

Valeria Duran

Math 6358

Homework 2

1a.

W	6	7	8	9	10	11	12	13	14	15	16	17	18
P(W)	1/35	1/35	2/35	3/35	4/35	4/35	5/35	4/35	4/35	3/35	2/35	1/35	1/35

$$1b. E(W) = 6*(1/35) + 7*(1/35) + 8*(2/35) + 9*(3/35) + 10*(4/35) + 11*(4/35) + 12*(5/35) + 13*(4/35) + 14*(4/35) + 15*(3/35) + 16*(2/35) + 17*(1/35) + 18*(1/35) = \mathbf{12}$$

1c. $Var(W) =$

$$((6-12)^2*(1/35)+(7-12)^2*(1/35)+(8-12)^2*(2/35)+(9-12)^2*(3/35)+(10-12)^2*(4/35)+(11-12)^2*(4/35)+(12-12)^2*(5/35)+(13-12)^2*(4/35)+(14-12)^2*(4/35)+(15-12)^2*(3/35)+(16-12)^2*(2/35)+(17-12)^2*(1/35)+(18-12)^2*(1/35)) = \mathbf{8.18 \approx 8}$$

$$2a. P(X=x) = \binom{20}{x} 0.70^x 0.30^{20-x}, x=1,2,\dots,20$$

$$2b. P(X>10) = 1 - \text{pbinom}(10,20,0.7) \quad \text{*not including 10*}$$

$$[1] \mathbf{0.9520381}$$

$$2c. P(6 \leq X \leq 10) = \text{pbinom}(10,20,0.7) - \text{pbinom}(5,20,0.7)$$

$$[1] \mathbf{0.04791896}$$

$$2d. E(X) = np = 20*0.7$$

$$[1] \mathbf{14}$$

$$SD(X) = \sqrt{20*0.7*0.3}$$

$$[1] \mathbf{2.04939}$$

2e. If 18 are sold, then between 2-10 chain driven models are sold.

$$P(2 \leq X \leq 10) = \text{pbinom}(10,20,0.7) - \text{pbinom}(1,20,0.7)$$

$$[1] \mathbf{0.0479619}$$

3a. $p = 0.5$; $P(7 \leq X \leq 18) = \text{pbinom}(18, 25, 0.5) - \text{pbinom}(6, 25, 0.5)$

[1] **0.9853667**

3b. $p = 0.8$; $P(7 \leq X \leq 18) = \text{pbinom}(18, 25, 0.8) - \text{pbinom}(6, 25, 0.8)$

[1] **0.2199647**

3c. *when $X \leq 7$ or $X \geq 18$ is $P(X \leq 7) \cup P(X \geq 18)^* = \text{pbinom}(7, 25, 0.5) + (1 - \text{pbinom}(17, 25, 0.5))$

[1] **0.04328525**

3d. $p = 0.6$; $P(8 \leq X \leq 17) = \text{pbinom}(17, 25, 0.6) - \text{pbinom}(7, 25, 0.6)$

[1] **0.8452428**

$p = 0.8$; $P(8 \leq X \leq 17) = \text{pbinom}(17, 25, 0.8) - \text{pbinom}(7, 25, 0.8)$

[1] **0.1091228**

3e. Try: $P(X \leq 6) \cup P(X \geq 19) = \text{pbinom}(6, 25, 0.5) * \text{including } 6^* + (1 - \text{pbinom}(18, 25, 0.5))$

including 19

[1] 0.0146333 *still greater than 0.01*

Try: $P(X \leq 5) \cup P(X \geq 20) = \text{pbinom}(5, 25, 0.5) + (1 - \text{pbinom}(19, 25, 0.5))$

[1] **0.004077315** *less than 0.01, so the decision rule chosen is $P(X \leq 5) \cup P(X \geq 20)^*$

4. $p = 0.005$; $n = 400$

repeated trials and independent, so assume Bernoulli

$P(X = x) = \binom{400}{x} 0.005^x (1 - 0.005)^{400-x}$, $x = 1, 2, \dots, 400$

$P(X = 1) = \binom{400}{1} 0.005^1 (1 - 0.005)^{400-1} = \text{dbinom}(1, 400, 0.005)$

[1] **0.2706694**

$P(X \leq 3) = \text{pbinom}(3, 400, 0.005)$ *including 3*

[1] **0.8575767**

5. $B = X - 120$ if $X > 120$ and $B = 0$ otherwise.

Profit: $250t - 500B$

```
a) passengers <- function(t){
  profit <- NULL
  for(i in 1:10000) {
    x <- rbinom(1,t,0.85)
    B <- max(x-120,0)
    profit[i] <- 250*t - 500*B
  }
  return(profit)
}

b) set.seed(12345) #set random seed to reproduce results
t <- 140:150
avg <- numeric(0)
for (i in 1:length(t)){
  avg[i] <- mean(passengers(t[i]))
  cat("The average profit for t=", t[i], "is", avg[i], "\n")
}
cat("The largest average profit of", max(avg), "is for t=", t[which.max(avg)], "\n")
```

output:

```
The average profit for t= 140 is 34400.65
The average profit for t= 141 is 34433.45
The average profit for t= 142 is 34469.4
The average profit for t= 143 is 34429.15
The average profit for t= 144 is 34415.45
The average profit for t= 145 is 34369.7
The average profit for t= 146 is 34254
The average profit for t= 147 is 34143.6
The average profit for t= 148 is 34012.1
The average profit for t= 149 is 33846.6
The average profit for t= 150 is 33658.25
```

```
> cat("The largest average profit of", max(avg), "is for t=", t[which.max(avg)], "\n")
```

The largest average profit of 34469.4 is for t= 142