

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0, \text{ with boundary and initial conditions}$$

$$\frac{\partial u}{\partial x}(0, t) = 0 = u(\pi, t), t > 0 \text{ and } u(x, 0) = f(x), 0 < x < \pi \text{ is}$$

a) $u(x, t) = \sum_{n=0}^{\infty} A_n \exp\left(-\left(\frac{2n+1}{2}\right)^2 t\right) \cos\left(\frac{2n+1}{2} x\right)$, with $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos\left(\frac{2n+1}{2} x\right) dx$

b) $u(x, t) = \sum_{n=0}^{\infty} A_n \exp(-n^2 t) \cos(nx)$, with $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos(nx) dx$

c) $u(x, t) = \sum_{n=1}^{\infty} A_n \exp\left(-\left(\frac{2n+1}{2}\right)^2 t\right) \sin\left(\frac{2n+1}{2} x\right)$, with $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin\left(\frac{2n+1}{2} x\right) dx$

d) $u(x, t) = \sum_{n=1}^{\infty} A_n \exp(-n^2 t) \sin(nx)$, with $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$

20) The function $f(x)$ defined by $f(x) = \begin{cases} 3 - x^2, & x \leq 1, \\ 3 - x, & 1 < x \leq 2, \\ x - 1, & x > 2. \end{cases}$ has

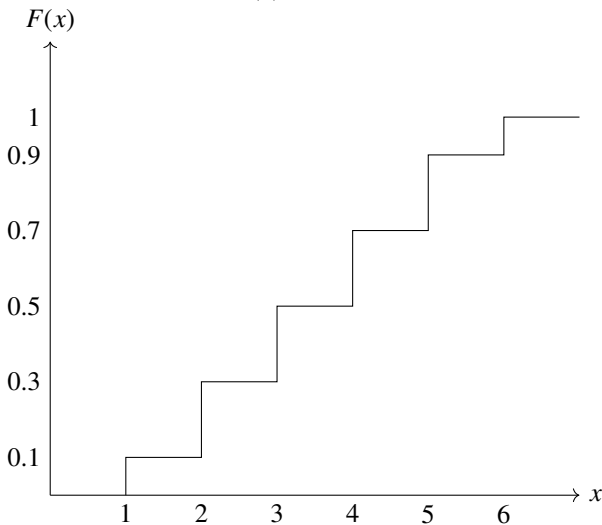
a) a local maxima at $x = 3$ and a local minima at $x = 0$

b) a local maxima at $x = 0$ and no local minima

c) a local maxima at $x = 0$ and a local minima at $x = 2$

d) no local maxima and a local minima at $x = 1$

21) In a biased die experiment, the random variable x of the outcome has the (cumulative) distribution function $F(x)$ shown below.



The variance of x is

a) 1.5

c) 3.5

b) 2.25

d) 4.25

22) For a boundary layer on a flat plate _____ forces and _____ forces are of the same order of magnitude

- a) body, inertia
- b) viscous, body
- c) inertia, viscous
- d) viscous, pressure

23) The temperature field in a fluid flow is given by $(60 - 0.2xy)^\circ\text{C}$. The velocity field is $\vec{V} = 2xy\hat{i} + ty\hat{j} \text{ m/s}$. The rate of change of the temperature measured by a thermometer moving along with the flow at $(2, -4) \text{ m}$ at $t = 4 \text{ s}$ is

- a) -12.8°C/s
- b) -10.6°C/s
- c) -6.4°C/s
- d) -4.8°C/s

24) Two tanks A and B , with the same height are filled with water till the top. The volume of tank A is 10 times the volume of tank B . What can you say about the pressures p_A and p_B at the bottom of the tanks A and B respectively?

- a) $p_A = 10p_B$
- b) $p_B = 10p_A$
- c) $p_A = p_B$
- d) Additional data is required to compare the two pressures

25) A velocity field in a plane flow is given by $\vec{V} = 2xy\hat{i} + 3y\hat{j}$. The vorticity at the point $(2, 4) \text{ m}$ is

- a) $-4\hat{k} \text{ rad/s}$
- b) $-3\hat{j} \text{ rad/s}$
- c) $-2\hat{k} \text{ rad/s}$
- d) $-3\hat{i} \text{ rad/s}$

26) Separation is said to occur at a wall when _____ at the wall becomes zero.

- a) internal energy
- b) pressure
- c) shear stress
- d) density