# 中级微观经济学

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# Contents

# 章节 1. Introduction

What do we cover?

- Consumer Theory (Preferences, utility)
- Equilibrium
- Firm technology, Profit maximization
- Firm supply
- Market structure (competition, monopoly, oligopoly)
- Externalities and public goods
- Game Theory
- Information

Problem Set: 15%, Group presentation: 10%, Midterm: 30%, Final: 45%.

我们将主要使用 Varian (2014) 作为教材.

Examples:

- Airfare. Relationship between the ticket price and the departure time. High demand → Charging high price.
- Streaming Media. 2 Strategies: Membership subscription and Individual purchase(Pay-per-view).
- Charity-linked products.
- Urban Green.
- Electric Vehicles. Subsidize the purchase of electric vehicles.

# 章节 2. Consumer Theory

Economic Modeling:

- Who are the participants?
- Some assumptions:
  - Rational Choice: A person chooses the best alternative available.
  - Equilibrium: The market is in equilibrium.

Consumer Choice. → Preference

Consumers are assumed to choose the best bundle of goods they can afford.

- Best:
- Can afford: Allocated budget.

**Consumption Choice Sets** A consumption choice set is the collection of *all* consumption choices available to the consumer.

What constraints consumption choice?

- Budget
- Time
- Other resource limitations

**Consumption Bundle** A consumption bundle containing $x_1$ units of commodity 1, $x_2$ units of commodity 2 and so on up to $x_n$ units of commodity $n$ is denoted by the vector $(x_1, x_2, ..., x_n)$.

Assume commodity prices are $p_1, p_2, ..., p_n$.

**Budget Constraints**

$$p_1 x_1 + p_{x_2} + .... + p_n x_n \leq m \tag{2.1}$$

where $m$ is the consumer's (disposable) income.

**Budget Set**

$$B(p_1, p_2, ..., p_n, m) \tag{2.2.1}$$

$$= \left\{(x_1, x_2, ..., x_n) |\ x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0, p_1 x_1 + p_{x_2} + .... + p_n x_n \leq m\right\} \tag{2.2.2}$$

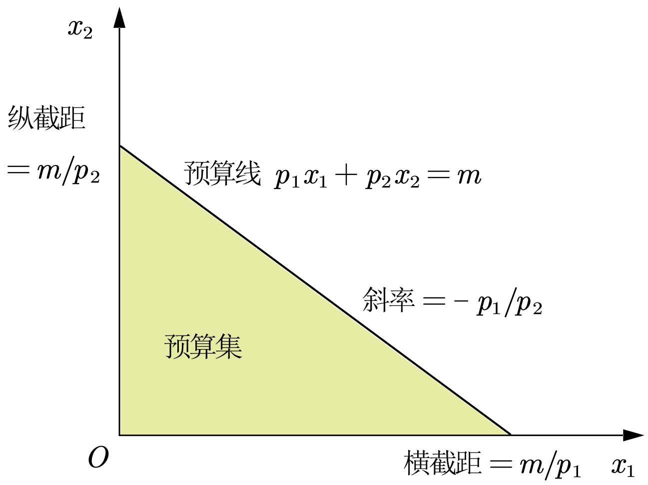


Figure 2.1: Budget Constraints

- Which is affordable? Unaffordable? Just affordable?

If $n = 3$ what do the budget constraints look like?

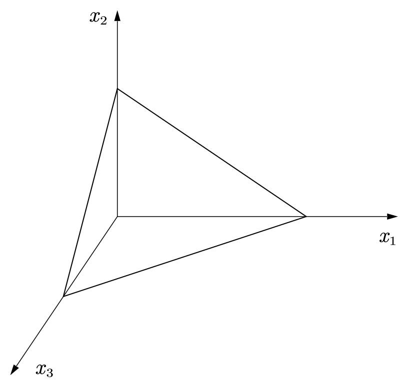


Figure 2.2: 3-dimensional Budget Constraints

In other words, opportunity cost of an extra unit of commodity 1 is $p_1/p_2$ units foregone of commodity 2.

