12 Photometry

The brightness of stars has, along with their locations, been studied by astronomers since ancient times. Prior to the 1860's observers necessarily estimated brightness using their eyes expressing the result in the *magnitude system* that Ptolemy introduced in the second century.

12.1 A short history

Modern instruments show that early measurements such as those made by Ptolemy and Tycho Brahe (who were both more interested in the position of objects) have an internal precision of about 0.5^{m}. Even a very skilled observer can do little better; al Sufi in the ninth century spent great effort on this problem and achieved a precision of some 0.4^{m}. With a telescope several observers, such as the Herschels, were able to produce results of $0.1-0.3^{\mathrm{m}}$ by using comparisons with known *sequences* of standard brightness stars.

Francois Arago suggested an optical/mechanical system by which adjusts the brightness of a comparison star until it matches the unknown star or dims the telescopic brightness of a star until it disappears. These systems are called *visual photometers*. Between 1879 and 1902 Harvard visual photometrists had measured the magnitudes of some 47 000 stars with a precision of about 0.08^{m} and an accuracy of better than 0.25^{m}. At this time in history (1900) the Pogson normal scale

$$\Delta m = -2.5 \log(b_1/b_2)$$

where b_1 and b_2 are the brightness of objectes 1 and 2 was standard amongst all astronomers.

1. The faintest stars visible to the naked eye, from the definition of the scale, are magnitude six. For point sources the brightness is increased by the use of a telescope by a factor G, called the light grasp. If the dark adapted human eye has a diameter of 7 mm, show that

   $$G \approx 2 \times 10^4 d^2$$

   where d is the telscope diameter in meters. Show further that the limiting magnitude through a visually used telescope, m_{L} is

   $$m_{\mathrm{L}} = 16.8 + 5 \log d.$$

In the same period photography progressed and astronomers were able to record the light of stars too faint to be seen by eye in any telscope. An international collaboration, the *Carte du Ciel* project, was started with the goal of photographing the entire sky and measuring the brightness of every star below 11.0^{m}. However, photographic plates do not measure light linearly and it took several years, until 1900–1910, before a reliable *photographic magnitude* system was established. The introduction of physical photometers in the period 1910–1920 to objectively measure images on photographic plates eventually led to magnitudes being measured with uncertainties in the range $0.015 - 0.03^{\mathrm{m}}$.

Experiments with photoelectric work began in the early part of the 20th century and in the 1930's with the introduction of vacuum-tube amplifiers detection limits on a 0.5 m telescope improved from 11.0^{m} to 13.0^{m}. Photomultiplier tubes, introduced during World War II, improved the situation greatly and quickly became the instrument of choice for measuring brightness, with uncertainties of 0.005^{m} in relative brightness. During the 1950 period to 1980 the RCA 1P21 photomultipliers were used by Harold Johnson to define the UBV system, which later was extended by the use of red sensitive photomultipliers into the infrared.

At present CCDs and other modern solid-state detectors have mostly superseded photomulitipliers. In the optical CCDs have superior efficiency, better stability, and a large multiplex advantage.

From space, for example the Kepler mission for detecting occultations by extrasolar planets, achieves uncertainties below 10 μm over time scales of several weeks.

12.2 The response function

A photometric device is sensitive over a restricted range of wavelengths called a bandpass. There are three types of bandpass photometry in astronomy.

12.2.1 Types of photometry

Single-band photometry For applications such as finding planets via occultations where one is only interested in measuring the fraction of light from the star blocked by a planet one needs only a single band. In this case one would generally want to construct a sequence of observations into a time series, *i.e.* a tabulation of brightness as a function of time, choosing a wide band to maximize signal and minimize the required exposure time and telescope size. An example of this is the Super Wasp telescope arrays placed on La Palma and in South Africa.

Broadband multi-color photometry Broadband multi-color photometry measures a very low resolution spectrum by sampling the brightness in several different bands. Broad band in this sense means that the spectroscopic resolving power $R = \lambda_{\mathrm{c}}/\Delta\lambda < 10-15$. These system attempt to choose bands that admit the maximum amount of light while still providing astrophysical information. The most typical example of such a system is the optical $UBVRI$ system which uses bandwidths in the range $65-160$ nm ($R = 4-7$). This system can provide information on surface temperature, as well as (more limited) information on luminosity, metal content, and interstellar reddening for a wide variety of stars.

