CS2400 : Assignment 1

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Introduction

In this assignment, we generate and sample two sinusoidal signals, and observe their properties in the frequency domain using the DFT. In this process, we study the properties and intricacies associated with the DFT.

Sinusoidal signals are real signals with two impulses in the frequency domain. The parameters include:

- A = The amplitue, the peak deviation of the function from zero
- f = the ordinary frequency, cycles per second
- $\omega = 2\pi f$, the angular frequency, the rate of change of the function argument in units of radians per second
- φ = the pase, specifies (in radians) where in its cycle the oscillation is at $t = 0$

Problem 1

Part a

In the sampling process, we take samples at regular intervals of length $\frac{1}{f_s}$ in the time domain where f_s is the sampling frequency.

$x[i] = A\sin(2\pi f\frac{i}{f_s})$ where i varies from 1 to N (Total number of samples)

Plots

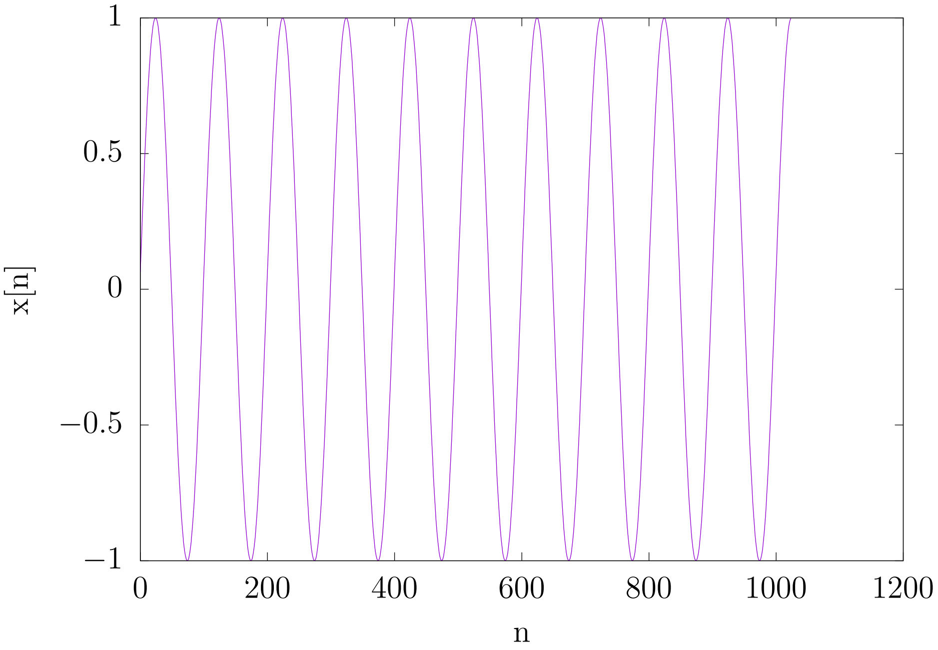

Figure 1: $x_1[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1, f = 500\text{Hz}, f_s = 50000\text{Hz}$

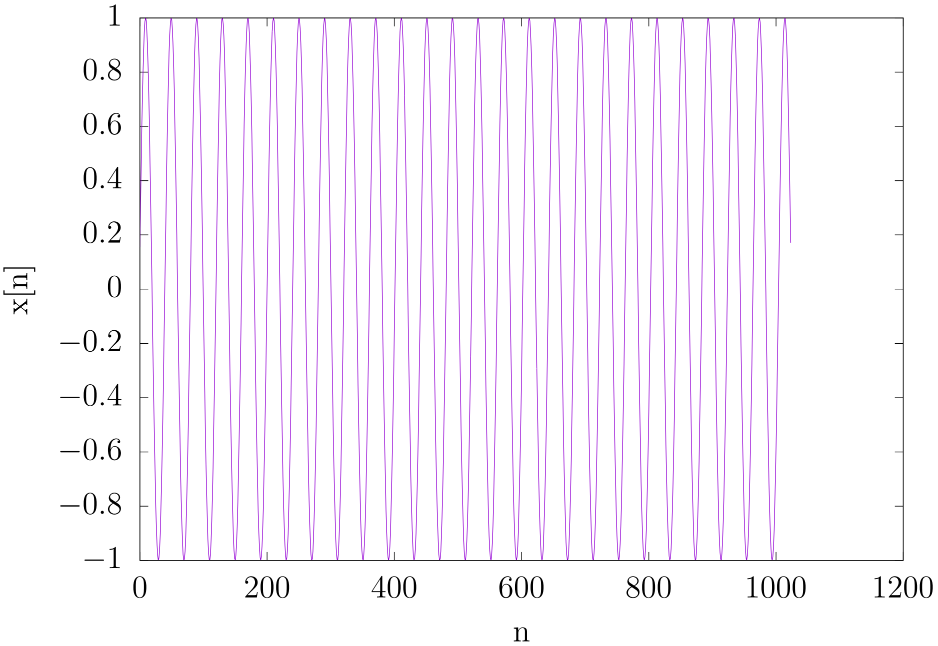

Figure 2: $x_2[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1.2, f = 500\text{Hz}, f_s = 20100\text{Hz}$

