

▼ Question 4 and 6

```
from numpy.random import rand
from numpy.random import randn

from numpy import hstack
from numpy import zeros
from numpy import ones

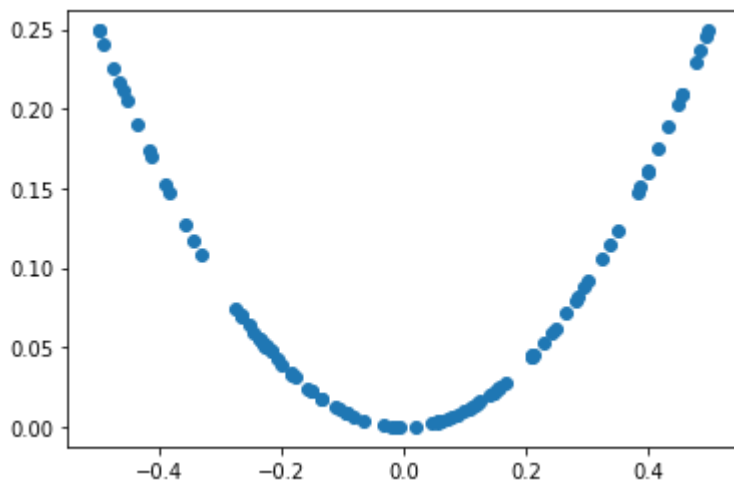
from matplotlib import pyplot

from keras.models import Sequential
from keras.layers import Dense
from keras.utils.vis_utils import plot_model
```

▼ Data-Set

```
# generate real randoms sample from  $x^2$ 
def generate_real_samples(n=100):
    # generate random inputs in  $[-0.5, 0.5]$ 
    X1 = rand(n) - 0.5
    # generate outputs  $X^2$  (quadratic)
    X2 = X1 * X1
    # stack arrays
    X1 = X1.reshape(n, 1)
    X2 = X2.reshape(n, 1)
    X = hstack((X1, X2))
    # generate class labels
    y = ones((n, 1))
    return X, y

# generate samples
data, y = generate_real_samples()
# plot samples
pyplot.scatter(data[:, 0], data[:, 1])
pyplot.show()
```



A simple dataset has been created in the form of x^2 and has been given a class value true

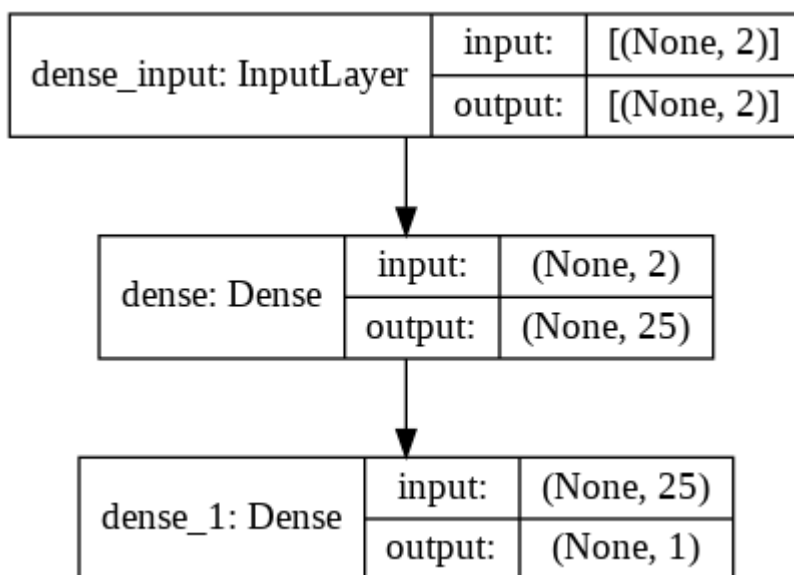
▼ Discriminator

```
# define the standalone discriminator model
def define_discriminator(n_inputs=2):
    model = Sequential()
    model.add(Dense(25, activation='relu', kernel_initializer='he_uniform', input_dim=n_inputs))
    model.add(Dense(1, activation='sigmoid'))
    # compile model
    model.compile(loss='binary_crossentropy', optimizer='adam', metrics=['accuracy'])
    return model

# define the discriminator model
model = define_discriminator()
# summarize the model
model.summary()
# plot the model
plot_model(model, to_file='discriminator_plot.png', show_shapes=True, show_layer_names=True)
```

Model: "sequential"

Layer (type)	Output Shape	Param #
dense (Dense)	(None, 25)	75
dense_1 (Dense)	(None, 1)	26
Total params: 101		
Trainable params: 101		
Non-trainable params: 0		



A simple MLP-3 model has been made for the discriminator with 2 inputs, a relu activation in the hidden layer (25 neurons) and sigmoid activation in the output layer.

▼ Training the discriminator with fake samples created by us

```

# generate n fake samples with class labels
def generate_fake_samples(n):
    # generate inputs in [-1, 1]
    X1 = -1 + rand(n) * 2
    # generate outputs in [-1, 1]
    X2 = -1 + rand(n) * 2
    # stack arrays
    X1 = X1.reshape(n, 1)
    X2 = X2.reshape(n, 1)
    X = hstack((X1, X2))
    # generate class labels
  
