## Derivata of Poisson Dista from Binomial Dista

Thm The Poisson (u) distribs the limit of the binomial (n, p) distn, w) E(binomial(n,p)) = u=np as n -> 0

Pf. Let X binomial (n,p)

$$P(X=X) = \binom{N}{X} P^{X} (1-P)^{N-X}$$

As stated above, u=np so we can replace PWI (which will be botum & & 1 for large n)

$$\lim_{n\to\infty} P(X=x) = \lim_{n\to\infty} \left[ \binom{n}{x} \binom{M}{n} (1-\frac{M}{n})^{n\to x} \right]$$

$$=\lim_{n\to\infty}\left[\frac{(u-x)_1x_1}{u_1}\cdot\frac{Nx}{n_x}\left(1-\frac{x}{n}\right)_{u-x}\right]$$

$$=\lim_{n\to\infty}\left[\frac{n(n-1)...(n-x-2)(n-x-1)}{n^x}\right]_{n\to\infty}^{lim}\left[\frac{M^x}{x!}(1-\frac{M}{n})^{n-\frac{1}{2}}\right]$$

$$\lim_{n \to \infty} P(x = x) = \frac{u^x}{x!} e^{-u}$$

I'm P(X=X) = pmf for the Poisson (u) distn.

why do we use the Poisson distr?

(from Knan Academy), Say X= # cars that pass /hr (1-60) (F) (1-60) (1-60) K

Problem is that if 2 cars pross

In one min ut, it would only

borning x hr = bo min we can be work a none success! so

P(X=k) = (3600) (3600) k (1-302) k but 2 cars can still

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## Vivian Duong

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Pf. Let X~binomial(n,p)

As stated above, u=np so we can replace PWI & (which will be bottom & & 1 for large m).

$$\lim_{N\to\infty}P(X=x)=\lim_{N\to\infty}\left[\binom{n}{x}\binom{m}{x}\binom{n-x}{n-x}\right]$$

Im P(X=X) = pmf for the Poisson (u) distn.

Why do we use the Poisson distn?

(from Knan Academy), Say X= # cars that pass /hr Xn binomial [PP(X:K) = (60) (70) (1-60) K

P(X=k)=(3600) (3600) Problem is that if 2 cars pass

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P(X=k)=(3600) Proble

Derivation d'innegative Binomial Distriction the Binomial Distri

Define Binomial Distn: The binomial distn gives
the probability of r successes out of N trials!

P(rin) = (N) Pr(1-p)N-r

Defor Negative Binomial Diston: The neg binomial diston gives the probability of (r-1) successes & Xfailures in XTV-1 thials, & success on the (XT) to trial.

Take 1st partolatefn: "probability of (r-1) Successed Out of (xtr-1) trials"

By clefn of Binomial Distr p(r-1/x+r-1) = (x-1x-1) pr-1 (1-p)x

The plauccess on last trial, the (xtr)th trialDisp.

=> by the multiplicate rule of probability

NB(x Ir, p)= (xtr-1)pri (1-p)x.p

 $Var(NB) = \begin{pmatrix} x+r-1 \\ y-1 \end{pmatrix} P^{r}(1-P) x$   $E(NB) = \frac{1}{P}$   $Var(NB) = \frac{r(1-P)}{P^{2}}$ 

Derivatin d'innegative Binomial Distri From

the Binomial Distri

Define Binomial Distn: The binomial distrigives

the probability of r successes out of N trials.  $p(rin) = \binom{N}{r} p^r (1-p)^{N-r}$ 

Defin Negative Binomial Distn: The neg binomial distributes the probability of (r-1) successes a Xfailures in X+r-1 thials, & success on the (X+r)+n trial.

Take 1st partoidefn: "probability of (r-1) Successus
out of (xtr-1) trials"

By defn of Binomial Distr p(r-1/x+r-1) = (x+r-1) pr-1 (1-p)x

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NB(x Ir, p)= (xtr-1)pri(1-p)x.p

Var(NB) = (x+r-1) Pr(1-P) x E(NB) = rad ((LP)) Var(NB) = r(1-P)

Derivation of Gamma from Poisson Dish

If  $X \sim Poisson(x)$ , the time until karrivals is  $T(k\frac{1}{2})$ The time until K arrivals of eventy.  $F(x) = P(T \leq x)$   $= 1 - P(X \leq K - 1)$   $= 1 - P(X \leq K - 1)$   $X = 1 - P(X \leq$ 

Compare W/ gamma polf: f(v)= xxx-1 e x/8 Pr(x)

TNGamma(x=K,0=)