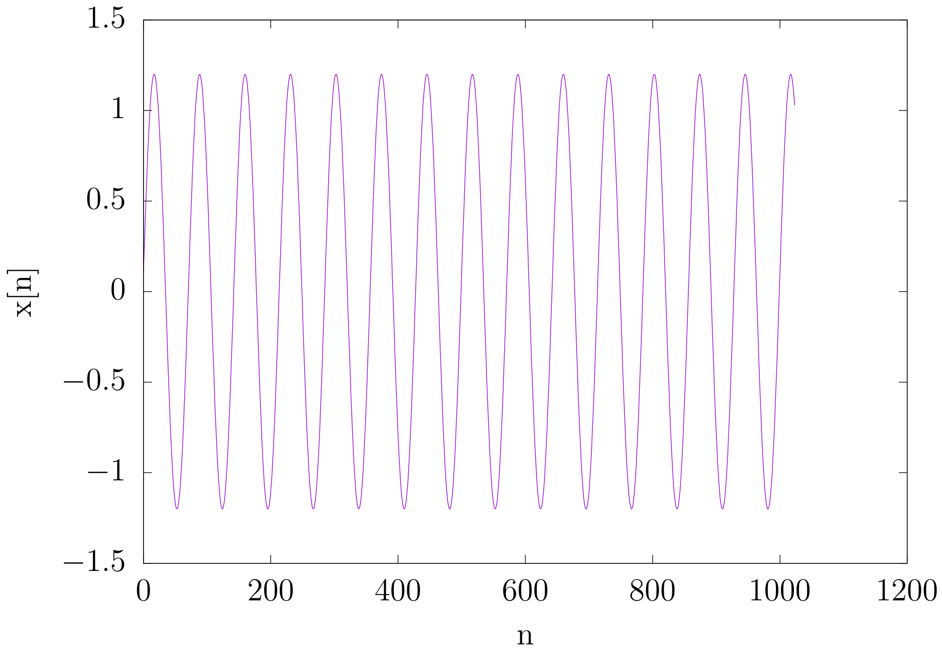

Figure 3: $x_3[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1.2, f = 700\text{Hz}, f_s = 50000\text{Hz}$

Observations

A sampled signal is observed. We observe that if we increase the sampling rate, the graph resembles more closely to the original signal. But at the same time as the data recorded/number of samples are constant, if we increase the sampling rate, the total duration of the sampled signal in the time domain decreases. Hence decreasing the total number of cycles observed.

We exploit the property that a sine signal has two zero crossings in a cycle. We first calculate the total number of zero crossings in our sampled signal. We do this by considering a zero crossing whenever the signal changes sign (i.e. $x[i] \cdot x[i+1] < 0$) or when the signal is zero (i.e. $x[i] = 0$).

2 zero crossings $\implies$ Total cycles is 1.
Hence Z zero crossings $\implies$ Total cycles is $\frac{Z}{2}$
Time length of the sampled signal $T = \frac{N}{f_s}$
So, frequency obtained experimentally is $\frac{\text{Total Cycles}}{\text{Time Length}}$, $f_{exp} = \frac{Zf_s}{2N}$
Frequency error $\epsilon = |f - f_{exp}|$

