中级微观经济学

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Contents

章节 1. Introduction

What do we cover?

- Consumer Theory (Preferences, utility)
- Equilibrium
- Firm technology, Profit maximization
- Firm supply
- Market structure (competition, monopoly, oligopoly)
- Externalities and public goods
- Game Theory
- Information

Problem Set: 15%, Group presentation: 10%, Midterm: 30%, Final: 45%.

我们将主要使用 Varian (2014) 作为教材.

Examples:

- Airfare. Relationship between the ticket price and the departure time. High demand → Charging high price.
- Streaming Media. 2 Strategies: Membership subscription and Individual purchase(Pay-per-view).
- Charity-linked products.
- Urban Green.
- Electric Vehicles. Subsidize the purchase of electric vehicles.

章节 2. Consumer Theory

Economic Modeling:

- Who are the participants?
- Some assumptions:
  - Rational Choice: A person chooses the best alternative available.
  - Equilibrium: The market is in equilibrium.

Consumer Choice. → Preference

Consumers are assumed to choose the best bundle of goods they can afford.

- Best:
- Can afford: Allocated budget.

Consumption Choice Sets A consumption choice set is the collection of *all* consumption choices available to the consumer.

What constraints consumption choice?

- Budget
- Time
- Other resource limitations

Consumption Bundle A consumption bundle containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector $(x_1, x_2, ..., x_n)$.

Assume commodity prices are $p_1, p_2, ..., p_n$.

Budget Constraints

$$p_1x_1 + p_{x_2} + ... + p_nx_n \leq m \tag{2.1}$$

where m is the consumer's (disposable) income.

Budget Set

$$B(p_1, p_2, ..., p_n, m) \tag{2.2.1}$$

$$= \Big\{(x_1, x_2, ..., x_n) |\ x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0, p_1x_1 + p_{x_2} + ... + p_nx_n \leq m\Big\} \tag{2.2.2}$$

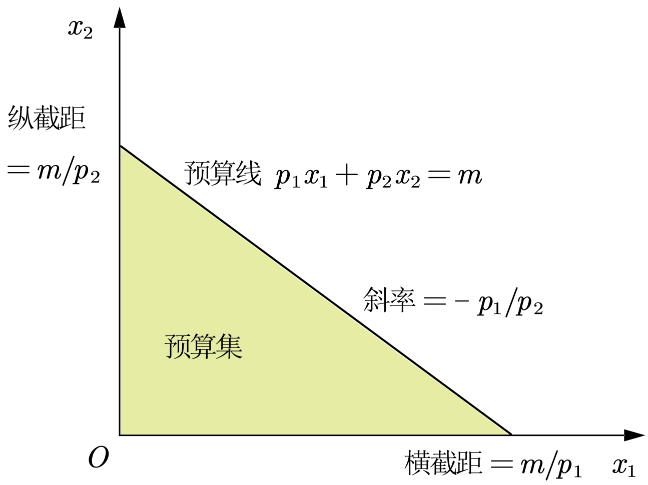


Figure 2.1: Budget Constraints

- Which is affordable? Unaffordable? Just affordable?

If $n = 3$ what do the budget constraints look like?

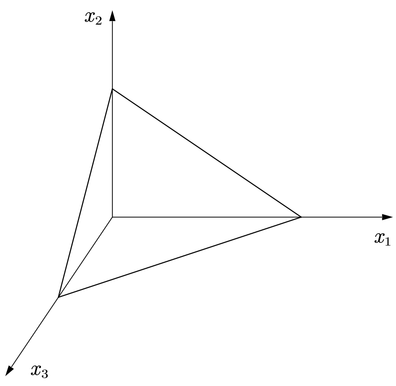


Figure 2.2: 3-dimensional Budget Constraints

In other words, opportunity cost of an extra unit of commodity 1 is p_1/p_2 units foregone of commodity 2.

Higher income gives more choice. improve consumer welfare.

enlarging, shrinking

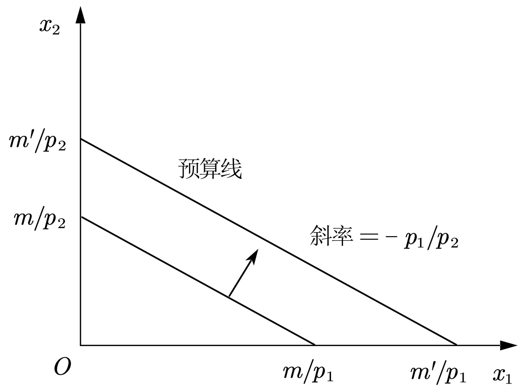


Figure 2.3: Higher Income

Increasing one price pivots the constraint inwards, reduces choice and will make the consumer worse off.

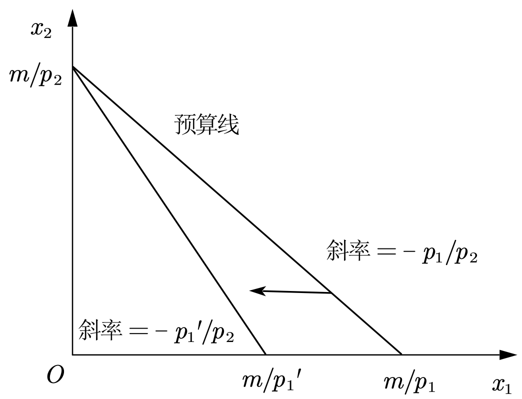


Figure 2.4: Increasing one price

- Q: If the economy is weak and so is consumer demand, what should the policy makers do to stimulate consumption?
- A: Reduce tax rate, sending cash to consumers. PRICE: subsidy.

Price changes: High demand, low supply.

2.1. *Ad Valorem* Sales Tax

Original price: $p \to$ New price: $(1+t)p$.

A uniform sales tax is applied uniformly to all goods.

$$(1+t)p_1x_1 + (1+t)p_2x_2 \le m \tag{2.3.1}$$

$$\Rightarrow p_1x_1 + p_2x_2 \le \frac{m}{1+t} \tag{2.3.2}$$

Remark: The tax essentially discount the income. And the equivalent income loss is

$$m - \frac{m}{1+t} = \frac{t}{1+t}m. \tag{2.4}$$

2.1.1. Example: The food stamp program

How does a commodity-specific gift such as a food stamp alters a family's budget constraint?

Suppose $m = 100$, $p_F = 1$ (food), "other goods" $p_G = 1$, the budget constrait is:

$$F + G \leq 100. \tag{2.5}$$

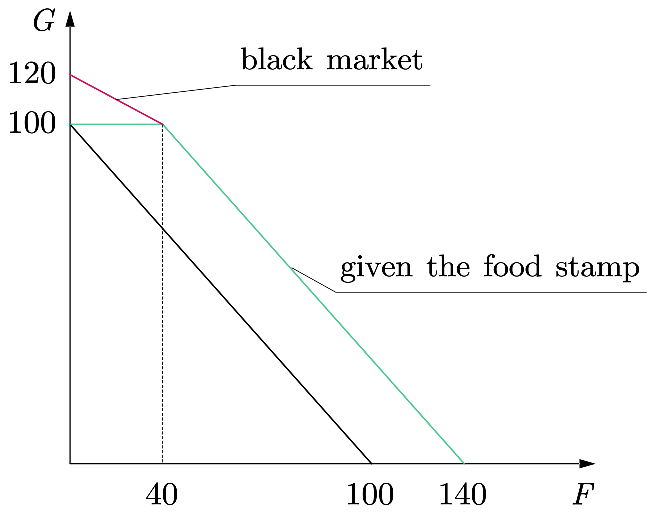


Figure 2.5: The food stamp program

Black market trading makes the budget set even larger. Black market improve consumer welfare.

2.2. Relative Price

Numeraire unit of account.

Changing the numeraire changes neither the budget constraint nor the budget set.

Any commodity can be chosen as the numeraire without changing the budget set or the budget constraint.

A straight line: constant relative price.

Quantity discounts:

Suppose $p_2 = 1$ is constant and $p_1 = 2$ when $0 \leq x_1 \leq 20$ and $p_1 = 1$ when $x_1 > 20$. The figure is like Figure 2.6.

Q: Is price always positive?

Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; $p_1 = -2, p_2 = 1$.

$$-2x_1 + x_2 \leq 10 \tag{2.6}$$

Like in Figure 2.7.

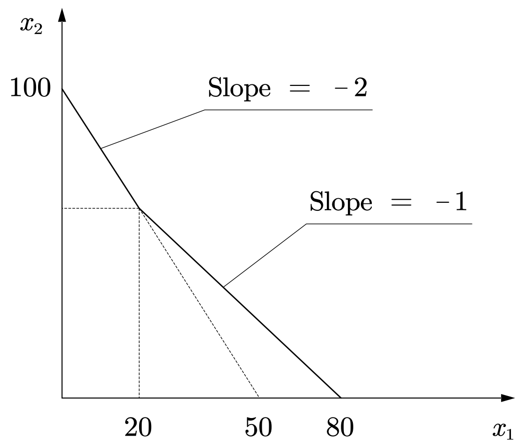


Figure 2.6: Quantity discounts

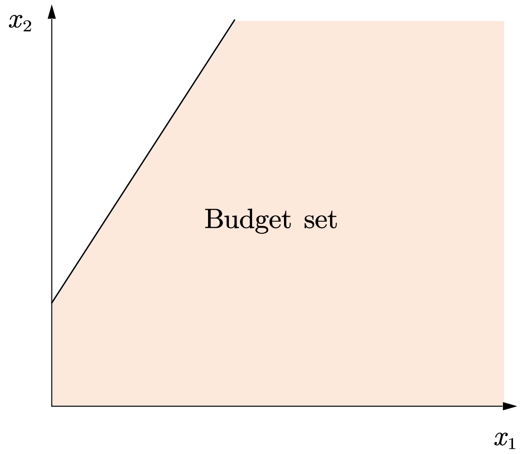


Figure 2.7: Negative prices

e.g. 示例 2.2.1

Why the consumption bundle in Figure 2.7 is unlimited?

双学位: A second degree in economics.

2.3. Multiple Constraints

Food Consumption vs Other Stuff.

- At least 10 units of food must be eaten to survive
- Budget constrained.
- Further restricted by a time constraint.

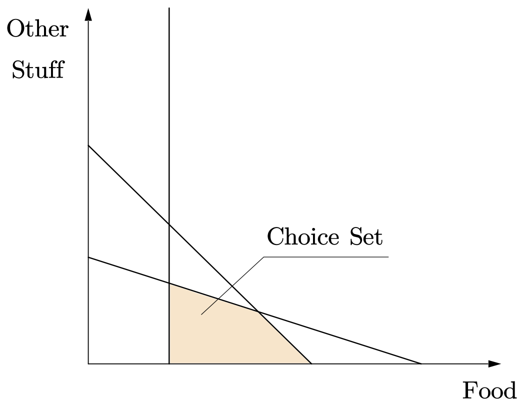


Figure 2.8: Multiple Constraints

章节 3. Preference and Utility

Consumers are assumed to choose the **best** bundle of goods they **can afford**.

Rationality in Economics: A decision maker always chooses its most preferred alternative from its set of available alternatives.

3.1. Preference Relations

strict preference x is more preferred than is y.
weak preference x is at least as preferred as is y.
indifference x is exactly as preferred as is y.

They are ordinal relations: only the order is important.

$\succ$ denotes strict preference. $x \succ y$ iff x is more preferred than y.

$\sim$ denotes indifference. $x \sim y$ iff x is exactly as preferred as y.

$\succsim$ denotes weak preference.

- $x \succsim y$ and $y \succsim x$ imply $x \sim y$.
- $x \succsim y$ and $\neg(y \succsim x)$ imply $x \succ y$.

Assumptions on preference relations:

1. Completeness. $\forall$ bundles x and y, either $x \succsim y$ or $y \succsim x$.
2. Reflexivity. $\forall$ bundles x, $x \succsim x$.
3. Transitivity. $x \succsim y$ and $y \succsim z$ imply $x \succsim z$.

3.2. Indifference Curve

Indifference curve: a set of bundles that are equally preferred.