Each band in a system such as this is is known as a *color*, so "two-color photometry" measures magnitudes in two separate bands. Usually one reports the results of n-color photometric measurements by giving one magnitude and $(n-1)$ color indices. The magnitude tells the apparent brightness, and the indices tell about other astrophysical variables such as the surface temperature. Color can also have a second meaning: the difference between two magnitudes. For example, the results of two-color photometry in B and V will be reported as a V magnitude and *one* $(B-V)$ color.

Narrow and intermediate-band photometry The intent of narrow band photometry ($R > 50$) is usually to isolate a specific line, molecular band, or other spectral feature. Common applications include the measurement of the strength of absorption features like Balmer-α or sodium D, or the ratio of the intensities of emission lines in gaseous nebulae. Intermediate-band photometry ($15 < R < 50$) measures spectroscopic features that cannot be resolved by broad bands but avoids the severe light loss of narrow-band photometry. Examples of such features include discontinuities in spectra, such as the Balmer discontinuity at 364.6 nm or very broad absorption features due to blended lines or molecular bands such as the band due to TiO in the spectra of M stars extending from 705 to 730 nm.

12.3 Magnitudes

We can write the *apparent magnitude* of the source as

$$m_{\mathrm{P}} = -2.5 \log (F_{\mathrm{P}}) + C_{\mathrm{P}} = -2.5 \log \int_0^\infty R_{\mathrm{P}}(\lambda) f_\lambda d\lambda + C_{\mathrm{P}}.$$

Where m_{P} is the bandpass magnitude, F_{P} is the energy flux (irradiance) within the band, f_λ is the monochromatic flux. The constant C_{P} is chosen to conform to some standard scale (*e.g.* the magnitude of Vega is zero in the visual system). The function $R_{\mathrm{P}}(\lambda)$ is called the *response function* of the entire observing system to the incident flux, it is the fraction of the energy of wavelength λ that will register on the photometer.

Note that photometers count photons and therefore do not measure the energy directly. Thus we write the *monochromatic photon flux*

$$\phi(\lambda) = \frac{\lambda}{hc} f_\lambda$$

and the quantity measured by by photon detectors is the *photon flux within the band*

$$\Phi_{\mathrm{P}} = \int_0^\infty R_{\mathrm{PP}}(\lambda)\phi(\lambda) d\lambda = \frac{1}{hc} \int_0^\infty R_{\mathrm{P}} f_\lambda d\lambda$$

where R_{PP} is the *photon response*: the fraction of photons of wavelength λ detected by the system. It is also possible possible to define the *monochromatic magnitude* defined from the monochromatic flux:

$$m_\lambda = -2.5 \log (f_\lambda) + C'(\lambda) = -2.5 \log \frac{hc\phi(\lambda)}{\lambda} + C'(\lambda)$$

$C'(\lambda)$ is arbitrary, and is often chosen so that the monochromatic magnitude of Vega or som other standard is a constant at every wavelength. In which case $C'(\lambda)$ is a strong function of λ. On the other hand $C'(\lambda)$ can also be chosen as a constant function and at the monochromatic magnitude reflects the spectrum in energy units.

12.3.1 Response function implementation

Both practical limits and intentional controls can determine the functional form of the response functions R_{P} and R_{PP}. The *sensitivity of the detector* limits the wavelength accessible. In some cases detector response alone sets the bandpass, in other cases the detector response defines only one edge of a given band.

A *filter* is the usual method for intentionally delimiting the band by blocking all wavelengths except for those in a specific range. Filters can also serve as *high-pass* or *low-pass* elements to by only defining the lower or upper cutoff of a band.

It is also possible to use a dispersing element to create a spectrum and then sampling discrete segments of the spectrum with one or more photometers. Such instruments are called *spectrophotometers*. These generally define bandpasses by using apertures, slots, or detectors of the proper size to select the desired segment of the spectrum.

For ground based observations *atmospheric transmission*, $S_{\mathrm{atm}}(\lambda)$, limits the wavelength that are accessible, and may define all or parts of a response function.

2. Find a plot of the Earth's atmospheric transmission as a function of wavelength λ.

Normally magnitudes are defined outside the Earth's atmosphere, and astronomers must remove atmospheric effects during data reduction.

