CS 2100: Discrete Mathematics for Computer Science

Tutorial 3

Instructor - John Augustine

November 15, 2012

- Your name and roll number must be clearly written on the first page of your answer booklet. It should also be written on the first page of each additional booklet.
- Write spaciously and legibly.
- The tutorial starts at 12PM and ends at 12:50PM, but I suspect you will be done sooner than that.

Let $(G, \circ)$ be a permutation group defined on a set S.

- Recall that the binary relation R on S induced by the permutation group G is the relation such that $\forall a, b \in S$, aRb iff there is a permutation in G that maps a to b.
- Recall that R is an equivalence relation.
- Recall that the number of elements that are invariant under a permutation π is the number of elements that are mapped onto themselves in π.
- Recall Burnside's theorem that states: The number of equivalence classes into which a set S is divided by the equivalence relation induced by a permutation group G of S is given by

$$\frac{1}{|G|} \sum_{\pi \in G} \Psi(\pi),$$

where $\Psi(\pi)$ is the number of elements that are invariant under the permutation π.

1. A bracelet is a circular arrangement of beads — the beads are placed uniformly around the circle. Consider bracelets comprising of five beads where each bead can be one of three colors (red, blue, and green). Two bracelets are not distinguishable if we can rotate one bracelet and match the colors of both bracelets. Otherwise they are distinct. Use Burnside's theorem to count the number of distinct bracelets.

2. You want to write all five digit numbers on slips of papers. This will require 10^5 slips of paper. Knowing that numbers 0, 1, 6, 8, 9, when seen upside down, read 0, 1, 9, 8, 6, you want to reuse the slips such that the number of slips is minimized but all five digit numbers are represented. How many slips must you write?