CHAPTER 3 Quadratic Equation and Inequations(Inequalities)

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Section-A JEE Advanced / IIT JEE

G : Comprehension Based Questions

$$PASSAGE - 1$$

Let p, q be integers and let α,β be the roots of the equation, $x^2 - x + 1$, where $\alpha \neq \beta$. For $n = 0, 1, 2 \ldots$, let $a_n = p\alpha^n + q\beta^n$

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b.s$

1) $a_{12} =$ (JEE Adv.2017)
   (a) $a_{11} - a_{10}$
   (b) $a_{11} + a_{10}$
   (c) $2a_{11} + a_{10}$
   (d) $a_{11} + 2a_{10}$
2) If $a_4 = 28$ and $p + 2q =$ (JEE Adv.2017)

(a) 21 (b) 14
(c) 7 (d) 12

H : Assertion & Reason Type Questions

1) Let a, b, c, p, q be real numbers. Suppose α,β are the roots of equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$ (2008)

STATEMENT-1 : $\left(p^2 - q\right)\left(b^2 - c\right) \geq 0$
and
STATEMENT-2 : $b \neq pa$ or $c \neq qa$

(a) STATEMENT-1 is true, STATEMENT-2 is true; STATEMENT-2 is a correct explanation for STATEMENT-1
(b) STATEMENT-1 is true, STATEMENT-2 is true; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(c) STATEMENT-1 is true,STATEMENT-2 is false
(d) STATEMENT-1 is false,STATEMENT-2 is true

I : INTEGER VALUE CORRECT TYPE

1) Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations :

$$3x - y - z = 0$$

$$-3x + z = 0$$

$$-3x + 2y + z = 0$$

Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is (2009)

2) The smallest value of k, for which both of the roots of the equation

$$x^2 - 8kx + 16\left(k^2 - k + 1\right) = 0$$

are real, distinct and have values at least 4, is (2009)

3) The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} where $a > 0$ is (2011)

4) The number of distinct real roots of

$$x^4 - 4x^3 + 12x^2 + x - 1 = 0$$

is (2011)

Section-B JEE Main / AIEEE

1) If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is (2002)

(a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$

2) Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then (2002)

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

3) Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ (2002)

(a) is always positive (b) is always negative
(c) does not exist (d) none of these

4) If p and q are the roots of the equation $x^2 + px + q = 0$, then (2002)

(a) $p = 1, q = 2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

5) If a, b, c are distinct $+ve$ real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is (2002)

(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real no.

6) If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in (2003)

(a) Arithmetic - Geometric Progression
(b) Arithmetic Progression
(c) Geometric Progression
(d) Harmonic Progression

7) The value of 'a' for which one root of the quadratic equation $\left(a^2 - 5a + 3\right) x^2 + (3a - 1)\, x + 2 = 0$ is twice as large as the other is (2003)

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $-\frac{2}{3}$ (d) $\frac{1}{3}$