3. Give reasons as to why it is better to define the response function $R(\lambda)$ *outside* the Earth's atmosphere. What are the advantages and disadvantages of doing this?

12.3.2 Response function description

There are a whole host of terms used to describe the response function $R(\lambda)$.

There is a single maximum value $R_{\max}$ which occurs at the *peak wavelength* λpeak. There are also two half-maximum points, often taken as specification of where the transmission band begin and ends, λ_{low} and λ_{high}

$$R(\lambda_{\mathrm{peak}}) = R_{\max}$$
$$R(\lambda_{\mathrm{low}}) = R(\lambda_{\mathrm{high}}) = R_{\max}/2$$

Given the maxima, the width of the response can be characterized by the *full width half maximum*

$$\mathrm{FWHM} = \lambda_{\mathrm{high}} - \lambda_{\mathrm{low}},$$

which in turn determines the *central wavelength* of the band

$$\lambda_{\rm cen} = (\lambda_{\rm low} + \lambda_{\rm high})/2$$

A perhaps more useful measure of the width of $R(\lambda)$ is provided by the *bandwidth*

$$W_0 = \frac{1}{R_{\max}} \int R(\lambda) d\lambda$$

which in turn suggest the definition of the *mean wavelength*

$$\lambda_0 = \frac{\int \lambda R(\lambda) d\lambda}{\int R(\lambda) d\lambda}$$

For a symmetric $R(\lambda)$ we have

$$\lambda_{\rm peak} = \lambda_{\rm cen} = \lambda_0$$

Quite informative is the *effective wavelength* of the response $R(\lambda)$ to a particular source. This is the weighted mean wavelength and indicates which photons influence a particular measurement:

$$\lambda_0 = \frac{\int f_\lambda \lambda R(\lambda) d\lambda}{\int f_\lambda R(\lambda) d\lambda}$$

A bandpass measurement is nearly equivalent to a measurement of the monochromatic flux at the wavelength $\lambda_{\rm eff}$ multiplied by the bandwidth W_0. Which is nearly correct in practice, and for broadband photometry of stars with sufficiently smoothed spectra using this equivalence only gives errors on the order of a few percent or less. But, to be strictly accurate with such an equivalence, another definition must be made of the middle of the band: the *isophotal wavelength* given by

$$W_0 f_{\rm iph} = \frac{1}{R_{\max}} \int f_\lambda R(\lambda) d\lambda$$

As for the effective wavelength the exact value of the isophotal wavelength will depend on the spectrum of the source.

12.3.3 Color indices

Multi band photometry can measure the shape of an object's spectrum. It is convenient to think of the bands as sampling the monochromatic flux of a smoothed spectrum at their isophotal wavelength. Figure 1 shows several blackbodies whose temperatures range from 1600 K to 16 000 K. The vertical scale of the figure shows the monochromatic magnitude in a system in which the constant C is set to be a constant independent of temperature. Remember that this is not usually the case in astronomical photometry, where the spectrum of some standard object (*e.g.* Vega, which is similar to a blackbody of temperature 9500 K) would be a horizontal line in a plot of m_λ as a function of λ.

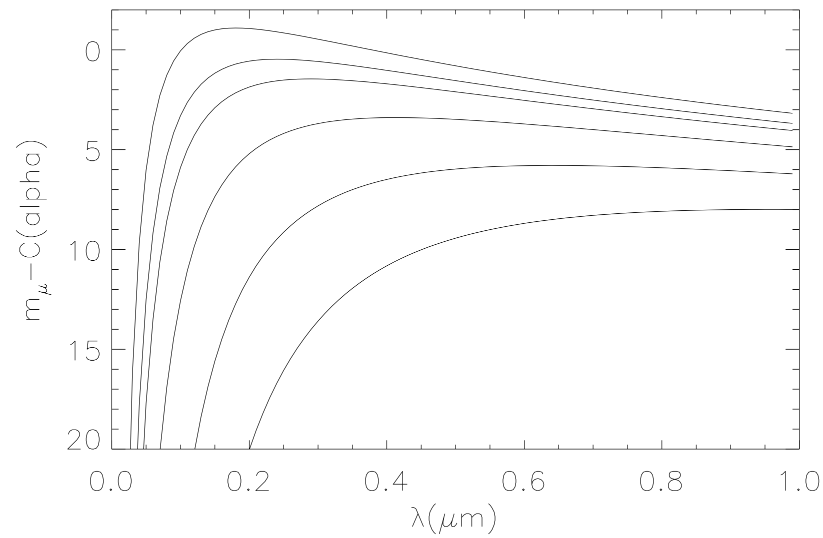

Figure 1: Blackbody curves for stars of temperature between 1600 K to 16 000 K. $C(alpha)$ is set arbitrarily to $C(alpha) = -15$.