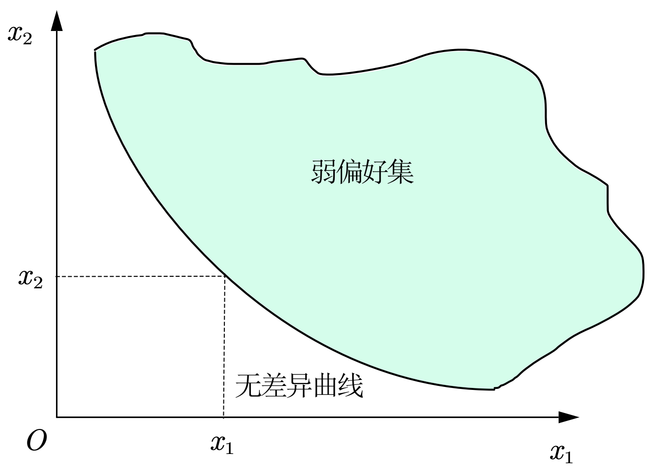


Figure 3.1: Indifference Curve

- $\mathrm{WP}(x)$: the set of all bundles that are weakly preferred to x.
- $\mathrm{I}(x)$: the set of all bundles that are indifferent to x.
- $\mathrm{I}(x) \subseteq \mathrm{WP}(x)$.

提示 3.2.1

Indifference curves cannot cross.

Proof: Suppose the intersection of two indifference curves is A. Then A is indifferent to B and C. But B and C are not indifferent to each other.

□

When more of a commodity is always preferred, the commodity is a good. If every commodity is a good then indifference curves are negatively sloped.

If less of a commodity is always preferred then the commodity is a bad. 1 Good and 1 bad: indifference curves are positively sloped.

3.3. Extreme Cases of Indifference Curves

3.3.1. Perfect Substitutes

If a consumer always regards units of commodities 1 and 2 as equivalent, the commodities are perfect substitutes and only the total amount of the two commodities in bundles determines their preference rank-order.

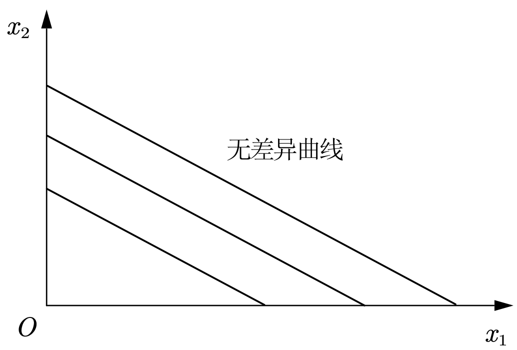


Figure 3.2: Perfect Substitutes

3.3.2. Perfect Complements

Fixed proportions of commodities 1 and 2 are required to provide utility.

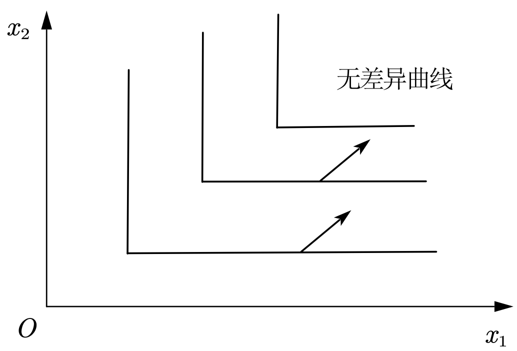


Figure 3.3: Perfect Complements

A L-shaped indifference curve. 45°.

3.3.3. Example: Good and Bad

You like pizzas but hate vegetables. You are only willing to eat an extra unit of vegetable if you get to eat an extra unit of pizza.

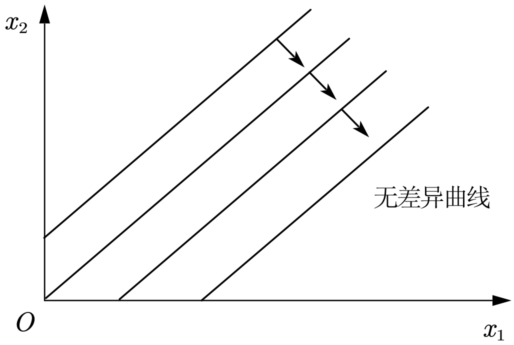


Figure 3.4: Good and Bad

3.3.4. Example: Satiation

You always want to study for 6 hours and workout for 2 hours. Any deviation from it gives you a lower utility level. Optimal bundle.

$\Longrightarrow$ A satiation point.

3.4. Preferences Exhibiting Satiation

A bundle strictly preferred to any other is a satiation point or a bliss point.

Why the shape is a circle? Because Any deviation from the satiation point gives you a lower utility level.

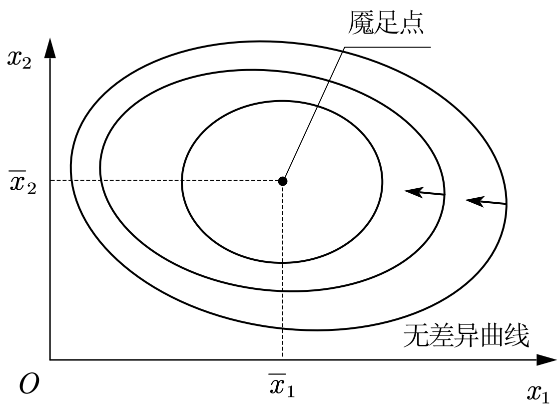


Figure 3.5: Satiation Point

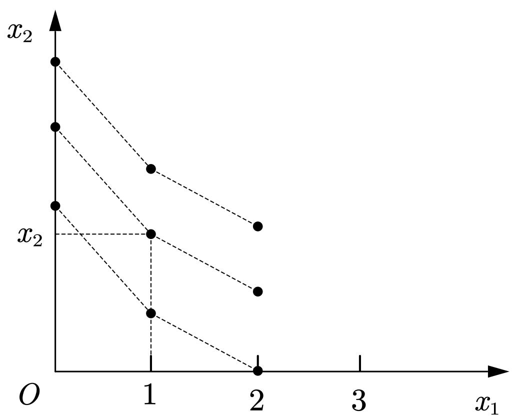


Figure 3.6: Indifferent Curves for Discrete Goods

3.5. Indifferent Curves for Discrete Goods

Like in Figure 3.6.

3.6. Well-Behaved Preferences

1. Monotonic. More is better or Less is better. (Satiation is a violation of this.)
2. Convexity. Mixtures are preferred to extremes. $\forall 0 < t < 1$,

$$tx + (1 - t)y \succ x = y. \tag{3.1}$$

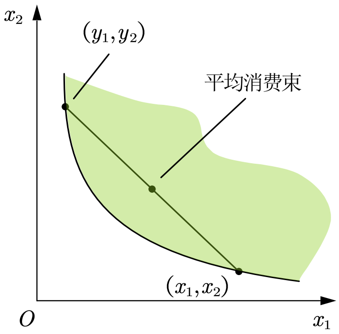


Figure 3.7: Convexity

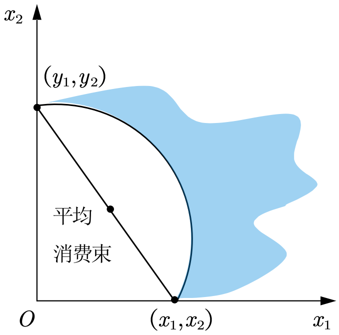


Figure 3.8: Concavity

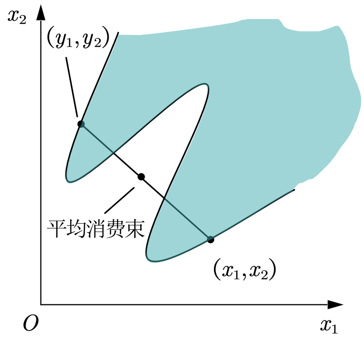


Figure 3.9: Mixture of Convexity and Concavity

For the most part, goods are consumed together. $\Rightarrow$ Convexity.

3.7. Slopes of Indifference Curves

The slope of an indifference curve (at a given point) is its marginal rate-ofsubstitution (MRS).

$$\text{MRS} = \frac{\mathrm{d}x_2}{\mathrm{d}x_1} \text{ at } x'. \tag{3.2}$$

Properties:

- Two goods $\Rightarrow$ MRS < 0.
- One good and one bad $\Rightarrow$ MRS > 0.
- Convex $\Leftrightarrow$ MRS always increases with x_1.

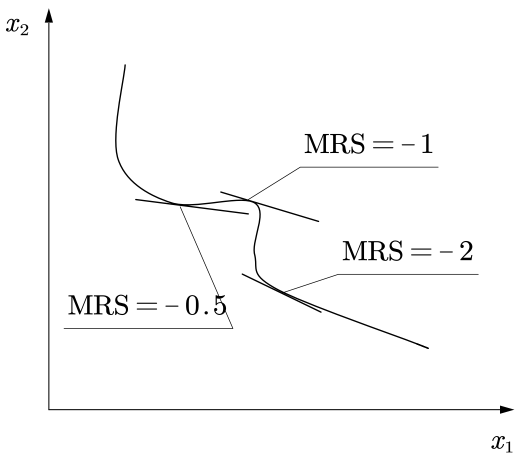


Figure 3.10: Violation of Convexity

3.8. Utility Function

Another way to represent preferences.

We can assign a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less preferred bundles.

The thing can be observed is the choices made by the consumer.

- Consumers make choices so as to maximize their utility to make themselves as happy as possible.

Continuity.

$$x' \succ x'' \Leftrightarrow U(x') > U(x'') \tag{3.3.1}$$

$$x' \prec x'' \Leftrightarrow U(x') < U(x'') \tag{3.3.2}$$

$$x' \sim x'' \Leftrightarrow U(x') = U(x'') \tag{3.3.3}$$

Utility is an ordinal concept. The value of utility is not important, only the order of the utility values. How much higher doesn't matter.

There is no unique utility function representation of a preference relation.

$U(x_1, x_2) = x_1 x_2$, and we can define $V(x_1, x_2) = x_1^2 x_2^2$, preserves the order of the utility values.

提示 3.8.1

U is a utility function represents a preference relation, and f is a strictly increasing function, then $f(U)$ also represents the same preference relation.

3.8.1. Perfect Substitution

$$U(x_1, x_2) = x_1 + x_2. \tag{3.4}$$

The pizza & vegetable example: $U(x_1, x_2) = x_1 - x_2$.

3.8.2. Perfect Complements

e.g. Left and right shoes, pencils and erasers, fries and ketchup, game consoles and games.

$$U(x_1, x_2) = \min\{x_1, x_2\}. \tag{3.5}$$

3.8.3. Quasi-linear Utility Function

Def **定义 3.8.1**

A utility function of the form

$$U(x_1, x_2) = f(x_1) + x_2 \tag{3.6}$$

is linear in just x_2 and is called a quasi-linear utility function.

3.8.4. Cobb-Douglas Utility Function

Def **定义 3.8.2**

A utility function of the form

$$U(x_1, x_2) = x_1^a x_2^b \tag{3.7}$$

is called a Cobb-Douglas utility function.

Well-behaved.

3.9. Marginal Utility

E.g. consider $U(x_1, x_2) = x_1^{1/2} x_2^2$, then

$$\mathrm{MU}_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2, \tag{3.8.1}$$

$$\mathrm{MU}_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2. \tag{3.8.2}$$

Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} \mathrm{d}x_1 + \frac{\partial U}{\partial x_2} \mathrm{d}x_2 = 0. \tag{3.9}$$

$$\Rightarrow \frac{\mathrm{d}x_2}{\mathrm{d}x_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}. \tag{3.10}$$

This is the MRS.

3.9.1. MRS for Quasi-linear Utility Function

$$\frac{\partial U}{\partial x_1} = f'(x_1), \frac{\partial U}{\partial x_2} = 1. \tag{3.11.1}$$

$$\Rightarrow \mathrm{MRS} = \frac{\mathrm{d}x_2}{\mathrm{d}x_2} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -f'(x_1). \tag{3.11.2}$$

MRS does not depend on x_2. So the slope of indifference curves for a quasi-linear utility function is constant along any line for which x_1 is constant.

3.9.2. Monotonic Transformation & MRS

If $V = f(U)$, we have

$$\mathrm{MRS} = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{f'(U) \times \partial U / \partial x_1}{f'(U) \times \partial U / \partial x_2} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}. \tag{3.12}$$

So MRS is **unchanged** by a positive monotonic transformation.

> e.g. **示例 3.9.1**
>
> Linda's preferences over magazines (M) and books (B) are given by:
>
> $$U(M, B) = 3M^{\frac{2}{3}} + 6B^{\frac{2}{3}}. \tag{3.13}$$
>
> Are Linda's preferences convex or not?

章节 4. Choice and Demand

A decision maker chooses its most preferred alternative from those available to it.