```

```
y = zeros((n, 1))
return X, y
```

Fake samples are created that closely resemble the dataset and are given the class value false

```
# train the discriminator model
def train_discriminator(model, n_epochs=1000, n_batch=128):
    half_batch = int(n_batch / 2)
    # run epochs manually
    for i in range(n_epochs):
        # generate real examples
        X_real, y_real = generate_real_samples(half_batch)
        # update model
        model.train_on_batch(X_real, y_real)
        # generate fake examples
        X_fake, y_fake = generate_fake_samples(half_batch)
        # update model
        model.train_on_batch(X_fake, y_fake)
        # evaluate the model
        _, acc_real = model.evaluate(X_real, y_real, verbose=0)
        _, acc_fake = model.evaluate(X_fake, y_fake, verbose=0)
        print(i, acc_real, acc_fake)
```

```
# define the discriminator model
model = define_discriminator()
# fit the model
train_discriminator(model)
```

```
941 1.0 0.859375
942 1.0 0.828125
943 1.0 0.875
944 1.0 0.90625
945 1.0 0.90625
946 1.0 0.890625
947 1.0 0.875
948 1.0 0.8125
949 1.0 0.875
950 1.0 0.84375
951 1.0 0.875
952 1.0 0.796875
953 1.0 0.9375
954 1.0 0.96875
955 1.0 0.921875
956 1.0 0.84375
957 1.0 0.890625
958 1.0 0.84375
959 1.0 0.90625
960 1.0 0.90625
961 1.0 0.875
962 1.0 0.890625
963 1.0 0.859375
964 1.0 0.859375
965 1.0 0.875
966 1.0 0.84375
```

```

967 1.0 0.890625
968 1.0 0.859375
969 1.0 0.90625
970 1.0 0.875
971 1.0 0.90625
972 1.0 0.875
973 1.0 0.90625
974 1.0 0.84375
975 1.0 0.859375
976 1.0 0.84375
977 1.0 0.859375
978 1.0 0.828125
979 1.0 0.921875
980 1.0 0.84375
981 1.0 0.828125
982 1.0 0.90625
983 1.0 0.828125
984 1.0 0.875
985 1.0 0.796875
986 1.0 0.828125
987 1.0 0.890625
988 1.0 0.859375
989 1.0 0.921875
990 1.0 0.84375
991 1.0 0.921875
992 1.0 0.8125

993 1.0 0.890625
994 1.0 0.84375
995 1.0 0.84375
996 1.0 0.953125
997 1.0 0.90625
998 1.0 0.9375
999 1.0 0.890625

```

From the training it can be seen that the model is 100% accurate with the dataset and can to a great extent guess when a fake sample is given to the model

▼ Generator

```

# define the standalone generator model
def define_generator(latent_dim, n_outputs=2):
    model = Sequential()
    model.add(Dense(15, activation='relu', kernel_initializer='he_uniform', input_dim=latent_dim))
    model.add(Dense(n_outputs, activation='linear'))
    return model

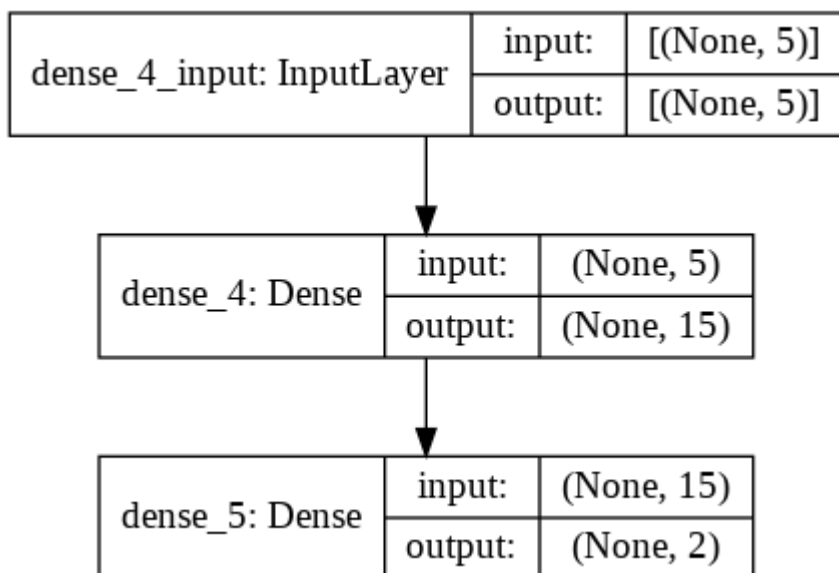
# define the generator model
model = define_generator(5)
# summarize the model

```

```
model.summary()
# plot the model
plot_model(model, to_file='generator_plot.png', show_shapes=True, show_layer_names=True)
```

Model: "sequential_2"

Layer (type)	Output Shape	Param #
dense_4 (Dense)	(None, 15)	90
dense_5 (Dense)	(None, 2)	32
Total params: 122		
Trainable params: 122		
Non-trainable params: 0		



A simple MLP-3 model has been made for the generator with 5 inputs, a relu activation in the hidden layer (15 neurons) and no non linearity in the output layer.

▼ generator used to generate data

```
# generate points in latent space as input for the generator
def generate_latent_points(latent_dim, n):
    # generate points in the latent space
    x_input = randn(latent_dim * n)
    # reshape into a batch of inputs for the network
    x_input = x_input.reshape(n, latent_dim)
    return x_input
```

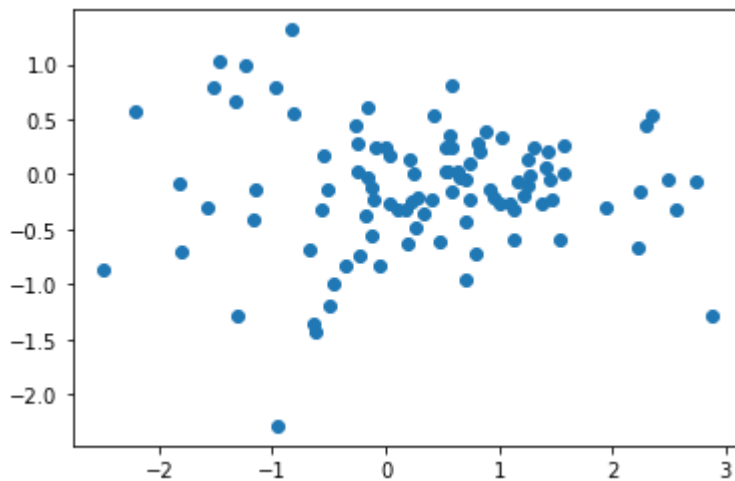
```
# use the generator to generate n fake examples and plot the results
def generate_fake_samples(generator, latent_dim, n):
    # generate points in latent space
```

```

x_input = generate_latent_points(latent_dim, n)
# predict outputs
X = generator.predict(x_input)
# plot the results
pyplot.scatter(X[:, 0], X[:, 1])
pyplot.show()

# size of the latent space
latent_dim = 5
# define the discriminator model
model = define_generator(latent_dim)
# generate and plot generated samples
generate_fake_samples(model, latent_dim, 100)

```



The generator will be trained with the help of the discriminator in the GAN which will be shown below.

