CS 3210 Design and Analysis of Algorithms, Programming Assignment 1

Given a set S of n points specified by their coordinates (x_i, y_i). we want to compute the *Maximal Layers* M_i of S. The Maximal layer M_1 corresponds to the usual definition maximal points, i.e., a point $p \in S$ is in M_1, if it is *not dominated* by another point $q \in S$. M_2 is the maximal points among $S - M_1$ and so on. Note that $\sum_i |M_i| = n$ since every point belongs to a unique layer.

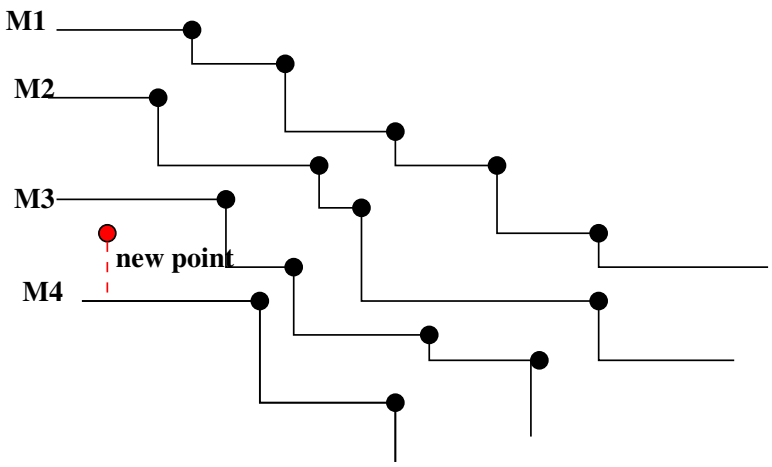
Describe and implement a line-sweep based method for this problem based on the line-sweep algorithms described in the class with the following modification. In a single sweep starting from the largest x coordinate, we will construct all the layers *simultaneously* instead of doing one at a time. If we compute M_{i+1} after M_i, it would be less efficient - for example, if there are $\sqrt{n}$ layers having $\sqrt{n}$ points in each layer, it will take us $O(\sqrt{n} \cdot n \log n)$ time. Note that, in the first $\sqrt{n}/2$ layers there will be at least $n/2$ points.

Instead, we will sweep all the layers simultaneously - for every new point, p_i, we will include it in the layer M_kj that it belongs (using some test) or start a new layer if it is below all the layers. This algorithm should run in $O(n \log n)$ time and output all the $k \leq n$ layers.

(i) Implement a data structure to represent the multiple staircase and to determine which staircase does a new point belongs to in $O(\log n)$ time.

Using a global counter, say T, count the total number of operations (comparisons, arithmetic) that the program executes and show a graph for different values of n. (ii) Using this data structure implement the line sweep and output all maximal layers.

(iii) Test your program on randomly chosen general points (avoid coincident points) within the positive orthant. You may be also provided some test cases for this.



Note: You can use C, C++ , Java to code but NOT Python