Higher income gives more choice. improve consumer welfare.

enlarging, shrinking

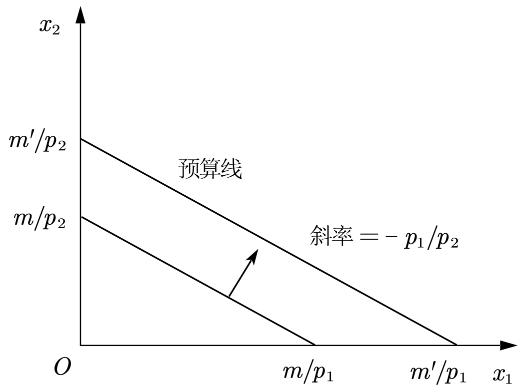


Figure 2.3: Higher Income

Increasing one price pivots the constraint inwards, reduces choice and will make the consumer worse off.

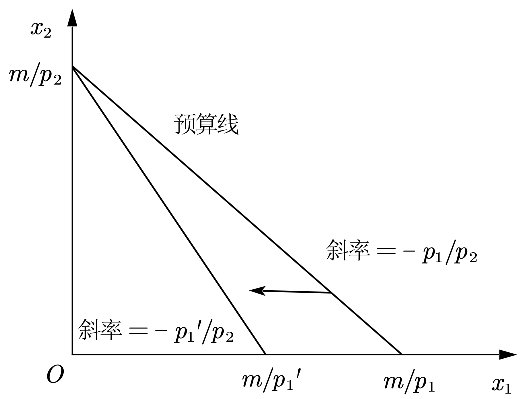


Figure 2.4: Increasing one price

- Q: If the economy is weak and so is consumer demand, what should the policy makers do to stimulate consumption?
- A: Reduce tax rate, sending cash to consumers. PRICE: subsidy.

Price changes: High demand, low supply.

## 2.1. *Ad Valorem* Sales Tax

Original price: $p \to$ New price: $(1+t)p$.

A uniform sales tax is applied uniformly to all goods.

$$(1+t)p_1x_1 + (1+t)p_2x_2 \leq m \tag{2.3.1}$$

$$\Rightarrow p_1x_1 + p_2x_2 \leq \frac{m}{1+t} \tag{2.3.2}$$

**Remark**: The tax essentially discount the income. And the equivalent income loss is

$$m - \frac{m}{1+t} = \frac{t}{1+t}m. \tag{2.4}$$

### 2.1.1. Example: The food stamp program

How does a commodity-specific gift such as a food stamp alters a family's budget constraint?

Suppose $m = 100$, $p_F = 1$ (food), "other goods" $p_G = 1$, the budget constrait is:

$$F + G \leq 100. \tag{2.5}$$

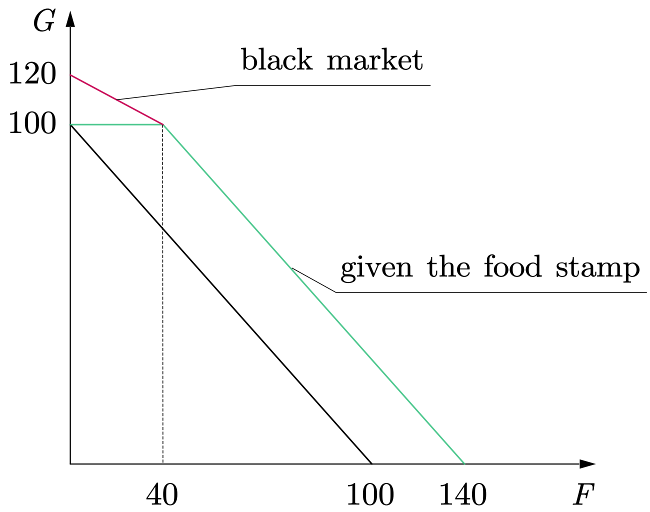


Figure 2.5: The food stamp program

Black market trading makes the budget set even larger. Black market improve consumer welfare.

## 2.2. Relative Price

**Numeraire** unit of account.

Changing the numeraire changes neither the budget constraint nor the budget set.

Any commodity can be chosen as the numeraire without changing the budget set or the budget constraint.

A straight line: constant relative price.

**Quantity discounts**:

Suppose $p_2 = 1$ is constant and $p_1 = 2$ when $0 \leq x_1 \leq 20$ and $p_1 = 1$ when $x_1 > 20$. The figure is like Figure 2.6.

**Q: Is price always positive?**

Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; $p_1 = -2, p_2 = 1$.

$$-2x_1 + x_2 \leq 10 \tag{2.6}$$

Like in Figure 2.7.