▼ GAN Model

```

def define_gan(generator, discriminator):
    # make weights in the discriminator not trainable
    discriminator.trainable = False
    # connect them
    model = Sequential()
    # add generator
    model.add(generator)
    # add the discriminator
    model.add(discriminator)
    # compile model
    model.compile(loss='binary_crossentropy', optimizer='adam')
    return model

```

```

# size of the latent space

```

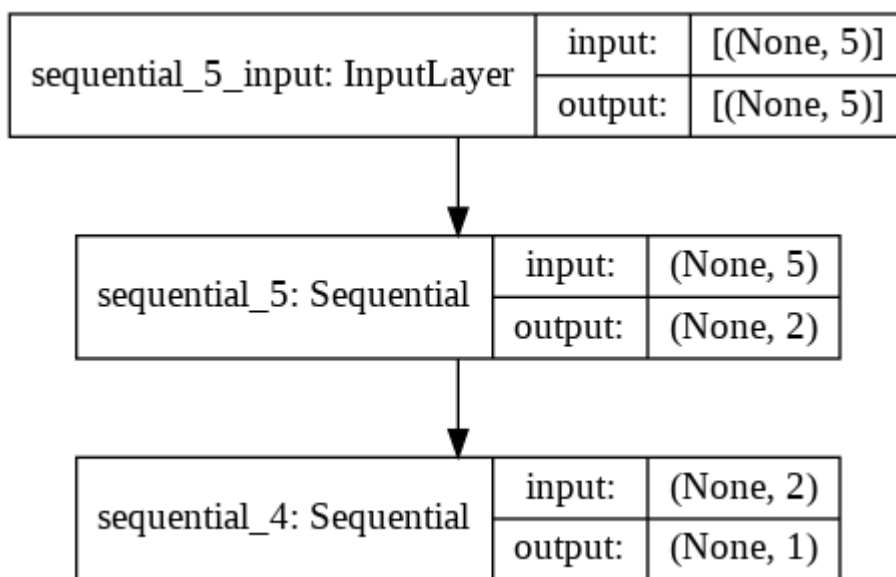
```

latent_dim = 5
# create the discriminator
discriminator = define_discriminator()
# create the generator
generator = define_generator(latent_dim)
# create the gan
gan_model = define_gan(generator, discriminator)
# summarize gan model
gan_model.summary()
# plot gan model
plot_model(gan_model, to_file='gan_plot.png', show_shapes=True, show_layer_names=True)

```

Model: "sequential_6"

Layer (type)	Output Shape	Param #
sequential_5 (Sequential)	(None, 2)	122
sequential_4 (Sequential)	(None, 1)	101
Total params: 223		
Trainable params: 122		
Non-trainable params: 101		



The GAN model is made by combining the generator and the discriminator from above

▼ Training the evaluating the GAN Model

```

# use the generator to generate n fake examples, with class labels
def generate_fake_samples(generator, latent_dim, n):
    # generate points in latent space
    x_input = generate_latent_points(latent_dim, n)
    # predict outputs

```



```

    # prepare outputs
    X = generator.predict(x_input)
    # create class labels
    y = zeros((n, 1))
    return X, y

# evaluate the discriminator and plot real and fake points
def summarize_performance(epoch, generator, discriminator, latent_dim, n=100):
    # prepare real samples
    x_real, y_real = generate_real_samples(n)
    # evaluate discriminator on real examples
    _, acc_real = discriminator.evaluate(x_real, y_real, verbose=0)
    # prepare fake examples
    x_fake, y_fake = generate_fake_samples(generator, latent_dim, n)
    # evaluate discriminator on fake examples
    _, acc_fake = discriminator.evaluate(x_fake, y_fake, verbose=0)
    # summarize discriminator performance
    print('epoch', epoch, 'real accuracy: ', acc_real, ' fake accuracy: ', acc_fake)
    # scatter plot real and fake data points
    pyplot.scatter(x_real[:, 0], x_real[:, 1], color='red')
    pyplot.scatter(x_fake[:, 0], x_fake[:, 1], color='blue')
    pyplot.show()

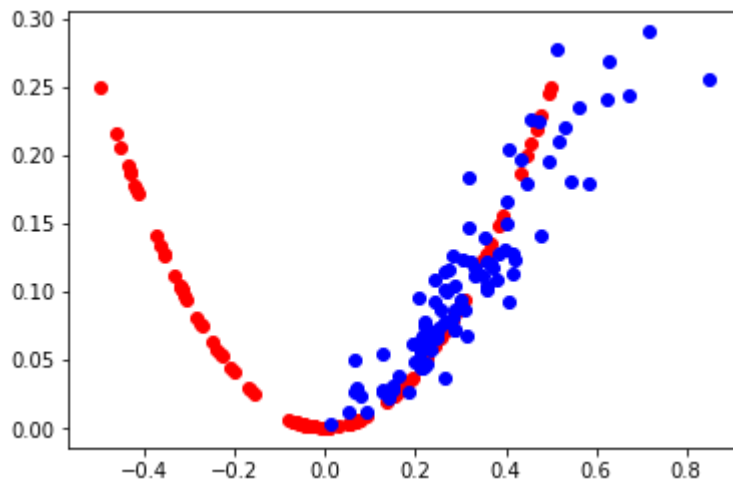
# train the generator and discriminator
def train(g_model, d_model, gan_model, latent_dim, n_epochs=10000, n_batch=128, n_eval=2000):
    # determine half the size of one batch, for updating the discriminator
    half_batch = int(n_batch / 2)
    # manually enumerate epochs
    for i in range(n_epochs):
        # prepare real samples
        x_real, y_real = generate_real_samples(half_batch)
        # prepare fake examples
        x_fake, y_fake = generate_fake_samples(g_model, latent_dim, half_batch)
        # update discriminator
        d_model.train_on_batch(x_real, y_real)
        d_model.train_on_batch(x_fake, y_fake)
        # prepare points in latent space as input for the generator
        x_gan = generate_latent_points(latent_dim, n_batch)
        # create inverted labels for the fake samples
        y_gan = ones((n_batch, 1))
        # update the generator via the discriminator's error
        gan_model.train_on_batch(x_gan, y_gan)
        # evaluate the model every n_eval epochs
        if (i+1) % n_eval == 0:
            summarize_performance(i, g_model, d_model, latent_dim)

# size of the latent space
latent_dim = 5
# create the discriminator
discriminator = define_discriminator()

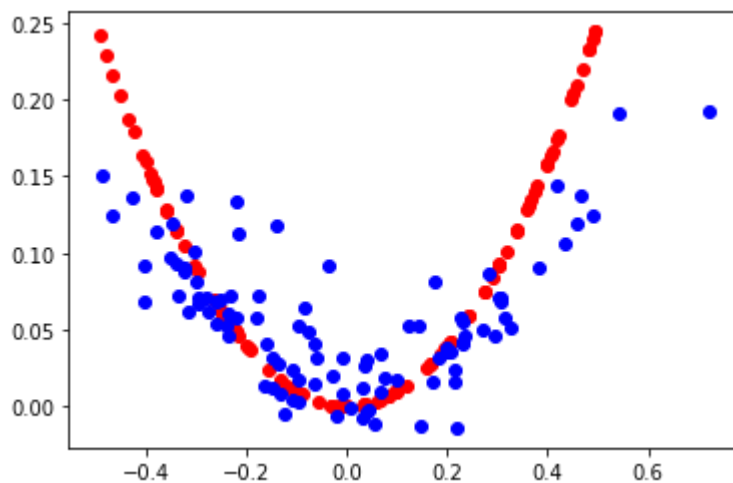
```

```
# create the generator
generator = define_generator(latent_dim)
# create the gan
gan_model = define_gan(generator, discriminator)
# train model
train(generator, discriminator, gan_model, latent_dim)
```