Plots

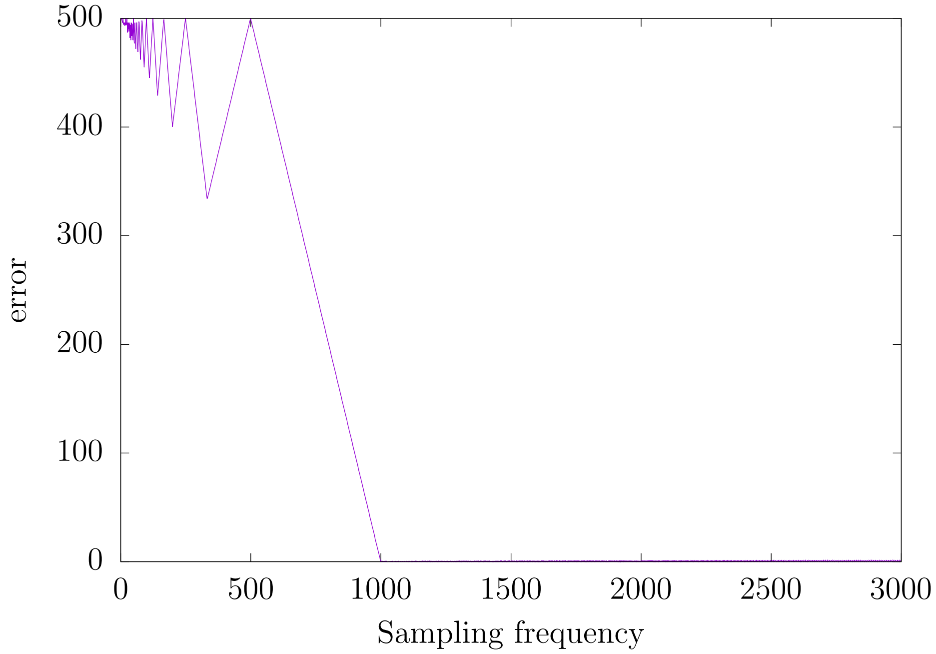

Figure 4: $\epsilon = |f - f_s|$ for $f = 500$ Hz

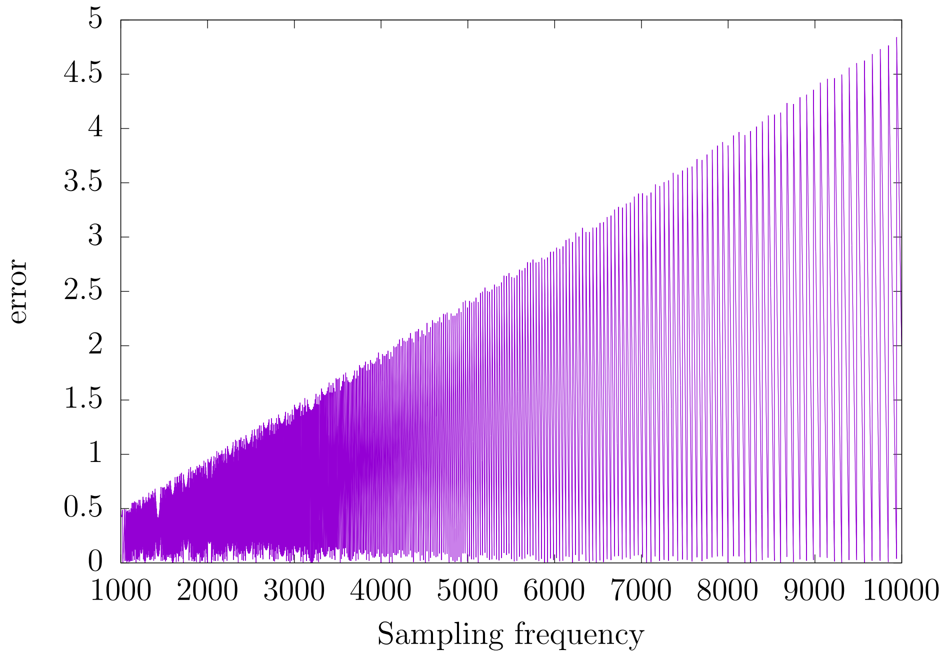

Figure 5: $\epsilon = |f - f_s|$ for $f = 500$ Hz

Observations

- The error is maximum if sampling rate is from 1 to $2f$. This is because the the number of zero crossings in one cycle is two. Hence the sampled signal should take one value in each half cycle to get accurate number of zero crossings. This implies time period is at most half, hence $f_s \geq 2f$
- Error increases with increase in sampling frequency after $f_s >> 2f$. The error is minute still, happens as the total time length of the signal is reducing, implying our sample space has reduced and hence accuracy of results.

Part b

Plots

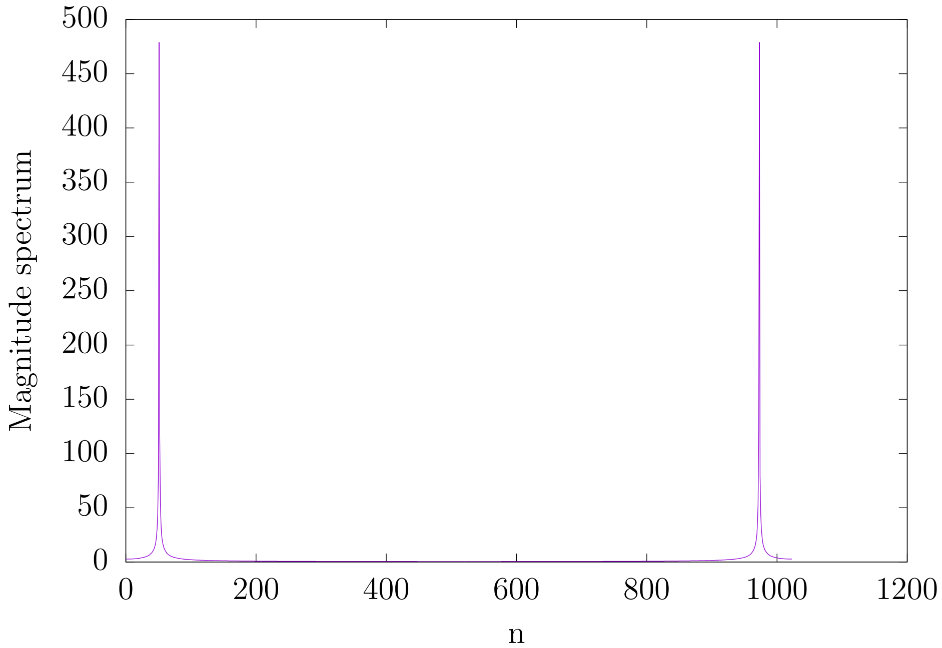

Figure 6: Magnitude spectrum of $x[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1, f = 500\text{Hz}, f_s = 10000\text{Hz}$