4. Reproduce figure 1 using *e.g* IDL.

For two (broad) bands centered at 0.4 μm and at 0.8 μm is clear that the arithmetical difference between these two magnitudes for a particular spectrum depends on the average slope of the spectrum, which in turn depends on the source's temperature. The convention is to speak of the difference between any two bandpass magnitudes used to sample the slope of the spectrum as a *color index*. By convention one computes the index in the sense

$$\text{index} = m(\text{shorter}\lambda) - m(\text{longer}\lambda)$$

The behavior of the color index at the long and short wavelength extremes of the Planck function are informative. In the Rayleigh-Jeans region (where $\lambda kT \gg hc$) we have

$$m_\lambda = \log T + C(\lambda)$$

so the color index is

$$(m_{\lambda_1} - m_{\lambda_2}) = C(\lambda_1) - C(\lambda_2) = \Delta C \tag{1}$$

which is a constant independent of temperature. At short wavelengths the *Wien approximation* applies and the surface brightness can be given by

$$B(\lambda, T) \approx \frac{2hc^2}{\lambda^5} \exp\left(-\frac{hc}{\lambda kT}\right).$$

The color index is then

$$(m_{\lambda_1} - m_{\lambda_2}) = \frac{a}{T}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) + C(\lambda_1) - C(\lambda_2) \tag{2}$$

Thus, at very low temperature or short wavelengths the index is a linear function of $1/T$.

5. Show that equations 1 and 2 are correct.

12.3.4 Line and feature index

Real objects such as stars will of course have more complex spectra than blackbodies with features of astrophysical significance such as absorption and emission lines, bands, and various discontinuities. Multi-band photometry can measure the strength of such features.

Two bands are often sufficient to measure the size of a discontinuity or the strength of a line.

The positioning of bands is important. The sensitivity of the index to the size of the break will diminish if either the bandpass response includes light from the opposite side of the break. Likewise, if a band is located too far away from the break, unrelated features can affect the index.

An alternative way of characterizing a line is to use two bands — one broad, one narrow — both centered on the line in question. While the broad band is relatively insensitive, the narrow band is quite sensitive. The index

$$\text{line index} = m_{\text{narrow}} - m_{\text{wide}}$$

tracks the strength of the absorption, in the sense that it becomes more positive with stronger absorption. One widely used index of this sort is the β index, which measures the strength of the Balmer beta line of hydrogen, usually useful for luminosity or temperature classification of stars.

Finally, note that three bands can be used to measure the *curvature* or second derivative of a spectrum. Curvature can arise on relatively short scale because of sharp absorption or emission lines, or on long scales because of broad or diffuse features such as molecular bands. A curvature index is defined by the differences

$$\text{curvature} = (m_S - m_C) - (m_C - m_L)$$

where S, C and L indicate the short, central, and long wavelength bands respectively.

12.4 Photometric systems

Photometric systems are defined by at least two specifications:

1. The wavelength response of each band — that is, the shape of $R_{\text{P}}(\lambda)$.

2. Some method for standardizing measurements made in those bands.

- Each observer needs to know the value of the constant C that will assure agreement of his magnitudes with those of all other observers.
- The different hardware produces some variety in the response functions in practice, so a method for standardization must allow correction of the inevitable systematic effects due to imperfect matching.

The fist specification, $R_{\mathrm{P}}(\lambda)$, determines the *instrumental* or *natural system*. The first and second together determine the *standard system*.

Almost all standard systems rely on some network of constant-brightness standard objects distributed around the sky. It is important to define a set of standards that include a wide variety of spectral types.