4.1. Rational Constrained Choice

To find the bundle in the budget set that is on the **highest** indifference curve.

Since preferences are well-behaved, (more is preferred to less), we can focus on the bundles that line on the budget line.

The Rational Constrained Choice is:

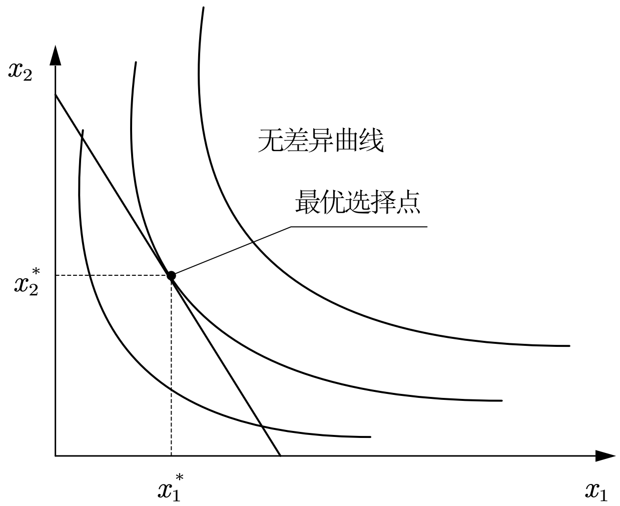


Figure 4.1: Rational Constrained Choice

(x_1^*, x_2^*) is the most preferred bundle on the budget line.

Ordinary Demand Ordinary demands will be denoted by $x_1^*(p_1, p_2, m)$ and $x_2^*(p_1, p_2, m)$.

The most preferred affordable bundle is the consumer's ORDINARY DEMAND at the given prices and budget.

$x_1^* > 0$ and $x_2^* > 0 \Rightarrow$ the demand bundle is INTERIOR. 注: *interior*, 内部的

If buying (x_1^*, x_2^*) consts m, then the budget is exhausted since

$$p_1 x_1^* + p_2 x_2^* = m. \tag{4.1}$$

所以, in Figure 4.1, (x_1^*, x_2^*) is interior and exhausts the budget.

The slope of the indifference curve at (x_1^*, x_2^*) is equal to the slope of the budget line.

1. the budget is exhausted:
$$p_1 x_1^* + p_2 x_2^* = m \tag{4.2}$$
2. the slope of the budget constraint $-\frac{p_1}{p_2}$ and the slope of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*).

4.2. Computing Ordinary Demands

Suppose that the consumer has Cobb-Douglas preferences:

$$U(x_1, x_2) = x_1^a x_2^b. \tag{4.3}$$

Then

$$\mathrm{MU}_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b, \tag{4.4.1}$$

$$\mathrm{MU}_2 = \frac{\partial U}{\partial x_2} = b x_1^a x_2^{b-1}. \tag{4.4.2}$$

So the MRS is

$$\mathrm{MRS} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}. \tag{4.5}$$

这将会得到:

$$x_2^* = \frac{bp_1}{ap_2} x_1^*. \tag{4.6}$$

接着联立 Equation (4.2), 得到:

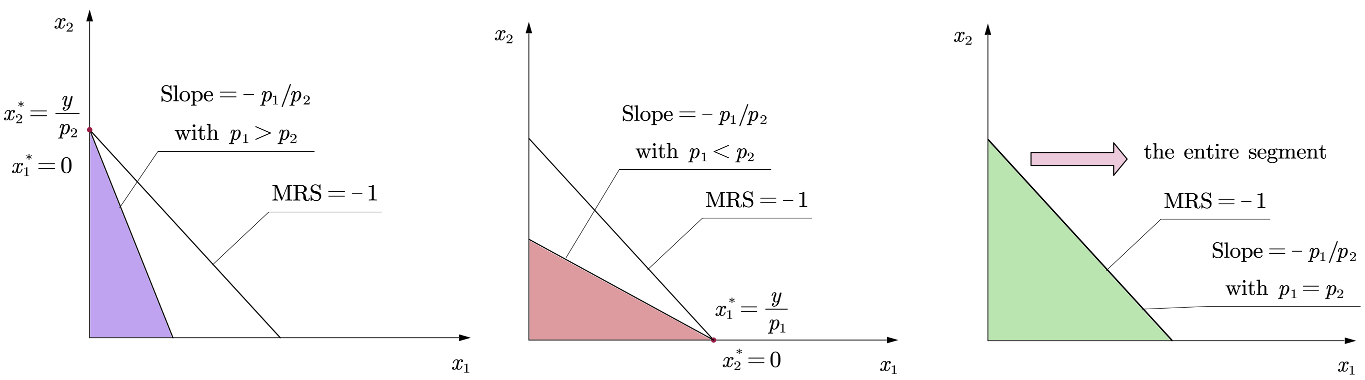
$$x_1^* = \frac{am}{(a+b)p_1}, x_2^* = \frac{bm}{(a+b)p_2}. \tag{4.7}$$

4.3. Corner Solutions

If either $x_1^* = 0$ or $x_2^* = 0$, then the ordinary demand (x_1^*, x_2^*) is at a corner solution to the problem.

4.3.1. Perfect Substitutes Case

Examples of Corner Solutions – the Perfect Substitutes Case



(a) Case 1 (b) Case 2 (c) Case 3[1]

Figure 2: the Perfect Substitutes Case

4.3.2. Non-Convex Preferences

Examples of Corner Solutions – Non-Convex Preferences

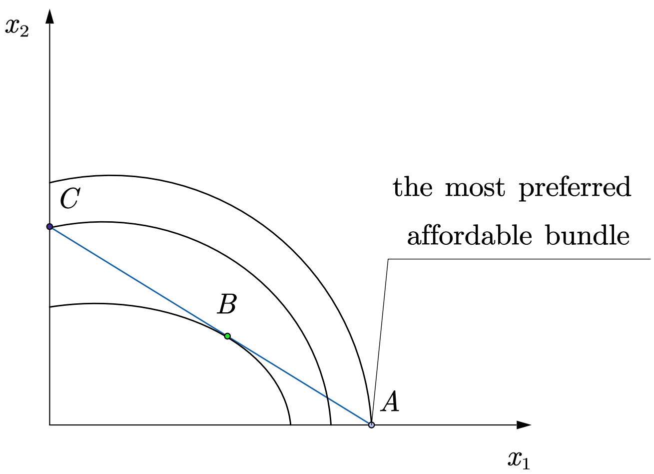


Figure 4.3: Non-Convex Preferences

4.4. 'Kinky' Solutions

[1]All the bundles in the constraint are equally the most preferred affordable when $p_1 = p_2$.

kinky: 奇怪的

4.4.1. Perfect Complements Case

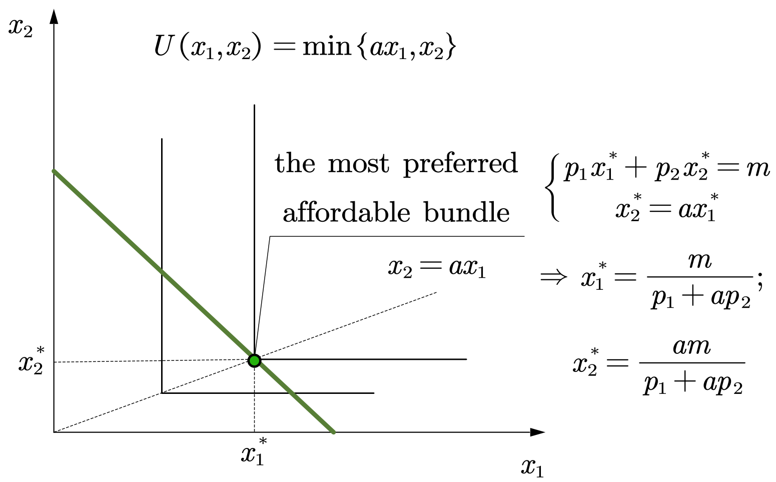


Figure 4.4: Perfect Complements Case

4.5. Properties of Demand Functions

Comparative statics analysis(静态分析) of ordinary demand functions:

- How do ordinary demands $x_1^*(p_1, p_2, y)$ and $x_2^*(p_1, p_2, y)$ change as p_1, p_2 and income y change?

4.5.1. Own-Price Changes

Fixed p_2 and y.

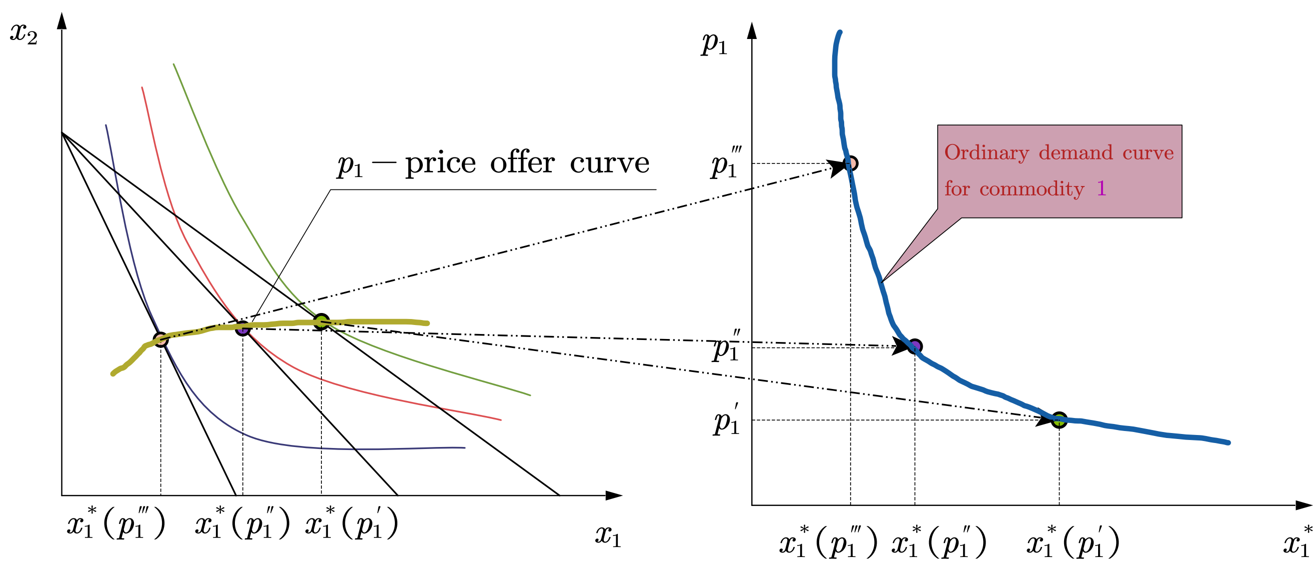


Figure 4.5: p_1-price offer curve & Ordinary demand curve for commodity 1

The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1-price offer curve. 实际上是x_2^*关于x_1^*的函数.

The plot of the x_1-coordinate of the p_1-price offer curve against p_1 is the ordinary demand curve for commodity 1.

4.5.1.1. Cobb-Douglas preferences

What does a p_1-price offer curve look like for Cobb-Douglas preferences?

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}, \tag{4.8.1}$$

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}. \tag{4.8.2}$$

这里的p_1-price offer curve 是一条平线, 而 Ordinary demand curve for commodity 1 是一个反函数的图象.

