Ordinary Least Squares (OLS)

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x + \varepsilon$ st $\min_{\beta_0,\beta_1} \varepsilon^2$ $\quad n$ = sample size

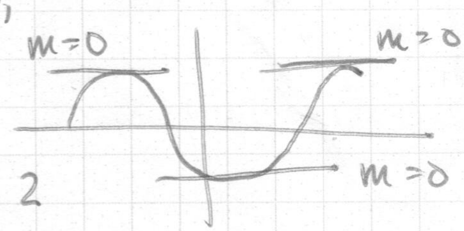
Assumg $\quad \vec{y} = A\vec{B} + \vec{\varepsilon}$ where $A = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$, $\vec{B} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$

ORDINARY : vanilla, first lin. regressn $\quad \vec{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$

Least-squares : we want to minimize ε^2

β_0, β_1 st $\min \|\varepsilon\|^2 : \min_{\beta_0,\beta_1} \|\varepsilon\|^2$

$$= \min_{\beta_0,\beta_1} \|\vec{y} - A\vec{\beta}\|^2$$



$$= \min_{\beta_0,\beta_1} \Sigma \left(\left[y_i - (\beta_0 + \beta_1 x_i)\right]\right)^2$$

Recall, the min & max of a fn is where slope = 0

$\Rightarrow$ one $\quad \dfrac{\partial \varepsilon}{\partial \beta_0} = -2\Sigma(y_i - \beta_0 - \beta_1 x_i)$

two $\quad \dfrac{\partial \varepsilon}{\partial \beta_1} = -2\Sigma(y_i - \beta_0 - \beta_1 x_i)x_i$

$\vdots$

– calculus –

$\vdots$

$$\boxed{\hat{\beta}_1^{OLS} = \frac{\Sigma y_i x_i - n\bar{y}\bar{x}}{\Sigma x_i^2 - n\bar{x}^2}, \quad \hat{\beta}_0^{OLS} = \bar{y} - \beta_1 \bar{x}, \quad \text{notatn: } \bar{y} = \text{mean}(y)}$$

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