Ordinary Least Squares (OLS) Linear Regression

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April 9, 2018

Ordinary least squares linear regression is linear regression using the method of least squares to estimate the parameters of a linear model. The method of least squares is a simple method, the first and probably only method, that students learn to estimate the parameters of a linear model.
The method of of least squares aims to estimate the parameters by minimizing (least) the square of the error ie the difference between the estimator and true value of the target/independent variable $y - \hat{y}$

We will look at the simplest case where there is only one independent variable. However, this is easily generalizable.

Assumptions:

1. $\exists$ n samples; $n = samplesize$
2. $y_1, ..., y_n$ are the target/response variables (are independent and identically distributed(iid))
3. $x_1, ..., x_n$ are the independent variables (independent not in the probabilistic sense)
4. $\exists$ a linear model:
$$\vec{y} = A\vec{\beta} + \vec{\varepsilon} \tag{1}$$

where $A = \begin{pmatrix} 1 & x_1 \\ . & . \\ . & . \\ . & . \\ 1 & x_n \end{pmatrix}$ and $\vec{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$

In OLS,
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x + \varepsilon \tag{2}$$

s.t. $\min\limits_{\beta_0, \beta_1} ||\varepsilon||^2$

β_0, β_1 st $\min ||\varepsilon||^2 : \min\limits_{\beta_0, \beta_1} ||\varepsilon||^2$
$= \min_{\beta_0, \beta_1} ||\vec{y} - A\vec{\beta}||^2$
$= \min_{\beta_0, \beta_1} \mathit{\Sigma}[y_i - (\beta_0 + \beta_1 x_i)]^2$

Recall, the min and max of a function is where slope = 0 =¿ solve for β_0 and β_1 where

$$\frac{\partial \varepsilon}{\partial \beta_0} = -2\mathit{\Sigma}(y_i\beta_0 - \beta_1 x_i) = 0 \tag{3}$$

and

$$\frac{\partial \varepsilon}{\partial \beta_0} = -2\mathit{\Sigma}(y_i\beta_0 - \beta_1 x_i)x_i = 0 \tag{4}$$

After solving using algebra

$$\hat{\beta_1}^{OLS} = \frac{\Sigma y_i x_i - n\bar{y}\bar{x}}{\Sigma x_i^- n\bar{x}^2} \tag{5}$$

and

$$\hat{\beta_0}^{OLS} = \bar{y} - \beta_1\bar{x} \tag{6}$$

notation: $\bar{x} = mean(x)$