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# Observer-Based Tracking Control for Suppressing Stick-Slip Vibration of Drillstring System*

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**Abstract:** A control method is proposed to suppress stick-slip vibration of drillstring system based on tracking control with zero steady-state error and state observer in this paper. A multiple degree-of-freedom (DOF) model of drillstring dynamic is presented first, which considers high-order modal of stick-slip vibration. Then, state observer is constructed to estimate the states of drillstring system, whose states are usually difficult to measure in the top of drillstring system. Finally, combing state feedback control and internal model principle, a tracking control with zero steady-state error is proposed to ensure the speed of rotary table and bit are consistent. The proposal only need top measurement, can eliminate multiple torsional modes, and has a strong robustness. Simulations show the effective of the proposal.

**Key Words:** Stick-slip vibration, Multi-DOF model, Tracking control, State observer

## 1 Introduction

A drilling system is developed for the production and the exploration of natural resources (e.g., oil and natural gas). As a major part of drilling, drillstring system is used to transfer driving torque from the top to the bottom for breaking rock. Drillstring system generally consists of rotary table, a set of drill pipes, drill collar and bit. Field measurements [2] show that the drillstring system is easy to produce various severe vibrations, namely, lateral vibration (bending), axial vibration (bit-bounce) and torsional vibration (stick-slip). Among these vibrations, stick-slip vibration is harmful and severe, which occurs for more than $50\%$ of the total drilling time. Stick-slip vibration increases tool failures, adversely affects borehole quality, and can lead to the coupling of different vibrations [3]. Therefore, the mechanism analysis and suppression control for stick-slip vibration are of great importance to the drilling industry.

Research on stick-slip vibration is mainly focused on modeling and controller design. Torsional pendulum models of drillstring with one or two degrees of freedom (DOF) have been proposed one after another [4]-[6]. Meanwhile, in order to improve the accuracy of drillstring dynamic model, some distributed parameter models are also proposed [7], [8]. However, on the one hand, although the low-order drillstring dynamic models are easy to design the controller, they ignore the high-order modal of stick-slip vibration. On the other hand, the distributed parameter models bring great modeling precision, but increase the computational complexity. Therefore, how to consider both the high-order modal of stick-slip vibration and modeling precision is a motivation of this paper. Considering the existing three-DOF model, four-DOF model proposed in [9], [10], which motivate us to establish a multi-DOF model of drillstring.

A large number of control methods can be found in many literatures, e.g., PID control [11], $H_\infty$ control [12], sliding mode control [10], [13], state-feedback and observer-based output feedback control [6], equivalent input disturbance method [14], [15]. As described in [16], a control system of drillstring under practical drilling process should meet the following requirements. I. Due to the downhole measurement for real-time control purposes is not realistic, so only surface measurement is available. II. The controller should be able to suppress multiple torsional vibration modes of drillstring. III. Robustness of control system should be guaranteed under nonlinear bit-rock interaction and complex environment. However, few methods can meet all the requirements mentioned above. Therefore, developing a control strategy which can suppress stick-slip vibration and has a strong robustness without depending on bottom measurement is the another motivation of this paper.

So, a control method based on tracking control with zero steady-state error and state observer is proposed. A full state observer is used first to estimate the state of drillstring system based on top measurement. Then, a tracking control system combined with state feedback, internal model and state observer is constructed to drive the rotary table and bit at a constant speed. Simulations show that the proposal can meet the requirements of practical drilling project.

The structure of this paper is arranged as follows. Section 2 introduces the drillstring dynamic model. Section 3 introduces the control system structure, then the solution of related control parameters is given. Section 4 presents the simulation results.

## 2 Drillstring system dynamic model

Drillstring dynamic model is introduced in this section, which includes drillstring model and bit-rock interaction model. In this section, we will introduce multiple degree of freedom (DOF) drillstring model and bit-rock interaction model, respectively.

