1.1.5.10

AI24BTECH11022 - Pabbuleti Venkata Charan Teja

Question:

Find the ratio in which the line segment joining the points $A\,(1,-5)$ and $B\,(-4,5)$ is divided by the x-axis. Also, find the coordinates of the point of division.

(10,2021)

Solution:

Variable	Value
A	$\begin{pmatrix}1\\-5\end{pmatrix}$
B	$\begin{pmatrix}-4\\5\end{pmatrix}$
$k:1$	Ratio in which the line AB is divided by x-axis
X	Point of division of A , B

Table 1: Variables Used

If X divides AB in the ratio $k:1$,

$$X = \frac{kB + A}{k+1} \tag{1}$$

$$X = \frac{1}{k+1}\begin{pmatrix}-4k+1\\5k-5\end{pmatrix} \tag{2}$$

$$X = \begin{pmatrix}x\\y\end{pmatrix} \tag{3}$$

$$X = x\begin{pmatrix}1\\0\end{pmatrix} + y\begin{pmatrix}0\\1\end{pmatrix} \tag{4}$$

$$X = \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} \tag{5}$$

$$\implies \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{k+1}\begin{pmatrix}-4k+1\\5k-5\end{pmatrix} \tag{6}$$

But as X is on x-axis,

$$y = 0 \tag{7}$$

$$\frac{5k-5}{k+1} = 0 \tag{8}$$

$$5k - 5 = 0 \tag{9}$$

$$k = 1 \tag{10}$$

$\therefore$ The ratio in which the line is divided by x-axis is $1:1$
The coordinates of the point of the division is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{k+1}\begin{pmatrix} -4k+1 \\ 5k-5 \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} \tag{13}$$

$\therefore$ The point of division $X = \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix}$

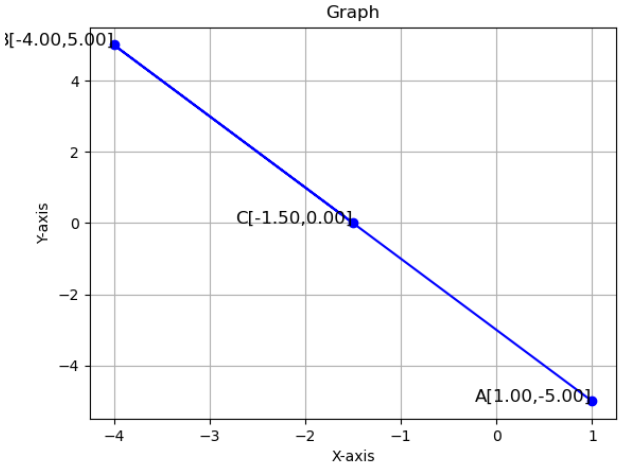

Fig. 1: Plot of the points