QUIZ 1

DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

(Maximum Marks: 40 Time allowed: ONE hour)

1. (a) State Well-Ordering Principle. (1 mark)
   (b) State the Principle of Mathematical Induction. (1 mark)
   (c) State the relationship between Well-Ordering Principle and Induction. (3 marks)
   (d) Using Well-Ordering Principle, show that, for positive integers a and b, $\exists$ integers q and r such that $a = qb + r$, where $0 \leq r \leq b-1$. (5 marks)

2. Give combinatorial proof for the following entities:
   (a) $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ (2 marks)
   (b) $\binom{n}{m}\binom{n-m}{k} = \binom{n}{k}\binom{n-k}{m}$ (2 marks)
   (c) $\sum_{j=0}^{m}\binom{m}{j}\binom{n}{k+j} = \binom{m+n}{m+k}$ (3 marks)
   (d) $\binom{n}{p}\binom{n}{q} = \sum_{k=0}^{n}\binom{n}{k}\binom{n-k}{p-k}\binom{n-p}{q-k}$ (3 marks)

3. (a) Use generating function technique to derive a closed form formula for the n^{th} CATALAN number. (4 marks)
   (b) Simplify $\sum_{k=0}^{n}\frac{1}{k+1}\binom{n}{k}$ using generating functions technique. (2 marks)
   (c) Show that any set of 10 two-digit numbers has two non empty disjoint sets with sums of their elements equal. (4 marks)

4. (a) Count the number of paths in XY-plane from (0,0) to (m,n) where m and n are positive integers. Here a path consists of series of steps where in each step you move one unit to the right or move one unit upward. (No moves to the left or down is allowed.) (3 marks)
   (b) How many solutions are there to the equation
   $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ where $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$ (2 marks)
   and to the inequality
   $x_1 + x_2 + x_3 \leq 11$ where x_i's are positive. (2 marks)
   (c) How many strings with 7 or more characters can be formed from the letters in EVERGREEN. (3 marks)