A *closed photometric system* is one in which a small group of observers carefully controls the instruments and data reduction, maximizing the internal consistency. Examples include the space borne HIPPARCHOS data and the Sloane Digital Sky Survey. An *open photometric system* is one in which all astronomers are encouraged to duplicate the defined natural system as best they can, and through references to a published list of standard stars add to the pool of observations in the system.

12.4.1 Common photometric systems

Visual and photographic systems The dark-adapted human eye determines the band of the *visual photometric system*. The introduction of optical/mechanical visual photometers led to the establishment of *standard sequences* of stars, including initially the *north polar sequence* and later many secondary sequences: amongst them importantly the 48 Harvard standard regions and the 115 Kapteyn selected areas.

In the early twentieth century, astronomers defined two bands based on the properties of photographic emulsion. The poor properties of emulsion as a photometric detector, and lack of very specific definitions, limited the success of this system. The *international photographic system* is sensitive in the near ultraviolet–blue region. The response of the *international photographic photovisual band* roughly corresponds to that of the visual band (*i.e.* the human eye, sensitive to green–yellow). The IAU in 1922 set the zero point of both magnitudes so that the 6^{m} magnitude A0 V stars in the north polar sequence would have (roughly) the same values as on the old Harvard visual system.

The $UBVRI$ system The most widely used photometric system prior to the present has been the Johnson-Cousins $UBVRI$ system. This system was originally based on the RCA 1P21 photomultiplier, a set of colored glass filters, and a list of magnitudes for a relatively small number of stars scattered on the celestial sphere. The V band is very close to the international photovisual band and its zero point was set so that $V = m\mathrm{pv}$ for standards in the north polar sequence. The U and B correspond to the short- and long-wavelength bands of the photographic band, and their zero points are set so that the colors $U - B$ and $B - V$ are zero for A0 V stars.

In the period 1960–1965 the system was extended to include bands in the red R_J and near infrared I_J, as well as the longer wavelength bands ($JHKLMNQ$) discussed below. Modern work with CCDs has tended to replace the original R_J and I_J with the R_C and I_C bands.

This multi band system was designed with the rough spectral classification of stars in mind. The $U-B$ index is sensitive to the Balmeer discontinuity (very obvious in A stars at 370 nm, much reduced for G stars). The discontinuity depends on luminosity for hot stars. The other indices are primarily sensitive to temperature. The $B-V$ index is more sensitive to metal abundance than $V-R$ or $R-I$. The $V-I$ index is the most purely temperature sensitive index in this system.

The Broadband infrared system: $JHKLMNQ$ This broadband system might be regarded as an extension of the $UBVRI$ system, it shares a common zero point so that the colors of an unreddened A0 V star are zero. Detectors in this region cannot be silicon CCDs but must be infrared arrays or single-channel infrared devices.

A large complication for these bands for ground based observations bandpass definitions can depend critically on atmospheric conditions (due to water vapor along the line of sight). Different observatories with identical hardware can experience different infrared window sizes and shapes if they are at different altitudes. The same observatory can experience similar bandpass variations due to changing humidity.

The IAU in 2000 recommended a preferred natural system — the Mauna Kea Observatory near-infrared system (MKO).

The intermediate band Strömgren system: $uvby\beta$ Bengt Strömgren designed this intermediate-band system in the late 1950s. The system avoids many of the shortcomings of the UBV system and aims to classify stars according to three characteristics: temperature, luminosity, and metal abundance. This works well for stars of spectral types B, A, F, and G provided the photometry is sufficiently accurate. The four intermediate band colors *uvby* are supplemented with a narrow band β index which tracks the strength of the Balmer beta line. This greatly improves the luminosity classification for hotter stars, and is a good temperature indicator for cooler stars.

Emission in the u and v bands is depressed by the presence of metals in a star's atmosphere. Also the u band is depressed by the Balmer discontinuity.

12.5 From source to telescope

At least four different effects can alter the photons on their way to the telescope:

- wavelength shifts
- extragalactic absorption
- Galactic and Solar System absorption

- atmospheric absorption

12.5.1 Wavelength shifts

The photons that leave the source is written $\phi_{\rm E}(\lambda_{\rm E} d\lambda_{\rm E}$, where the subscript 'E' stands for "emitted".

