Problem: A Patient test positive of rare diease. The Probability of having the disease is 0.01 (1%). The test is 99% sensitive (true positive rate) & 99% specific (true negative) what is the probability that patient actually has the disease given a positive test result?

→ D be event of having disease
$-D$ be event of not having disease.
T^+ be the event of testing positive

Given
$P(D) = 0.01$
$P(-D) = 0.99$
$P(T^+ | D) = 0.99$
$P(T^+ | -D) = 0.02$

we have to find $P(D | T^+)$

Bayes Theorm*

$$P(D|T^+) = \frac{P(T^+|D).P(D)}{P(T^+)}$$

where
$P(T^+) = P(T^+|D).P(D) + P(T^+|-D).P(-D)$

Plugging in the values.

$P(T^+) = (0.99 . 0.01) + (0.02 . 0.99)$
$P(T^+) = 0.0099 + 0.0198$.
$P(T^+) = 0.0297$.

using Bayes Theorm

$$P(D|T^+) = \frac{0.0099}{0.0297}$$

$P(D|T^+) \simeq 0.3333$

so probability is 33%.

finding Eigenvalues & Eigen vector of a Matrix.

$$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

→ Finding Eigenvalues.

$(A - \lambda I) = 0$.

$$\det\begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix} = (4-\lambda)(3-\lambda) - 2.1 = \lambda^2 - 7\lambda + 10 = 0.$$

solving $\lambda^2 - 7\lambda + 10 = 0$.

$$\lambda = \frac{7 \pm \sqrt{49-40}}{2} = \frac{7 \pm 3}{2}$$

eigen values are $\lambda_1 = 5$ $\lambda_2 = 2$.

Finding Eigenvectors.
for $\lambda_1 = 5$

$$(A - 5I)x = 0 \quad \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$-x_1 + x_2 = 0$; $x_1 = x_2$

eigen vector corresponding to $\lambda_1 = 5$ is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

for $\lambda_2 = 2$:

$$(A - 2I)x = 0 \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$2x_1 + x_2 = 0$ $x_2 = -2x_1$

$\lambda_2 = 2$ is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Calculating the determinant of a 3x3 matrix.

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$$

→ Determinant.

$$\det(B) = 1 \cdot \begin{pmatrix} 1 & 4 \\ 6 & 0 \end{pmatrix} - 2 \cdot \begin{pmatrix} 0 & 4 \\ 5 & 0 \end{pmatrix} + 3 \cdot \begin{pmatrix} 0 & 1 \\ 5 & 6 \end{pmatrix}$$

$$\det(B) = 1 \cdot (1.0 - 4.6) - 2\,(0.0 - 5.4) + 3 \cdot (0.6 - 1.5)$$

$$= 1 \cdot (-24) - 2 \cdot (-20) + 3 \cdot (-5)$$

$$\det(B) = -24 + 40 - 15 = 1$$

so $\det(B) = 1$

Properties ~~of~~ & Application of Normal Distribution

Properties

→ Shape : Symmetrical & bell Shaped.

→ mean mode median all are equal.

→ Standard deviation Determines the width of the bell curve.

→ 68-95-99 Rule : About 68% of data lies within 1 standard deviation, 95% within 2, 99.7% within 3.

Application.

→ Used in natural and social sciences for real valued random variables

→ In hypothesis testing and confidence interval

→ In fiance for asset return.

→ In quality control for process variability.

Calculating Probability.

Problem : If x is normally distributed with mean $\mu = 50$ & standard deviation $\sigma = 10$ what is the probability that x is between 40 & 60?

→ Convert to standard normal distribution Z.

$$Z = \frac{X - \mu}{\sigma}$$

for $x = 40$

$$Z = \frac{40 - 50}{10} = -1$$

for $x = 60$

$$Z = \frac{60 - 50}{10} = 1$$

Using standard ~~d~~ normal distribution tables or a calculator the probability that Z lies between -1 & 1 is approx 0.6826.