrays

- **Combined indexing:** masking+slicing, fancy+slicing
  - Output has the same numer of dimensions as input

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
```

```
x[[True,False,True], 1:]
# Masking + Slicing: [[1.0,2.0],[7.0,8.0]]
```

| | | |
|---|---|---|
| 0.0 | 1.0 | 2.0 |
| 3.0 | 4.0 | 5.0 |
| 6.0 | 7.0 | 8.0 |

```
x[[0,2], :2]
# Fancy + Slicing: [[0.0,1.0],[6.0,7.0]]
```

| | | |
|---|---|---|
| 0.0 | 1.0 | 2.0 |
| 3.0 | 4.0 | 5.0 |
| 6.0 | 7.0 | 8.0 |

# Accessing Numpy Arrays

- **Combined indexing:** simple+slicing, simple+masking
  - Simple indexing **reduces** the number of dimensions

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
```

```
x[0, 1:]
# Simple + Slicing: [1.0, 2.0]
```

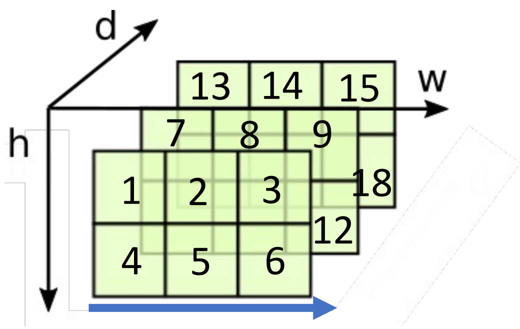
| | | |
|---|---|---|
| 0.0 | 1.0 | 2.0 |
| 3.0 | 4.0 | 5.0 |
| 6.0 | 7.0 | 8.0 |

```
x[[True, False, True], 0]
# Simple + Masking: [0.0, 6.0]
```

| | | |
|---|---|---|
| 0.0 | 1.0 | 2.0 |
| 3.0 | 4.0 | 5.0 |
| 6.0 | 7.0 | 8.0 |

- **Simple indexing + slicing**
  - The dimension selected with simple indexing is **removed** from the output

arr[:,:,0]



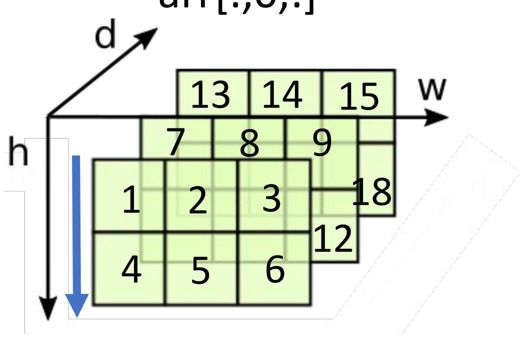

shape = (3, 2, 1)
[[[1], [4]],
[[7], [10]],
[[13], [16]]]

→

**Final output**

shape = (3, 2)
[[1, 4],
 [7, 10],
 [13, 16]]

arr[:,0,:]




shape = (3, 1, 3)
[[[1,2,3]],
 [[7,8,9]],
 [[13,14,15]]]

→

shape = (3, 3)
[[1, 2, 3],
 [7, 8, 9],
 [13, 14, 15]]

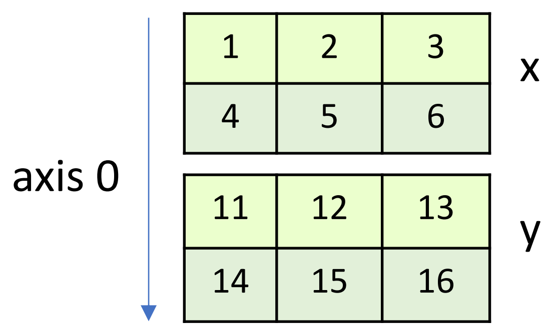
# Notebook Examples

- **2-Numpy Examples.ipynb**
  - **3) Accessing Numpy Arrays**

**Summary**:

- Array concatenation
- Array splitting
- Array reshaping
- Adding new dimensions

- Array concatenation along **existing axis**
  - The result has the **same number of dimensions** of the input arrays



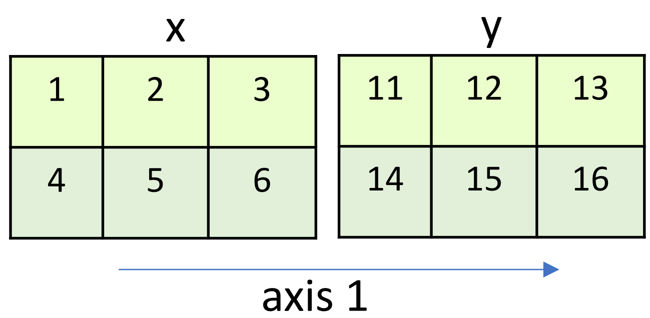

In [1]:

```
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
np.concatenate((x, y))      # Default axis: 0
```

Out[1]:

```
[[1,2,3],[4,5,6],[11,12,13],[14,15,16]]
```

- **Array concatenation along existing axis**
  - Concatenation along **rows (axis=1)**