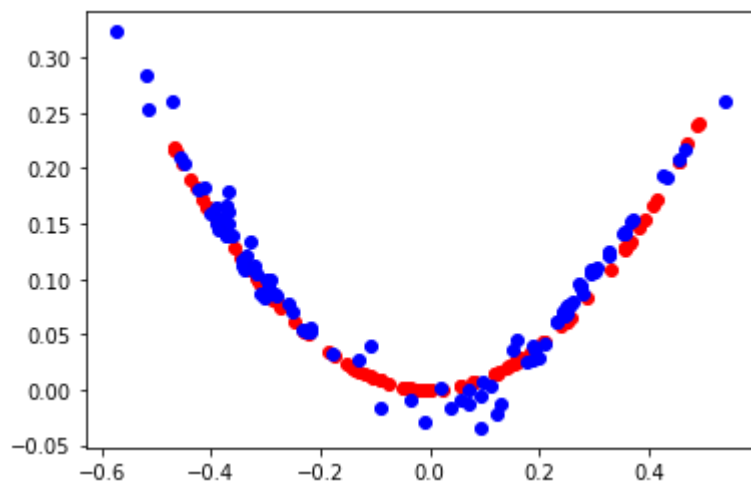
epoch 1999 real accuracy: 0.4699999988079071 fake accuracy: 0.8199999928474426



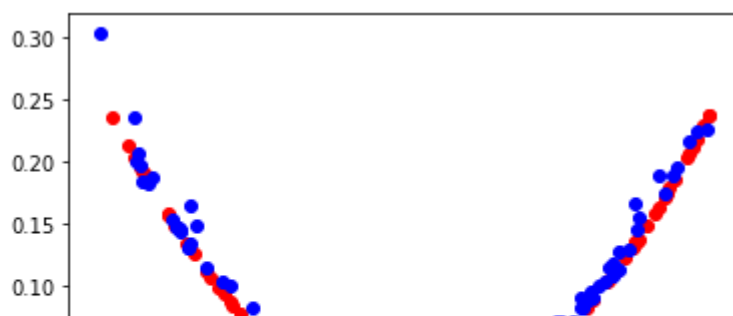
epoch 3999 real accuracy: 0.4300000071525574 fake accuracy: 0.6600000262260437

