Institute for Aerospace Studies
UNIVERSITY OF TORONTO

AER1516: Robot Motion Planning

Assignment #1: Planning in Theory

Winter 2022

Assignment Overview

This assignment is focused on the background theory that underlies much of modern motion planning (paths, manifolds, homeomorphisms and diffeomorphisms, etc.). The assignment is due on **Monday, February 28, 2022** by **11:59 p.m.**. Submission will be via Quercus. The number of points available are shown next to each problem; there are **50 points** in total. Importantly, you will submit your solutions as a single PDF file and *your answers must by typeset* (written answers are not acceptable).

Problems

1. Prove that a 'bug' executing the Bug2 algorithm to move from a start position to a goal position never travels a distance greater than

$$d_{\text{tot}} = d_{\text{goal}} + \frac{1}{2}\sum_{i=1}^{M} n_i p_i$$

where d_{goal} is the Euclidean distance from the start to the goal, p_i is the perimeter length of obstacle i, and n_i is the number of times obstacle i 'crosses' (intersects) the line segment from the start to the goal. You may assume that the goal is always reachable. (**10 points**)

2. Consider the Z-Y-X Euler angle sequence, used to represent an element of $SO(3)$ (as shown on Slide 19 of Lecture 2 and described here). What happens when $Y = \pi/2$ rads or $Y = -\pi/2$ rads, and why is this a concern? (**6 points**)

3. The unit circle S^1,

$$S^1 = \{(x, y) \,|\, x^2 + y^2 = 1\},$$

cannot be covered by a single chart (mapping to an open set in $\mathbb{R}$). However, it can be covered by two charts. Define two charts, (U, ϕ) and (V, ψ), such that $U \subset S^1$, $\phi : U \to \mathbb{R}$ and $V \subset S^1$, $\psi : V \to \mathbb{R}$. Show that the two charts cover S^1 and that the map $\phi \circ \psi^{-1}$ is a diffeomorphism. Be sure to carefully indicate the the points in U and in V. (**12 points**)

4. Give the dimension of the configuration spaces of the following systems and briefly explain your answers. (**4 points**)
   (a) Two mobile robots rotating and translating in the plane.
   (b) Two mobile robots tied together by a rope rotating and translating in the plane. .
   (c) A train on train tracks, including the wheel angles? (The wheels roll without slipping.)
   (d) Your legs as you pedal a bicycle (remaining seated with feet fixed to the pedals).

5. Prove that the union operator propagates from the workspace to the configuration space. That is, the union of two configuration space obstacles is the configuration space obstacle of the union of two workspace obstacles. In other words, assuming $\mathcal{C}$ is a configuration space operator, show that

$$\mathcal{C}\left(\mathcal{WO}_i \bigcup \mathcal{WO}_j\right) = \mathcal{CO}_i \bigcup \mathcal{CO}_j$$

using the notation from LaValle (slightly modified). If helpful, you may assume that you are working with the two-link planar manipulator shown on Slide 3 of Lecture 3. (**10 points**)

6. Does the wavefront planner on a discrete grid yield the shortest distance to the goal? Why or why not, briefly? If so, what metric is the wavefront planner using? (**8 points**)