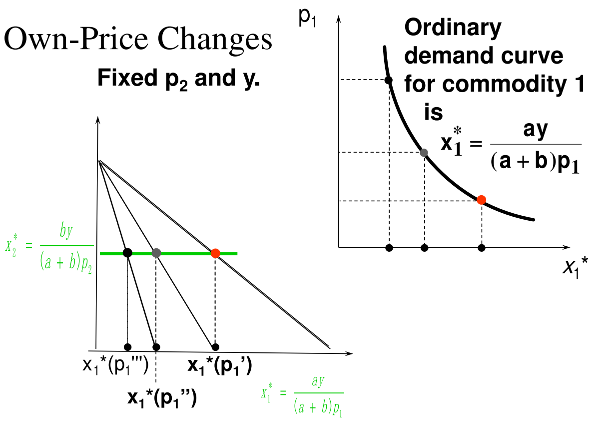


Figure 4.6: p_1-price offer curve for Cobb-Douglas preferences

4.5.1.2. Perfect Complements

What does a p_1-price offer curve look like for a perfect-complements utility function?

$$x_1^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}, \tag{4.9.1}$$

$$x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}. \tag{4.9.2}$$

所以就有:

$$x_2^* = x_1^*. \tag{4.10}$$

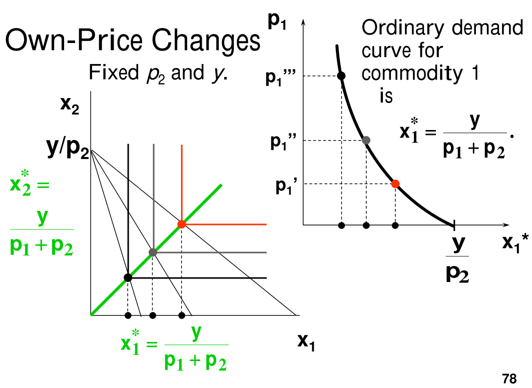


Figure 4.7: p_1-price offer curve for a perfect-complements utility function

4.5.1.3. Perfect Substitutes

首先我们写出 Ordinary demand:

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & \text{,if } p_1 > p_2 \\ y/p_1 & \text{,if } p_1 < p_2 \end{cases}, \tag{4.11.1}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & \text{,if } p_1 < p_2 \\ y/p_2 & \text{,if } p_1 > p_2 \end{cases}. \tag{4.11.2}$$

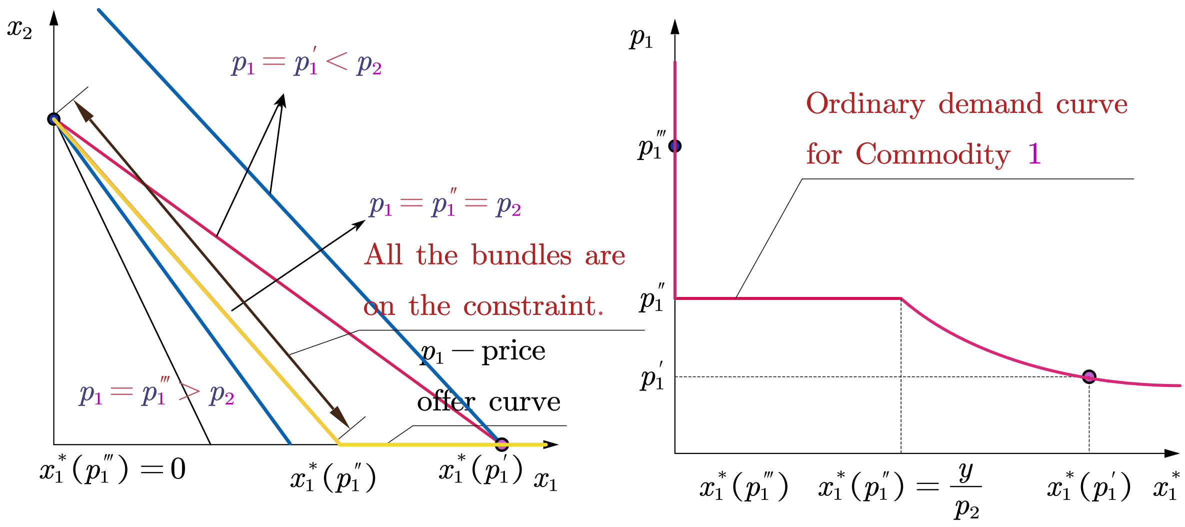


Figure 4.8: p_1-price offer curve for a perfect-substitutes utility function

4.6. Inverse Demand

Inverse Demand: At what price for commodity 1 will the consumer buy x_1 units of commodity 1?

Cobb-Douglas:

$$x_1^* = \frac{ay}{(a+b)p_1} \to p_1 = \frac{ay}{(a+b)x_1^*}. \tag{4.12}$$

Perfect Complements:

$$x_1^* = \frac{y}{p_1 + p_2} \to p_1 = \frac{y}{x_1^*} - p_2. \tag{4.13}$$

4.7. Income Changes

Income Offer Curve:

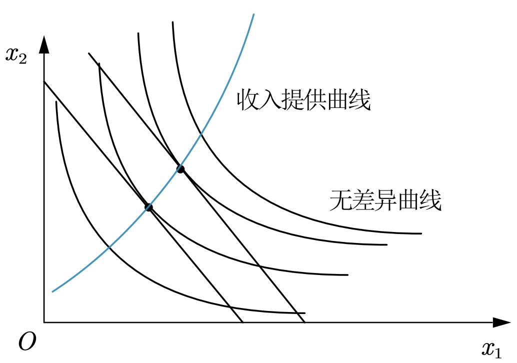


Figure 4.9: Income Offer Curve

A plot of quantity demanded against income is called an **Engel curve**.

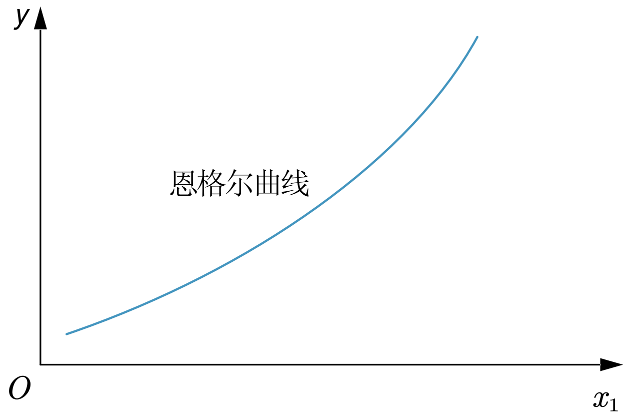


Figure 4.10: Engel Curve

Cobb-Douglas:

$$x_1^* = \frac{ay}{(a+b)p_1}; x_2^* = \frac{by}{(a+b)p_2}. \tag{4.14.1}$$

$$\Rightarrow y = \frac{(a+b)p_1}{a} x_1^*; y = \frac{(a+b)p_2}{b} x_2^*. \tag{4.14.2}$$

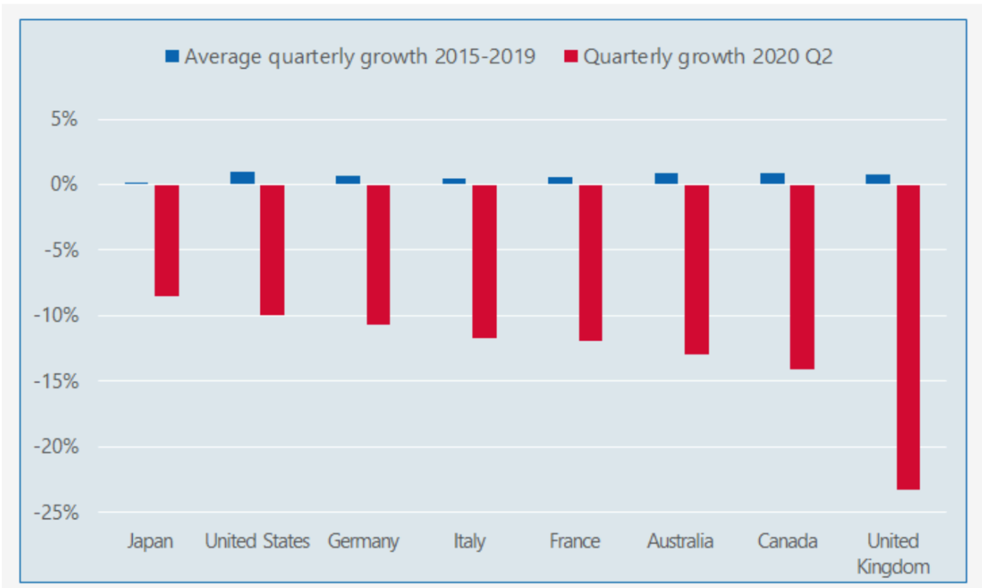
Perfectly-Substitutable Preferences: 在假设$p_1 < p_2$等情况下讨论.

Perfectly-Complementary Preferences:

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}. \Rightarrow y = (p_1 + p_2)x_1^* = (p_1 + p_2)x_2^*. \tag{4.15}$$

4.8. Discussion

The Covid outbreak resulted in significant decline in household spending.



Source: OECD Statistics Directorate

Figure 4.11: Significant decline in household spending

- How to achieve a swift recovery of consumption post-Covid?
- Could you use the tools we have learned to support your reasoning?

4.8.1. Stimulating Tools

刺激工具有:

- Vouchers(代金券): XX yuan to spend on retail or service outlets
- Shopping Coupons(消费券): x-yuan off if purchasing y-yuan or more.

4.9. Homotheticity

Homotheticity(齐次性): A consumer's preferences are homothetic if and only if

$$(x_1, x_2) \prec (y_1, y_2) \Leftrightarrow (kx_1, kx_2) \prec (ky_1, ky_2) \tag{4.16}$$

for every $k > 0$.

That is, Consumer's preferences only depend on the ratio of good 1 to good 2.

Quasilinear preferences are not homothetic.

$$U(x_1, x_2) = f(x_1) + x_2. \tag{4.17}$$

4.10. Normal and Inferior

Does the demand for a good always increase as the income increases?

Normal good: the demand for a normal good would increase when income increases.

Inferior good: an increase in income results in a reduction in its consumption (low-quality good).

4.11. Ordinary Goods, Giffen Goods

Does the demand for a good always increase as its own price reduces?

A good is called **ordinary** if the quantity demanded of it always increases as its own **price** decreases.

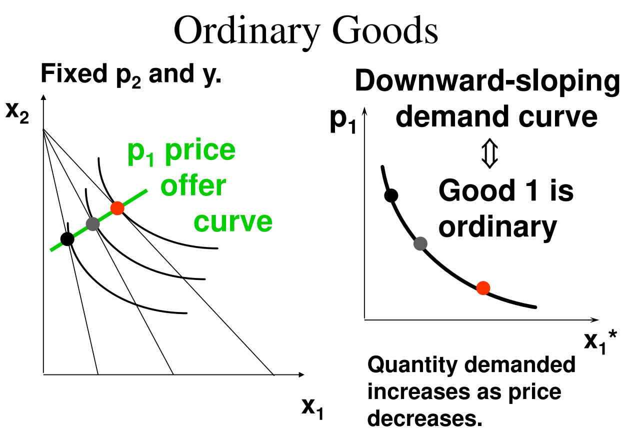


Figure 4.12: Ordinary Goods

If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called **Giffen**. 价格增加, 需求也增加, 称为吉芬商品.

The income effect of the price change more than offsets the substitution effect (will discuss more later).

4.12. Cross-Price Effects

$p_2 \uparrow$

- demand for commodity 1 $\uparrow$, then commodity 1 is a substitute for commodity 2.
- demand for commodity 1 $\downarrow$, then commodity 1 is a complement for commodity 2.

Perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}, \tag{4.18}$$

so,

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0. \tag{4.19}$$

Therefore commodity 2 is a complement for commodity 1.

A **Cobb-Douglas** example:

$$x_2^* = \frac{by}{(a+b)p_2}, \tag{4.20}$$

so,

$$\frac{\partial x_2^*}{\partial p_1} = 0. \tag{4.21}$$

Therefore commodity 1 is neither a complement nor a substitute for commodity 2.

4.13. Effects of a Price Change

1. **Substitution effect**: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.
2. **Income effect**: the consumer's budget of $\$y$ can purchase more than before, as if the consumer's income rose, with consequent income effects on quantities demanded.

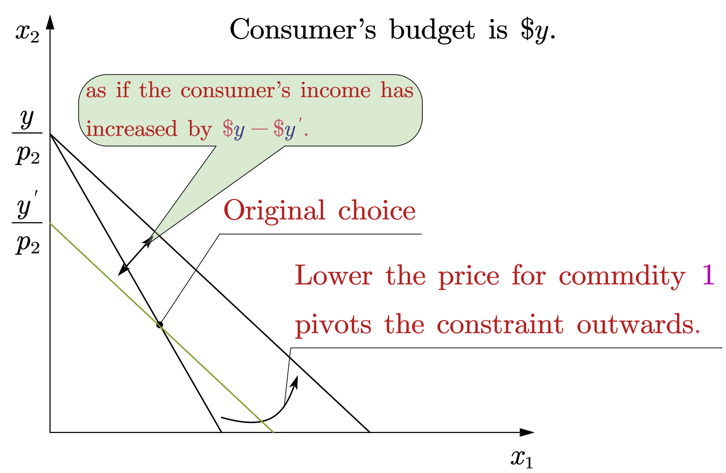


Figure 4.13: Income effect

Changes to quantities demanded due to this 'extra' income are the income effect of the price change.

Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.