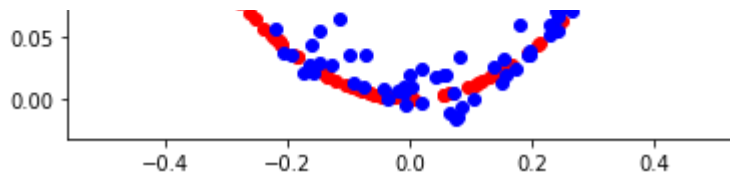


epoch 5999 real accuracy: 0.4099999964237213 fake accuracy: 0.47999998927116394

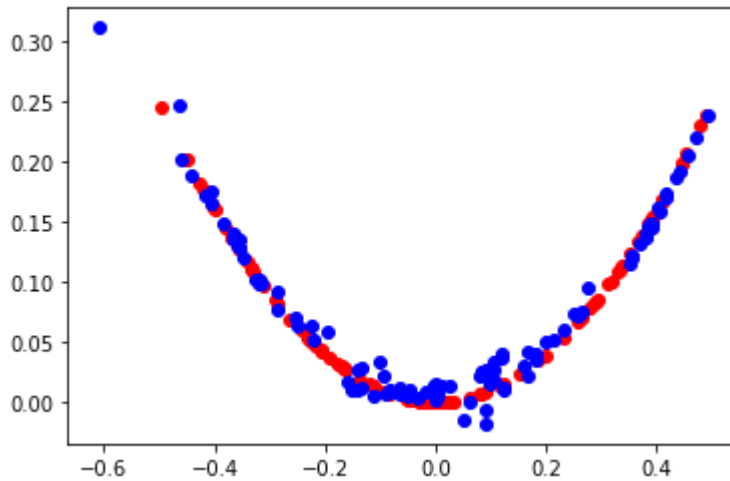


epoch 7999 real accuracy: 0.6600000262260437 fake accuracy: 0.4000000059604645





epoch 9999 real accuracy: 0.5600000023841858 fake accuracy: 0.3700000047683716



from the results of training we can see-

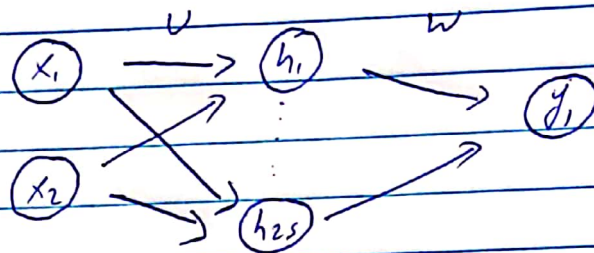
1. The plot shows us the the output of the generator plotted along side the dataset. We can see that in the start of training the generator was produces data that looked nothing like the dataset. As training proceeded we can see the incrmal improvements the generator made in producing data that behaved more like our dataset.
2. The accuracy of the discriminator is also printed above the plots with the respective epoch. We cen see at the start the discriminator is fairly able to categorize the fake data generated by the generator correctly. But as training proceeds and the generator is tuned better we can see a dip in accuracy of the discriminator being able categorize the fake data even though the accuracy of the real dataset increases.

✓ 9m 10s completed at 8:46 PM



5. GAN Model.

→ Phase-I - Discriminator.



Input layer hidden layer Output layer

2 Neuron 25 Neuron 1 Neuron

→ $x \in \mathbb{R}^2 \quad t \in \mathbb{Z}^1$

→ $U \in \mathbb{R}^{25 \times 2}, \quad b \in \mathbb{R}^{25}, \quad W \in \mathbb{R}^{1 \times 25}, \quad c \in \mathbb{R}^1$

→ Equation.

$$g = UX + b.$$

$$h = \phi'(g)$$

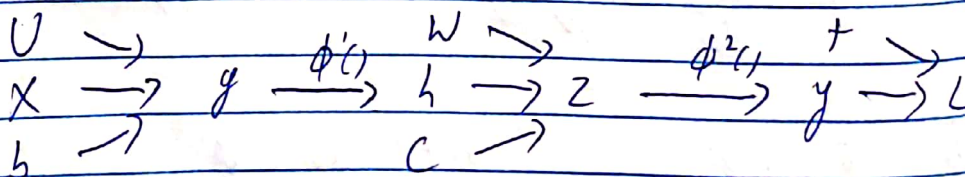
$$z = Wh + c$$

$$y = \phi^z(z)$$

$\phi'(t) \rightarrow \text{ReLU F'n.}$

$\phi(t) \rightarrow \text{Sigmoid.}$

→ CFG



$$L = \frac{1}{N} \sum_{i=1}^N \left[t \log(1 + \exp(-z)) + (1-t) \log(1 + \exp(z)) \right]$$

→ Back Propagation

$$\bar{L} = \frac{dL}{dt} = 1.$$

$$\bar{z} = \bar{L} \cdot \frac{dz}{dt} = 1 \cdot \frac{1}{N} (y-t)$$

$$\bar{w} = \bar{z} \cdot \frac{dz}{dw} = \frac{1}{N} (y-t) \cdot h.$$

$$\bar{c} = \bar{z} \cdot \frac{dz}{dc} = \frac{1}{N} (y-t) \cdot 1$$

$$\bar{h} = \bar{z} \cdot \frac{dz}{dh} = \frac{1}{N} (y-t) \cdot w.$$

$$\bar{g} = \bar{h} \cdot \frac{dh}{dg} = \frac{1}{N} (y-t) \cdot w \cdot \phi''(g) \quad \text{with } \frac{d^2 \phi}{dg^2} = \frac{1}{w}.$$

$$\phi''(g) = \begin{cases} 1 & g \geq 0. \\ 0 & \text{otherwise.} \end{cases}$$

$$\bar{u} = \bar{g} \cdot \frac{dg}{du} = \frac{1}{N} (y-t) \cdot w \cdot \phi'(g) \cdot x$$

$$\bar{b} = \bar{g} \cdot \frac{dg}{db} = \frac{1}{N} (y-t) \cdot w \cdot \phi'(g)$$

-> note -> $t = 1$ if input from dataset.
 $t = 0$ if input from generator.

-> Weight update.

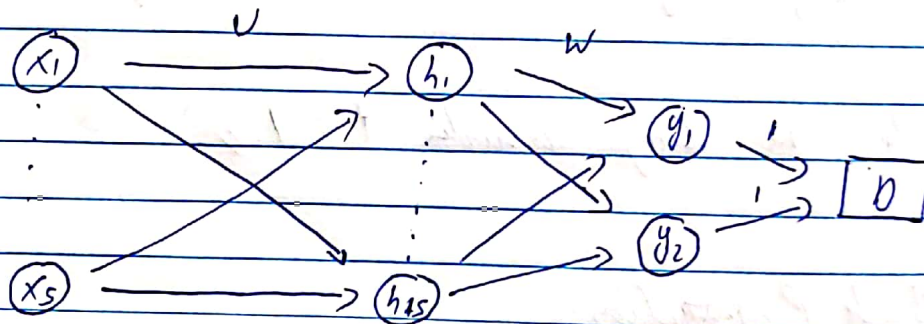
$$U' = U - \alpha \bar{U}$$

$$b' = b - \alpha \bar{b}.$$

$$W' = W - \alpha \bar{W}.$$

$$C' = C - \alpha \bar{C}.$$

-> Phase - 2 - Generator.



Input layer.

hidden layer.

Output layer.

Discriminator

5 Neurons

15 Neurons

2 Neurons

$$\Rightarrow x \in \mathbb{R}^5, y \in \mathbb{R}^2$$

$$\Rightarrow U \in \mathbb{R}^{15 \times 5}, b \in \mathbb{R}^{15}, W \in \mathbb{R}^{2 \times 15}, C \in \mathbb{R}^2$$

⊗ -> The outputs of the generator are fed into the discriminator with no weights.

