University of Oslo

AST3310 - Astrophysical plasma and stellar interiors
Project 1

Modeling the solar core

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Abstract

I here discuss the properties of the interior of the Sun based on models and simplifications addressed in [1] and [2]. Numerical calculations are used to solve the governing equations of the interior of the Sun. Using the simplifications, one should expect the temperature to end at ~ 15 MK. It is found that the results heavily depends on initial properties, and in particular the mass step ∂m.

1 Introduction

2 Theory

The equations that regulate the physical properties of the interior of the Sun are expressed as

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$
$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \tag{2}$$
$$\frac{\partial L}{\partial m} = \varepsilon \tag{3}$$
$$\frac{\partial T}{\partial m} = -\frac{3\kappa L}{256\pi^2 \sigma r^4 T^3}. \tag{4}$$

The variable ε in (3) is the total energy generation per unit mass. It is found by looking at the energy generation from nuclear reactions. It depends on the abundancy of different elements, temperature and density. The variable κ is the opacity, which is an average of frequency of photons. The pressure in eq. (2) is a sum of the gas pressure P_{G}, and the radiative pressure P_{R}.

$$P = P_{\mathrm{G}} + P_{\mathrm{R}}$$
$$P = \frac{\rho}{\mu m_{\mathrm{u}}} kT + \frac{a}{3} T^4$$
$$\Rightarrow \rho = \frac{\mu m_{\mathrm{u}}}{kT}\left(P - \frac{a}{3}T^4\right), \tag{5}$$

which yields the density derived from the equation of state. Here, μ is the average molecular weight, m_{u} is the atomic mass unit and k is the Boltzmann constant. The constant a is defined as $a = 4\sigma/c$, where σ is the Stefan-Boltzmann constant, and c is the speed of light.

The total energy generation per unit mass ε, is found by

$$\varepsilon = \sum Q'_{ik} r_{ik}, \tag{6}$$

where i, k represents two elements, Q'_{ik} is the energy released from the fusion of two elements, r_{ik} is the reaction rates per unit mass for two elements. The energies Q'_{ik} from the pp chains are listed in [1, p. 39, Table 2.1]. The reaction rates per unit mass is defined by

$$r_{ik} = \frac{n_i n_k}{\rho(1+\delta_{ik})}\lambda_{ik}, \tag{7}$$

	Unit system			
	CGS		SI	
Physical property	Unit name	Unit abbr.	Unit name	Unit abbr.
Length	centimetre	cm	metre	m
Weight	gram	g	kilogram	kg
Time	second	s	second	s
Temperature	kelvin	K	kelvin	K
Energy	erg	erg	joule	J
Pressure	barye	Ba	pascal	Pa

Table 1: Overview of the difference between CGS units and SI units.

where n_i, n_k is the number density for an element, δ_{ik} is the Kronecker delta and λ_{ik} is the reaction rate of a fusion. The number density of an element is easily defined as

$$n = \frac{\rho \chi_a}{a m_\mathrm{u}}, \tag{8}$$

where χ is the number fraction of an element, and a is the atomic number of the element. We denote X, Y, Z to be the number fractions of hydrogen, helium and heavier metals, respectively. Finally, the reaction rates λ_{ik} for two elements i, k can be found in [1, p. 46, Table 2.3].

3 The code

3.1 Simplifications

In order to make a simple model I needed some simplifications. A list of assumptions and simplifications follows.

- There is no change in the composition of elements as a function of radius.
- I assume there is no change in the density of deuterium, so that the rate of destruction of deuterium is the same as the production, and that any reaction involving deuterium happens instantaneous.
- All nuclear energy comes from the three PP-chains. I have not included the CNO cycle.
- I assume all elements to be fully inonised.

3.2 Units

Given that κ is in units of $\left[\mathrm{cm}^2\ \mathrm{g}^{-1}\right]$, λ in units of $\left[\mathrm{cm}^3\ \mathrm{s}^{-}1\right]$ and so on, I chose to adapt the CGS unit system (centimetre-gram-second). Table 1 shows an overview of the differences.

3.3 Structure

We see from eq. (4) that $\partial T/\partial m$ depends on the opacity, κ. A table of opacities that correspond to different values of temperature and density has been provided. I wrote a function that reads the file and stores values of T, R and κ in separate arrays. Here, $R = R(T,\rho) = \rho/T_6$. The function compares the table temperature with the actual temperature, and the same with the variable R. It returns the κ with table values of T and R that most closely resembles the present values.

Next, I wrote a function that calculates the energy generation per mass unit from nuclear reactions. This means entering the energy releases Q'_{ik} from [1, p. 39, Table 2.1], the reaction rates λ_{ik} from [1, p. 46, Table 2.3] and finding the number densities n for all particles that are involved in a nuclear reaction. Given the number fractions X, Y, Z and sub-fractions of these for different elements, I was able to calculate the energy generation per mass by using eq. (6), (7) and (8).

I have a function that calculates the density at present time, given the temperature T and total pressure P. This is found from the equation of state, as given in eq. (5). This required me to calculate the molecular weight, μ. It is given on the form

$$\mu = \frac{1}{\mu}\frac{\rho}{n_{\text{tot}}}.$$

We can find the total particle density with

$$\begin{aligned} n_{\text{tot}} &= n_X + n_Y + n_Z + n_e \\ &= \frac{X\rho}{m_{\text{u}}} + \frac{Y\rho}{4m_{\text{u}}} + \frac{Z\rho}{Am_{\text{u}}} + \left(\frac{X\rho}{m_{\text{u}}} + \frac{2Y\rho}{4m_{\text{u}}} + \frac{Z\rho}{2m_{\text{u}}}\right), \end{aligned}$$

where A is the average atomic weight of the heavier elements present, which we assume to be $A = 7$. This assumption is based on that we only know of two heavier elements present, which are ^{7}Be and ^{7}Li. We then get

$$\begin{aligned} \mu &= \frac{\rho m_{\text{u}}}{\rho m_{\text{u}}}\left(\frac{1}{2X + \frac{3}{4}Y + \frac{9}{14}Z}\right) \\ &= \frac{1}{2X + \frac{3}{4}Y + \frac{9}{14}Z}. \end{aligned}$$

We have assumed all elements to be fully ionised.

Further, I have four functions that return the right-hand side in eq. (1), (2), (3) and (4). These are called upon in another function that integrates all the equations. For simplicity, I have chosen the forward Euler integration scheme as numerical integration method.

3.4 The Euler integration scheme

References

[1] Michael Stix, *The Sun*. Springer, New York, 2nd Edition, 2002.

[2] Boris Gudiksen *AST3310: Astrophysical plasma and stellar interiors*. 2014.