Because of the Doppler effect, or because of the expansion of the Universe, or because of other relativistic effects, the wavelength of the observed photon will differ from its original value. The new value is given by

$$\lambda_o = (1+z)\lambda_{\rm E}$$

where z is the redshift parameter $z = (\lambda_o - \lambda_{\rm E})/\lambda_{\rm E}$ of the source. The number of photons is conserved in these processes so

$$\phi(\lambda) d\lambda = \phi_{\rm E}(\lambda_{\rm E} d\lambda_{\rm E}$$

Thus, the observed and emitted monochromatic photon flux are related by

$$\phi(\lambda) = \frac{1}{1+z}\phi_{\rm E}\left(\frac{\lambda}{1+z}\right) = f_\lambda \frac{\lambda}{hc}$$

and the monochromatic flux density is then

$$f_{\rm E}(\lambda_{\rm E}) = f_{\rm E}\left(\frac{\lambda}{1+z}\right) = \frac{hc\phi_{\rm E}(\lambda_{\rm E})}{\lambda_{\rm E}} = (1+z)^2 f_\lambda$$

Note that the monochromatic flux density is *not* conserved.

An observer who uses a bandpass with photon response $R(\lambda)$ will measure the magnitude

$$\begin{aligned} m_{\rm R} &= -2.5 \log \int R(\lambda)\frac{\phi(\lambda)}{\lambda} d\lambda + C_{\rm R} \\ C_{\rm R} &= -2.5 \log \int R(\lambda \frac{g_\lambda}{hc} d\lambda \end{aligned}$$

where g_λ is the spectrum of a photometric standard magnitude zero. We need to find how $m_{\rm R}$ relates to a magnitude measured for these same photons before their wavelength shift. Call the unshifted band the photons began their journey in Q, different from R.

$$\begin{aligned} m_{\rm Q} &= -2.5 \log \int Q(\lambda)\frac{\phi_{\rm E}(\lambda)}{\lambda} d\lambda + C_{\rm Q} \\ &= -2.5 \log \left[(1+z) \int Q(\lambda)\frac{\phi(\lambda(1+z))}{\lambda} d\lambda\right] + C_{\rm Q} \\ C_{\rm Q} &= -2.5 \log \int Q(\lambda \frac{g_\lambda}{hc} d\lambda \end{aligned}$$

We must consider the difference

$$m_{\mathrm{R}} - m_{\mathrm{Q}} = 2.5\log(1+z) + C_{\mathrm{R}} - C_{\mathrm{Q}} + 2.5\log\left[\frac{\int Q(\lambda)\frac{\phi(\lambda(1+z))}{\lambda}d\lambda}{\int R(\lambda)\frac{\phi(\lambda)}{\lambda}d\lambda}\right]$$

For objects in our galaxy, z is small, and one can use $R(\lambda) = Q(\lambda)$, $C_{\mathrm{R}} = C_{\mathrm{Q}}$, in which case the first three terms add up to zero. The last term describes the effect of photons shifting into and out of the band. In the case of narrow bands near sharp spectral features, even small Doppler shifts can produce large differences between $\phi((1+z)\lambda)$ and $\phi(\lambda)$.

For distant objects z becomes large because of the expansion of the Universe. Given knowledge of $\phi(\lambda)$ and z it is possible to use an observed bandpass magnitude to compute the magnitude that would be observed if the source had a redshift $z = 0$. Hubble called this kind of correction the *K correction*.

12.5.2 Absorption outside the atmosphere

Interstellar gas and dust absorb and scatter light. It is common to refer to both processes as "absorption". Absorption not only reduces the number of photons arriving at the telescope, *extinction*, but also alters the shape of the spectrum.

Diffuse gas absorbs photons to produce *interstellar absorption lines and bands*. In the optical sodium D is usually the strongest interstellar line, in the UV the Lyman-alpha line is usually strongest. At short wavelengths gas will also produce continuous absorption and absorption edges due to ionization *e.g.* at 91.2 nm due to the Lyman continuum. Absorption by dust will generally alter the overall shape of the spectrum. In the region $0.22 - 5.0\ \mu$m, dust scatters short wavelength photons more than long wavelength photons, so the resulting change in shape of the spectrum is called *interstellar reddening*.

