CS 2100: Discrete Mathematics for Computer Science

Quiz 1

Instructor - John Augustine

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Notes

- Your name and roll number must be clearly written on the first page of your answer booklet. It should also be written on the first page of each additional booklet.
- Write spaciously and legibly.
- There are <u>six</u> questions. Each question is worth 10 points. However, they are all NOT of the same difficulty.
- The quiz starts at 8AM and ends at 8:50AM.

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Problem Definition

1. **(10 pts)** State the name of the law applied at each numbered step below.

$$
\begin{aligned}
&(p \wedge \neg q) \vee (\neg p \wedge \neg q) \\
&\qquad \equiv (\neg q \vee p) \wedge (\neg q \vee \neg p) && (1) \\
&\qquad \equiv \neg q \vee (p \wedge \neg p) && (2) \\
&\qquad \equiv \neg q \vee \mathbb{F} && (3) \\
&\qquad \equiv \neg q. && (4)
\end{aligned}
$$

2. **(10 pts)** Each of the following arguments are either valid or invalid. For each of valid arguments, state the rule of inference that guarantees its validity. For each of the invalid arguments, state the type of error.

   a. `Madhu knows karate and Madhu knows swimming.`
      `Therefore, Madhu knows karate.`

   b. `If the network is stable, DNS will work correctly.`
      `The network is stable.`
      `Therefore, DNS will work correctly.`

   c. `If the program is correct, there will be no compiler error.`
      `There were no compiler errors.`
      `Therefore, the program is correct.`

d. If the proof is correct, every case must be addressed.
Every case is not addressed.
Therefore, the proof is not correct.

e. If you can design an efficient algorithm for integer factoring,
you can write an efficient program to factor
the product of two primes.
If you can factor the product of two primes,
you can break an RSA encryption.
Therefore, if you can design an efficient algorithm for integer factoring,
you can break an RSA encryption.

3. (10 pts) Indicate which of the following statements are true and which are false.

a. $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, x = y + 1$.

b. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x = y + 1$.

c. $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+$ such that $xy = 1$.

d. $\forall x \in \mathbb{Z}^+$ and $\forall y \in \mathbb{Z}^+, \exists z \in \mathbb{Z}^+$ such that $z = x - y$.

e. $\forall x \in \mathbb{Z}$ and $\forall y \in \mathbb{Z}, \exists z \in \mathbb{Z}$ such that $z = x - y$.

4. (10 pts) What is wrong with the following proof.

"Theorem:" For all integers $n \geq 1$, $3^n - 2$ is even.

"Proof:" Assume that the theorem is true for an integer k, where $k \geq 1$. That is, suppose that $3^k - 2$ is even. We must show that $3^{k+1} - 2$ is even. To this end, note that

$3^{k+1} - 2 = 3^k \cdot 3 - 2$
$3^k(1+2) - 2$
$3^k - 2$

$$\begin{aligned} 3^{k+1} - 23^k \cdot 3 - 2 & \\ &= 3^k(1+2) - 2 \\ &= (3^k - 2) + 3^k \cdot 2. \end{aligned}$$

Now $3^k - 2$ is even by the inductive hypothesis and $3^k \cdot 2$ is even by inspection. Hence, the sum of the two quantities is even[1]. It follows that $(3^k - 2) + 3^k \cdot 2$ is even implying that $3^{k+1} - 2$ is even. $\square$

5. (10 pts) This question has two parts.

a. Briefly describe Russell's paradox.

b. Suppose U is a universal set and we require every other set to be a subset of U. Do we still encounter Russell's paradox under this setting? Explain.