→ ~~CFG~~

→ Equation:

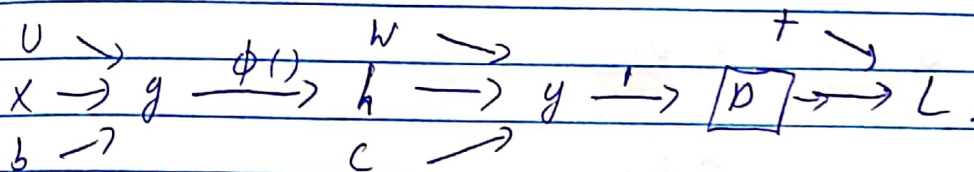
$$g = Ux + b.$$

$$h = \phi(g)$$

$\phi() \rightarrow \text{ReLU F'n.}$

$$y = Wh + c.$$

→ CFG.



$$L = \frac{1}{N} \sum_i^N [t \log(1 + \exp(-2z_i)) + (1-t) \log(1 + \exp(2z_i))]$$

→ note → we always consider $t = 1$.

→ Back Propagation.

→ Considering the results we got in phase - 1
Back - Propagation.

$$\bar{x}_0 = \bar{g}_0 \cdot \frac{dg_0}{dx_0} = \bar{g}_0 \cdot U_0 \quad (\text{from discriminative equations}).$$

$$\bar{y} = \bar{x}_0.$$

$$\bar{h} = \bar{y} \cdot \frac{dy}{dh} = \bar{y} \cdot W.$$

$$\bar{w} = \bar{y} \cdot \frac{dy}{dw} = \bar{y} \cdot h.$$

$$\bar{c} = \bar{y} \cdot \frac{dy}{dc} = \bar{y}$$

$$\bar{g} = \bar{h} \cdot \frac{dh}{dy} = \bar{h} \cdot \phi'(y).$$

$$\phi'(y) = \begin{cases} 1 & y \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\bar{u} = \bar{g} \cdot \frac{dg}{du} = \bar{g} \cdot x.$$

$$\bar{b} = \bar{g} \cdot \frac{dg}{db} = \bar{g}$$

→ Weight Updates

$$w' = w - \alpha \bar{w}$$

$$c' = c - \alpha \bar{c}$$

$$u' = u - \alpha \bar{u}$$

$$b' = b - \alpha \bar{b}.$$

▼ Question 7

```

from numpy.random import rand
from numpy.random import randn

import numpy as np

from numpy import hstack
from numpy import zeros
from numpy import ones

from matplotlib import pyplot
from sklearn.metrics import mean_squared_error
from scipy.special import expit as sigmoid

from keras.models import Sequential
from keras.layers import Dense
from keras.utils.vis_utils import plot_model
from keras import backend as k
from keras import losses

def loss_stable(z,t,N):
    return (1./N) * np.sum(t*np.logaddexp(0,-z) + (1-t)*np.logaddexp(0,z))

def drelu(z):
    z[z<=0] = 0
    z[z>0] = 1
    return z

```

▼ Dataset

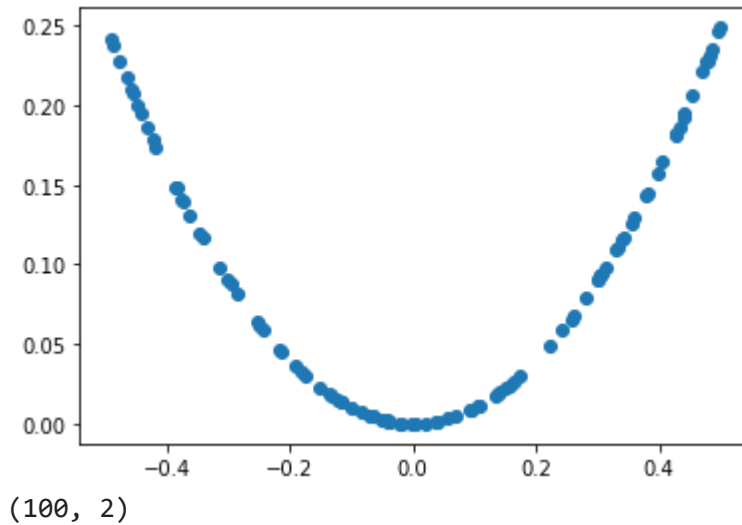
```

# generate real randoms sample from x^2
def generate_real_samples(n=100):
    # generate random inputs in [-0.5, 0.5]
    X1 = rand(n) - 0.5
    # generate outputs X^2 (quadratic)
    X2 = X1 * X1
    # stack arrays
    X1 = X1.reshape(n, 1)
    X2 = X2.reshape(n, 1)
    X = hstack((X1, X2))
    # generate class labels
    y = ones(n)

```

```
return x, y
```

```
# generate samples
data , y = generate_real_samples()
# plot samples
pyplot.scatter(data[:, 0], data[:, 1])
pyplot.show()
print(data.shape)
```



▼ Discriminator model

```
def forward_dis(X,U,b,W,c):
    G = np.dot(X, U.T) + b
    H = G* (G>0)
    z = np.dot(H,W.T) + c
    y = sigmoid(z)

    return y,z,H,G
```

▼ Generator model

```
def forward_gen(X,U,b,W,c):
    G = np.dot(X, U.T) + b
    H = G* (G>0)
    y = np.dot(H,W.T) + c

    return y,H,G
```

▼ Discriminator Backpropagation

```
def grad_decent_dis(x,t,U,b,W,c):
    N=x.shape[0]
    #U = np.random.randn(25,2)
    #b = np.zeros(25)
    #W = np.random.randn(25)
    #c = 0
    #num_steps = 50000
    alpha = 0.01
    #thresh=0.02
    #for step in range(num_steps):
    y,z,H,G = forward_dis(x,U,b,W,c)
    l= loss_stable(z,t,N)
    #if (l<thresh):
    #    #print('converged at step: ',step)
    #    #break
    #    #if (step % 1000==0):
    #        #print (step,' loss = ',l)

    E_bar = 1
    z_bar = (1./N) * (y - t)
    #y_bar = (1./N) * (y.T - t)
    #z_bar = y_bar * (y.T*(1-y.T))
    W_bar = np.dot(H.T,z_bar)
    c_bar = np.dot(z_bar, np.ones(N))
    H_bar = np.outer(z_bar, W.T )
    G_bar = H_bar * drelu(G)
    U_bar = np.dot(G_bar.T, x)
    b_bar = np.dot(G_bar.T , np.ones(N))

    U -= alpha * U_bar
    b -= alpha * b_bar
    W -= alpha * W_bar
    c -= alpha * c_bar

    return U,b,W,c,l
```

