

1.1.9.23

AI24BTECH11022 - Pabbuleti Venkata Charan Teja

Question:

If the point $P \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ is equidistant from the points $Q \begin{pmatrix} 3 \\ k \end{pmatrix}$ and $R \begin{pmatrix} k \\ 5 \end{pmatrix}$, find the value of k .
(10, 2018)

Solution:

Variable	Value
P	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
Q	$\begin{pmatrix} 3 \\ k \end{pmatrix}$
R	$\begin{pmatrix} k \\ 5 \end{pmatrix}$

Table 1: Variables Used

If P is equidistant from the points Q and R ,

$$\|P - Q\| = \|P - R\| \quad (1)$$

$$\implies \|P - Q\|^2 = \|P - R\|^2 \quad (2)$$

$$\implies \|P\|^2 - 2P^T Q + \|Q\|^2 = \|P\|^2 - 2P^T R + \|R\|^2 \quad (3)$$

which can be simplified to obtain,

$$(Q - R)^T P = \frac{\|Q\|^2 - \|R\|^2}{2} \quad (4)$$

$$(Q - R)^T = \begin{pmatrix} 3 - k \\ k - 5 \end{pmatrix} \quad (5)$$

$$(Q - R)^T P = (3 - k \quad k - 5) \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (6)$$

$$(Q - R)^T P = 2k - 10 \quad (7)$$

$$\|Q\|^2 = Q^T Q = 9 + k^2 \quad (8)$$

$$\|R\|^2 = R^T R = 25 + k^2 \quad (9)$$

$$\frac{\|Q\|^2 - \|R\|^2}{2} = -8 \quad (10)$$

Substituting the equations gives,

$$2k - 10 = -8 \quad (11)$$

$$\implies k = 1 \quad (12)$$

∴ The points Q and R are,

$$Q = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (13)$$

$$R = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (14)$$

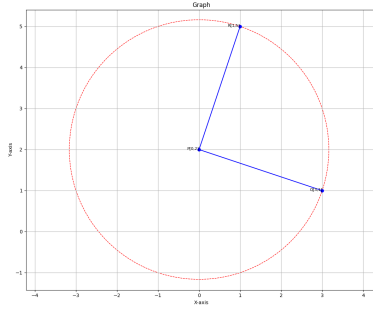


Fig. 1: Plot of the points