### 2.1 Multi-DOF Drillstring model

A multi-DOF model for drillstring consists of a rotary table, a set of drill pipes, a drill collar, and a drill bit. The

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equation of n-DOF model is written as

$$J\ddot{\theta} + (C_1 + C_2)\dot{\theta} + K\theta = S_r T_r + S_b T_b \tag{1}$$

where $\theta = [\theta_r, \theta_{p1}, ..., \theta_{p(n-3)}, \theta_c, \theta_b]^T \in \mathbb{R}^n$ is the angular position of n elements, corresponding to a rotary table, n-3 drill pipes, a drill collar and a drill bit. $T_r \in \mathbb{R}$ is torque input of top drive motor, and $T_b \in \mathbb{R}$ is friction torque from bit-rock interaction. $S_r = [1, 0, ..., 0]^T \in \mathbb{R}^n$, $S_b = [0, 0, ..., -1]^T \in \mathbb{R}^n$. $J = \mathrm{diag}\{J_r, J_{p1}, ..., J_{p(n-3)}, J_c, J_b\} \in \mathbb{R}^{n\times n}$ is inertia matrix, $C_1 = \mathrm{diag}\{d_r, d_{p1}, ..., d_{p(n-3)}, d_c, d_b\} \in \mathbb{R}^{n\times n}$ is local damping matrix, $C_2$ and $K$ are mutual damping and torsional stiffness matrices represented as.

$$C_2 = \begin{bmatrix} c_p & -c_p & 0 & 0 & ... & 0 & 0 & 0 & 0 \\ -c_p & 2c_p & -c_p & 0 & ... & 0 & 0 & 0 & 0 \\ 0 & c_p & 2c_p & -c_p & ... & 0 & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & -c_p & c_p + c_c & -c_c & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & -c_c & c_c + c_b & -c_b \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & -c_b & c_b \end{bmatrix}$$

$$K = \begin{bmatrix} k_p & -k_p & 0 & 0 & ... & 0 & 0 & 0 & 0 \\ -k_p & 2k_p & -k_p & 0 & ... & 0 & 0 & 0 & 0 \\ 0 & k_p & 2k_p & -k_p & ... & 0 & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & -k_p & k_p + k_c & -k_c & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & -k_c & k_c + k_b & -k_b \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & -k_b & k_b \end{bmatrix}$$

Other symbols (e.g., $J_r, c_r, d_r, k_p$) represent the specific value of one element in the mass, damping, and stiffness matrices.

### 2.2 Bit-rock interaction model

The bit-rock interaction model is given as

$$T_b(\dot{\theta}_b) = \begin{cases} T_1(\dot{\theta}_b) & \text{if} |\dot{\theta}_b| < \varphi, |T_1| \le |T_2| \\ T_2 sgn(T_1(\dot{\theta}_b)) & \text{if} |\dot{\theta}_b| < \varphi, |T_1| > |T_2| \\ \mu_b(\dot{\theta}_b) W_{ob} R_b sgn(\dot{\theta}_b) & \text{if} |\dot{\theta}_b| \ge \varphi \end{cases} \tag{2}$$

where $W_{ob}$ is weight on bit (WoB), $R_b$ is bit radius, $T_1$ is reaction torque and $T_2 = W_{ob} R_b \mu_b$ is static friction torque. $\varphi$ is a small positive value. The equation of $T_1$ is

$$T_1 = c_b(\dot{\theta}_c - \dot{\theta}_b) + k_b(\theta_c - \theta_b) - d_b\dot{\theta}_b \tag{3}$$

And $\mu_b$ is dry friction coefficient described as

$$\mu_b(\dot{\theta}_b) = \mu_{cb} + (\mu_{sb} - \mu_{cb}) e^{-\frac{\gamma_b}{v_f}|\dot{\theta}_b|} \tag{4}$$

where $\mu_{sb}$ and $\mu_{cb}$ are static friction coefficient and coulomb friction coefficient, respectively, $\gamma_b$ is velocity decrease rate, and $v_f$ is a constant.

The bit-rock interaction model (2) introduces a velocity weakening effect and is suitable to describe stick-slip motion.

### 2.3 State space form of drillstring dynamic model

Through a transformation, (1) can be re-described in a form of standard state space equation, which is easy to analyse and design controller.

Choose $x = [\dot{\theta}_r, \dot{\theta}_{p1}, ..., \dot{\theta}_{p(n-3)}, \dot{\theta}_c, \dot{\theta}_b, \theta_{p1} - \theta_r, \theta_{p2} - \theta_{p1}, ..., \theta_{p(n-3)} - \theta_{p(n-2)}, \theta_c - \theta_{p(n-3)}, \theta_b - \theta_c]^T \in \mathbb{R}^{2n-1} = \mathbb{R}^m$ as state variable, then we can get a state space equation of n-DOF drillstring model as follows