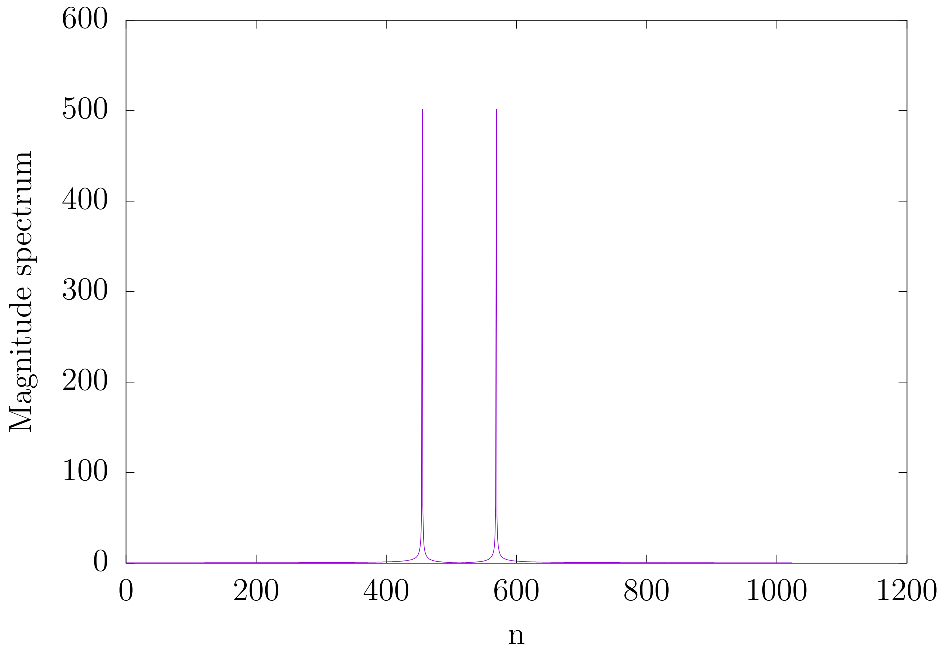

Figure 7: Magnitude spectrum of $x[n] = A\sin(2\pi f \frac{n}{f_s})A = 1, f = 500\text{Hz}, f_s = 900\text{Hz}$

Observations

- The maximum in the FFT magnitude spectrum is observed at two indices. One os the complex conjugate of the other. The index at which the first max occurs is $n = N\frac{f}{f_s}$. The second index is $N - n$.
- Frequency Spectrum is erroneous when $f_s < 2f$. (Due to Sampling Theorem).
- For figure 6
  $R = 10000$ Hz
  Expirmental index at which max occurs $= 50$
  $f_{exp} = \frac{51}{1024} \cdot 10000 = 498$ Hz
- For figure 7
  $R = 900$ Hz
  Expirmental index at which max occurs $= 456$
  $f_{exp} = \frac{456}{1024} \cdot 900 = 401$ Hz (Error huge because $f_s < 2f$).

Problem 2

Part a

Addition of the signals occurs at the same sampling rate. Each sample is added individually.