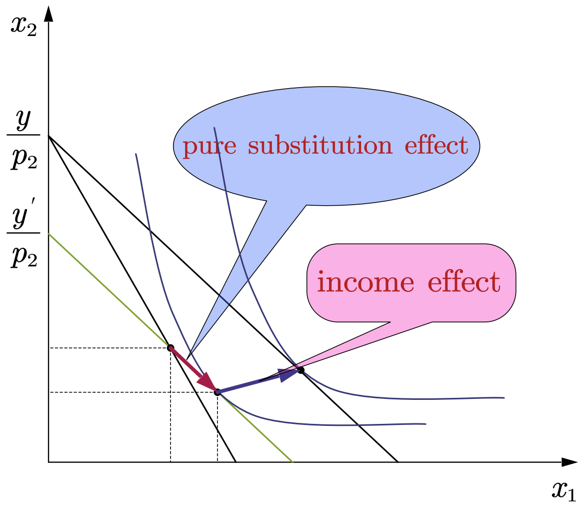


Figure 4.14: Pure Substitution Effect & Income Effect

章节 5. Revealed Preference and Intertemporal Choice

Utility is not observed, but chooses are.

5.1. Revealed Preference Analysis

Consumers' consumption choices can reveal their preferences.

Suppose we observe the demands (consumption choices) that a consumer makes for different budgets.

Direct Preference Revelation Suppose that the bundle x^* is chosen when the bundle y is affordable. Then x^* is revealed directly as preferred to y. x is revealed directly as preferred to y will be written as

$$x \underset{D}{\succ} y. \tag{5.1}$$

Indirect Preference Revelation Suppose x is revealed directly preferred to y, and y is revealed directly preferred to z. By transitivity, x is revealed indirectly preferred to z.

$$x \underset{I}{\succ} z. \tag{5.2}$$

$$x \underset{D}{\succ} y, y \underset{D}{\succ} z \Rightarrow x \underset{I}{\succ} z. \tag{5.3}$$

5.1.1. Two Axioms of Revealed Preference

The Weak Axiom of Revealed Preference (WARP) If the bundle x is revealed directly as preferred to the bundle y then it is never the case that y is revealed directly as preferred to x; i.e.

$$x \underset{D}{\succ} y \Rightarrow \neg\left(y \underset{D}{\succ} x\right). \tag{5.4}$$

The other bundles that are affordable but not chosen must be worse than what is chosen.

你能购买y但是买了x, 说明x更好, 那么你决定购买y的时候, 必然意味着x不能被购买.

Checking if Data Violate the WARP

计算当前的选择下，其他 affordable 的 bundle，这就说明了当前的选择被直接显示偏好于其他 affordable 的 bundle.

那么观察这些直接的显示偏好关系是否有冲突.

The Strong Axiom of Revealed Preference (SARP) If the bundle x is revealed (directly or indirectly) as preferred to the bundle y and $x \neq y$, then it is never the case that the y is revealed (directly or indirectly) as preferred to x; i.e.

$$x \underset{D}{\succ} y \text{ or } x \underset{I}{\succ} y \Rightarrow \neg\left(y \underset{D}{\succ} x \text{ or } y \underset{I}{\succ} x\right). \tag{5.5}$$

What choice data would satisfy the WARP but violate the SARP?

那么观察这些直接的和间接的显示偏好关系是否有冲突.

That the observed choice data satisfy the SARP is a condition necessary and sufficient for there to be a well-behaved preference relation that "rationalizes" the data.

5.2. Intertemporal Choice

Persons often receive income in "lumps"; e.g. monthly salary.

Begin with some simple example:

- 2 Periods, 1, 2.
- r denotes the interest rate per period.
- The future value one period from now of $\$m$ is

$$\text{FV} = m(1+r). \tag{5.6}$$

Present Value How much money would have to be saved now, in the present, to obtain \$1 at the start of the next period?

$$m(1+r) = 1, \Rightarrow m = \frac{1}{1+r}. \tag{5.7}$$

The present value of $\$m$ available at the start of the next period is

$$\text{PV} = \frac{m}{1+r}. \tag{5.8}$$

- Let m_1 and m_2 be incomes received in periods 1 and 2.
- Let c_1 and c_2 be consumptions in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.
- The intertemporal choice problem: 求解 most preferred 消费束.

这个问题需要知道:

- the intertemporal budget constraint

- intertemporal consumption preferences

1. Suppose that the consumer chooses not to save or to borrow.

- Period 1: $c_1 = m_1$.
- Period 2: $c_2 = m_2$.

2. Now suppose that the consumer spends nothing on consumption in period 1; that is, $c_1 = 0$ and the consumer saves $s_1 = m_1$.

- Period 1: $c_1 = 0$.
- Period 2: $c_2 = m_1(1 + r) + m_2$.

3. Now suppose that the consumer spends everything possible on consumption in period 1, so $c_2 = 0$.

- Period 1: $c_1 = m_1 + \frac{m_2}{1+r}$.
- Period 2: $c_2 = 0$.

4. Suppose consumed c_1 in Period 1.

- Period 1: $c_1 = c_1$.
- Period 2: $c_2 = m_2 + (1 + r)(m_1 - c_1)$.

$$c_2 = -(1 + r)c_1 + m_2 + (1 + r)m_1. \tag{5.9}$$

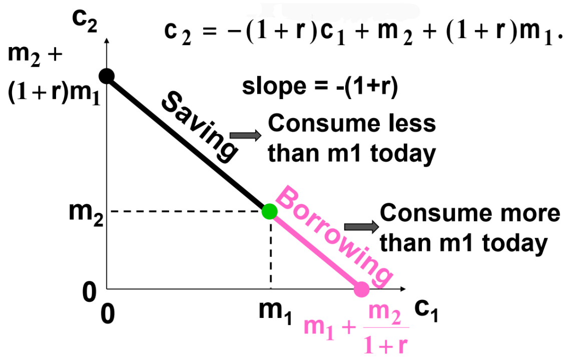


Figure 5.1: The Intertemporal Budget Constraint

现在加上了价格:

$$(1 + r)p_1c_1 + p_2c_2 = (1 + r)m_1 + m_2. \tag{5.10}$$

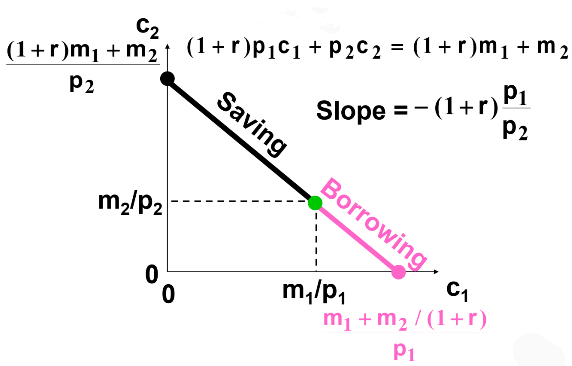


Figure 5.2: The Intertemporal Budget Constraint

5.2.1. Price Inflation

现在假设通货膨胀率为π, 那么$p_2 = p_1(1+\pi)$.

$$(1+r)p_1c_1 + (1+\pi)p_1c_2 = (1+r)m_1 + m_2. \tag{5.11}$$

$$c_2 = -\frac{1+r}{1+\pi}c_2 + \left(\frac{(1+r)m_1}{p_1(1+\pi)} + \frac{m_2}{p_1(1+\pi)}\right). \tag{5.12}$$

Real Interest Rate

$$-(1+\rho) = -\frac{1+r}{1+\pi} \Rightarrow \rho = \frac{r-\pi}{1+\pi}. \tag{5.13}$$

e.g. **示例 5.2.1**

Jeff is deciding how much to consume now (period 1) and how much to save for retirement (period 2). Suppose his preference is represented by:

$$u = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}, \tag{5.14}$$

where $\sigma > 0$.

Suppose $p_1 = p_2 = 1$, $m_1 = W$, $m_2 = 0$, and the interest rate is r.

- Q1: What is his budget constraint in the present value form?
- Q2: How much of income will Jeff consume and how much will he save?
- Q3: Suppose $\sigma = \frac{1}{2}$. Does increasing in r increase or decrease savings?

Answer.

- Q1:

$$c_1 + \frac{c_2}{1+r} = W. \tag{5.15}$$

- Q2: We have:

$$\frac{\partial u}{\partial c_1} = c_1^{-\sigma}, \frac{\partial u}{\partial c_2} = c_2^{-\sigma} \tag{5.16.1}$$

$$\Rightarrow \text{MRS} = -\frac{c_2^{\sigma}}{c_1^{\sigma}}. \tag{5.16.2}$$

And we have:

$$1+r = \frac{c_2^{\sigma}}{c_1^{\sigma}}, c_2 = (1+r)^{\frac{1}{\sigma}}c_1. \tag{5.17}$$

And we have:

$$c_1 = \frac{W}{1+(1+r)^{1/\sigma-1}}. \tag{5.18}$$

- Q3:

$$c_1 = \frac{W}{2+r}, \text{sav} = W\left(1 - \frac{1}{r+2}\right). \tag{5.19}$$

$r \uparrow \rightarrow \text{sav} \uparrow$.

5.3. Valuing Securities

A financial security is a financial instrument that promises to deliver an income stream.

E.g., A security that pays:

- m_1 at the end of year 1;
- m_2 at the end of year 2;
- m_3 at the end of year 3.
- PV of the security is

$$\frac{m_1}{1+r} + \frac{m_2}{(1+r)^2} + \frac{m_3}{(1+r)^3}. \tag{5.20}$$

5.3.1. Valuing Bonds

A bond is a special security: pays a fixed amount $\$x$ for T years (its maturity date) and then pays its face value $\$F$.

$$\mathrm{PV} = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \cdots + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}. \tag{5.21}$$

章节 6. Uncertainty and Consumer Surplus

6.1. Insurance & Uncertainty

Why do people buy insurance?

Uncertainty

- tomorrow's price
- future wealth
- future availability of goods
- present and future actions of other people

应对方法:

- buy insurance
- a portfolio of contingent. 资产组合

Suppose accident occurs with probability π_{a}, and does not with probability π_{na}, and accident causes a loss of L.

A contract implemented is state-contingent, which means the payment is contingent on the state of the world.

- Pay γ for \$1 of insurance
- Consumer has m wealth
- c_{na} is consumption without accident
- c_{a} is consumption with accident

Without insurance, $c_{\mathrm{na}} = m, c_{\mathrm{a}} = m - L$.

Buy K worth of insurance, $c_{\text{na}} = m - \gamma K, c_{\text{a}} = m - \gamma K - L + K = m - L + (1-\gamma)K$.

$$K = \frac{c_{\text{a}} - m + L}{1-\gamma}, c_{\text{na}} = m - \frac{\gamma(c_{\text{a}} - m + L)}{1-\gamma} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} c_{\text{a}}. \tag{6.1}$$

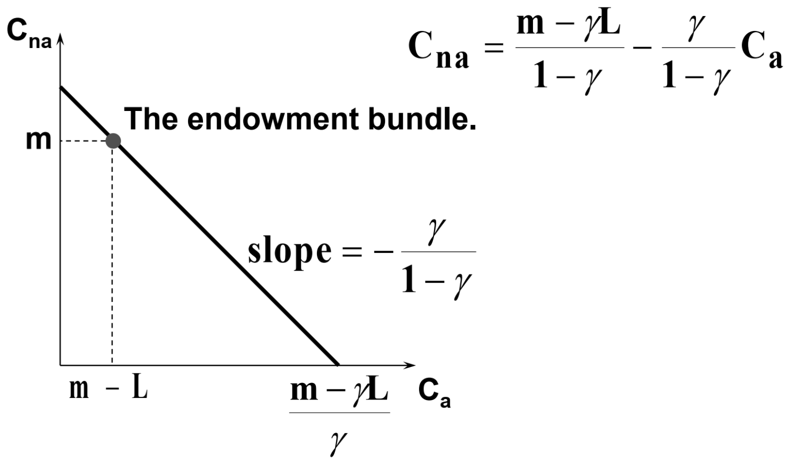


Figure 6.1: State-Contingent Budget Constraints

6.2. Preferences Under Uncertainty

Lottery:

- Win \$90 with probability $\frac{1}{2}$, and \$0 with probability $\frac{1}{2}$.
- $U(\$90) = 12$, $U(\$0) = 2$.
- $\text{EU} = \frac{1}{2} \times 12 + \frac{1}{2} \times 2 = 7$.
- $\text{EM} = \frac{1}{2} \times \$90 + \frac{1}{2} \times \$0 = 45$.
- Now $U(\$45)$和7的关系.

