1st ODE Integrating Factor

Vivian Duong

November 19, 2012

Claim: Suppose the First Order Differential Equation

$$\frac{dy}{dt} + p(t)y = g \tag{1}$$

where $p(t)$ and $g(t)$ are **continuous** functions. Continuous functions have no holes or breaks in it. If you were to draw it, you would never need to pick up your pencil.

Definition 1 (continuous function)**.** *A function f(x) is continuous at x=a if* $\lim_{x\to a} f(x) = f(a)$ *A function is continuous on the interval [a,b] if it is continuous at each point in the interval.*

And suppose there exists an integrating factor, $\mu(t)$ where

$$\mu(t) = e^{\int p(t)\mathrm{d}t} \tag{2}$$

Then the solution of (1) is

$$y = \frac{\int \mu(t)g(t)\mathrm{d}t + c}{\mu(t)} \tag{3}$$

Proof. This proof can also function as step-by-step instructions on how to solve a First Order Differential Equation.

1. Multiply both sides of 1 by μ

   $\mu(t)\frac{dy}{dt} + \mu(t)p(t)y = \mu(t)g$

   $\mu(t)p(t) = e^{\int p(t)\mathrm{d}t}p(t) = \mu(t)'$

   $\mu(t)y' + \mu'(t)y = \mu(t)g$

   $(\mu y)' = \mu g$

   $\int(\mu y)'\mathrm{d}t = \int \mu g \mathrm{d}t$

   $\mu y + c = \int \mu g \mathrm{d}t$

   $y(t) = \dfrac{\int \mu(t)g(t)\mathrm{d}t + c}{\mu(t)}$

□