Plots

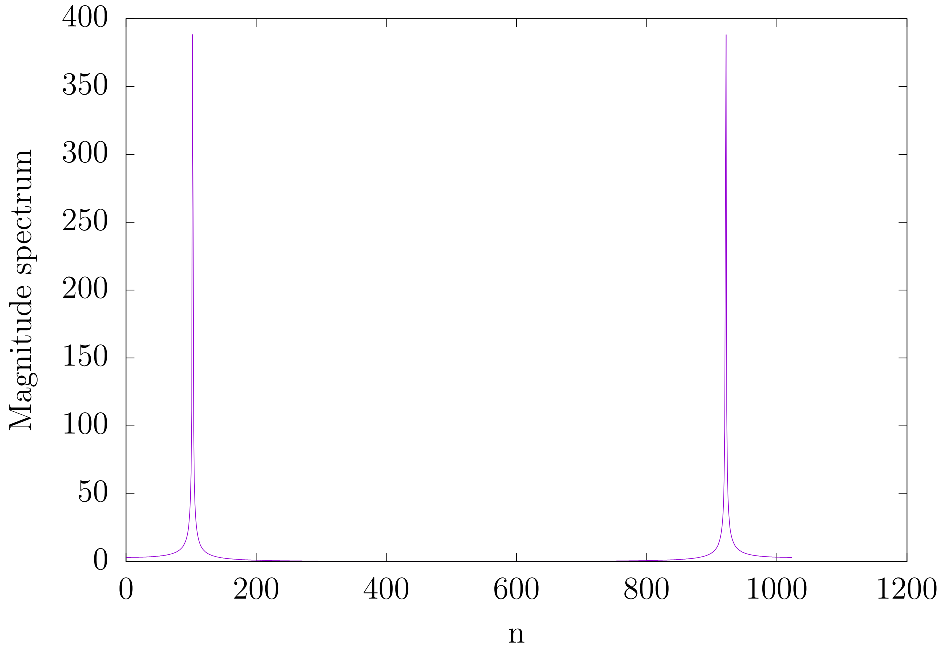

Figure 8: Magnitude spectrum of $x_1[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1, f = 500\text{Hz}, f_s = 5000\text{Hz}$

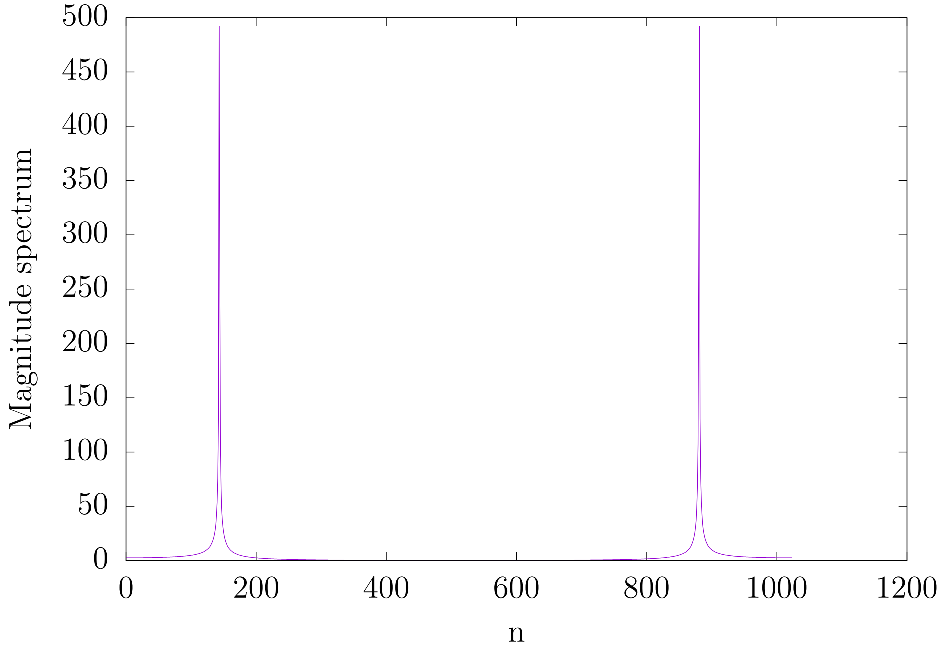

Figure 9: Magnitude spectrum of $x_2[n] = A\sin(2\pi f \frac{n}{f_s})A = 1.2, f = 700\text{Hz}, f_s = 5000\text{Hz}$

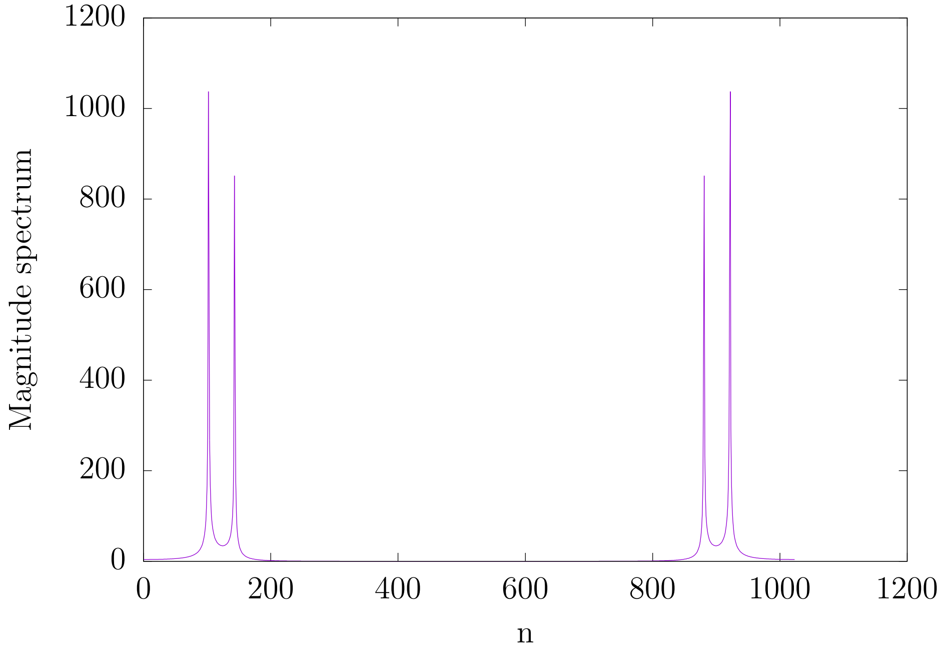

Figure 10: Magnitude spectrum of $x_1[n] + x_2[n]$

Observations

The resulting DFT is the sum of the DFTs of the individual signals.