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_1 d(t) \\ y(t) = Cx(t) \end{cases} \tag{5}$$

herein, the speed of rotary table and bit are $x_1 = \dot{\theta}_r$ and $x_n = \dot{\theta}_b$, respectively. Because only top measurement is available (i.e. $x_1$), so $y(t)$ is the speed of rotary table. The control input $u(t) = T_r$, disturbance $d(t) = T_b$. The matrices $A, B, B_1, C$ are described by

$$A = \begin{bmatrix} -J^{-1}(C_1 + C_2) & J^{-1}a_1 \\ a_2 & O_{n-1\times n-1} \end{bmatrix}$$

$$a_1 = \begin{bmatrix} -k_p & 0 & ... & 0 \\ k_p & -k_p & ... & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & ... & k_r & -k_b \\ 0 & ... & 0 & k_b \end{bmatrix}$$

$$a_2 = \begin{bmatrix} 1 & -1 & 0 & ... & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & ... & 0 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} J^{-1}S_r \\ O_{n-1\times 1} \end{bmatrix} B_1 = \begin{bmatrix} J^{-1}S_b \\ O_{n-1\times 1} \end{bmatrix} C = \begin{bmatrix} 1 \\ O_{2n-2\times 1} \end{bmatrix}^T$$

where $O$ represents zero matrix.

According to (5), it can be seen that the order of the state space model is 2n. For an practical drillstring system, n tends to infinity. Therefore, a multi-DOF model can guarantee the modeling precision for drillstring system.

## 3 Tracking control of drillstring system

The suppression problem of stick-slip vibration can be regarded as a tracking control problem of bit-speed. Meanwhile, because only surface measurement is available, a observer-based control method is suitable to cope with this situation. Therefore, a control system based on tracking control with zero steady-state error and state observer is presented in this section.

The structure of control system is shown in Fig. 1, which includes the drillstring dynamic model, internal model, state feedback and state observer, respectively. Drillsting dynamic model has already been described in (5).

The internal model of reference input and disturbance is shown as follows

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c e(t) \\ y_c(t) = C_c x_c(t) \end{cases} \tag{6}$$

Then, a augmented-state ($\bar{x}(t) = \begin{bmatrix} x(t) & x_c(t) \end{bmatrix}^T$) representation for the plant and the internal model is given as

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{B}_1 d(t) \\ &= \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix} \bar{x}(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} d(t) \end{aligned} \tag{7}$$

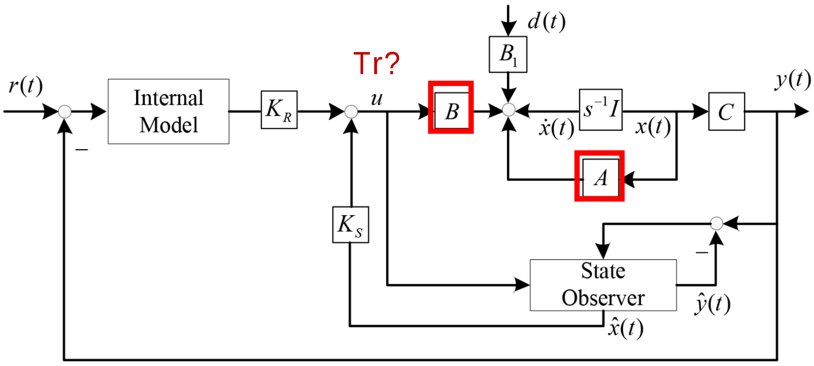



Fig. 1: Structure of control system

with control law

$$u(t) = -\bar{K}\bar{x}(t) = \begin{bmatrix} -K_S & K_R \end{bmatrix} \bar{x}(t) \tag{8}$$

where $K_S$ and $K_R$ are the gain of state feedback and internal model, respectively.

However, in practical drilling process, only top measurement is available, which makes the control based on full state feedback impossible. Therefore, a full-dimensional state observer is used to estimate the state in (5) and is described as

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{9}$$

where $\hat{x}(t)$ is the observed state of $x(t)$, and matrix $L$ is the gain matrix to be determined.

Combined with the state observer, then the control law of the proposal is redescribed as

$$u(t) = \begin{bmatrix} -K_S & K_R \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ x_c(t) \end{bmatrix} \tag{10}$$

## 4 Simulation and analysis

This section gives a simulation and analysis for stick-slip vibration and a comparison between the proposal and the sliding mode control (SMC) proposed in [10]. As described before, the order of n tends to infinity. On the one hand, low-frequency torsional vibration dominates the vibrations of the entire drillstring system. On the other hand, higher-order models are usually reduced in order to design the control strategies easily. Then, a 4-DOF model of drillstring dynamic is introduced in this paper, which is a good approximation for drillstring and can be find in [10], [13]. Parameters for simulation are shown in Table 1 [10].