Now define $S_{\mathrm{ism}}(\lambda)$ as the fraction of photons of wavelength λ that are transmitted by the interstellar medium within our galaxy and $S_{\mathrm{exg}}(\lambda)$ as the fraction of photons arriving at λ that are transmitted by the interstellar medium outside our galaxy. Note that because of cosmological redshift, absorption described by $S_{\mathrm{exg}}(\lambda)$ involve photons that had wavelength $\lambda/(1+z')$ when they were absorbed by material with redshift parameter z'. (This produces the phenomenon of the *Lyman-alpha forest* in the spectra of distant objects: multiple absorption lines due to Ly α at multiple red-shifts.) The photon flux that reaches the top of the Earth's atmosphere is

$$\phi(\lambda) = S_{\mathrm{ism}}(\lambda)S_{\mathrm{exg}}(\lambda)\phi_0(\lambda) = S_{\mathrm{ism}}(\lambda)S_{\mathrm{exg}}(\lambda)\frac{1}{1+z}\phi_{\mathrm{E}}((1+z)\lambda) = f_\lambda\frac{\lambda}{hc}$$

where $\phi(\lambda)$ is the photon flux outside the atmosphere and $\phi_0(\lambda)$ is the photon flux outside the atmosphere corrected for interstellar absorption.

12.5.3 Absorption by the atmosphere

Extinction in the Earth's atmosphere is a strong function of wavelength. At sea level, three opaque regions define two transmitting windows. Rayleigh scattering and absorption by atoms and molecules cause a complete loss of transparency at all wavelengths shorter than about 300 nm. This sets the short end of the *optical infrared window*. The second opaque region, from absorption in molecular bands — primarily due to H_2O and CO_2 — begins at roughly 0.94 μm, has a few breaks in the infrared and mid infrared, and extends from 30 mm to the start of the *microwave radio window* at around 0.6 cm. The radio window ends at around 20 m because of ionospheric absorption and reflection.

Qualitatively we can set up an atmospheric transmission function $S_{\rm atm}(\lambda, t, e, a)$ as the fraction of photons of wavelength λ that are transmitted by the Earth's atmosphere at time t, elevation angle e and azimuth a. The photon flux that reaches the telescope is then

$$\phi_{\rm A}(\lambda) = S_{\rm atm}(\lambda, t, e, a) S_{\rm ism}(\lambda) S_{\rm exg}(\lambda) \frac{1}{1+z} \phi_{\rm E}((1+z)\lambda) = f_\lambda^{\rm A} \frac{\lambda}{hc}$$

and the rate at which energy gets detected in an infinitesimal band is

$$dE_{\rm sig} = a T'_{\rm P}(\lambda) f_\lambda^{\rm A} d\lambda = a T'_{\rm P}(\lambda) \frac{\phi_{\rm A}(\lambda)}{\lambda} d\lambda$$

Where a here is the effective collecting area of the telescope, and $T'_{\rm P}(\lambda)$ is a function of the overall wavelength dependent efficiency of the instrument. Integrating this equation gives us the *instrumental magnitude*

$$\begin{aligned} m_{\rm P}^{\rm A} &= -2.5 \log \int T'_{\rm P} S_{atm}(\lambda) \frac{\phi(\lambda)}{\lambda} d\lambda + C'_{\rm P} \\ &= m_{\rm P}^{\rm O} + A_{\rm atm} + A_{\rm ism} + A_{\rm exg} + C_{\rm P}^{\rm z} \\ &= m_{\rm P} + A_{\rm atm} \end{aligned}$$

Here the A parameters represent the atmospheric, Galactic, and extragalactic absorption, in magnitudes; $C_{\rm P}^{\rm z}$ is the correction for wavelength shift; and $C'_{\rm P}$ is the constant that sets the zero point of the instrumental magnitude scale. The quantity $m_{\rm P}$, the *instrumental magnitude outside the atmosphere*, depends on the telescope and photometer but is independent of the atmosphere. The quantity $m_{\rm P}^{\rm O}$ is the instrumental magnitude in the emitted frame corrected for all absorption effects. $m_{\rm P}$ can be written as

$$m_{\rm P} = -2.5 \log \int T_{\rm P} \frac{\phi(\lambda)}{\lambda} d\lambda + C_{\rm P}$$

where $T_{\rm P}(\lambda)$ and $C_{\rm P}$ characterize the instrumental system located outside the atmosphere.