2 Proofs that the Derivative of x to the nth power equals nx to the n-1 power

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Two proofs that $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$

Proof. Here is the first one.

$$\text{Let } y = x^n$$
$$ln(y) = ln(x^n)$$
$$ln(y) = nln(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}[ln(y(x))] = \frac{\mathrm{d}}{\mathrm{d}x}[nln(x)]$$
$$\frac{\mathrm{d}x}{\mathrm{d}y}.\frac{\mathrm{d}y}{\mathrm{d}x} = n\frac{1}{x}$$
$$\frac{1}{y}.\frac{\mathrm{d}y}{\mathrm{d}x} = n\frac{1}{x}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{yn}{x}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}y = \frac{nx^n}{x}$$
$$\frac{\mathrm{d}}{\mathrm{d}x}(x^n) = nx^{n-1}$$

□

Here is a another way.

Proof. Let $f(x) = x^n$

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$
$$= \lim_{x\to a} \frac{x^n - a^n}{x - a}$$
$$= \lim_{x\to a} \frac{\underline{(x-a)}(x^{n-1} + ax^{n-2} + a^2x^{n-3} + ... + a^{n-3}x^2 + a^{n-2}x + a^{n-1}}{\underline{x-a}}$$
$$= \lim_{x\to a} x^{n-1} + ax^{n-2} + a^2x^{n-3} + ... + a^{n-3}x^2 + a^{n-2}x + a^{n-1}$$
$$= \lim_{x\to a} a^{n-1} + aa^{n-2} + a^2a^{n-3} + ... + a^{n-3}a^2 + a^{n-2}a + a^{n-1}$$
$$= \lim_{x\to a} a^{n-1} + a^{n-1} + a^{n-1} + ... + a^{n-1} + a^{n-1} + a^{n-1}$$
$$f'(a) = na^{n-1}$$
$$f'(x) = nx^{n-1}$$

□