$$\mathcal{F}(x[n] + h[n]) = \mathcal{F}(x[n]) + \mathcal{F}(h[n]) \tag{1}$$

Part b

$$y[n] = x[n] * h[n]$$
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Plots

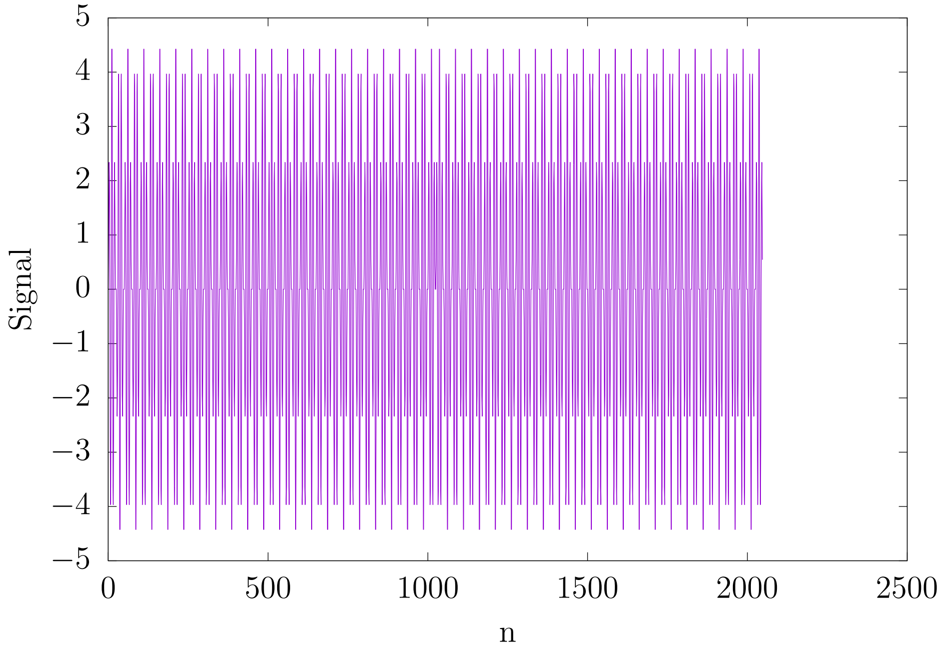

Figure 11: $x_1[n] * x_2[n]$

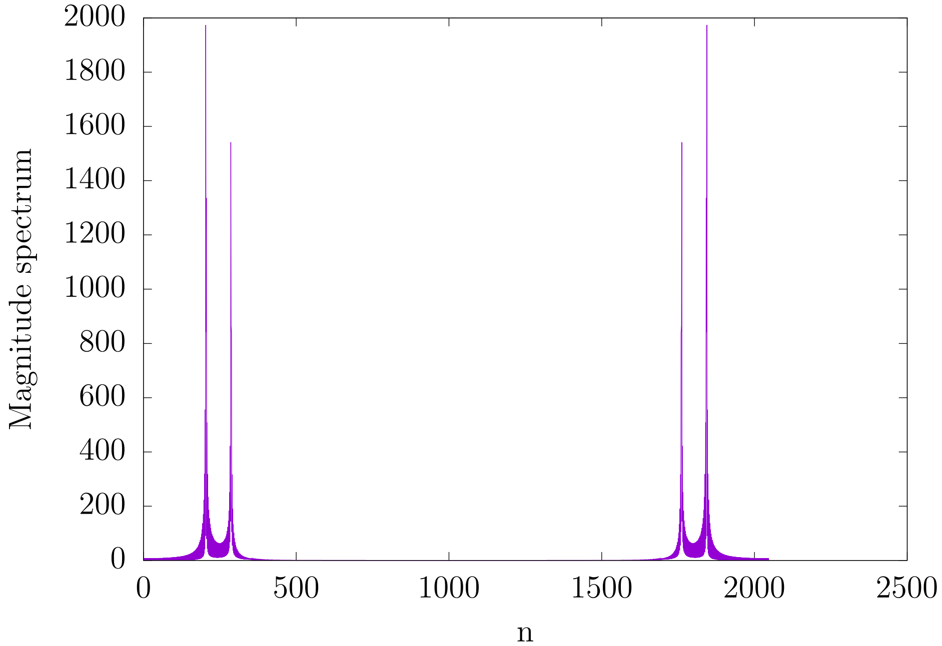

Figure 12: Magnitude spectrum of $x_1[n] * x_2[n]$

Observations

Fourier transform of the convolved signal is the multiplication of the Fourier transforms of the individual signals scaled down by a factor of 2π.