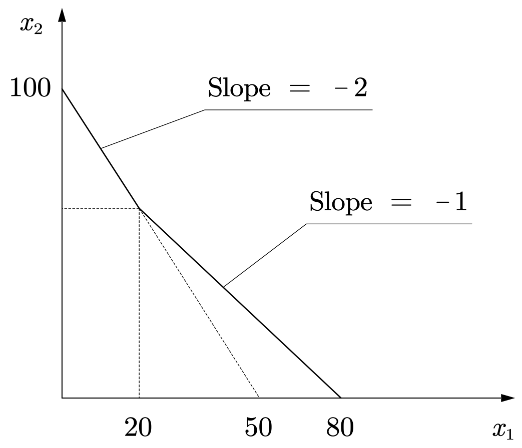


Figure 2.6: Quantity discounts

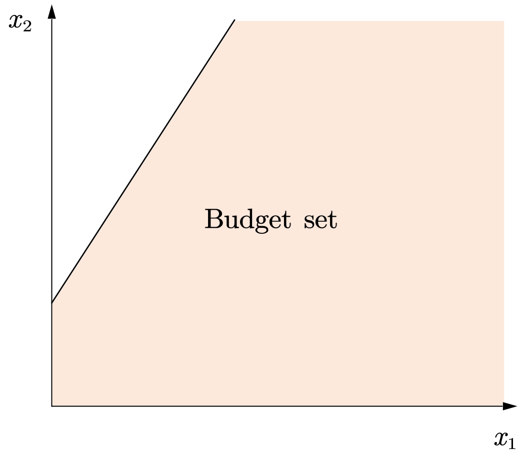


Figure 2.7: Negative prices

e.g. 示例 **2.2.1**

Why the consumption bundle in Figure 2.7 is unlimited?

双学位: A second degree in economics.

## 2.3. Multiple Constraints

Food Consumption vs Other Stuff.

- At least 10 units of food must be eaten to survive
- Budget constrained.
- Further restricted by a time constraint.

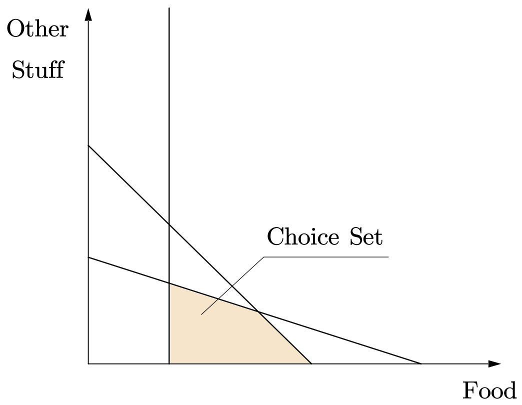


Figure 2.8: Multiple Constraints

# 章节 3. Preference and Utility

Consumers are assumed to choose the **best** bundle of goods they **can afford**.

**Rationality in Economics**: A decision maker always chooses its most preferred alternative from its set of available alternatives.

## 3.1. Preference Relations

**strict preference** $x$ is more preferred than is $y$.
**weak preference** $x$ is at least as preferred as is $y$.
**indifference** $x$ is exactly as preferred as is $y$.

They are ordinal relations: only the order is important.

$\succ$ denotes strict preference. $x \succ y$ iff $x$ is more preferred than $y$.

$\sim$ denotes indifference. $x \sim y$ iff $x$ is exactly as preferred as $y$.

$\succsim$ denotes weak preference.

- $x \succsim y$ and $y \succsim x$ imply $x \sim y$.
- $x \succsim y$ and $\neg(y \succsim x)$ imply $x \succ y$.

Assumptions on preference relations:

1. Completeness. $\forall$ bundles $x$ and $y$, either $x \succsim y$ or $y \succsim x$.
2. Reflexivity. $\forall$ bundles $x$, $x \succsim x$.
3. Transitivity. $x \succsim y$ and $y \succsim z$ imply $x \succsim z$.

## 3.2. Indifference Curve

Indifference curve: a set of bundles that are equally preferred.

