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Math 6358

Homework 4

Problem 1.

When asking mom:
Y = 0.2
N = 0.3
Ask dad = 0.5
When asking dad:
Y = 0.1
N = 0.2
Ask mom = 0.7

a.

```
> teen = matrix(c(0,0.7,0,0,0.5,0,0,0,0.2,0.1,1,0,0.3,0.2,0,1), nrow = 4)
> teen
     [,1] [,2] [,3] [,4]
[1,]  0.0  0.5  0.2  0.3
[2,]  0.7  0.0  0.1  0.2
[3,]  0.0  0.0  1.0  0.0
[4,]  0.0  0.0  0.0  1.0
```

$$P = \begin{pmatrix} 0 & 0.5 & 0.2 & 0.3 \\ 0.7 & 0 & 0.1 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b. The absorbing states are “Yes” (3) and “No” (4)

c.
```
> A = matrix(c(0, 0.7, 0.5, 0), nrow = 2)
> A
     [,1] [,2]
[1,]  0.0  0.5
[2,]  0.7  0.0
> solve(diag(2) - A)%*%matrix(c(1,1),nrow=2)
         [,1]
[1,] 2.307692
[2,] 2.615385
```

The mean time to absorption from state 1 (talk to Mom) is 2.31 and the mean time to absorption from state 2 (talk to Dad) is 2.62

d. > C = matrix(c(0.2, 0.1, 0.3, 0.2), nrow = 2)
> solve(diag(2) - A)%*%C
[,1] [,2]
[1,] 0.3846154 0.6153846
[2,] 0.3692308 0.6307692

If he asks his mom first, he has a 0.3846 probability of getting the car. If he asks his dad first, he has a 0.3692 probability of getting the car. Therefore, he should ask his mom first.

2.
a. > D = matrix(c(0,0.3,0,0,0,0,0,0.7,0.3,0,0,0,0,0,0.7,0.3,0,0,0,0,0,0.7,0.3,0,0,0,0,0,0.7,0.3,
+ 1,0,0,0,0,0.7), nrow = 6)
> D
[,1] [,2] [,3] [,4] [,5] [,6]
[1,] 0.0 0.0 0.0 0.0 0.0 1.0
[2,] 0.3 0.7 0.0 0.0 0.0 0.0
[3,] 0.0 0.3 0.7 0.0 0.0 0.0
[4,] 0.0 0.0 0.3 0.7 0.0 0.0
[5,] 0.0 0.0 0.0 0.3 0.7 0.0
[6,] 0.0 0.0 0.0 0.0 0.3 0.7

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0.7 \end{pmatrix}$$

b.
> DT_I = t(D) - diag(6)
> SSD = qr.solve(rbind(matrix(c(1,1,1,1,1,1),ncol = 6),DT_I),
+ matrix(c(1,0,0,0,0,0,0),nrow = 7),tol = 1e-7)
> SSD
[,1]
[1,] 0.05660377
[2,] 0.18867925
[3,] 0.18867925
[4,] 0.18867925
[5,] 0.18867925
[6,] 0.18867925

$$\pi = \begin{pmatrix} 0.05660377 \\ 0.18867925 \\ 0.18867925 \\ 0.18867925 \\ 0.18867925 \\ 0.18867925 \end{pmatrix}$$

c.

$$State\ 0 = \frac{1}{\pi_0} = \frac{1}{0.05660377} = 17.66667$$

On average, approximately 17.67 weeks separate successive orders of five E550s.

d.

steady state probability for m:

$$x_0 + x_1 + \cdots + x_m = 1$$

Steady state transition equations:

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ p & 1-p & 0 & 0 & 0 & 0 \\ 0 & p & 1-p & 0 & 0 & 0 \\ 0 & 0 & p & 1-p & 0 & 0 \\ 0 & 0 & 0 & p & 1-p & 0 \\ 0 & 0 & 0 & 0 & p & 1-p \end{pmatrix}$$

$$\begin{gathered} x_0 = x_m \\ x_1 = (1-p)x_1 + px_0 \\ x_2 = (1-p)x_2 + px_1 \\ x_{m-1} = (1-p)x_{m-1} + px_{m-2} \\ x_m = (1-p)x_m + px_{m-1} \end{gathered}$$

From 2 part b SSD is 3/53, 10/53, 10/53,…, 10/53

$$\frac{10 * p}{10(m+p)}$$

So

$$SSD\ x_m : \frac{1}{m+p}$$

$$\text{And } SSD\ x_0 = \frac{p}{p+m}$$

The expected number of weeks between orders is:

$$\frac{1}{x_0} = \frac{1}{\frac{p}{m+p}} = \frac{m+p}{p} = \frac{m}{p} + 1$$