In [1]:

```
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
np.concatenate((x, y), axis=1)
```

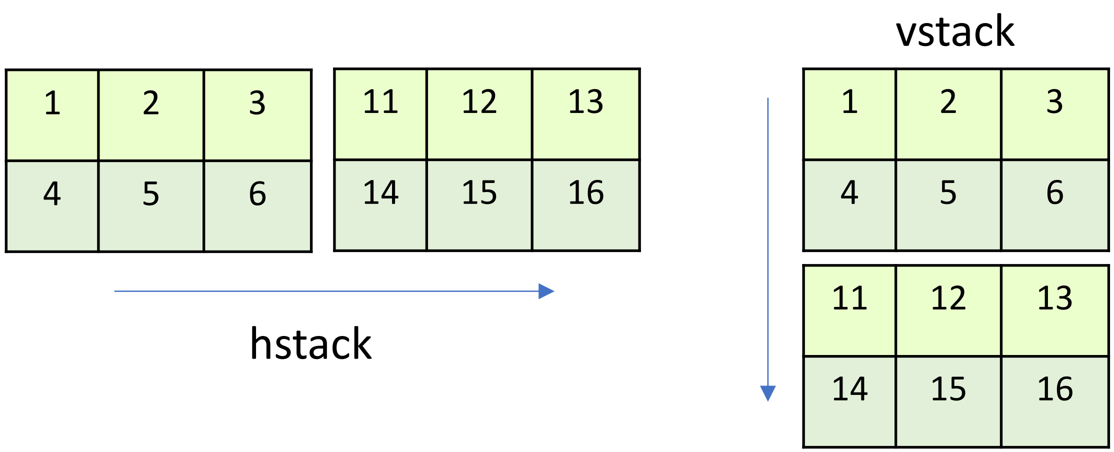
Out[1]:

```
[[1,2,3,11,12,13],[4,5,6,14,15,16]]
```

# Working with arrays

## Array concatenation: hstack, vstack

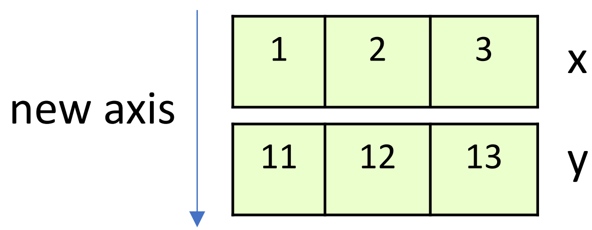
- Similar to np.concatenate()




In [1]:

```
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
h = np.hstack((x, y))       # along rows (horizontal)
v = np.vstack((x, y))       # along columns (vertical)
```

- **Array concatenation: hstack, vstack**
  - **vstack** allows concatenating 1-D vectors along **new axis** (not possible with np.concatenate)




In [1]:
```
x = np.array([1,2,3])
y = np.array([11,12,13])
v = np.vstack((x, y))        # vertically
```

# Working with arrays

- **Splitting arrays (split, hsplit, vsplit)**
  - **np.split(arr, N, axis=0)**
    - outputs a **list** of Numpy arrays
    - If N is integer: divide *arr* into *N* equal arrays (along axis), if possible!
    - if N is a 1d array: specify the entries where the array is split (along *axis*)

give the list of the index where u want to split

x

| | | | | | | |
|---|---|---|---|---|---|---|
| index | 0 | 1 | **2** | 3 | **4** | 5 |
| values | 7 | 7 | 9 | 9 | 8 | 8 |

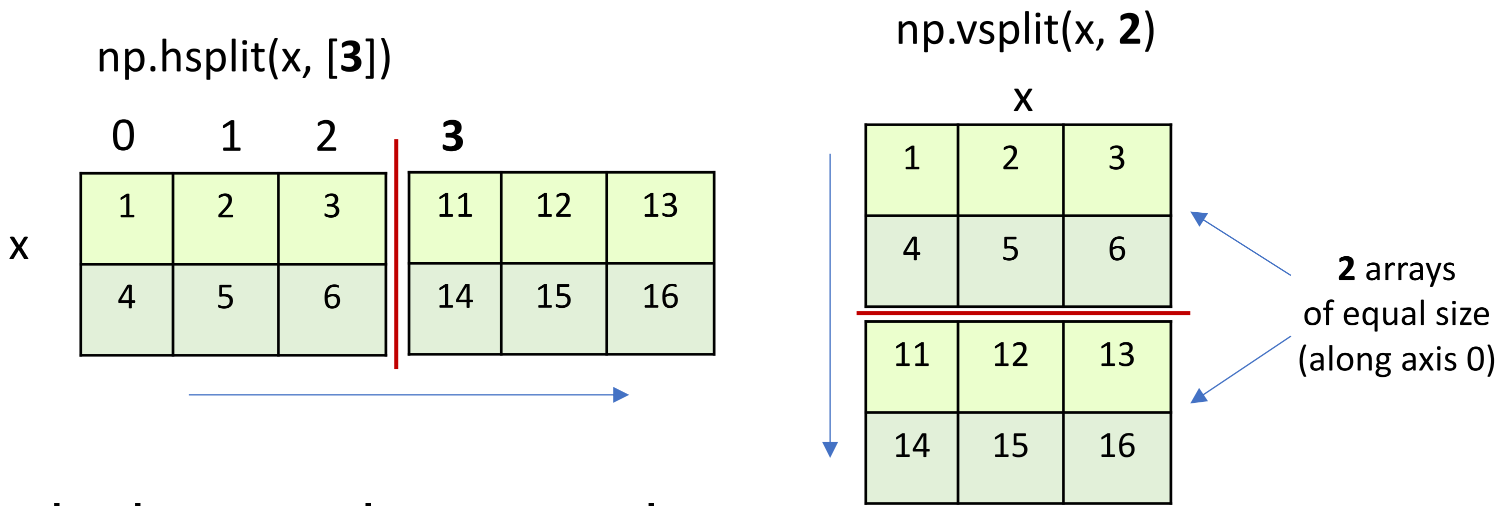
In [1]:

```
x = np.array([7, 7, 9, 9, 8, 8])
np.split(x,[2,4])          # split before element 2 and 4
                           # same as passing N = 3
```

