9-9.2-43

AI24BTECH11022 - Pabbuleti Venkata Charan Teja

Question:
Find the area of the region bounded by the curve $y = x^2$ and $y = x + 6$ and $x = 0$.

Solution:

Variable	**Value**
V	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}$
f	0
h	$\begin{pmatrix} -6 \\ 0 \end{pmatrix}$
m	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Table 1: Variables Used

The given curve can be expresssed as a conic with parameters

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, u = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}, f = 0 \tag{1}$$

The given line parameters are

$$h = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, m = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$

Substituting from the above in

$$k_i = \frac{1}{m^\top Vm} \left(-m^\top (Vh + u) \pm \sqrt{\left[m^\top (Vh + u)\right]^2 - g(h)\left(m^\top Vm\right)} \right)$$

gives

$$k_1 = 4, k_2 = 9 \tag{3}$$

Substituting values of k_i in $x_i = h + k_i m$ gives points of intersection of the line and the curve

$$\implies x_1 = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, x_2 = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \tag{4}$$

According to the plot,

The required area in 2nd Quadrant is

$$Area = \int_{-2}^{0} (x+6) - x^2 dx \tag{5}$$

$$Area = \frac{22}{3} \tag{6}$$

The required area in 1st Quadrant is

$$Area = \int_{0}^{3} (x+6) - x^2 dx \tag{7}$$

$$Area = \frac{27}{2} \tag{8}$$

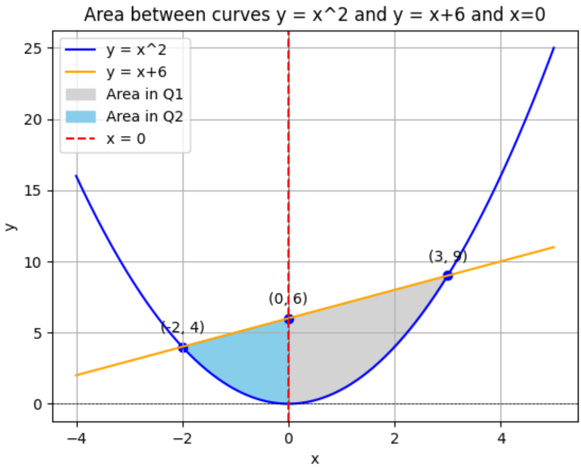

Fig. 1: Plot of the points