$$\mathcal{F}(x[n] * h[n]) = \frac{1}{2\pi}\mathcal{F}(x[n]) \cdot \mathcal{F}(h[n]) \tag{2}$$

Part c

Modulation Property

$$\mathcal{F}(x[n]) = X[k]$$
$$\mathcal{F}(x[n]e^{\frac{2\pi j}{N}nm}) = X[k-m]$$

Plots

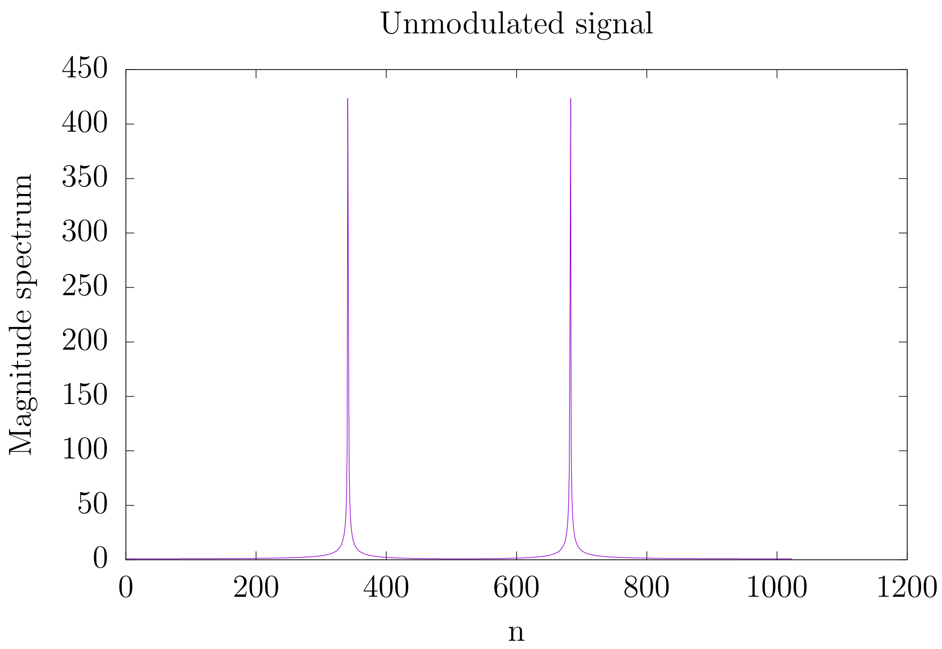

Figure 13: Magnitude spectrum of $x[n] = A\sin(2\pi f \frac{n}{f_s}) A = 1, f = 500\text{Hz}, f_s = 10000\text{Hz}$

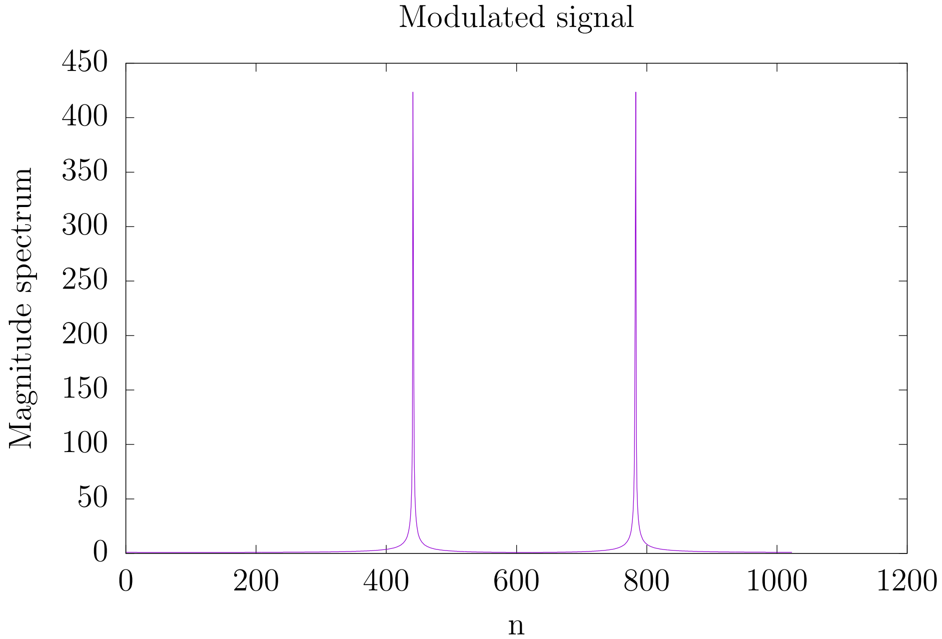

Figure 14: Magnitude spectrum of $x[n] = A\sin(2\pi f \frac{n}{f_s})e^{\frac{2\pi j}{N}nm} A = 1, f = 500\text{Hz}, f_s = 10000\text{Hz}, m = 100$

Observations

The DFT of the modulated signal was the DFT of the original signal shifted by $m = 100$ units.