Out[1]:

```
[array([7, 7]), array([9, 9]), array([8, 8])]
```

- **Splitting arrays (split, hsplit, vsplit)**
  - **hsplit, vsplit** with 2D arrays
    - return a **list** with the arrays after the split




- In both examples output is:

Out: [array([[1,2,3],[4,5,6]]), array([[11,12,13],[14,15,16]])]

## Reshaping arrays

In [1]:
```
x = np.arange(6)
y = x.reshape((2,3))
```

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 |

→

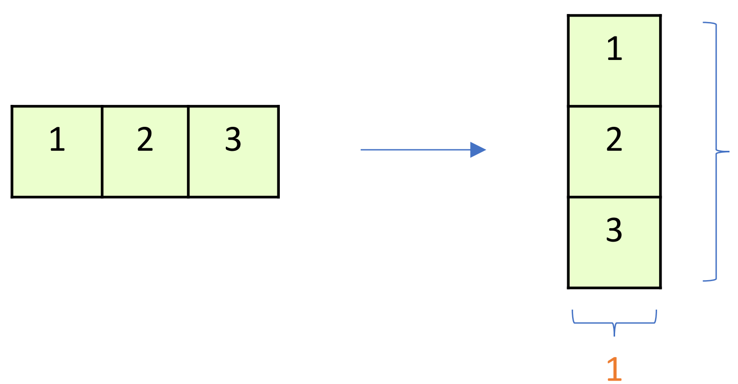
| | | |
|---|---|---|
| 0 | 1 | 2 |
| 3 | 4 | 5 |

- y is filled following the index order:
  - y[0,0] = x[0], y[0,1] = x[1], y[0,2] = x[2]
  - y[1,0] = x[3], y[1,1] = x[4], y[1,2] = x[5]

- **Reshaping arrays**
  - At most one dimension can be −1 (“unknown”)
  - If present, the size is inferred from
    - The source array
    - The other dimensions

In [1]:

```
x = np.array([1,2,3])
y = x.reshape(-1,1)
```




The first dimension (rows) is inferred to be 3, considering that the second dimension (columns) is 1 and x.size = 3

## Adding new dimensions

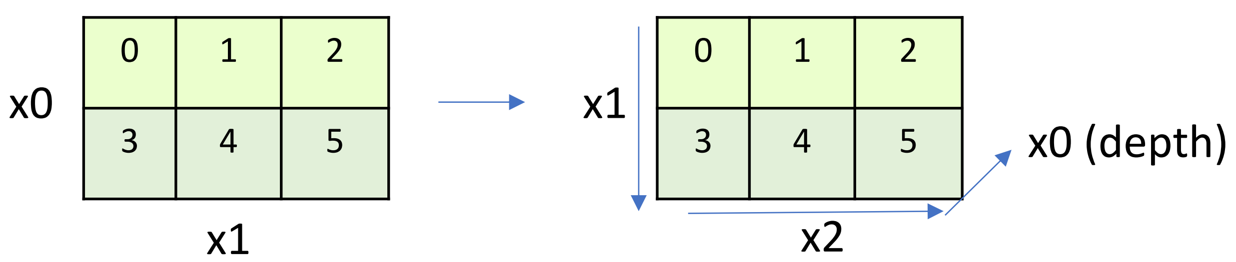
- **np.newaxis** adds a new dimension with **shape=1** at the specified position

In [1]:

```
arr = np.array([[1,2,3],[4,5,6]])
res = arr[np.newaxis, :, :] # output shape = (1,2,3)
print(res)
```

Out[1]:

```
[[[1,2,3],[4,5,6]]]
```



# Adding new dimensions

- **Application:** row vector to column vector
  - Alternative approach to .reshape(−1,1)

In [1]:

```
arr = np.array([1,2,3])
res = arr[:, np.newaxis]    # output shape = (3,1)
print(res)
```

Out[1]:

```
[[1],[2],[3]]
```