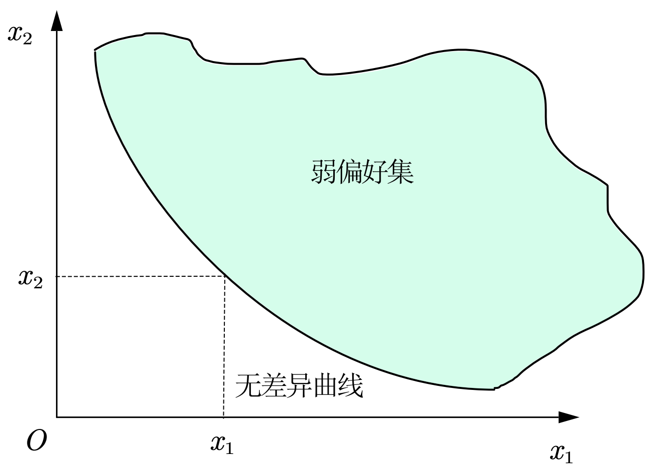


Figure 3.1: Indifference Curve

- $\mathrm{WP}(x)$: the set of all bundles that are weakly preferred to $x$.
- $I(x)$: the set of all bundles that are indifferent to $x$.
- $I(x) \subseteq \mathrm{WP}(x)$.

提示 **3.2.1**

Indifference curves cannot cross.

**Proof**: Suppose the intersection of two indifference curves is $A$. Then $A$ is indifferent to $B$ and $C$. But $B$ and $C$ are not indifferent to each other.

□

When more of a commodity is always preferred, the commodity is a good. If every commodity is a good then indifference curves are negatively sloped.

If less of a commodity is always preferred then the commodity is a bad. 1 Good and 1 bad: indifference curves are positively sloped.

## 3.3. Extreme Cases of Indifference Curves

### 3.3.1. Perfect Substitutes

If a consumer always regards units of commodities 1 and 2 as equivalent, the commodities are perfect substitutes and only the total amount of the two commodities in bundles determines their preference rank-order.

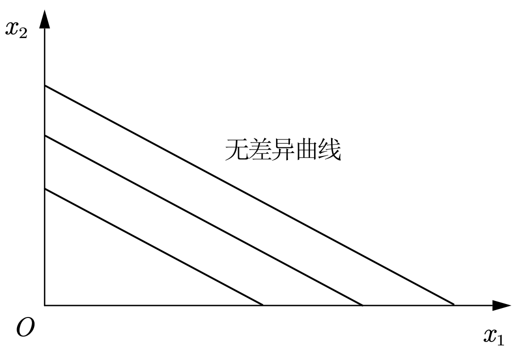


Figure 3.2: Perfect Substitutes

### 3.3.2. Perfect Complements

Fixed proportions of commodities 1 and 2 are required to provide utility.

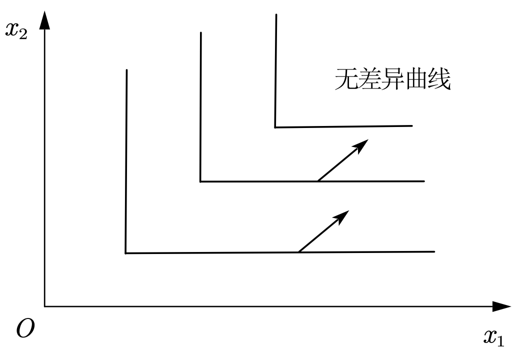


Figure 3.3: Perfect Complements

A L-shaped indifference curve. 45°.

### 3.3.3. Example: Good and Bad

You like pizzas but hate vegetables. You are only willing to eat an extra unit of vegetable if you get to eat an extra unit of pizza.

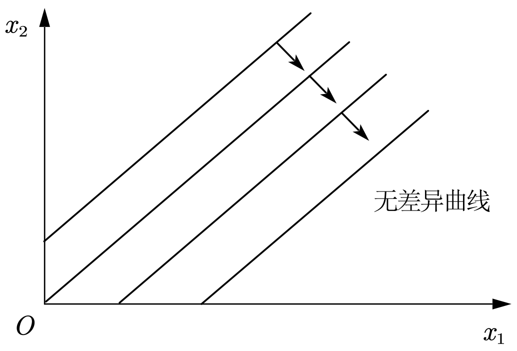


Figure 3.4: Good and Bad

### 3.3.4. Example: Satiation

You always want to study for 6 hours and workout for 2 hours. Any deviation from it gives you a lower utility level. Optimal bundle.

$\Longrightarrow$ A satiation point.

## 3.4. Preferences Exhibiting Satiation

A bundle strictly preferred to any other is a satiation point or a bliss point.