▼ Generator data generation

```
# generate points in latent space as input for the generator
def generate_latent_points(latent_dim, n):
    # generate points in the latent space
    x_input = randn(latent_dim * n)
    # reshape into a batch of inputs for the network
    x_input = x_input.reshape((n, latent_dim))
```

```

x_input = x_input.reshape(n, latent_dim)
return x_input

def generate_fake_samples_gen(latent_dim, n, U,b,W,c):
    # generate points in latent space
    x_input = generate_latent_points(latent_dim, n)
    # predict outputs
    X,H,G = forward_gen(x_input,U,b,W,c)
    # create class labels
    y = zeros(n)
    return X, y, H,G,x_input

```

▼ Generator Backpropagation

```

def grad_decent_gen(t,U,b,W,c,Ug,bg,Wg,cg,latent_dim,n):
    x,_,Hg,Gg, latent_x= generate_fake_samples_gen(latent_dim, n, Ug,bg,Wg,cg)
    N=x.shape[0]
    #U = np.random.randn(25,2)
    #b = np.zeros(25)
    #W = np.random.randn(25)
    #c = 0
    #num_steps = 50000
    alpha = 0.001
    #thresh=0.02
    #for step in range(num_steps):
    y,z,H,G = forward_dis(x,U,b,W,c)
    l= loss_stable(z,t,N)
    #if (l<thresh):
        #print('converged at step: ',step)
        #break
    #if (step % 1000==0):
        #print (step,' loss = ',l)

    z_bar = (1./N) * (y - t)
    H_bar = np.outer(z_bar, W.T )
    G_bar = H_bar * drelu(G)
    x_bar = np.dot(G_bar,U)
    z_bar = x_bar
    W_bar = np.dot(z_bar.T,Hg)
    c_bar = np.dot(np.ones(N),z_bar)
    H_bar = np.dot(z_bar, Wg )
    G_bar = H_bar * drelu(Gg)
    U_bar = np.dot(G_bar.T, latent_x)
    b_bar = np.dot(G_bar.T , np.ones(N))

    Ug -= alpha * U_bar
    bg -= alpha * b_bar
    Wg -= alpha * W_bar

```

```
cg -= alpha * c_bar

return Ug,bg,Wg,cg,l
```

▼ Final GAN model and training

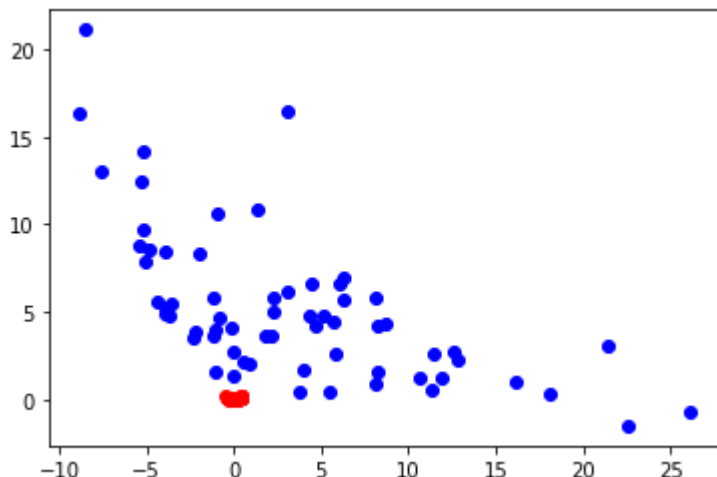
```
n_epochs=20001
n_batch=128
l=0
latent_dim = 5

U = np.random.randn(25,2)
b = np.zeros(25)
W = np.random.randn(25)
c = 0

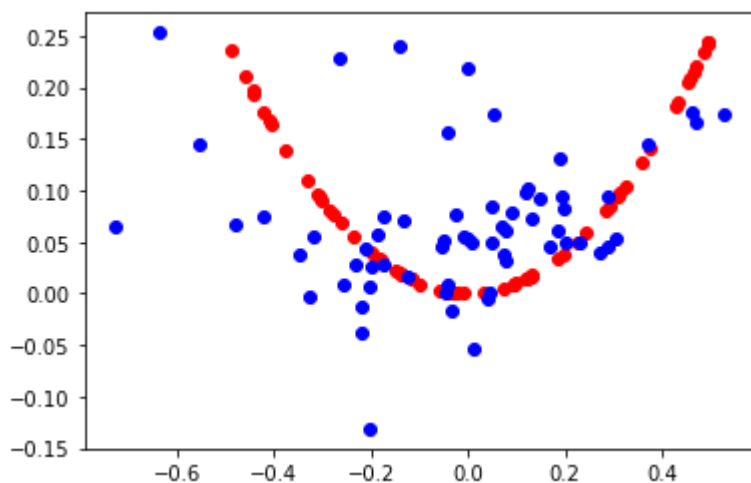
Ug = np.random.randn(15,5)
bg = np.zeros(15)
Wg = np.random.randn(2,15)
cg = np.zeros(2)

for i in range(n_epochs):
    # generate real examples
    X_real, y_real = generate_real_samples(half_batch)
    # update discriminator model
    U,b,W,c,l=grad_decent_dis(X_real,y_real,U,b,W,c)
    if (i%4000==0):
        print('Discriminator loss real',i,':',l)
    # generate fake examples
    X_fake, y_fake,Hg,Gg,latent_x = generate_fake_samples_gen(latent_dim,half_batch,Ug,bg,Wg,cg)
    # update discriminator model
    U,b,W,c,l=grad_decent_dis(X_fake,y_fake,U,b,W,c)
    if (i%4000==0):
        print('Discriminator loss fake',i,':',l)
    #train the generator model
    Ug,bg,Wg,cg,l = grad_decent_gen(np.ones(n_batch),U,b,W,c,Ug,bg,Wg,cg,latent_dim,n_batch)
    if (i%4000==0):
        print('Generator loss',i,':',l)
        pyplot.scatter(X_real[:, 0], X_real[:, 1], color='red')
        pyplot.scatter(X_fake[:, 0], X_fake[:, 1], color='blue')
        pyplot.show()
```