$x = 2$

1, 2, 3, 4, 5
6, 7, 8, 9, 10

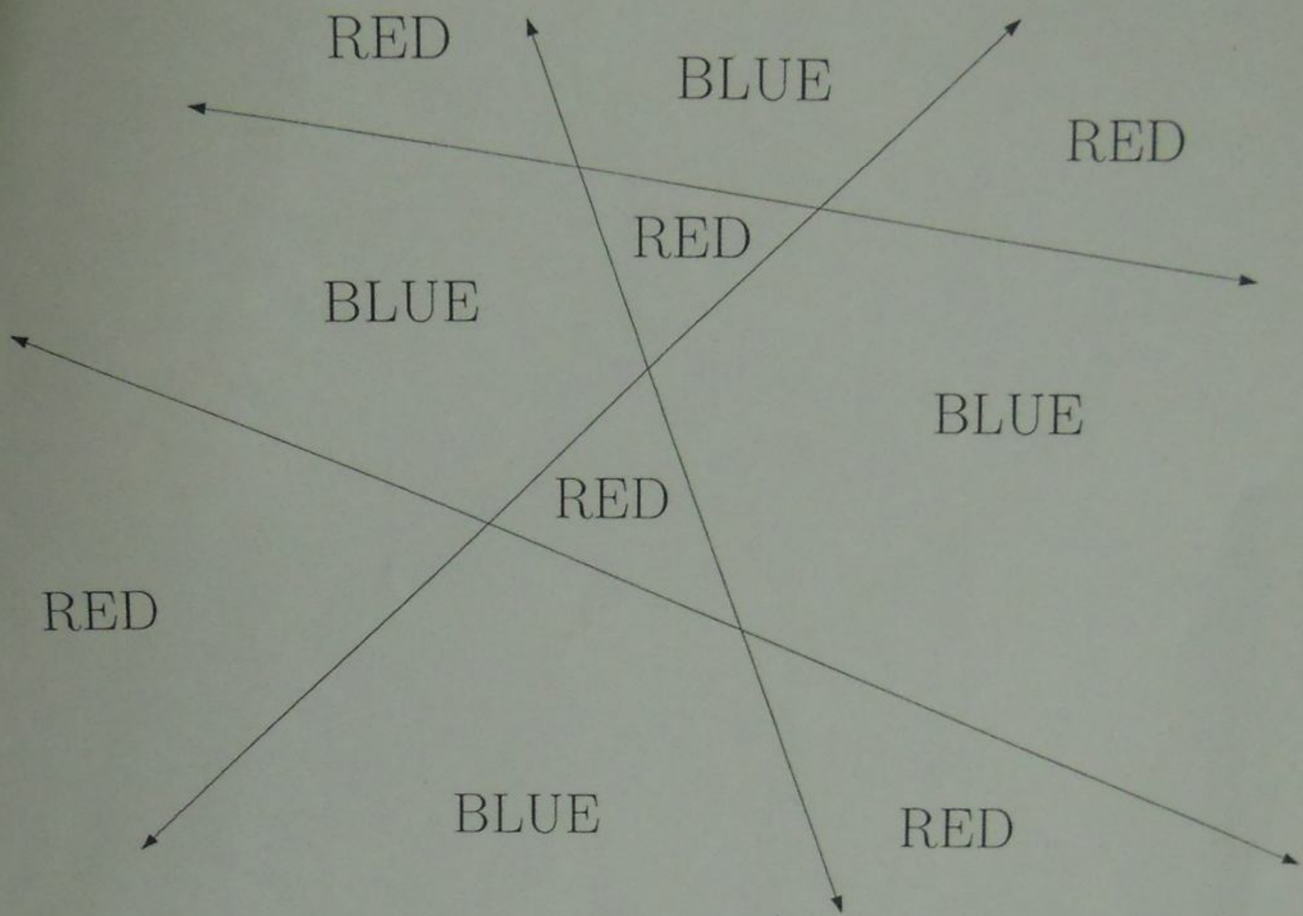
$3 = \cancel{5}\,8$

... $3^3 - 2$
25

[1] The sum of two even integers is even. For our purposes, you may take this for granted.

6. **(10 pts)** Consider a set of n lines[2] (also known as an arrangement of lines) on a 2D plane as shown below. The lines will subdivide the 2D space into regions or faces. The faces are said to be coloured if we assign a colour to each face such that no two faces that share a line segment boundary have the same colour. The figure below shows such a colouring that moreover only requires two colour.

Show that we can always colour the faces induced by an arrangement of lines with two colours.



[2]You may assume for simplicity that no two lines are parallel and no three lines pass through the same point.