Preferences Under Uncertainty

- risk-aversion: $U(\text{EM}) > \text{EU}$.
- risk-loving: $U(\text{EM}) < \text{EU}$.
- risk-neutrality: $U(\text{EM}) = \text{EU}$.

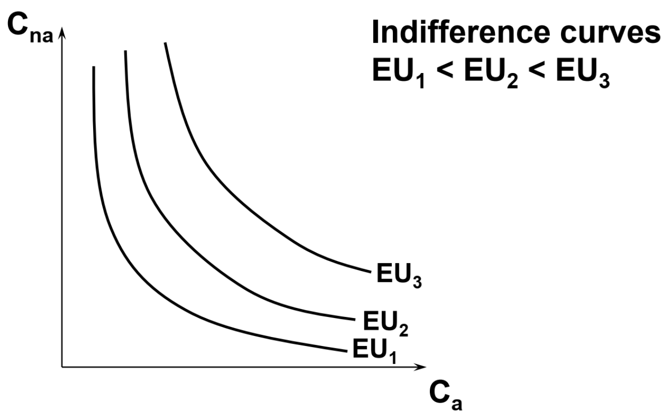


Figure 6.2: Indifference Curves Under Uncertainty

6.2.1. MRS of an Indifference Curve

Get consumption c_1 with probability π_1 and c_2 with probability π_2 $(\pi_1 + \pi_2 = 1)$. 所以我们就有:

$$\mathrm{EU} = \pi_1 U(c_1) + \pi_2 U(c_2). \tag{6.2}$$

接着我们可以得到:

$$\mathrm{dEU} = \pi_1 \ \mathrm{MU}(c_1)\,\mathrm{d}c_1 + \pi_2 \ \mathrm{MU}(c_2)\,\mathrm{d}c_2 = 0 \tag{6.3.1}$$

$$\Rightarrow \frac{\mathrm{d}c_2}{\mathrm{d}c_1} = -\frac{\pi_1 \ \mathrm{MU}(c_1)}{\pi_2 \ \mathrm{MU}(c_2)}. \tag{6.3.2}$$

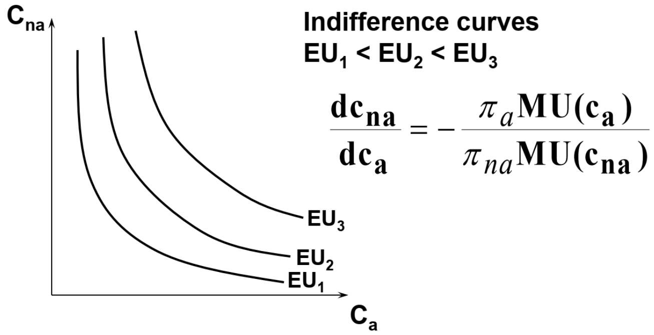


Figure 6.3: Indifference Curves Under Uncertainty

6.2.2. Most Preferred Affordable State-Contingent Consumption Plan

MRS = slope of budget constraint

$$\frac{\gamma}{1-\gamma} = \frac{\pi_{\mathrm{a}} \ \mathrm{MU}(c_{\mathrm{a}})}{\pi_{\mathrm{na}} \ \mathrm{MU}(c_{\mathrm{na}})}. \tag{6.4}$$

6.3. Competitive Insurance

How much would the insurance premium γ be?

Suppose entry to the insurance industry is free.

Expected economic profit:

$$\gamma K - \pi_{\mathrm{a}} K - (1-\pi_{\mathrm{a}}) \times 0 = (\gamma - \pi_{\mathrm{a}})K. \tag{6.5}$$

考虑到, Expected economic profit = 0, 那么:

$$\gamma = \pi_{\mathrm{a}}. \tag{6.6}$$

也就是: free entry $\Rightarrow \gamma = \pi_{\mathrm{a}}$.

When insurance is fair, rational insurance choices satisfy

$$\frac{\gamma}{1-\gamma} = \frac{\pi_{\mathrm{a}}}{1-\pi_{\mathrm{a}}} = \frac{\pi_{\mathrm{a}} \ \mathrm{MU}(c_{\mathrm{a}})}{\pi_{\mathrm{na}} \ \mathrm{MU}(c_{\mathrm{na}})} \tag{6.7.1}$$

$$\Rightarrow \mathrm{MU}(c_{\mathrm{a}}) = \mathrm{MU}(c_{\mathrm{na}}). \tag{6.7.2}$$

Marginal utility of income must be the same in both states.

6.4. Diversification

Typically, diversification lowers expected earnings in exchange for lowered risk.

6.5. Consumer's Surplus

对于某项政策的影响, 我们需要估计消费者的福祉.

Benefit-Cost Analysis.

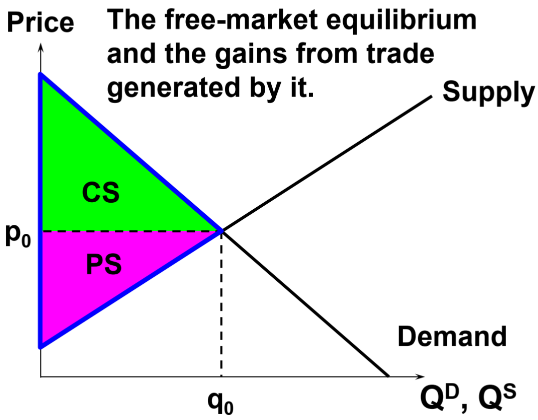


Figure 6.4: The Consumer Surplus

6.6. Monetary Measures of Gains-to-Trade

Gains-to-Trade: 交易带来的收益量.

- Consumer's Surplus
- Equivalent Variation
- Compensating Variation

假设你一次只能买一个单位的汽油. r_1表示购买第一个单位的保留价格(reservation price).

- The difference between the consumer's reservation-price and ordinary demand curves is due to income effects. The price at which a consumer is willing to purchase some good 1 depends on how much money he or she has for consuming other good.
- If the consumer's utility function is *quasilinear* in income then there are no income effects and Consumer's Surplus is an exact $ measure of gains-to-trade.

补偿变动 (Compensating Variation, CV): 在价格变化后，需补偿(或剥夺)的货币量，使其效用回到变化前的水平.

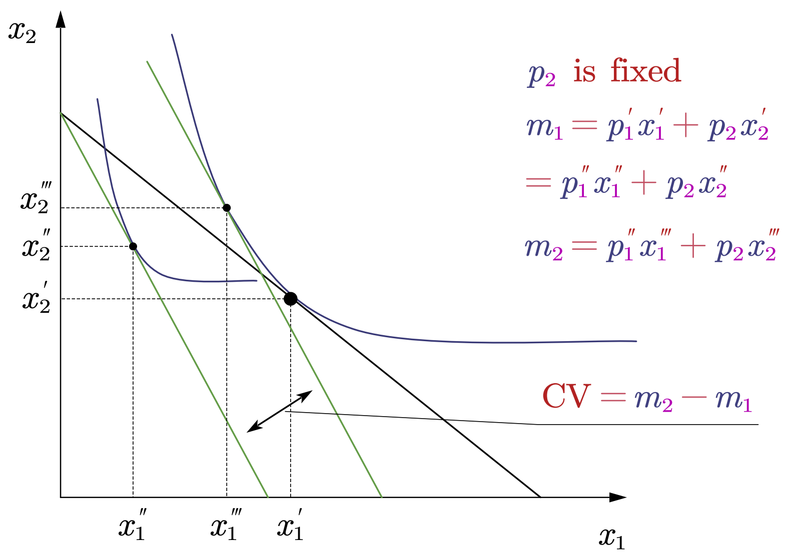


Figure 6.5: Compensating Variation

注意: 可以发现图中是$m_2 = p_1'' x_1''' + p_2 x_2'''$, 这里的$p_1$是变化过后的.

等价变动 (Equivalent Variation, EV): 在价格变化前, 需补偿消费者的货币量, 使其效用达到价格变化后的水平.

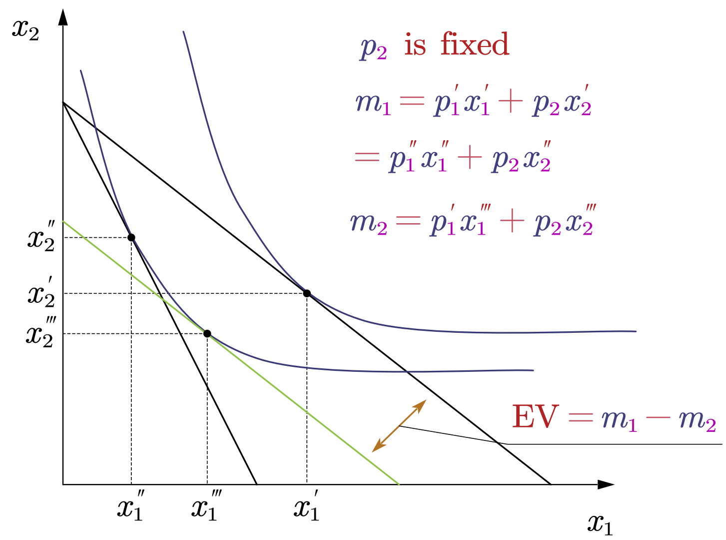


Figure 6.6: Equivalent Variation

注意: 可以发现图中是$m_2 = p_1' x_1''' + p_2 x_2'''$, 这里的$p_1$是变化前的.

- CV measures the amount of money that the consumer would have to be paid to compensate him/her for a price change.
- EV measures the amount of money that the consumer would be willing to pay to avoid a price change.

CV and EV are two ways of measuring "how far apart" two indifference curves are.

In general, the measure of distance will depend on the slope of the tangent lines - on the prices that we choose to determine the budget lines.

如果消费者的偏好是半线性(quasilinear)的, 这三种方式是完全一样的.

也就是我们假设$U(x_1, x_2) = v(x_1) + x_2$, 同时$p_1$从$p_1'$上升到$p_1''$.

那么消费者剩余(Consumer's Surplus):

$$\mathrm{CS}(p_1') = v(x_1') - v(0) - p_1' x_1' \tag{6.8.1}$$

$$\mathrm{CS}(p_1'') = v(x_1'') - v(0) - p_1'' x_1'' \tag{6.8.2}$$

$$\Delta\mathrm{CS} = \mathrm{CS}(p_1') - \mathrm{CS}(p_1'') \tag{6.8.3}$$

$$= v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'') \tag{6.8.4}$$

现在考虑 Compensating Variation, 前后的 utility 不变:

$$v(x_1') + m - p_1' x_1' = v(x_1'') + m + \mathrm{CV} - p_1'' x_1'' \tag{6.9.1}$$

$$\Rightarrow \mathrm{CV} = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'') = \Delta\mathrm{CS} \tag{6.9.2}$$

现在考虑 Equivalent Variation, 前后的 utility 不变:

$$v(x_1') + m - \mathrm{EV} - p_1' x_1' = v(x_1'') + m - p_1'' x_1'' \tag{6.10.1}$$

$$\Rightarrow \mathrm{EV} = v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'') = \Delta\mathrm{CS} \tag{6.10.2}$$

所以, when the consumer has quasilinear utility,

$$\mathrm{CV} = \mathrm{EV} = \Delta\mathrm{CS}. \tag{6.11}$$

6.7. Producer's Surplus

生产者剩余 = 获得收益(Revenue) - 成本(Variable Cost).

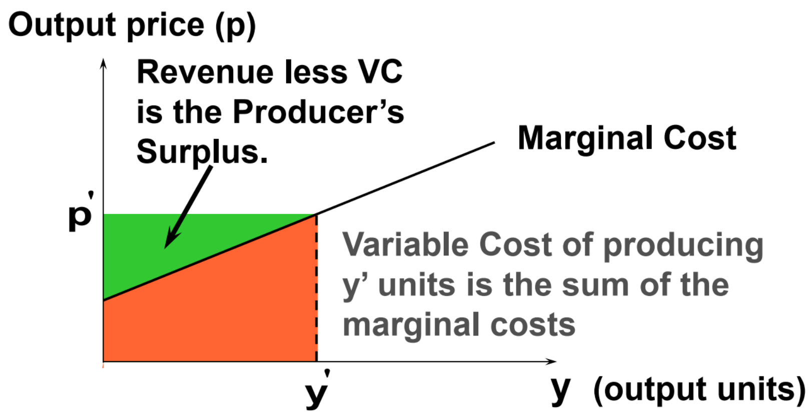


Figure 6.7: Producer's Surplus

6.8. Benefit-Cost Analysis

检测 market intervention 的影响.