Why the shape is a circle? Because Any deviation from the satiation point gives you a lower utility level.

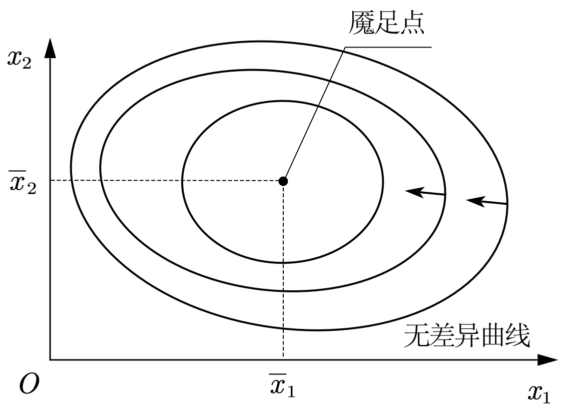


Figure 3.5: Satiation Point

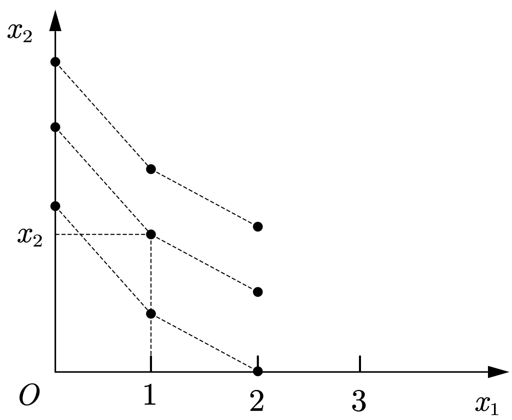


Figure 3.6: Indifferent Curves for Discrete Goods

## 3.5. Indifferent Curves for Discrete Goods

Like in Figure 3.6.

## 3.6. Well-Behaved Preferences

1. Monotonic. More is better or Less is better. (Satiation is a violation of this.)
2. Convexity. Mixtures are preferred to extremes. $\forall 0 < t < 1$,

$$tx + (1-t)y \succ x = y. \tag{3.1}$$

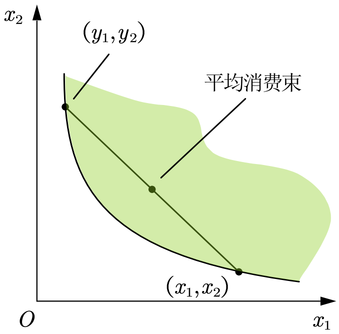


Figure 3.7: Convexity

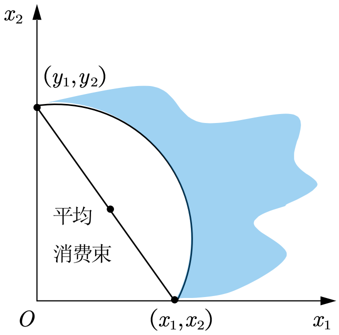


Figure 3.8: Concavity

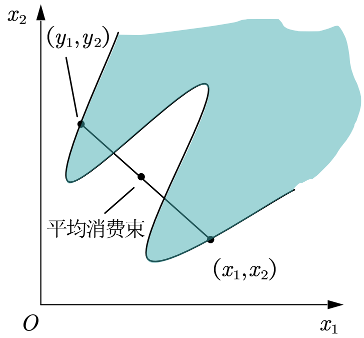


Figure 3.9: Mixture of Convexity and Concavity

For the most part, goods are consumed together. $\Rightarrow$ Convexity.

## 3.7. Slopes of Indifference Curves

The slope of an indifference curve (at a given point) is its marginal rate-ofsubstitution (MRS).

$$\text{MRS} = \frac{\mathrm{d}x_2}{\mathrm{d}x_1} \text{ at } x'. \tag{3.2}$$

Properties:

- Two goods $\Rightarrow$ MRS $< 0$.
- One good and one bad $\Rightarrow$ MRS $> 0$.
- Convex $\Leftrightarrow$ MRS always increases with $x_1$.

