Valeria Duran

Math 6358

September 14, 2019

Homework 1

1. a) factorial(20)/(factorial(12)*factorial(8))

   (choose(12,12) + (choose(8,8)*choose(12,4)))/125970

   0.003937445

   b) wswswsws + swswswsw

   (((12/20)*(8/19)*(11/18)*(7/17)*(10/16)*(6/15)*(9/14)*(5/13)) +

   ((8/20)*(12/19)*(7/18)*(11/17)*(6/16)*(10/15)*(5/14)*(9/13)))

   0.007859014

2. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $P(A) = 0.40$

   $0.63 = 0.40 + 0.55 - P(A\cap B)$ $P(B) = 0.55$

   $P(A\cup C) = P(A) + P(C) - P(A\cap C)$ $P(C) = 0.70$

   $0.77 = 0.40 + 0.70 - P(A\cap C)$ $P(A\cup B) = 0.63$

   $P(B\cup C) = P(B) + P(C) - P(B\cap C)$ $P(A\cup C) = 0.77$

   $0.80 = 0.55 + 0.70 - P(B\cap C)$ $P(B\cup C) = 0.80$

   $P(A\cup B\cup C) = P(A) + P(B) + P(C) + P(A\cap B\cap C) - P(A\cap B) - P(A\cap C) - P(B\cap C)$

   $P(A\cap B\cap C) = 0.85 - 0.4 - 0.55 - 0.7 + 0.32 + 0.33 + 0.45$ $P(A\cup B\cup C) = 0.85$

   a) **$P(A\cup B\cup C) = 0.85$** $P(A\cap B) = 0.32$

   b) **$P(A\cup B\cup C)' =$** $1 - 0.85 =$ **0.15** $P(A\cap C) = 0.33$

   c) **$P(A'\cap B'\cap C)$** $= P(C) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$ $P(B\cap C) = 0.45$

   $= 0.70 - 0.33 - 0.45 + 0.30 =$ **0.22** $P(A\cap B\cap C) = 0.30$

   d) **$P\{(A\cap B'\cap C')\cup(A'\cap B\cap C')\cup(A'\cap B'\cap C)\} =$**

   $(A'\cap B'\cap C) + P(B) - P(A\cap B) - P(B\cap C) + P(A\cap B\cap C) + P(A) - P(A\cap B) - P(A\cap C)$

   $+ P(A\cap B\cap C) =$ **0.35**

3. $P(A|B) = \frac{P(A\cap B)}{P(B)} = P(A)$ $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.625 = P(A) + P(B) - 0.144$

$P(B) = 0.770 - P(A)$

$P(A) * P(B) = P(A \cap B)$

$P(A)[0.770 - P(A)] = P(A \cap B)$

$P(A)^2 - 0.77P(A) + 0.144 = 0$

$P(A) = 0.45, 0.32$ *choose P(A) = 0.45 since P(A)>P(B)

$P(B) = \frac{P(A \cap B)}{P(A)} = \frac{0.144}{0.45}$ **P(A) = 0.45, P(B) = 0.32**

4. a) 10^{14} = **1e+14**

   b) $\frac{2^{14} - 2}{10^{14}}$ **=1.6382e-10**

5. P(A) = 0.6 P(A)* P(B) = P(A∩B) P(A)*P(C) = P(A∩C)

   P(B) = 0.5 P(A∩B) = 0.3 P(A∩C) = 0.24

   P(C) = 0.4

   P(B) * P(C) = P(B∩C)

   P(B∩C) = (0.5)(0.4) = 0.2

   a) (i) P(A∩B∩C) = 0.6 * 0.5 * 0.4 = **0.12**

      (ii) P(A∩B∩C)' = 1 – 0.12 = **0.88**

   b) (i) P(A∪B'∪C') = 0.6*0.5*0.6 = **0.18**

      (ii) P{(A∪B'∪C') ∪(A'∪B'∪C) ∪(A∪B'∪C') =

      = 0.18 + 0.4*0.5*0.6 + 0.4*0.5*0.4 = **0.38**

6. (i) $P(M \cap S1' \cap S2') = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$

