Claim: If $y = \phi(t)$ is a solution of the non-homogenous 2nd order differential equation

$$y" + p(t)y' + q(t)y = g(t) \tag{1}$$

where $g(t)$ is *not* always zero, then

$$y = c\phi(t) \tag{2}$$

is not a solution.

Proof. Because (2) is a solution of (1), then

$$\phi"(t) + p(t)\phi'(t) + q(t)\phi(t) = g(t) \tag{3}$$

Deriving (2)

$$\begin{aligned} y &= c\phi(t) \\ y' &= c\phi'(t) \\ y" &= c\phi"(t) \end{aligned}$$

Plugging it into (1)

$$c\phi"(t) + p(t)c\phi'(t) + q(t)c\phi(t) \neq g(t) \tag{4}$$

$$c(\phi"(t) + p(t)\phi'(t) + q(t)phi(t)) \neq g(t)$$
$$c(g(t)) \neq g(t)$$

$\square$