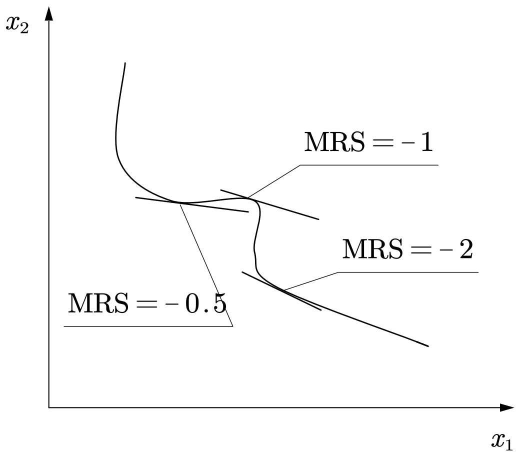


Figure 3.10: Violation of Convexity

## 3.8. Utility Function

Another way to represent preferences.

We can assign a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less preferred bundles.

The thing can be observed is the choices made by the consumer.

- Consumers make choices so as to maximize their utility to make themselves as happy as possible.

Continuity.

$$x' \succ x'' \Leftrightarrow U(x') > U(x'') \tag{3.3.1}$$

$$x' \prec x'' \Leftrightarrow U(x') < U(x'') \tag{3.3.2}$$

$$x' \sim x'' \Leftrightarrow U(x') = U(x'') \tag{3.3.3}$$

Utility is an ordinal concept. The value of utility is not important, only the order of the utility values. How much higher doesn't matter.

There is no unique utility function representation of a preference relation.

$U(x_1, x_2) = x_1 x_2$, and we can define $V(x_1, x_2) = x_1^2 x_2^2$, preserves the order of the utility values.

提示 **3.8.1**

$U$ is a utility function represents a preference relation, and $f$ is a strictly increasing function, then $f(U)$ also represents the same preference relation.

### 3.8.1. Perfect Substitution

$$U(x_1, x_2) = x_1 + x_2. \tag{3.4}$$

The pizza & vegetable example: $U(x_1, x_2) = x_1 - x_2$.

### 3.8.2. Perfect Complements

e.g. Left and right shoes, pencils and erasers, fries and ketchup, game consoles and games.

$$U(x_1, x_2) = \min\{x_1, x_2\}. \tag{3.5}$$

### 3.8.3. Quasi-linear Utility Function

Def 定义 **3.8.1**

A utility function of the form

$$U(x_1, x_2) = f(x_1) + x_2 \tag{3.6}$$

is linear in just $x_2$ and is called a quasi-linear utility function.

### 3.8.4. Cobb-Douglas Utility Function

Def 定义 **3.8.2**

A utility function of the form

$$U(x_1, x_2) = x_1^a x_2^b \tag{3.7}$$

is called a Cobb-Douglas utility function.

Well-behaved.

## 3.9. Marginal Utility

E.g. consider $U(x_1, x_2) = x_1^{1/2} x_2^2$, then

$$\mathrm{MU}_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2, \tag{3.8.1}$$

$$\mathrm{MU}_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2. \tag{3.8.2}$$

Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} \mathrm{d}x_1 + \frac{\partial U}{\partial x_2} \mathrm{d}x_2 = 0. \tag{3.9}$$

$$\Rightarrow \frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}. \tag{3.10}$$

This is the MRS.

### 3.9.1. MRS for Quasi-linear Utility Function

$$\frac{\partial U}{\partial x_1} = f'(x_1), \frac{\partial U}{\partial x_2} = 1. \tag{3.11.1}$$

$$\Rightarrow \mathrm{MRS} = \frac{\mathrm{d}x_2}{\mathrm{d}x_2} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -f'(x_1). \tag{3.11.2}$$

MRS does not depend on $x_2$. So the slope of indifference curves for a quasi-linear utility function is constant along any line for which $x_1$ is constant.

### 3.9.2. Monotonic Transformation & MRS

If $V = f(U)$, we have

$$\mathrm{MRS} = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{f'(U) \times \partial U / \partial x_1}{f'(U) \times \partial U / \partial x_2} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}. \tag{3.12}$$

So MRS is **unchanged** by a positive monotonic transformation.

e.g. **示例 3.9.1**

Linda's preferences over magazines $(M)$ and books $(B)$ are given by:

$$U(M, B) = 3M^{\frac{2}{3}} + 6B^{\frac{2}{3}}. \tag{3.13}$$

Are Linda's preferences convex or not?

## References

Varian, H. R. (2014) *Intermediate Microeconomics: A Modern Approach*. 9th ed. New York: W.W. Norton & Company