   $P(S1' \cap S2') = (\frac{1}{2} * \frac{1}{2} * \frac{1}{2}) + (\frac{1}{2} * 1 * 1) = \frac{5}{8}$

   $P(M|(S1' \cap S2')) = \frac{1}{8} * \frac{8}{5} = \mathbf{\frac{1}{5}}$

   (ii) $P(S3'|(S1' \cap S2')) = P(S3' \cap (S1' \cap S2'))/ P(S1' \cap S2') = \frac{\frac{1}{2}*\frac{1}{2}*\frac{1}{2}*\frac{1}{2}}{\frac{1}{16}+\frac{1}{2}}$ = **0.111**

7. $P(A_1) = 0.55$ $P(A_1 \cup A_2) = 0.80$

$P(A_2) = 0.65$ $P(A_2 \cap A_3) = 0.40$

$P(A_3) = 0.70$ $P(A_1 \cup A_2 \cup A_3) = 0.88$

a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$0.80 = 0.55 + 0.65 - P(A_1 \cap A_2)$

$P(A_1 \cap A_2) = 0.40$

b) $P(A_2|A_3) = P(A_2 \cap A_3)/ P(A_3) = \frac{0.40}{0.70} = \mathbf{0.5714}$

$P(A_2|A_3)$: The probability that a customer likes vehicle #2 given that they liked vehicle #3

c) (i) $P(A_2|A_3) = P(A_2) \therefore 0.5714 \neq 0.55$

(ii) $P(A_2 \cap A_3) = P(A_2)^* P(A_3) \therefore 0.40 \neq 0.65 * 0.70$

A_2 and A_3 are not independent

d) $P((A_2 \cup A_2)| A_1') = \frac{P(A_1' \cap (A_2 \cup A_2))}{P(A_1')} = \frac{0.88-0.55}{1-0.55} = \mathbf{0.733}$

8. a) $a_0 = P(\text{Allan wins}|\text{ he starts with \$0}) = 0$

$a_5 = P(\text{Allan wins}|\text{ he starts with \$5}) = 1$

b) $a_2 = 0.5\, a_1 + 0.5\, a_3$

c) $a_i = 0.5(a_{i-1} + a_{i+1})$

$a_i - a_{i-1} = 0.5(a_{i+1} - a_{i-1})$

$a_1 = 0.5a_2$

$a_2 - a_1 = 0.5(a_3 - a_1) = 0.5(a_3 - 0.5a_2) \therefore a_2 = 2/3\, a_3$

$a_3 - a_2 = 0.5(a_4 - a_2) \therefore 1/3a_3 = 0.5(a_4 - 2/3a_3) \therefore a_3 = 3/4a_4$

$a_4 - a_3 = 0.5(a_5 - a_3) \therefore 1/4\, a_4 = 0.5(a_5 - 3/4\, a_4) \therefore a_4 = 4/5a_5$

$a_5 = 1$

$a_4 = 0.8$

$a_3 = 0.6$

$a_2 = 0.4$

$a_1 = 0.2$

d) $P(\text{Allan wins}|\text{ they start with \$a and \$b}) = \frac{a}{a+b}$

9.

```
win = 0
ten = 0
for(i in 1:10000){
  dollars=20; flips=0
  while (dollars>0 && dollars<100) {
    flips = flips + 1
    cont = sample(c(-10,10),1)
    dollars = dollars + cont}
  if(dollars==100) {
    win = win + 1
  }
  if(flips <= 10){
    ten = ten + 1
  }

}
```

a) > win/10000

[1] **0.2016**

b) > ten/10000

[1] **0.566**

10.

```
> #Generate 500 birthdays with replacement from 1:365 days in a year
> birthdays = sample(1:365,500,replace=TRUE)
> #frequency counts of table
> freq = as.numeric(table(birthdays))
> #estimate of probability
```

a) > length(which(freq>=3))/365

[1] **0.1890411**

b) > length(which(freq>=5))/365

[1] **0.01643836**