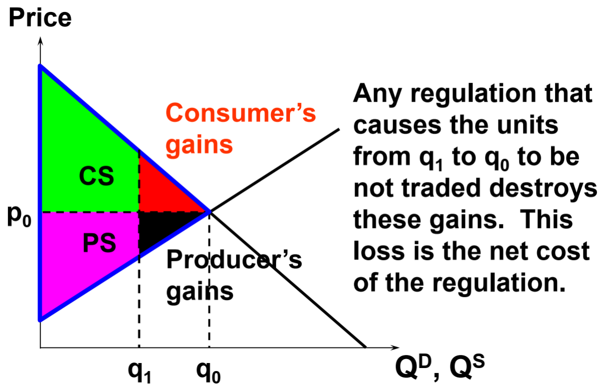


Figure 6.8: Quantity Limits

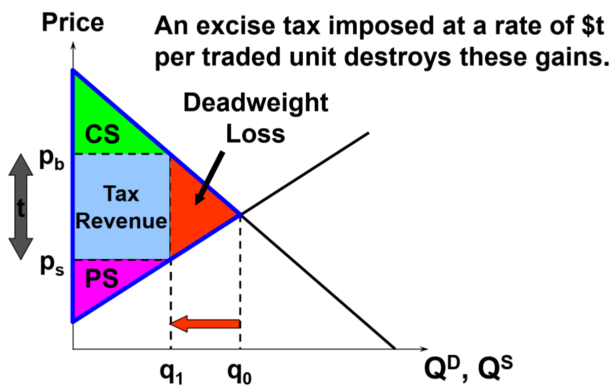


Figure 6.9: Imposing Tax

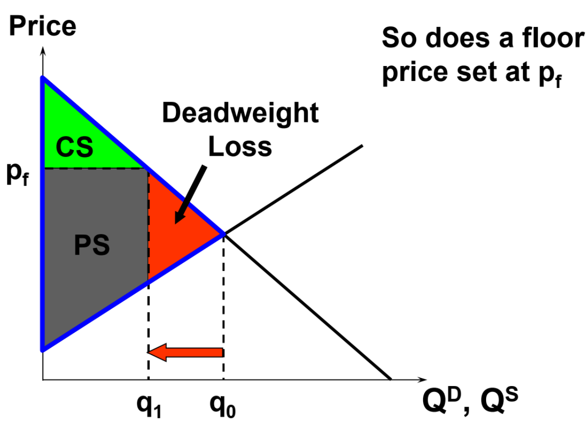


Figure 6.10: Floor Price

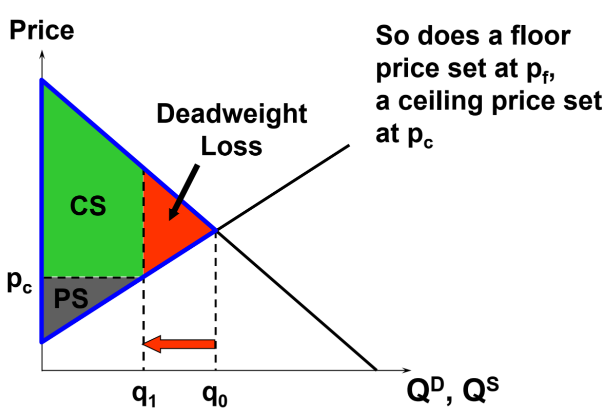


Figure 6.11: Ceil Price

e.g. 示例 6.8.1

Suppose the demand for books is given by the inverse demand function

$$p_D(B) = 10 - \frac{1}{10}B. \tag{6.12}$$

The inverse supply function for books is

$$p_S(B) = \frac{1}{10}B. \tag{6.13}$$

1. What is the equilibrium price and quantity of books?
2. If the city imposes a \$2 tax on books, what is the new equilibrium price and quantity?
3. What is the tax revenue received by the city?
4. What is the deadweight loss associated with this tax?

章节 7. Market Demand and Equilibrium

7.1. Market Demand

- **RQ1**: How do we get the market-level demand curve?
- **RQ2**: How do we estimate the entire class's demand of movies?

An economy containing n consumers, denoted by $i = 1, ..., n$. Consumer i's ordinary demand function for commodity j is

$$x_j^{*i}(p_1, p_2, m^i). \tag{7.1}$$

When all consumers are price-takers, the market demand function for commodity j is

$$X_j(p_1, p_2, m^1, ..., m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i). \tag{7.2}$$

If all consumers are identical then

$$X_j(p_1, p_2, M) = nx_j^*(p_1, p_2, m), \tag{7.3}$$

where $M = nm$.

The market demand curve is the "horizontal sum" of the individual consumers' demand curves.

比如下面我们只考虑两个消费者A和B.

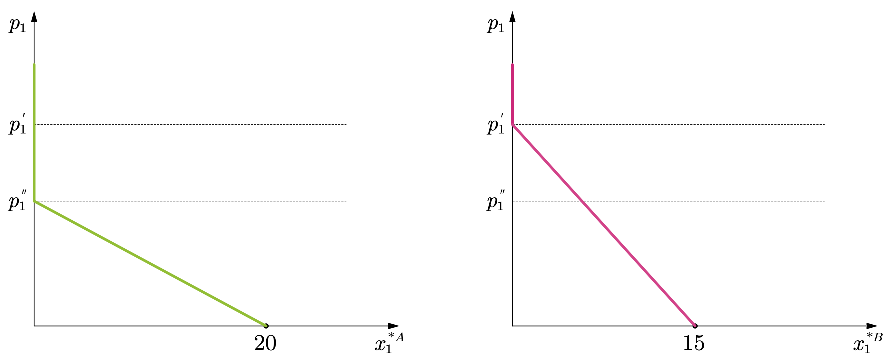


Figure 7.1: Individual Consumers' Demand Curves

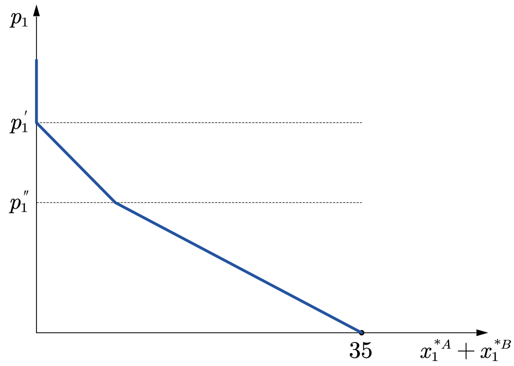


Figure 7.2: The "Horizontal Sum"

7.2. Elasticities

Elasticity measures the "sensitivity" of one variable with respect to another. (unit-free)

The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta X}{\%\Delta Y} = \frac{\Delta X / X}{\Delta Y / Y}. \tag{7.4}$$

own-price elasticity of demand quantity demanded of commodity i with respect to the price of commodity i.

$$\frac{\Delta q_i^D / q_i^D}{\Delta p_i / p_i}. \tag{7.5}$$

cross-price elasticity of demand demand for commodity i with respect to the price of commodity j.

$$\frac{\Delta q_i^D / q_i^D}{\Delta p_j / p_j}. \tag{7.6}$$

income elasticity of demand demand for commodity i with respect to income.

own-price elasticity of supply quantity supplied of commodity i with respect to the price of commodity i.

elasticity of supply with respect to the price of labor quantity supplied of commodity i with respect to the wage rate.

7.2.1. Arc and Point Elasticities

An "average" own-price elasticity of demand for commodity i over an interval of values for p_i is an arc elasticity, usually computed by a mid-point formula. As in Figure 7.3.

$$\varepsilon_{x_i^*, p_i} = \frac{\%\Delta x_i^*}{\%\Delta p_i} \tag{7.7.1}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'} \tag{7.7.2}$$

$$\%\Delta x_i^* = 100 \times \frac{x_i'' - x_i'''}{(x_i'' + x_i''')/2}. \tag{7.7.3}$$

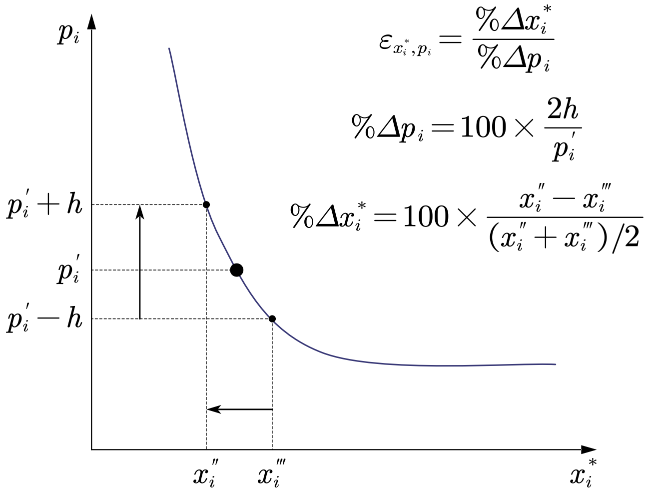


Figure 7.3: Arc Own-price Elasticity of Demand

Elasticity computed for a single value of p_i is a point elasticity.

$$\varepsilon_{x_i^*,p_i} = \frac{p_i'}{x_i'} \times \frac{\mathrm{d}x_i^*}{\mathrm{d}p_i} \tag{7.8}$$

is the elasticity at the point (x_i', p_i').

Suppose $p_i = a - bx_i^*$.

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{(a-p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a-p_i}. \tag{7.9}$$

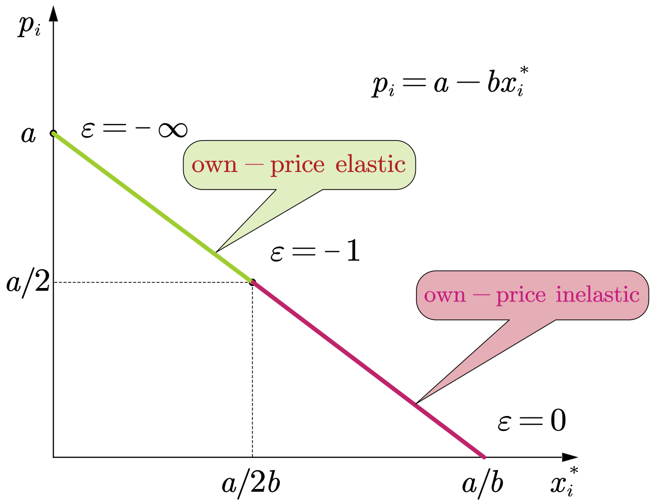


Figure 7.4: Point Own-Price Elasticity with $p_i = a - bx_i^*$

Suppose $x_i^* = kp_i^a$.

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{kp_i^a} \times kap_i^{a-1} = a. \tag{7.10}$$

7.2.2. Revenue and Own-Price Elasticity Demand

Sellers: prefer consumers' inelastic demand.

If raising a commodity's price causes little decrease in quantity demanded (inelastic), then sellers' revenues rise.

Sellers' revenue is: $R(p) = p \times X^*(p)$.

$$\frac{\mathrm{d}R}{\mathrm{d}p} = X^*(p) + p\frac{\mathrm{d}X^*}{\mathrm{d}p} = X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{\mathrm{d}X^*}{\mathrm{d}p}\right] = X^*(p)\left[1 + \varepsilon_{x^*,p}\right]. \tag{7.11}$$

接下来讨论$\varepsilon_{x^*,p}$和-1的大小即可.

