CHAPTER 12 Differentiation

ai24btech11022 - Pabbuleti Venkata Charan Teja

Section-A JEE Advanced / IIT-JEE

A : Fill in the Blanks

7) If $f(x)$ is a twice differentiable function and given that $f(1)=1, f(2)=4, f(3)=9$, then (2005S)
   (a) $f''(x)=2\ ,\forall\ x\in(1,3)$
   (b) $f''(x)=f'(x)=5$ for some $x\in(2,3)$
   (c) $f''(x)=3\ ,\forall\ x\in(2,3)$
   (d) $f''(x)=2$ for some $x\in(1,3)$
8) $\frac{d^2x}{dy^2}$ (2007-3 marks)
   (a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
   (c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$
9) Let $g(x)=\log f(x)$ is twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=xf(x)$. Then, for $N=1,2,3,\ldots$ (2008)
   $g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=$
   (a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N-1)^2}\right\}$
   (b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N-1)^2}\right\}$
   (c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N+1)^2}\right\}$
   (d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N+1)^2}\right\}$
10) Let $f:[0,2]\to\mathbb{R}$ be a function which is continuous on $[0,2]$ and is differentiable on $(0,2)$ with $f(0)=1$. Let $F(x)=\int_0^{x^2} f\left(\sqrt{t}\right)dt$ for $x\in[0,2]$. If $F'(x)=f'(x)$ for all $x\in(0,2)$ then $F(2)$ equals (JEE Adv. 2014)
   (a) e^2-1 (b) e^4-1
   (c) $e-1$ (d) e^4

D : MCQs with One or More than One Correct

1) Let $f:\mathbb{R}\to\mathbb{R}$, $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ be differentiable functions such that $f(x)=x^3+3x+2$, $g(f(x))=x$ and $h(g(g(x)))=x$, $\forall x\in\mathbb{R}$. Then (JEE Adv. 2016)
   (a) $g'(2)=\frac{1}{15}$ (b) $h'(1)=666$
   (c) $h(0)=16$ (d) $h(g(3))=36$
2) For every twice differential function $f:\mathbb{R}\to[-2,2]$ with $(f(0))^2+(f'(0))^2=85$, which of the following statement(s) is(are) TRUE? (JEE Adv. 2018)
   (a) There exist $r,s\in\mathbb{R}$, where $r<s$, such that f is one-one on the open interval (r,s)
   (b) There exists $x_0\in(-4,0)$ such that $|f'(x_0)|\le 1$
   (c) $\lim_{x\to\infty} f(x)=1$
   (d) There exists $\alpha\in(-4,4)$ such that $f(\alpha)+f''(\alpha)=0$ and $f'(\alpha)\neq 0$
3) For any positive integer n, define $f_n(x)=\sum_{j=1}^{n}\tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right), \forall\, x\in(0,\infty)$
   Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.
   Then, which of the following statement(s) is (are) TRUE? (JEE Adv. 2018)
   (a) $\sum_{j=1}^{5}\tan^2\left(f_j(0)\right)=55$
   (b) $\sum_{j=1}^{10}\left(1+f_j'(0)\right)\left(\sec^2\left(f_j(0)\right)\right)=10$
   (c) For any fixed positive integer n, $\lim_{x\to\infty}\tan(f_n(x))=\frac{1}{n}$
   (d) For any fixed positive integer n, $\lim_{x\to\infty}\sec^2(f_n(x))=1$
4) Let $f:(0,\pi)\to\mathbb{R}$ be a twice differentiable function such that $\lim_{t\to x}\frac{f(x)\sin t-f(t)\sin x}{t-x}=\sin^2 x$, $\forall x\in(0,\pi)$. If $f\left(\frac{\pi}{6}\right)=-\frac{\pi}{12}$, then which of the following statement(s) is(are) TRUE? (JEE Adv. 2018)
   (a) $f\left(\frac{\pi}{4}\right)=\frac{\pi}{4\sqrt{2}}$
   (b) $f(x)<\frac{x^4}{6}-x^2$, $\forall x\in(0,\pi)$
   (c) There exists $\alpha\in(0,\pi)$ such that $f'(\alpha)=0$
   (d) $f''\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=0$

E : Subjective Problems

1) Find the derivative of $\sin\left(x^2+1\right)$ with respect to x from first principle. (1978)

2) Find the derivative of

$$f(x)=\begin{cases}\frac{x-1}{2x^2-7x+5} & when\ x\neq 1\\ -\frac{1}{3} & when\ x=1\end{cases} \qquad (1)$$

at $x=1$ (1979)

3) Given $y=\frac{5x}{3\sqrt{(1-x^2)}}+\cos^2(2x+1)$. Find $\frac{dy}{dx}$ (1980)

4) Let $y=e^{x\sin x^3}+(\tan x)^x$. Find $\frac{dy}{dx}$ (1981 - 2 Marks)