1.1.9.23

AI24BTECH11022 - Pabbuleti Venkata Charan Teja

Question:

If the point $P\begin{pmatrix}0\\2\end{pmatrix}$ is equidistant from the points $Q\begin{pmatrix}3\\k\end{pmatrix}$ and $R\begin{pmatrix}k\\5\end{pmatrix}$, find the value of k.

(10, 2018)

Solution:

Variable	Value
P	$\begin{pmatrix}0\\2\end{pmatrix}$
Q	$\begin{pmatrix}3\\k\end{pmatrix}$
R	$\begin{pmatrix}k\\5\end{pmatrix}$

Table 1: Variables Used

If P is equidistant from the points Q and R,

$$\|P-Q\| = \|P-R\| \tag{1}$$

$$\implies \|P-Q\|^2 = \|P-R\|^2 \tag{2}$$

$$\implies \|P\|^2 - 2P^\top Q + \|Q\|^2 = \|P\|^2 - 2P^\top R + \|R\|^2 \tag{3}$$

which can be simplified to obtain,

$$(Q-R)^\top P = \frac{\|Q\|^2 - \|R\|^2}{2} \tag{4}$$

$$(Q-R)^\top = \begin{pmatrix}3-k\\k-5\end{pmatrix} \tag{5}$$

$$(Q-R)^\top P = \begin{pmatrix}3-k & k-5\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \tag{6}$$

$$(Q-R)^\top P = 2k-10 \tag{7}$$

$$\|Q\|^2 = Q^\top Q = 9+k^2 \tag{8}$$

$$\|R\|^2 = R^\top R = 25+k^2 \tag{9}$$

$$\frac{\|Q\|^2 - \|R\|^2}{2} = -8 \tag{10}$$

Substituting the equations gives,

$$2k-10 = -8 \tag{11}$$

$$\implies k = 1 \tag{12}$$

$\therefore$The points Q and R are,

$$Q = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{13}$$

$$R = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{14}$$

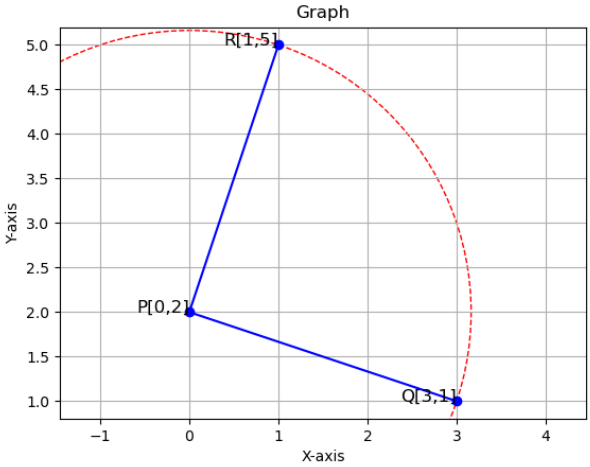

Fig. 1: Plot of the points