7.3. Market Equilibrium

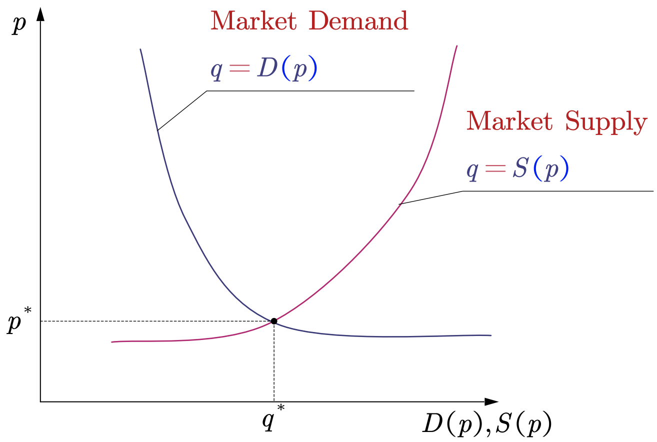


Figure 7.5: Market Equilibrium

Linear:

$$D(p) = a - bp \tag{7.12.1}$$

$$S(p) = c + dp \tag{7.12.2}$$

At the equilibrium price p^*, $D(p^*) = S(p^*)$. Thus,

$$p^* = \frac{a-c}{b+d} \tag{7.13.1}$$

$$q^* = \frac{ad+bc}{b+d} \tag{7.13.2}$$

Fixed Quantity of Supply:

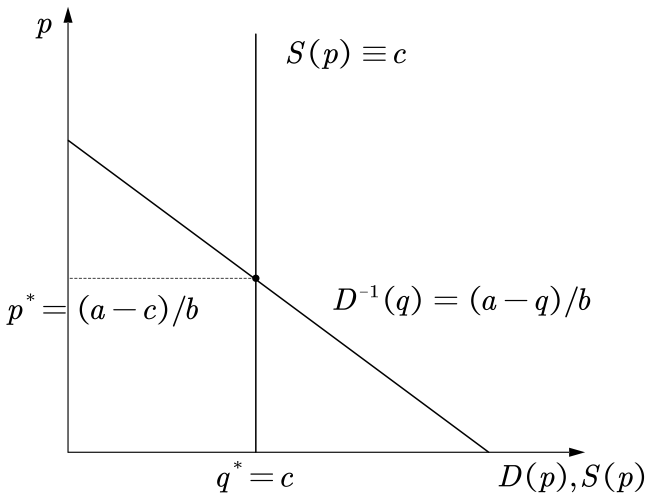


Figure 7.6: Fixed Quantity of Supply

e.g. Cigarette, Sports event tickets, Concert tickets, etc.

Market quantity supplied is extremely sensitive to price.

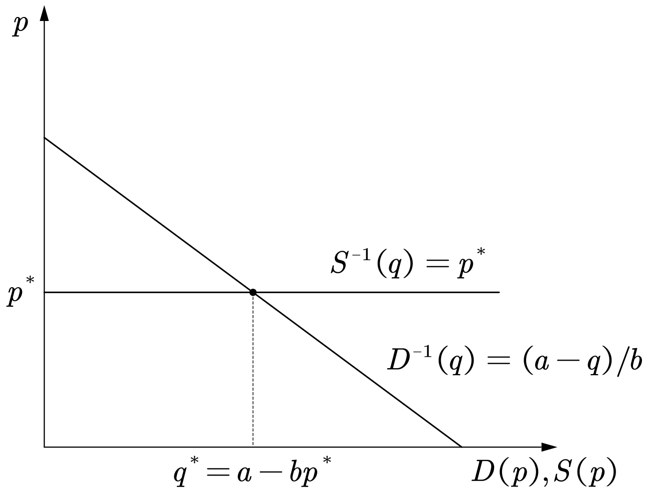


Figure 7.7: Supply Quantity Sensitive to Price

7.4. Quantity Taxes

A quantity tax levied at a rate of \$$t$ is a tax of \$$t$ paid on each unit traded (based on quantity traded).

If the tax is levied on sellers then it is an excise tax.

- Sellers will increase their prices to avoid paying the tax.

If the tax is levied on buyers then it is a sales tax.

A tax rate t makes the price paid by buyers, p_b, higher by t from the price received by sellers, p_s.

$$p_b = p_s + t. \tag{7.14}$$

> 提示 **7.4.1**
>
> The market must be clear.
>
> $$D(p_b) = S(p_s). \tag{7.15}$$

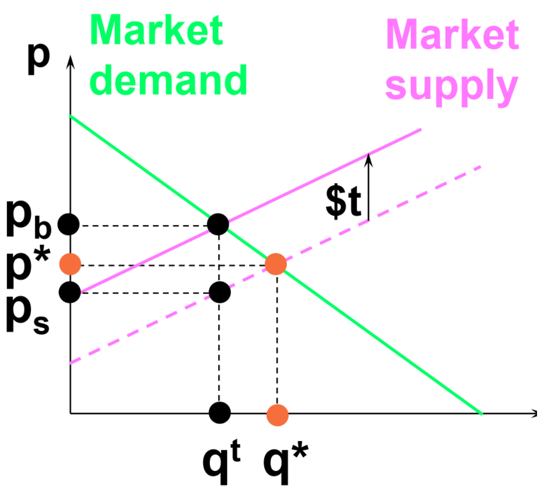


Figure 7.8: Excise Tax

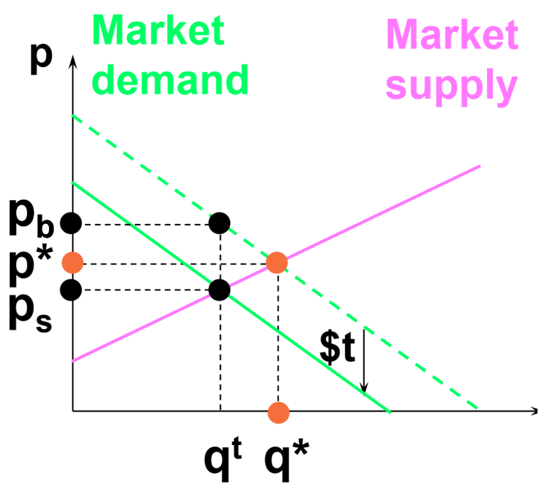


Figure 7.9: Sales Tax

- An excise tax raises the market supply curve by \$$t$.
  - And sellers receive only $p_s = p_b - t$.
- A sales tax lowers the market demand curve by \$$t$.
  - And buyers pay $p_b = p_s + t$.

The division of the \$$t$ between buyers and sellers is the incidence of the tax.

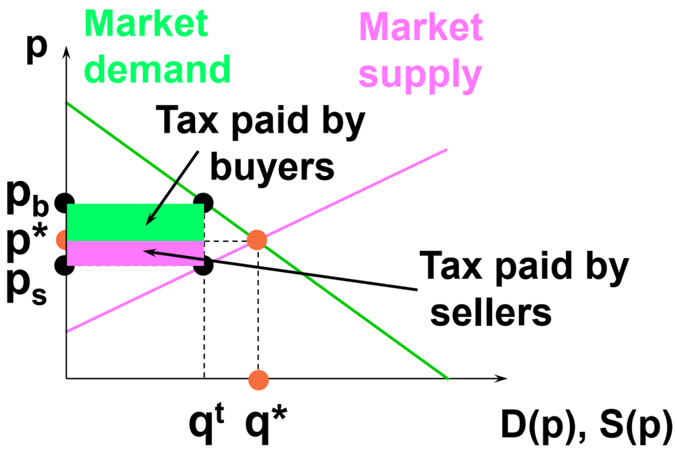


Figure 7.10: Tax Incidence

解 Tax Burden:

- 使用$p_b = p_s + t$和$D(p_b) = S(p_s)$, 解出p_b和p_s.
- 设p^*是没有税的时候平衡的价格.
- Tax paid by buyer is $p_b - p^*$.
- Tax paid by seller is $p^* - p_s$.

The incidence of a quantity tax depends upon the own-price **elasticities** of demand and supply.

$$\varepsilon_D \approx \frac{\Delta q / q^*}{(p_b - p^*)/p^*} \Rightarrow p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}. \tag{7.16}$$

$$\varepsilon_S \approx \frac{\Delta q / q^*}{(p^* - p_s)/p^*} \Rightarrow p^* - p_s \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}. \tag{7.17}$$

Tax incidence is $\dfrac{p_b - p^*}{p^* - p_s} \approx \dfrac{\varepsilon_S}{\varepsilon_D}$.

Deadweight loss: The cost of implementing a quantity tax.

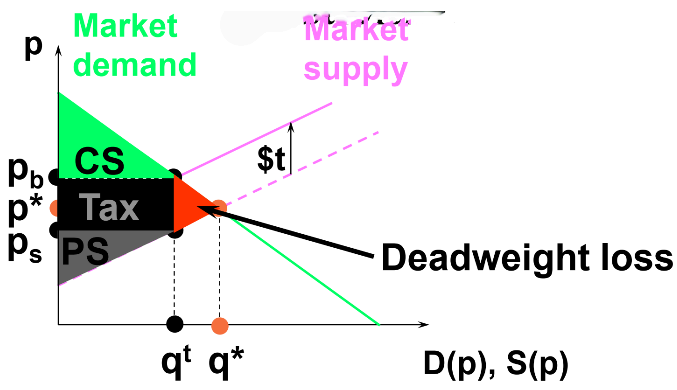


Figure 7.11: Deadweight Loss

Deadweight Loss 来自于贸易的减少量. 由于 inelastic, 贸易的减少量就比较小, 因此 Deadweight Loss 就比较小.

章节 8. Technology and Profit Maximization

Firms: Firms maximize profits.

8.1. Technologies

Technology A technology is a process by which inputs are converted to an output.

Input Bundle x_i denotes the amount used of input i, i.e., the level of input i. An input bundle is a vector of the input levels $(x_1, x_2, ..., x_n)$.

Production Function y denotes the output level. The technology's production function states the maximum amount of output possible from an input bundle.

$$y = f(x_1, x_2, ..., x_n). \tag{8.1}$$

Def 定义 8.1.1

A production plan is an input bundle and an output level $(x_1, x_2, ..., x_n, y)$.

A production plan is feasible if $y \leq f(x_1, x_2, ..., x_n)$.

Technology Set The collection of all feasible production plans is the technology set.

$$T = \{(x_1, x_2, ..., x_n, y) \mid y \leq f(x_1, x_2, ..., x_n) \text{ and } x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0\} \tag{8.2}$$

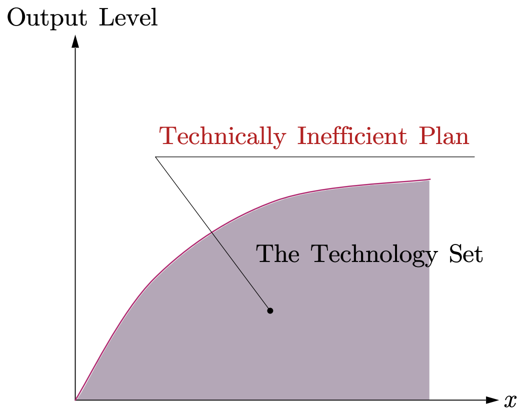


Figure 8.1: Technology Set

Isoquant The y output unit isoquant is the set of all input bundles that yield at most the same output level y.

8.1.1. Cobb-Douglas Technologies

A Cobb-Douglas production function is of the form

$$y = Ax_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}. \tag{8.3}$$

8.1.2. Fixed-Proportions Technologies

A fixed-proportions production function is of the form

$$y = \min\{a_1x_1, a_2x_2, ..., a_nx_n\}. \tag{8.4}$$

8.1.3. Perfect-Substitutes Technologies

A perfect-substitutes production function is of the form

$$y = a_1x_1 + a_2x_2 + \cdots + a_nx_n. \tag{8.5}$$

8.1.4. Marginal (Physical) Products

The marginal product of input i is the rate-of-change of the output level as the level of input i changes, holding all other input levels fixed.

$$\mathrm{MP}_i = \frac{\partial y}{\partial x_i}. \tag{8.6}$$

Diminishing The marginal product of input i is diminishing if it becomes smaller as the level of input i increases. That is, if

$$\frac{\partial \mathrm{MP}_i}{\partial x_i} = \frac{\partial^2 y}{\partial x_i^2} < 0. \tag{8.7}$$

8.1.5. Returns-to-Scale

- If, for any input bundle $(x_1, x_2, ..., x_n)$,

$$f(kx_1, kx_2, ..., kx_n) = kf(x_1, x_2, ..., x_n), \tag{8.8}$$

then the technology described by the production function f exhibits constant returns-to-scale.

- If, for any input bundle $(x_1, x_2, ..., x_n)$,

$$f(kx_1, kx_2, ..., kx_n) < kf(x_1, x_2, ..., x_n), \tag{8.9}$$

then the technology described by the production function f exhibits diminishing returns-to-scale.

- If, for any input bundle $(x_1, x_2, ..., x_n)$,

$$f(kx_1, kx_2, ..., kx_n) > kf(x_1, x_2, ..., x_n), \tag{8.10}$$

then the technology described by the production function f exhibits increasing returns-to-scale.

References

Varian, H.R. (2014) *Intermediate Microeconomics: A Modern Approach*. 9th ed. New York: W.W. Norton & Company.