Table 1: Parameters for simulation

| | | | | | |
|---|---|---|---|---|---|
| $J_r$ | 930 | $J_p$ | 2782.25 | $J_c$ | 750 |
| $J_b$ | 471.97 | $k_p$ | 698.06 | $k_c$ | 1080 |
| $k_b$ | 907.48 | $d_r$ | 425 | $d_p$ | 0 |
| $d_c$ | 0 | $d_b$ | 50 | $c_p$ | 139.61 |
| $c_c$ | 190 | $c_b$ | 181.49 | WoB | 97347 |
| $R_b$ | 0.1556 | $\mu_{cb}$ | 0.5 | $\mu_{sb}$ | 0.8 |
| $\varphi$ | $10^{-}6$ | $\gamma_b$ | 0.9 | $\nu_f$ | 1 |

### 4.1 Phenomenon of stick-slip vibration

The constant matrices $A, B, B_1, C$ in (5) can be calculated by setting $n = 4$ based on Table 1. So the dimension of (5) equals to 7 in this section. First, the frequency response characteristics of the drillstring system is considered. By ignoring the local damping, we can obtain the natural frequency and Bode diagram of the drillstring system from bit torque $T_b$ to bit speed $\dot{\theta}_b$ as shown in Fig. 2. It can be seen that the magnitude of $\dot{\theta}_b$ caused by $T_b$ dominates at the first-order natural frequency (about 0.852 rad/s).

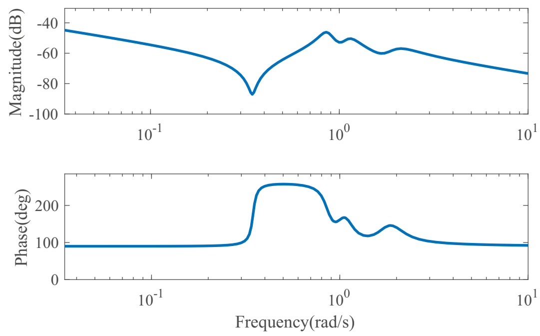



Fig. 2: Frequency response of 4-DOF drillstring dynamic model from bit torque $T_b$ to bit speed $\dot{\theta}_b$

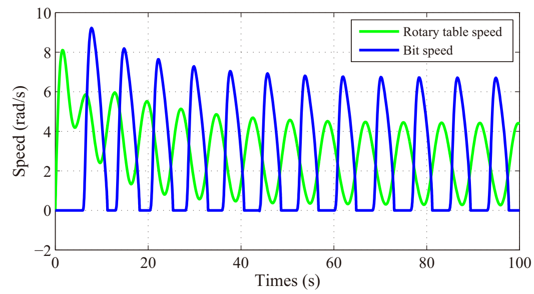



Fig. 3: Stick-slip vibration

Then, time-domain simulation of stick-slip vibration is given. The torque input $T_r$ of top drive motor is set to 10000 N. The two phases of stick-slip vibration can be observed clearly in Fig. 3. In stick phase, the speed of bit is almost equal to zero, while in slip phase, the speed of both rotary table and bit fluctuates severely. By calculating the time interval between the two peaks, the frequency of stick-slip vibration is obtained ($1/8.0 \times 2\pi = 0.785$ rad/s), which is lower than the first-order natural frequency of drillstring system. This phenomenon is consistent with the existing literatures and the field observation, and easily exacerbates the stick-slip vibration. Therefore, a constant speed of rotary table and bit is necessary for improving the effective of drilling process and ensuring the safety.

### 4.2 Determination of controller parameters

A reference speed for rotary table is set as $r(t) = 12\,\mathrm{rad/s}$ and the parameters of internal model (6) for the reference and step disturbance is $A_c = 0, B_c = 1, C_c = 1$.

Based on linear quadratic regulator optimal control algorithm [17], choosing error weighting matrix and control weighting matrix as follows

$$Q = \mathrm{diag}\{\ 1\ \ 1\ \ 1\ \ 10\ \ 1\ \ 1\ \ 1\ \ 1000\ \}, R = 0.001 \tag{11}$$

then the gain of state feedback and internal model are obtained as follows

$$\begin{aligned} K_S|K_R = [&735.3\ \ 2490.5\ \ 504.9\ \ 247.6 \\ &611.6\ \ 909.0\ \ 959.5|\ \ 1000] \end{aligned} \tag{12}$$

In order to estimate the state of drillstring dynamic, an approximate pole placement for state observer (9) is chosen as

$$\left[\ -0.93\ \ -2 \pm 0.84i\ \ -1.5 \pm 1i\ \ -3 \pm 1.95i\ \right] \tag{13}$$

Then, the gain of state observer is calculated as

$$\begin{aligned} L = [&12.22\ \ 178.38\ \ -301.66 \\ &-80.86\ \ -52.89\ \ -580.81\ \ 633.20]^T \end{aligned} \tag{14}$$

### 4.3 Validation of control system

In this section, the proposal is applied to eliminate the stick-slip vibration and is compared with a existing SMC.

