Proof.

Theorem 0.1. *Given* $M(x,y) + N(x,y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $M_y = N_x$, *and* $\psi(x, y(x)) = \int M \mathrm{d}y = \int N \mathrm{d}x$, $\psi(x,y) = c$, *which is the implicit solution.*

$$\psi_x = M$$
$$\psi_y = N$$
$$\psi_x + \psi_y \frac{\mathrm{d}y}{\mathrm{d}x} = 0$$
$$\psi_x \mathrm{d}x + \psi_y \mathrm{d}y = 0$$
$$\frac{\partial \psi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}x} + \frac{\partial \psi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$
$$\int \frac{\mathrm{d}}{\mathrm{d}x}\psi(x, y(x)) = \int 0$$

Definition 1 (Partial Derivatives Chain Rule)**.** $\frac{\mathrm{d}}{\mathrm{d}t}(z(x_1, ..., x_n)) = \frac{\partial z}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial z}{\partial x_2}\frac{\partial x_2}{\partial t} + ... + \frac{\partial z}{\partial x_n}\frac{\partial x_n}{\partial t}$

$$\psi(x,y) = C$$

$\square$