Time shift property

$$\mathcal{F}(x[n]) = X[k]$$
$$\mathcal{F}(x[(n-m)_N]) = X[k] \cdot e^{-\frac{2\pi j}{N}km}$$

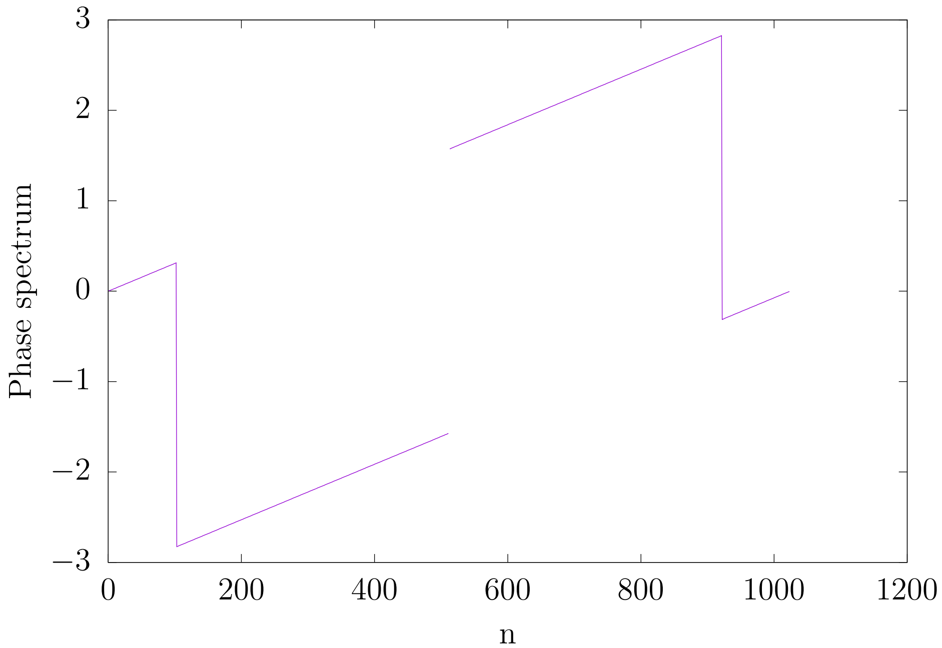

Figure 15: Phase spectrum of $x[n] = A\sin(2\pi f \frac{n}{f_s})A = 1, f = 500\text{Hz}, f_s = 10000\text{Hz}$

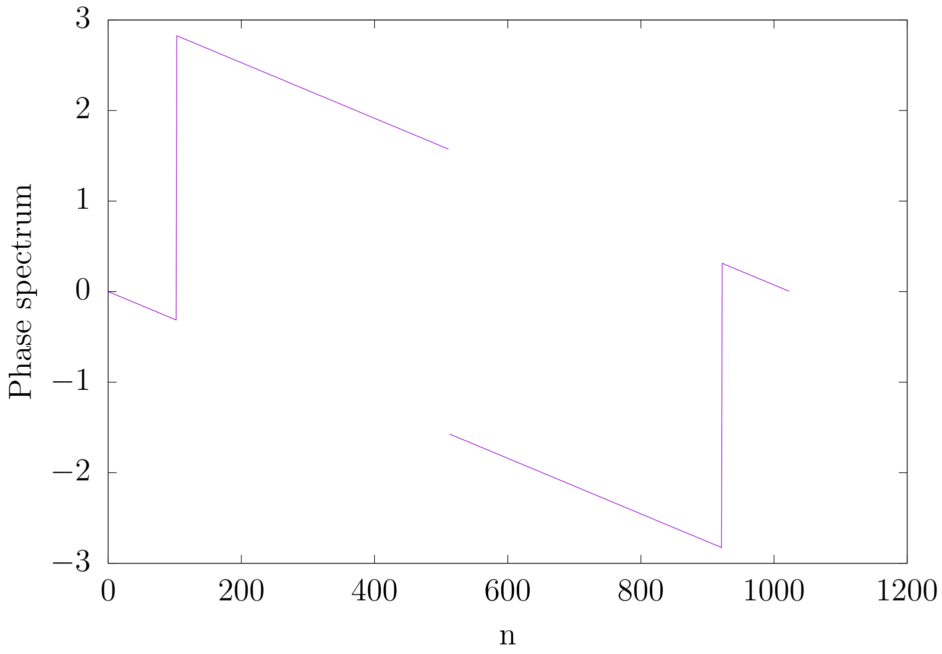

Figure 16: Phase spectrum of $x[(n-m)_N]$ with $m = 1$

Observations

The phase spectrum of the DFT of the time shifted signal was the phase spectrum of the original signal shifted appropriately.