In [10], a SMC is proposed by introducing a surface along which the system trajectories enter a sliding regime and the control goal is met ($\dot{\theta}_b \to \dot{\theta}_r \to r(t)$). Then, a comparison for the proposal and SMC is carried out.

In $0 \sim 50\,\mathrm{s}$, as can be seen from Fig. 4(a), when the controller is applied to the drillstring system, the drill bit is first in the stick phase for a few seconds (about $8\,\mathrm{s}$), then the bit speed gradually increases to around reference speed ($12\,\mathrm{rad/s}$), and the drillstring system is finally stabilized at $30\,\mathrm{s}$. Fig. 4(b) shows that the observed bit speed implements the tracking for bit speed at about $12\,\mathrm{s}$, and performance of the state observer is satisfied.During this period, the overshoot and adjustment time of the proposal ($14.73\,\mathrm{rad/s}$ and $30\,\mathrm{s}$) in Fig. 4(a) are superior to SMC ($19\,\mathrm{rad/s}$ and $40\,\mathrm{s}$) in Fig. 4(c).

At $t = 50\,\mathrm{s}$, the WoB is suddenly set from $97347\,\mathrm{N}$ to $110\,\mathrm{KN}$, which represents a common case in bottom part of drillstring system. In $50 \sim 100\,\mathrm{s}$, the fluctuation amplitude of drillstring system with the proposal is smaller than that with SMC, and the time of the system entering steady state is also shorter.

Therefore, simulations show that the proposal is effective in suppressing stick-slip vibration.

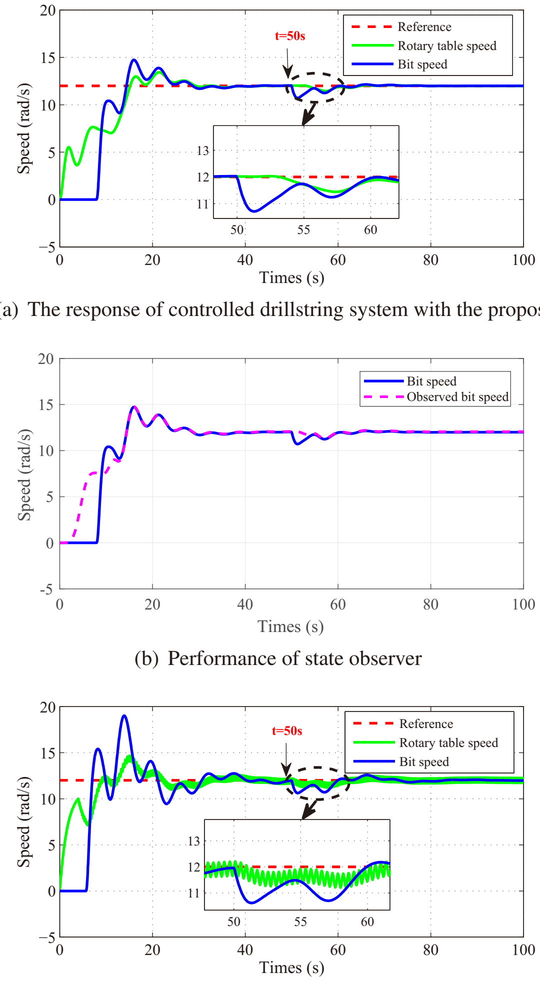
## 5 Conclusion

A control method is proposed in this paper to suppress stick-slip vibration of drillstring system based on tracking control and state observer. The main contribution of this paper lies in following points,

(1) A multi-DOF model of drillstring system is proposed to consider multiple torsional vibration modes.

(2) The proposed control method does not require bottom measurement and has a strong robustness under complex environment.

In the next stage, we will continue to improve the proposal and hope to provide guidance for practical drilling control systems.



(a) The response of controlled drillstring system with the proposal

(b) Performance of state observer

(c) The response of controlled drillstring system with SMC

Fig. 4: Comparison between the proposal and SMC

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