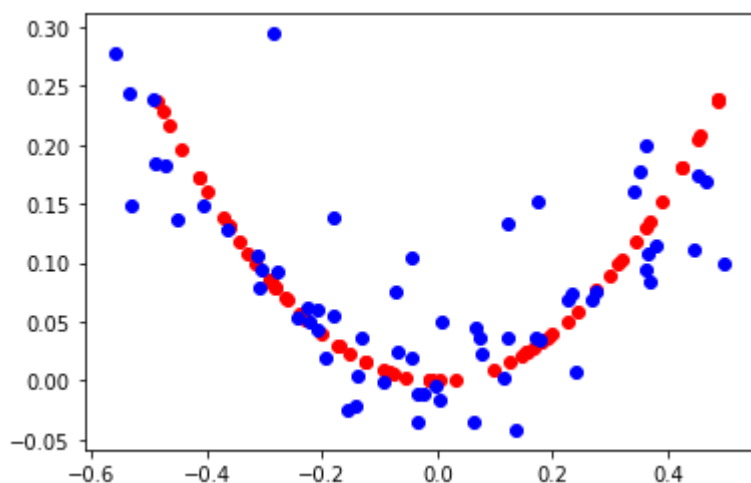
↳ Discriminator loss real 0 : 0.4747555792634539
Discriminator loss fake 0 : 33.52022145779241
Generator loss 0 : 0.0009854603279658707



Discriminator loss real 4000 : 0.6742679308560113
Discriminator loss fake 4000 : 0.6622314971280567
Generator loss 4000 : 0.7711444227533917

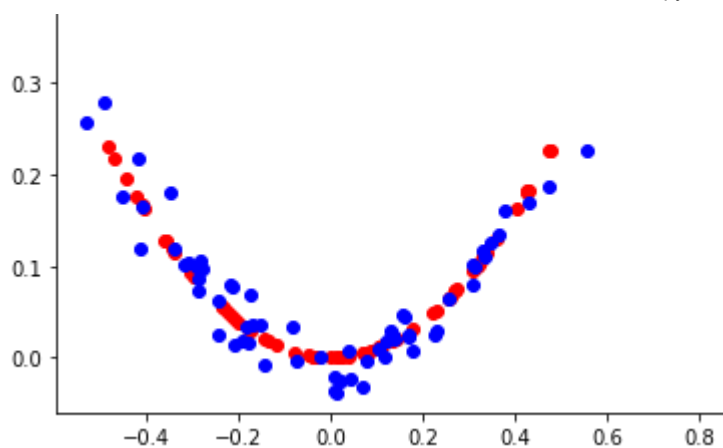


Discriminator loss real 8000 : 0.6719639277806612
Discriminator loss fake 8000 : 0.6901468099080383
Generator loss 8000 : 0.7899212772449655

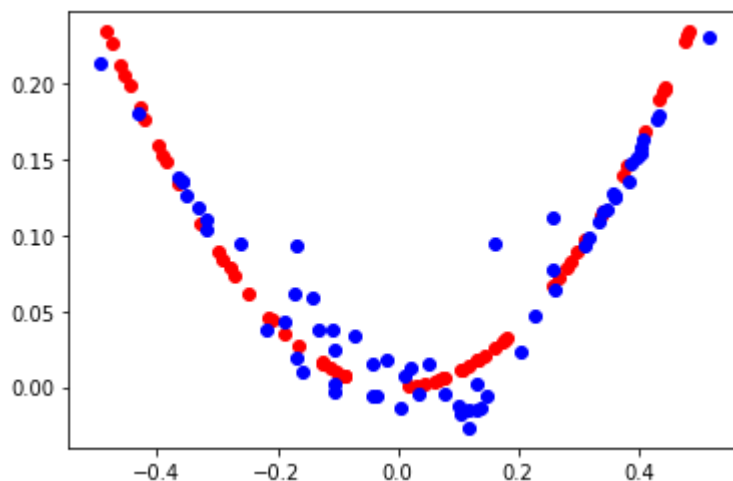


Discriminator loss real 12000 : 0.7074233669054396
Discriminator loss fake 12000 : 0.7004637099317828
Generator loss 12000 : 0.7304616141340543

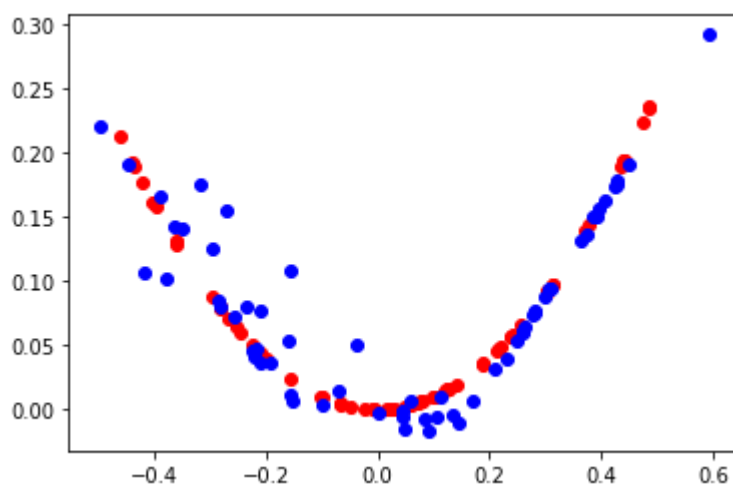




Discriminator loss real 16000 : 0.7022284859361609
Discriminator loss fake 16000 : 0.7034814613068479
Generator loss 16000 : 0.7128372253233979



Discriminator loss real 20000 : 0.7072081649916047
Discriminator loss fake 20000 : 0.6987434467645222
Generator loss 20000 : 0.7161312557414106



We can see as we train the generator gets more competent in producing data that resembles the dataset

