Complex Roots of the Characteristic Equation

Vivian Duong

Prove that $y_1 = cost$ and $y_2 = sint$ are a fundamental set of solutions of $y" + y = 0$ and that the Wronskian isn't zero.

Definition 1 (characteristic equation). *The characteristic equation of a 2^{nd} order differential equation is* $ay" + by' + cy = 0$

$$y" + y = 0 \tag{1}$$

is a characteristic equation where a = 1, b = 0, c = 1.

Theorem 0.1. *Suppose the characteristic equation:* $ay" + by' + cy = 0$
Then, $y = e^{rt}$ *where r is defined as*
$ar^2 + br + c = 0$

Solving for r and finding the solutions for equation (1)

$$\begin{gathered} y" + y = 0 \\ r^2 + r = 0 \\ r^2 = -1 \\ r_1 = i, r_2 = -i \\ y_1 = ce^{it} \text{ and } y_2 = ce^{-it} \end{gathered}$$

Theorem 0.2 (Euler's Formula). $e^{it} = cost + isint$

$y_1 = c_1cost + c_2isint$ and $y_2 = c_3cos(-t) + c_4sin(-t)$

Theorem 0.3 (Trigonometric Formulas). $sin(-t) = -sin(t)$
$cos(-t) = cos(t)$

Definition 2 (General Solution of a Homogeneous Differential Equation). *The general solution of a homgeneous differential equation is the sum of its solutions. So if y_1 and y_2 is a solution, the general solution, y, is $y = y_1 + y_2$*

$y = c_1cost + c_2sint$
c_3 and c_4 is absorbed by c_1 and c_2